High-dimensional inference using nested particle filters

— Nested sequential Monte Carlo methods

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Joint work with Fredrik Lindsten and Thomas B. Schön

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1 Background – sequential Monte Carlo
2 2D MRF – nested SMC applied
3 Experiments – spatio-temporal MRFs

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Nonlinear filtering

The filtering problem for a nonlinear state space model,

$$x_{t+1} \mid x_t \sim f(x_{t+1} \mid x_t),$$
$$y_t \mid x_t \sim g(y_t \mid x_t),$$

amounts to computing

$$p(x_t \mid y_{1:t}) = \frac{g(y_t \mid x_t) \int f(x_t \mid x_{t-1}) p(x_{t-1} \mid y_{1:t-1}) dx_{t-1}}{p(y_t \mid y_{1:t-1})}$$

for $t = 1, 2, \ldots$

The bootstrap filter

The *bootstrap particle filter* approximates $p(x_t \mid y_{1:t})$ by

$$\widehat{p}^N(x_t \mid y_{1:t}) := \sum_{i=1}^{N} \frac{W_t^i}{\sum_\ell W_t^\ell} \delta_{X_t^i}(x_t).$$

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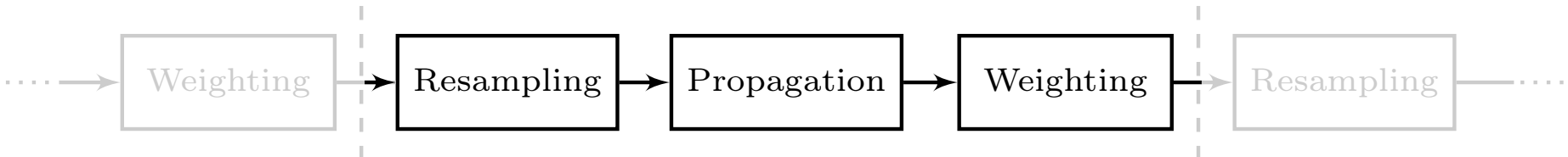

- *Resampling:* $\{(X_{t-1}^i, W_{t-1}^i)\}_{i=1}^N \to \{(\tilde{X}_{t-1}^i, 1/N)\}_{i=1}^N$.

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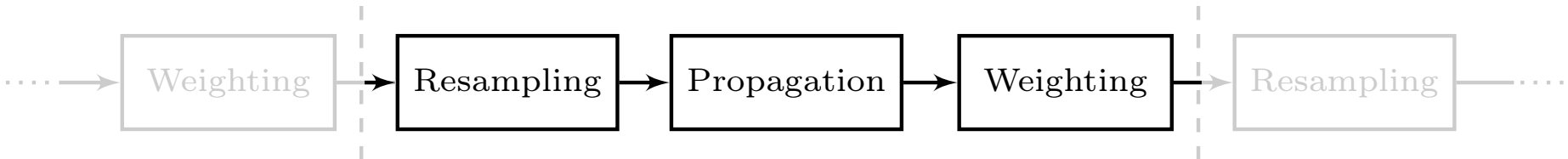

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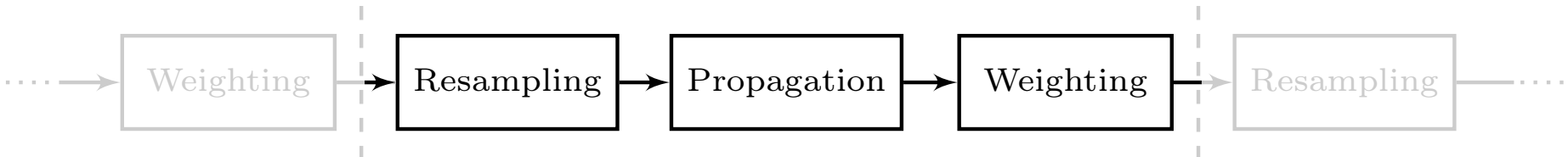

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- *Propagation:* $X_t^i \sim f(x_t \mid \tilde{X}_{t-1}^i)$.
- *Weighting:* $W_t^i = g(y_t \mid X_t^i)$.

$$\Rightarrow \{(X_t^i, W_t^i)\}_{i=1}^N$$

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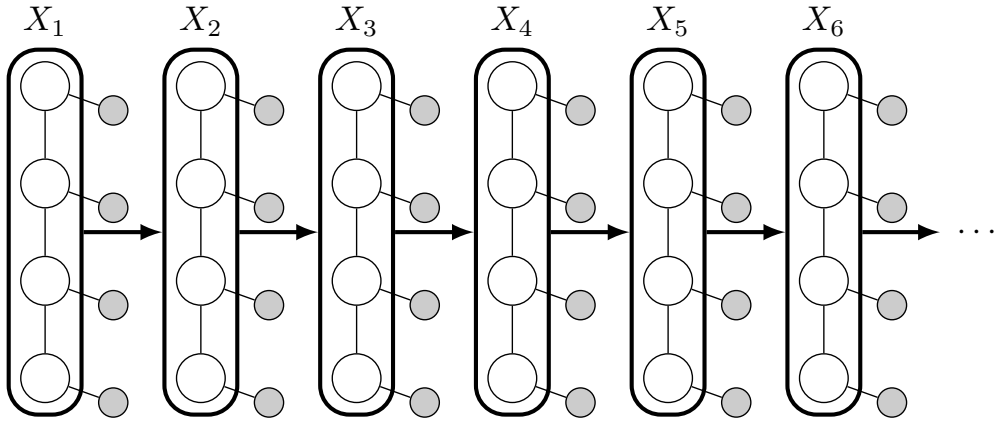

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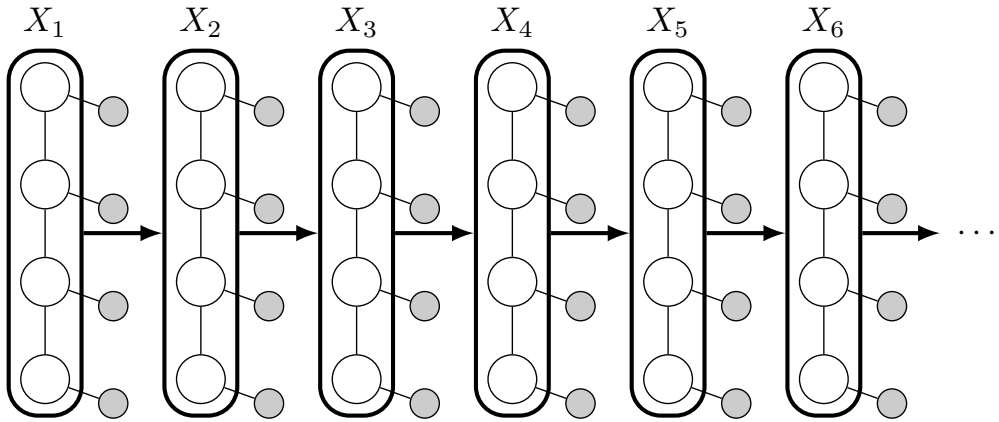

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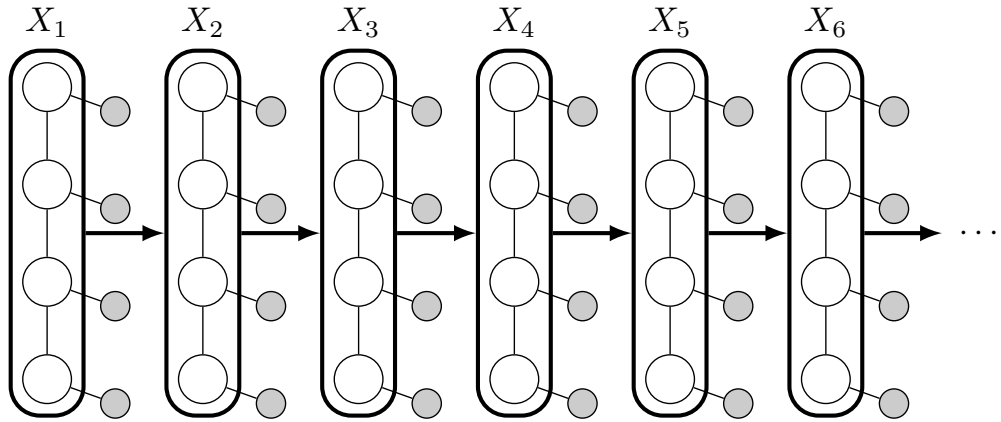

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Does a better proposal distribution improve our result?

Preview of the idea

- Optimal *proposals* and *resampling weights* — known conceptually but intractable to compute.
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- Use *SMC* to approximate the optimal proposals and resampling weights.
- Sampling distribution not available on closed form — still possible to obtain a valid algorithm!
- Nested SMC satisfies the conditions on the proposal approximation $\Rightarrow$ possible to use within itself (nesting to arbitrary degree).

Fully adapted auxiliary SMC sampler

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Results in an *unweighted* set of particles $\{X_{1:t}^i\}_{i=1}^N$, such that $\bar{\pi}_t^N(x_{1:t}) = \frac{1}{N}\sum_{i=1}^N \delta_{X_{1:t}^i}(x_{1:t})$ approximates $\bar{\pi}_t$.

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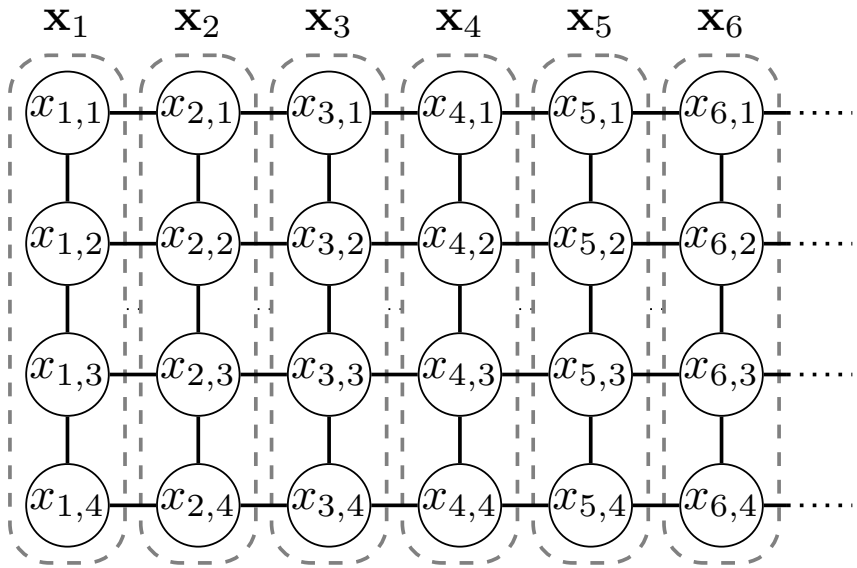
- ☐ CLT with std MC rate $1/\sqrt{N}$, asymptotic variance depends on M.
- ☐ Unbiased estimator of $\mathcal{Z}_t$: $\widehat{\mathcal{Z}}_t = \widehat{\mathcal{Z}}_{t-1} \times \frac{1}{N} \sum_{i=1} \widehat{Z}_t^i$.

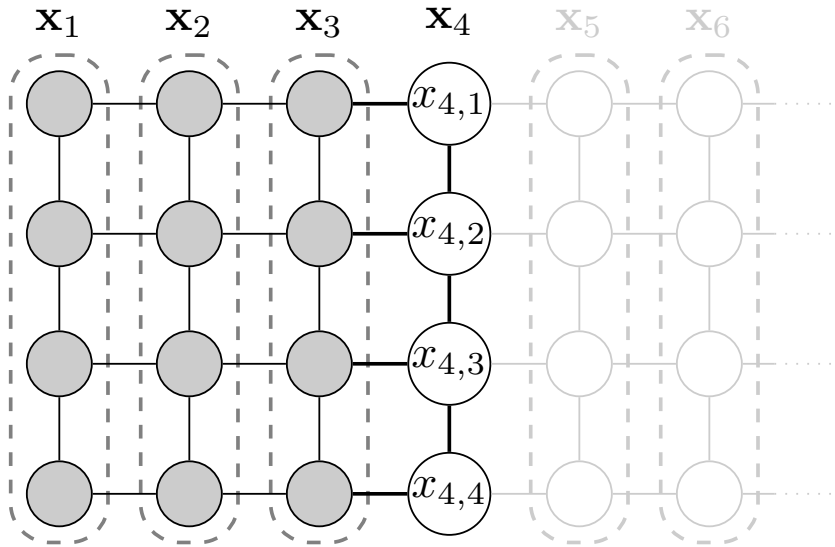
2D Markov Random Field

1 spatial + 1 temporal dimension



$$\bar{\pi}_t(\mathbf{x}_{1:t}) = \frac{1}{\mathcal{Z}_t}\boldsymbol{\phi}_1(\mathbf{x}_1)\prod_{s=2}^{t}\left\{\boldsymbol{\phi}_s(\mathbf{x}_s)\boldsymbol{\psi}_s(\mathbf{x}_{s-1},\mathbf{x}_s)\right\}.$$

2D MRF – Nested SMC implementation (I/III)



Optimal proposals given by:

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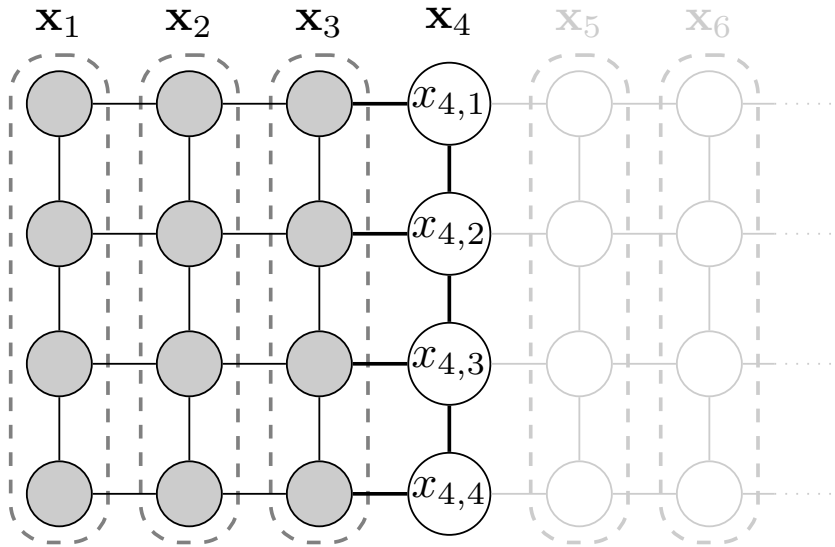

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$$= \left\{ \prod_{k=1}^{d} G_{t,k}(x_{t,k}) \prod_{k=2}^{d} m(x_{t,k-1}, x_{t,k}) \right\} \left\{ \prod_{k=1}^{d} \psi(x_{t-1,k}, x_{t,k}) \right\}$$

2D MRF – Nested SMC implementation (II/III)

Proposed algorithm:

Step 1: Resampling

(Optimal weights $\widetilde{W}_{t-1}^i = Z_t(\mathbf{X}_{t-1}^i)$ with $Z_t(\mathbf{x}_{t-1}) = \int q_t(\mathbf{x}_t \mid \mathbf{x}_{t-1}) \mathrm{d}\mathbf{x}_t$.)

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  - Run PF with M particles for target $q_t(\mathbf{x}_t \mid \mathbf{X}_{t-1}^i)$.

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  - Estimate normalising constant: $\widehat{Z}_t^i = \prod_{k=1}^d \left\{ \frac{1}{N} \sum_{j=1}^M W_k^{i,j} \right\}$.

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- Resample $\{\mathbf{X}_{t-1}^i\}_{i=1}^N$ *and corresponding PFs* based on $\{\widehat{Z}_t^i\}_{i=1}^N$.

2D MRF – Nested SMC implementation (III/III)

Step 2: Propagation

- Assume particle $\mathbf{X}_{t-1}^i$ resampled n_t^i times.
- For $i = 1, \ldots, N$, generate n_t^i descendants of $\mathbf{X}_{t-1}^i$ by *backward simulation*:

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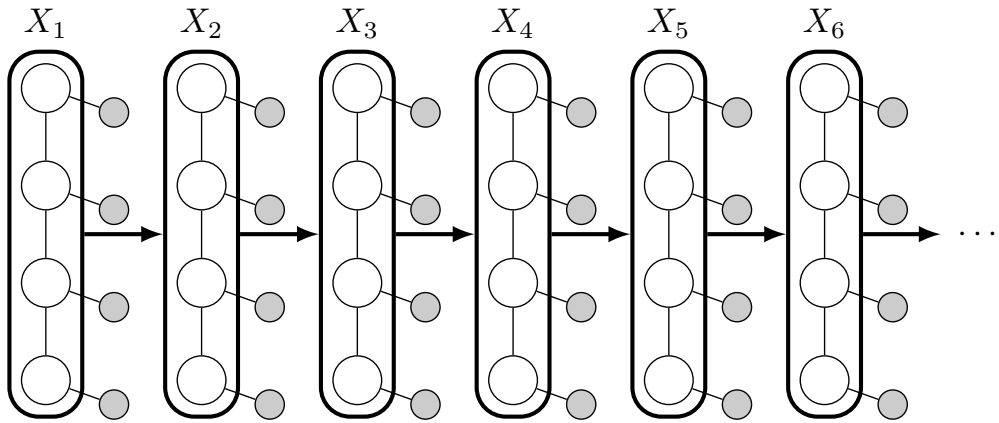
$$\Rightarrow \mathbf{X}'_t = X'_{t,1:d} \overset{\text{approx.}}{\sim} \bar{q}_t(\cdot \mid \mathbf{X}_{t-1}^i).$$

Results in N *unweighted* particles: $\{\mathbf{X}_t^i\}_{i=1}^N$

ex) Gaussian spatio-temporal model



Gaussian spatio-temporal model in the form of a 2D MRF, $d \times t$, i.e. $\dim x_t = d$.

$$p(x_{1:t}, y_{1:t}) \propto \prod_{s=1}^{t} \underbrace{\mathcal{N}(y_s; x_s, \tau^{-1} I)}_{G} \underbrace{\mathcal{N}(x_s; a x_{s-1}, I)}_{\psi} \underbrace{\mathcal{N}(x_s; 0, \Sigma)}_{m}$$

where Σ^{-1} is a banded matrix (reflecting local dependencies).

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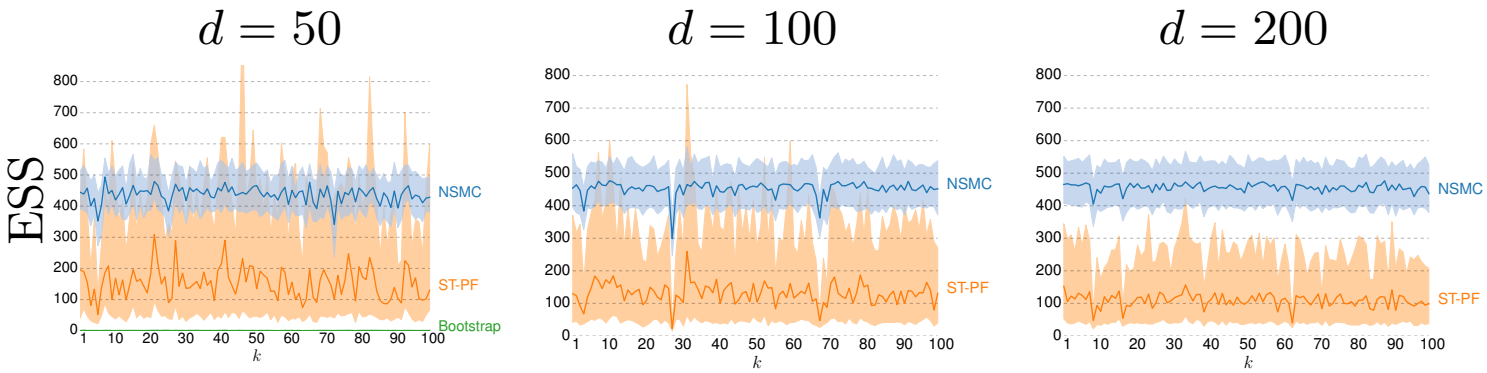
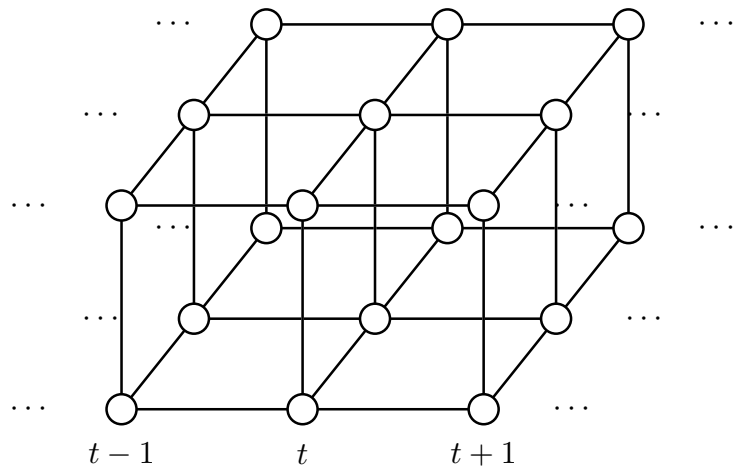


Figure: Median (over dimension) effective sample size (ESS) and 15–85% percentiles. $N = 500$ and $M = 2d$. (Results for 100 independent runs.)

$$\mathsf{ESS}_{t,k} := \left(\mathbb{E} \left[\frac{(\hat{x}_{t,k} - \mu_{t,k})^2}{\sigma_{t,k}^2} \right] \right)^{-1}$$

Beskos, A., Crisan, D., Jasra, A., Kamatani, K. and Zhou, Y. *A Stable Particle Filter in High-Dimensions*. arXiv:1412.3501, Dec. 2014.

ex) Spatio-temporal model for drought prediction



- System state $x_t = \{x_{t,k,\ell}\}_{k=1,\ell=1}^{K,L}$, i.e., dimension is $d = K \times L$.
- Binary variables: $x_{t,k,\ell} = 0$ (normal state) or $x_{t,k,\ell} = 1$ (drought).
- Yearly Gaussian observations of precipitation at each site.

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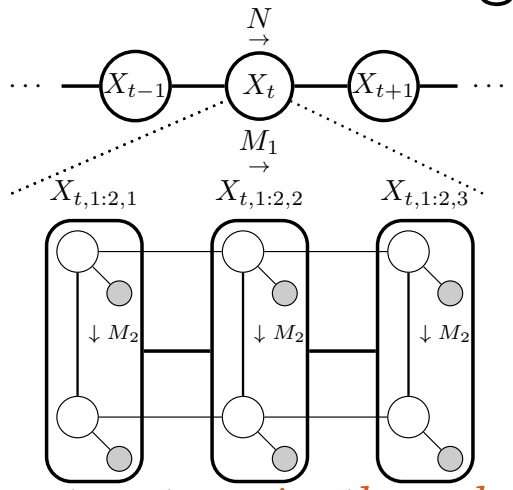


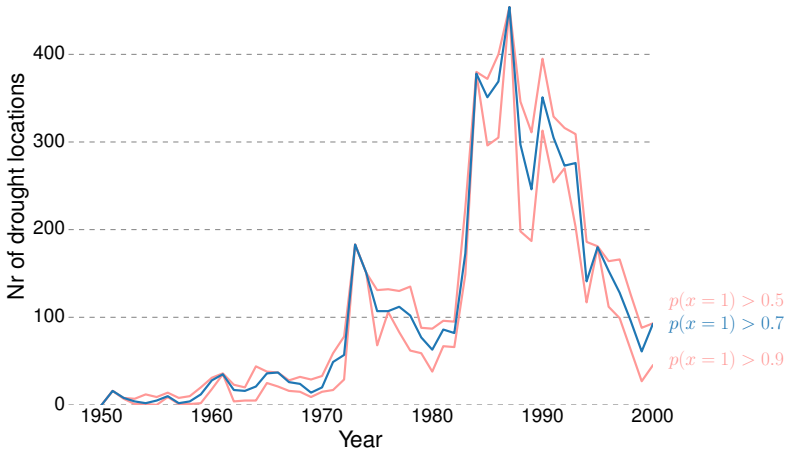
Exploit the rectangular structure in three levels:

Level 1: Instantiate a Nested SMC sampler targeting the full posterior filtering distribution.

Level 2: To sample x_t, we run a Nested SMC sampler, operating on the "columns" $x_{t,1:K,\ell}$, $\ell = 1, \ldots, L$.

Level 3: To sample each column $x_{t,1:K,\ell}$ we run a third level of SMC, operating on the individual components $x_{t,k,\ell}$, $k = 1, \ldots, K$.

ex) Spatio-temporal model for drought prediction



- Data from the Sahel region in Africa for years 1950–2000.
- $\{K, L\} = \{24, 44\}$ $(\Rightarrow d = 1\,056)$.
- $\{N, M_1, M_2\} = \{100, 40, 20\}$.

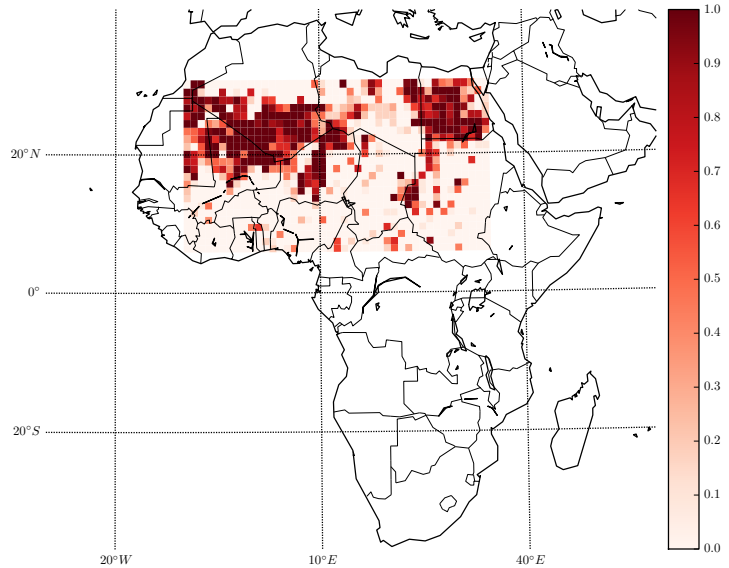


Figure: Sahel region in 1989.

Wrapping up

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- NSMC allows us to *"exactly approximate"* a fully adapted SMC sampler.
- Forward-backward strategy for lattice models.
- Provably correct for any number(s) M of particles in the "internal" filter(s).
- Modular to an arbitrary degree.
- Pushes the dimension-limit for SMC from "tens" to "hundreds" (?).

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Worth to note:

- Can straightforwardly be used with, e.g., Particle MCMC for learning.
- Computational complexity of the method is $N \times M$ (for two layers). However, *much more efficient* than a bootstrap PF with $N \times M$ particles!
- NSMC *does not* beat the curse of dimensionality!
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Thank you!

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