Inference for dynamics of continuous variables: the Extended Plefka Expansion with hidden nodes

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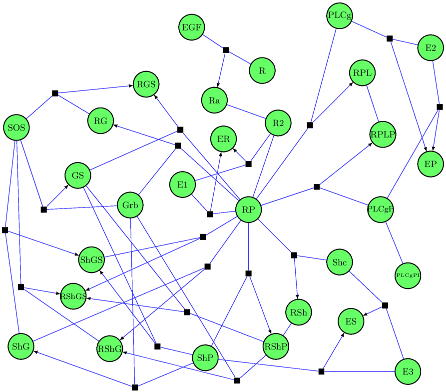
Introduction and Motivation

Fundamental and practical **limitations**:



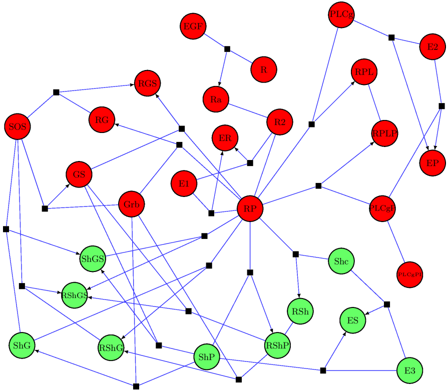
EGFR network from *Kholodenko et al. (1999)*

- Non-linear equations
- Vast amount of information ($\sim 10^4$ eqs)
- **Uncertainty** $\rightarrow$ No complete qualitative understanding

SIZE, COMPLEXITY, UNCERTAINTY

$\Downarrow$

Statistical Physics for Model Reduction

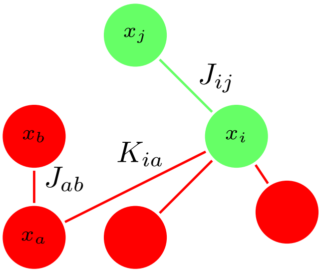


Let us assume it is possible to characterize **only some** nodes

Small subset of variables:
Subnetwork $\rightarrow$ "observed"
Embedded in a larger network:
Bulk $\rightarrow$ "hidden" (unknown)

Question

What can we say in general about the **INFERENCE of hidden dynamics**?



LINEAR DYNAMICS

$i, j \to$ hidden (Bulk)
$a, b \to$ observed (Subnetwork)

ξ_i, ξ_a Gaussian white noises

$$\langle \xi_i(t)\xi_j(t') \rangle = \sigma_b^2 \delta_{ij} \delta(t-t')$$

$$\langle \xi_a(t)\xi_b(t') \rangle = \sigma_s^2 \delta_{ab} \delta(t-t')$$

$$\partial_t x_i(t) = -\lambda x_i(t) + \sum_j J_{ij} x_j(t) + \sum_a K_{ia} x_a(t) + \xi_i(t)$$

$$\partial_t x_a(t) = -\lambda x_a(t) + \sum_b J_{ab} x_b(t) + \sum_j K_{aj} x_j(t) + \xi_a(t)$$

Extended Plefka Expansion

B. Bravi, P. Sollich, M. Opper, J. Phys. A., submitted, arXiv:1509.07066 (2015)

DYNAMICAL MEAN FIELD APPROXIMATION

1. Effectively **non-interacting** dynamics:
   Couplings are replaced by **memory** + **coloured** noise
2. Extension: 1st and 2nd moments are constrained
3. **Gaussian** statistics conditioned on observations

Posterior mean $\mu_i(t)$ = best estimate of hidden dynamics
Forward-Backward propagation

Posterior variance $C_i(t,t)$ = estimate of **error** of prediction

Expect Plefka to give exact $\mu_i(t)$ and $C_i(t,t)$ (=errors) when **Mean Field** interactions + **Thermodynamic limit** $N^B \to \infty$

Stationary Regime $\to$ Time Translation Invariant
Average $\tilde{C}^B(\omega) = \frac{1}{N^B}\sum_i \tilde{C}_i(\omega)$ in **Fourier** space

$$\tilde{C}^B(\omega) = \underbrace{\frac{\sigma_s^2}{k^2}}_{\text{Amplitude}} \underbrace{\mathcal{C}^B_{\alpha,\gamma,\eta,p}(\Omega)}_{\text{Dimensionless}}$$

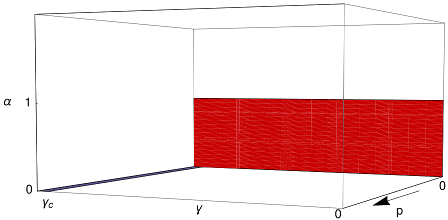
Dimensionless parameters

- $\alpha = \frac{N^S}{N^B}$ Ratio observations/hidden states
- $\gamma = \frac{j}{\lambda}$ Stability of hidden dynamics $\gamma < \gamma_c = \frac{1}{1+\eta}$
- $p = \frac{\lambda}{\sigma}$ Decay constant & hidden-to-observed coupling

$\sigma = \frac{\sigma_b k}{\sigma_s}$ Defines the frequency scale: $\Omega = \frac{\omega}{\sigma}$

Critical regime

We analyze in the parameter space α, γ, p the **singularities** of $\mathcal{C}^B(0)$



Critical regions

1. $\forall p$, $\alpha = 0$ and $\gamma > \gamma_c$

   No observations: internal stability

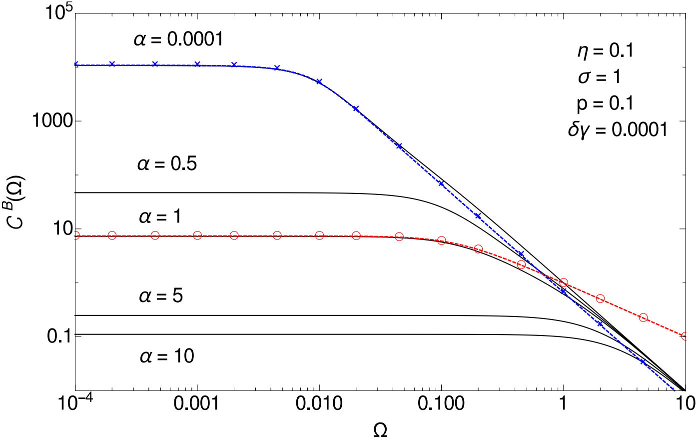
2. $\forall \gamma$, $p = 0$ and $0 < \alpha < 1$
   (i.e. $k \gg \lambda$ at fixed $\frac{\sigma_s}{\sigma_b}$)

   "Underconstrained" hidden system: strong constraints from observations, but too few

Power-law dependence of $\mathcal{C}^B(0)$ on $\delta\alpha$, $\delta\gamma$, p:

Scaling analysis $\Rightarrow$ **Master curves**

Information on **relaxation** times and inference **error**



$$\Omega^* \ll \Omega \ll 1 \qquad \alpha \to 0 \quad \mathcal{C}^B \sim \frac{1}{\Omega^2} \qquad \alpha \to 1 \quad \mathcal{C}^B \sim \frac{1}{\Omega}$$

Thank you for your attention!