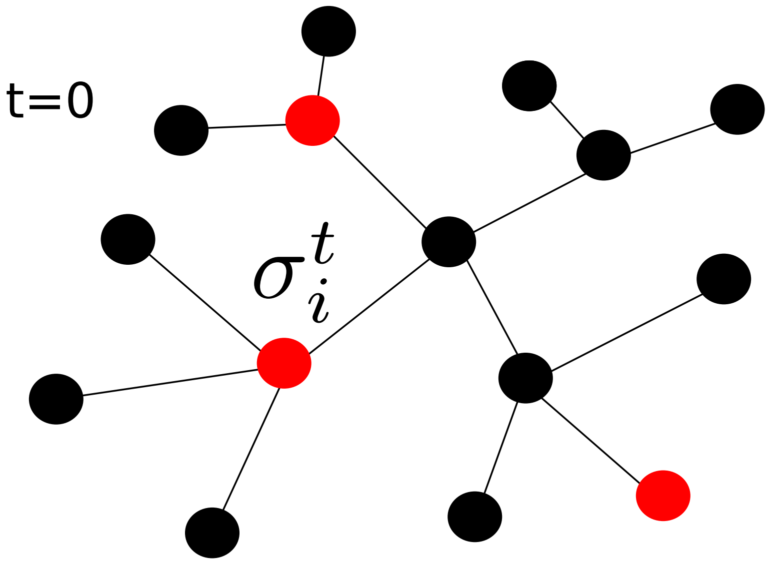
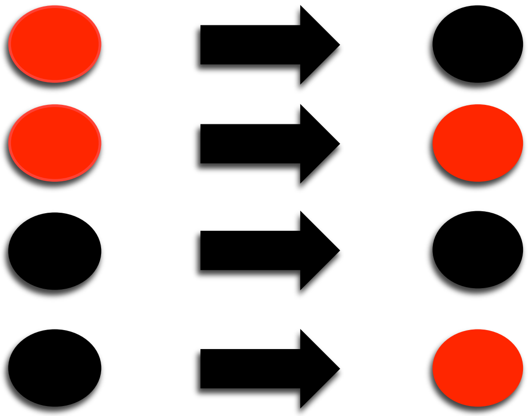
A matrix product representation for dynamical processes on networks

Caterina De Bacco, Thomas Barthel and Silvio Franz,
Université Paris Sud 11



$$W(\sigma_i^{t+1}|\sigma_i^t, \{\sigma_j^t\}_{j\in\partial i})$$



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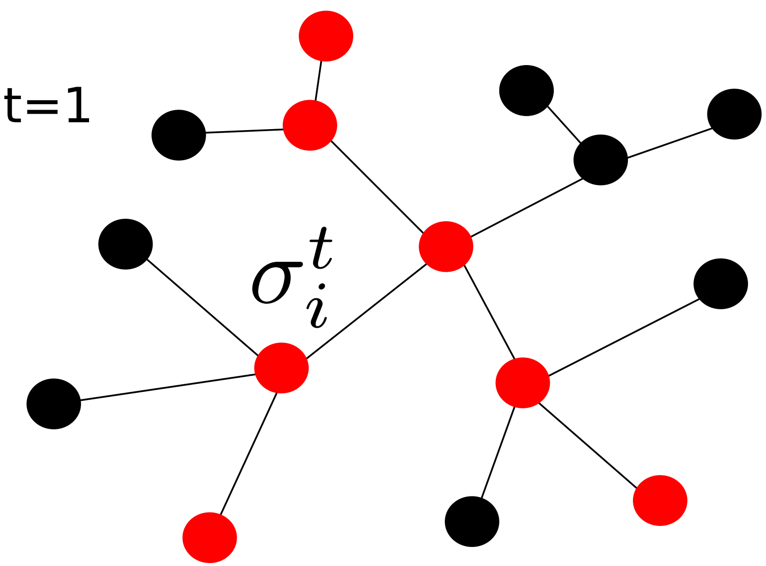

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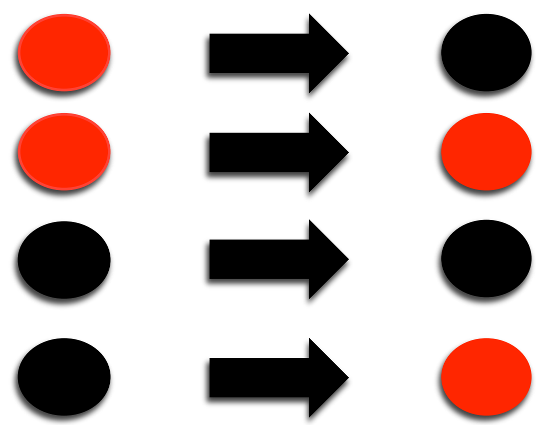

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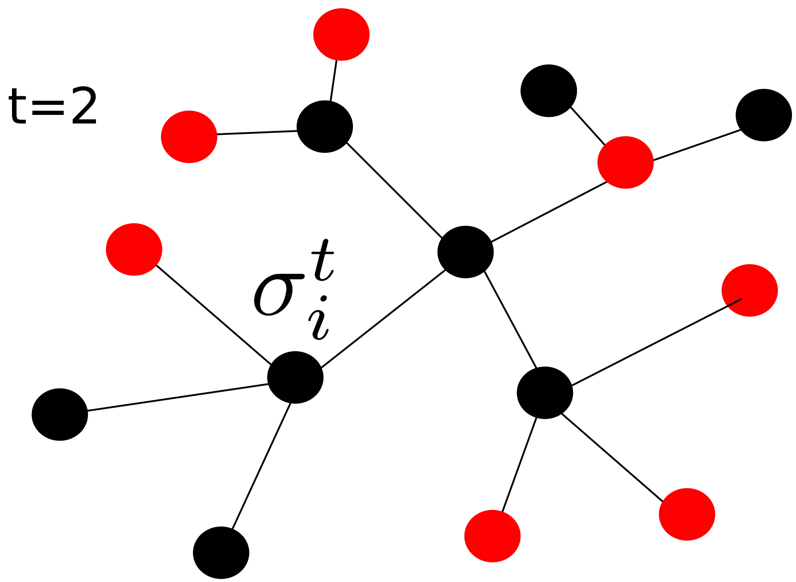

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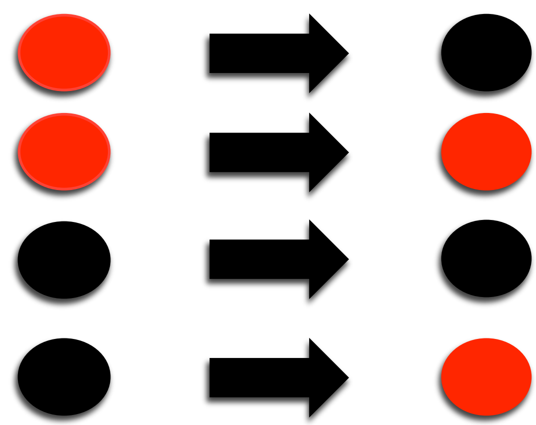

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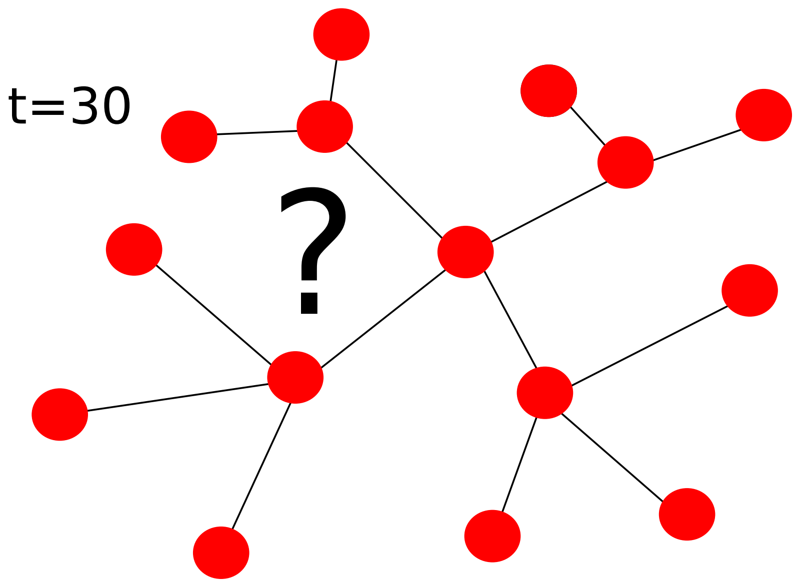

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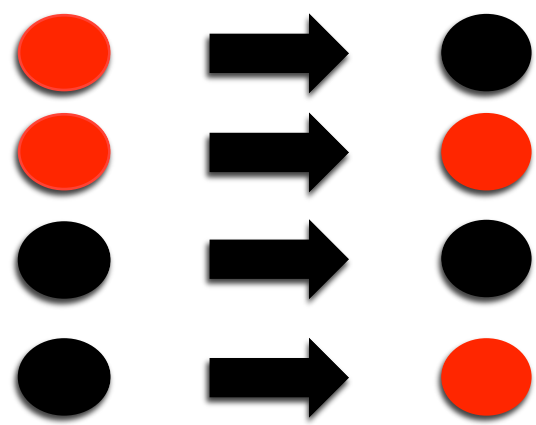

N-nodes joint distribution:

$$P(\bar{\sigma}^T) = P_0(\{\sigma_i^0\}) \prod_{i=1}^{N} \prod_{t=0}^{T-1} W(\sigma_i^{t+1} | \sigma_i^t, \{\sigma_j^t\}_{j \in \partial i})$$

Exponential complexity in N and T …

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$$\mu_{ij}(\bar{\sigma}_i^t | \bar{\sigma}_j^{t-1}) = \frac{1}{Z_{ij}} P_i(\sigma_i^0) \times$$

$$\sum_{\{\bar{\sigma}_k^{t-1}\}} \prod_{s=0}^{t-1} W(\sigma_i^{s+1} | \sigma_i^s, \{\sigma_j^s\}_{j\in\partial i}) \prod_{k\in\partial i\setminus j} \mu_{ki}(\bar{\sigma}_k^{t-1} | \bar{\sigma}_i^{t-2})$$

Dynamic cavity equation

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Dynamic cavity equation

Exponential complexity in N and T ...

Matrix product ansatz

$$\mu_{ij}(\bar{\sigma}_i^t|\bar{\sigma}_j^{t-1}) =$$

$$A_{ij}^{(t+1)}(\sigma_i^t)A_{ij}^{(t)}(\sigma_i^{t-1})\left[\prod_{s=1}^t A_{ij}^{(s)}(\sigma_i^{s-1}|\sigma_j^s)\right]A_{ij}^{(0)}(\sigma_j^0)$$

Exponential complexity in N and T …

Matrix product ansatz

$$\mu_{ij}(\bar{\sigma}_i^t|\bar{\sigma}_j^{t-1}) =$$

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SVD + truncation

Exponential complexity in N and T ...

Matrix product ansatz

$$\mu_{ij}(\bar{\sigma}_i^t|\bar{\sigma}_j^{t-1}) =$$

$$A_{ij}^{(t+1)}(\sigma_i^t)A_{ij}^{(t)}(\sigma_i^{t-1})\left[\prod_{s=1}^t A_{ij}^{(s)}(\sigma_i^{s-1}|\sigma_j^s)\right]A_{ij}^{(0)}(\sigma_j^0)$$

SVD + truncation

Exponential complexity in N and T...

$$\mu_{ij}(\bar{\sigma}_i^{t+1}|\bar{\sigma}_j^t) = B_{ij}^{(t+2)}(\sigma_i^{t+1}) B_{ij}^{(t+1)}(\sigma_i^t) \left[\prod_{s=1}^t B_{ij}^{(s)}(\sigma_i^{s-1}|\sigma_j^s)\right] B_{ij}^{(0)}(\sigma_j^0)$$

=

$$\frac{1}{Z_{ij}} P_i(\sigma_i^0) \sum_{\{\bar{\sigma}_k^{t-1}\}} \prod_{s=0}^t W(\sigma_i^{s+1}|\sigma_i^s, \{\sigma_j^s\}_{j\in\partial i}) \times$$

$$\prod_{k\in\partial i\setminus j} \left\{ A_{ki}^{(t+1)}(\sigma_k^t) A_{ki}^{(t)}(\sigma_k^{t-1}) \left[\prod_{s=1}^t A_{ki}^{(s)}(\sigma_k^{s-1}|\sigma_i^s)\right] A_{ki}^{(0)}(\sigma_i^0) \right\}$$

A matrix product algorithm for the far-from-equilibrium evolution of dynamical processes on networks

Thomas Barthel,[1,2] Caterina De Bacco,[2] and Silvio Franz[2]

[1]Department of Physics, Duke University, Durham, NC 27708, USA

[2]LPTMS, CNRS, Univ. Paris-Sud, Universit Paris-Saclay, 91405 Orsay, France

(Dated: July 27, 2015)

We propose and test a novel algorithm to address the long-standing problem of capturing the far-from-equilibrium dynamics of dynamical processes on networks. Our model combines two successful ideas developed in different contexts of statistical physics: in the classical framework the cavity method, or message-passing algorithm, in its dynamical version; in quantum many-body theory the idea of matrix product approximation of a state function. This unusual combination of statistical formalisms allows to effectively approximate a dynamical process on networks were variables evolve

http://arxiv.org/abs/1508.03295