Neural Networks with Few Multiplications

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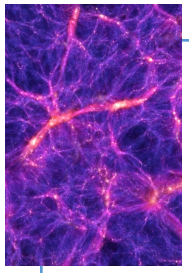
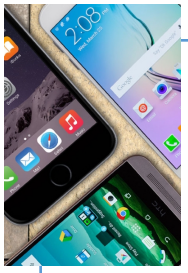
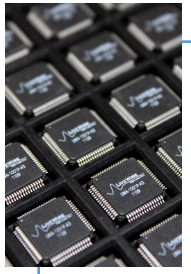
Why we don’t want massive multiplications?



Computationally expensive



Faster computation is likely to be crucial for further progress and for consumer applications on low-power devices.



A multiplier-free network could pave the way to fast, hardware friendly training of neural networks.

Various trials in the past decades…

- Quantize weight values (Kwan & Tang, 1993; Marchesi et al., 1993).
- Quantize states, learning rates, and gradients. (Simard & Graf, 1994)
- Completely Boolean network at test time (Kim & Paris, 2015
- Replace all floating-point multiplications by integer shifts. (Machado et al., 2015)
- Bit-stream networks (Burge et al., 1999) substituting weight connections with logical gates.
- ……

Binarization as regularization?

In many cases, neural networks only need very low precision

Low precision

Stochasticity comes with benefits

- Dropout, Blackout. [4][5]
- Noisy Gradients [3]
- Noisy activation functions [2]

stochasticity

Stochastic binarization?

Can we take advantage of the impreciseness of a binarization process so that we can have reduced computation load and extra regularization at the same time?

Our approach

Binarize weight values

- *BinaryConnect[*Courbariaux, et al., 2015*]* and *TernaryConnect*
- Binarize weights in the forward/backward propagations, but store a full-precision version of them in the backend.

Quantize backprop

- *Exponential quantization*
- Employ quantization of the representations while computing down-flowing error signals in the backward pass.

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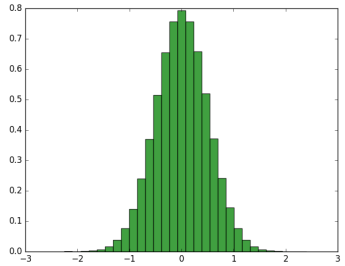
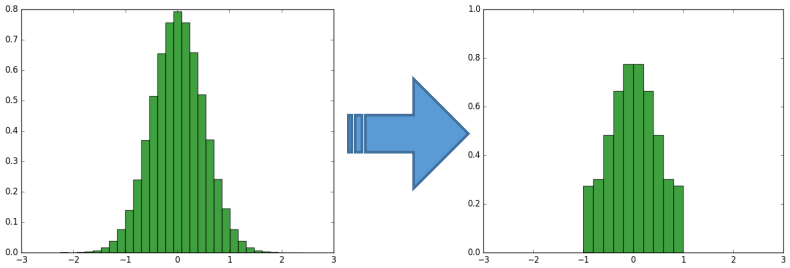
Binarize Weight Values



Original weight histogram

Binarize Weight Values



Original weight histogram

weight clipping

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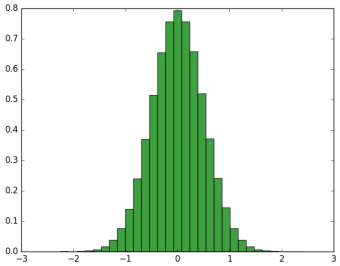

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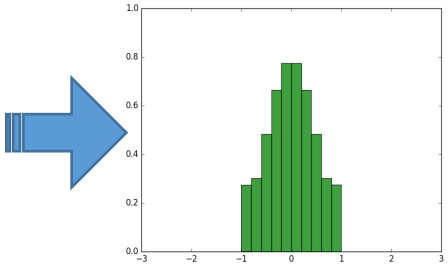
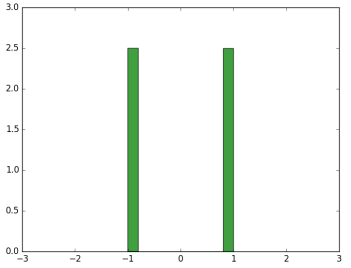
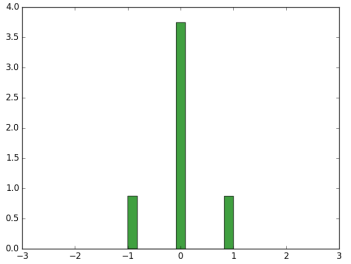


weight clipping

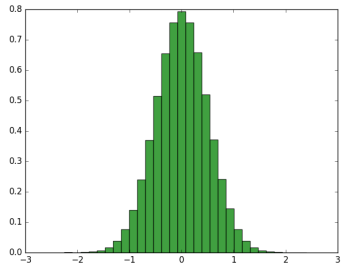
BinaryConnect



TernaryConnect



Binarize Weight Values



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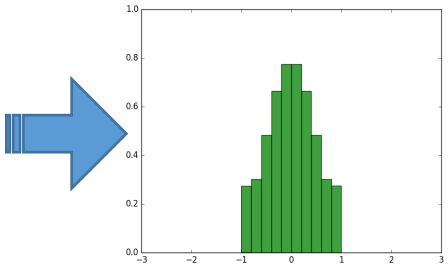
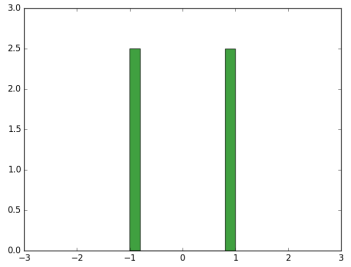


weight clipping

BinaryConnect



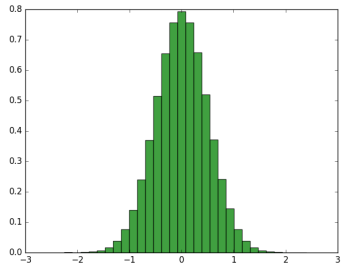
Stochastic

- $P(W_{ij} = 1) = \frac{w_{ij}+1}{2}$
- $P(W_{ij} = -1) = 1 - P(W_{ij} = 1)$

Deterministic

- $W_{ij} = \begin{cases} 1 & w_{ij} > 0 \\ -1 & otherwise \end{cases}$

Binarize Weight Values



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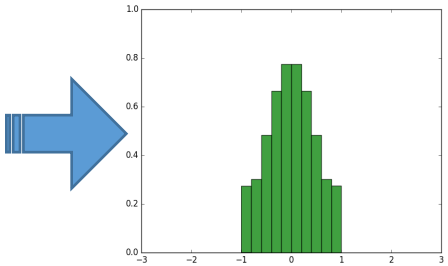
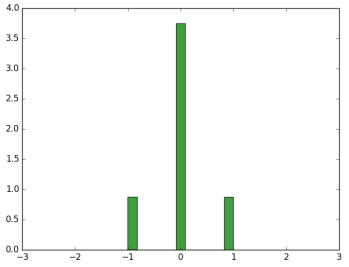


weight clipping

TernaryConnect



Stochastic

- If $w_{ij} > 0$:
  - $P(W_{ij} = 1) = w_{ij}$
  - $P(W_{ij} = 0) = 1 - w_{ij}$
- Else:
  - $P(W_{ij} = -1) = -w_{ij}$
  - $P(W_{ij} = 0) = 1 + w_{ij}$

Deterministic

- $W_{ij} = \begin{cases} 1 & w_{ij} > 0.5 \\ 0 & -0.5 < w_{ij} \leq 0.5 \\ -1 & w_{ij} \leq -0.5 \end{cases}$

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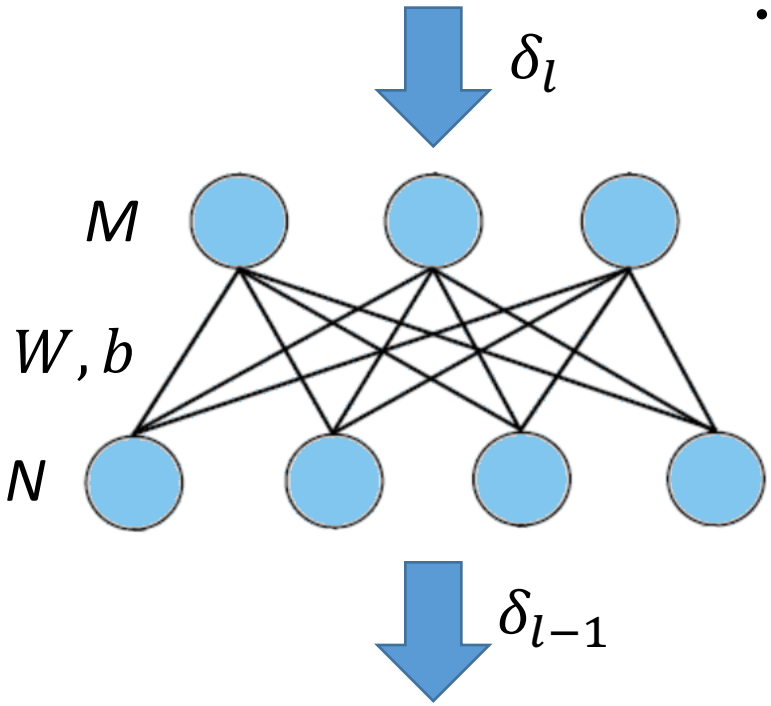
Quantized Backprop



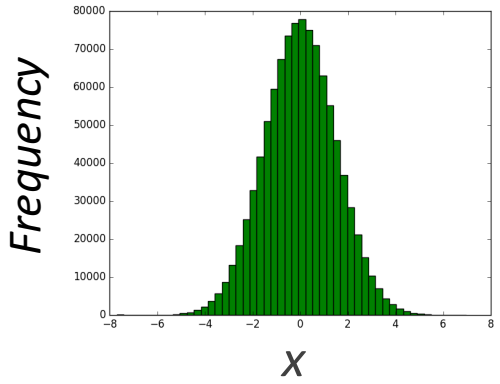
- Consider the update you need to take in the backward pass of a given layer, with *N* input units and *M* outputs:

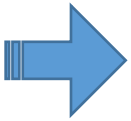
$$\Delta W = [\eta \delta_l \circ h'(Wx + b)] \cdot x^T \quad (1)$$

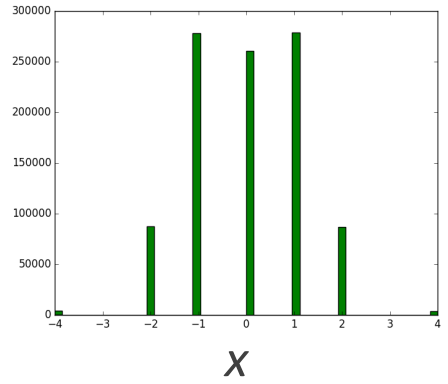
$$\Delta b = \eta \delta_l \circ h'(Wx + b) \quad (2)$$

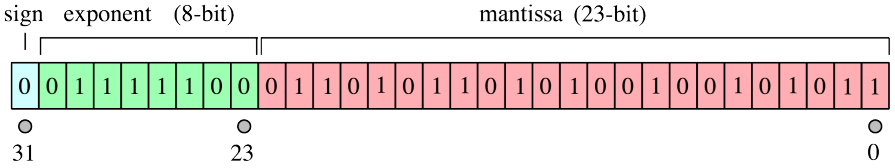
$$\delta_{l-1} = [W^T \cdot \delta_l] \circ h'(Wx + b) \quad (3)$$

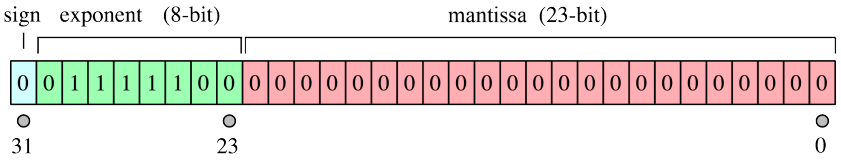
Exponential Quantization



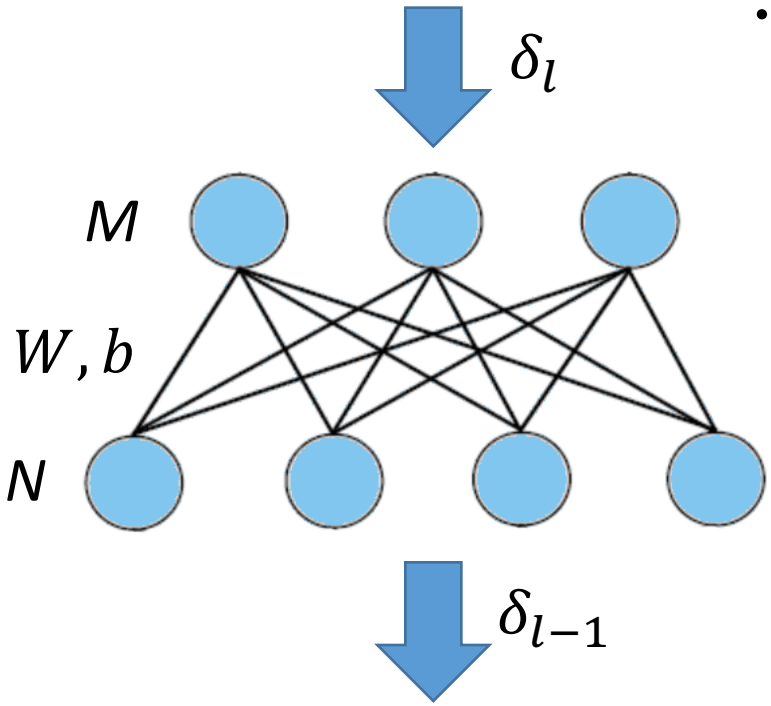








Quantized Backprop



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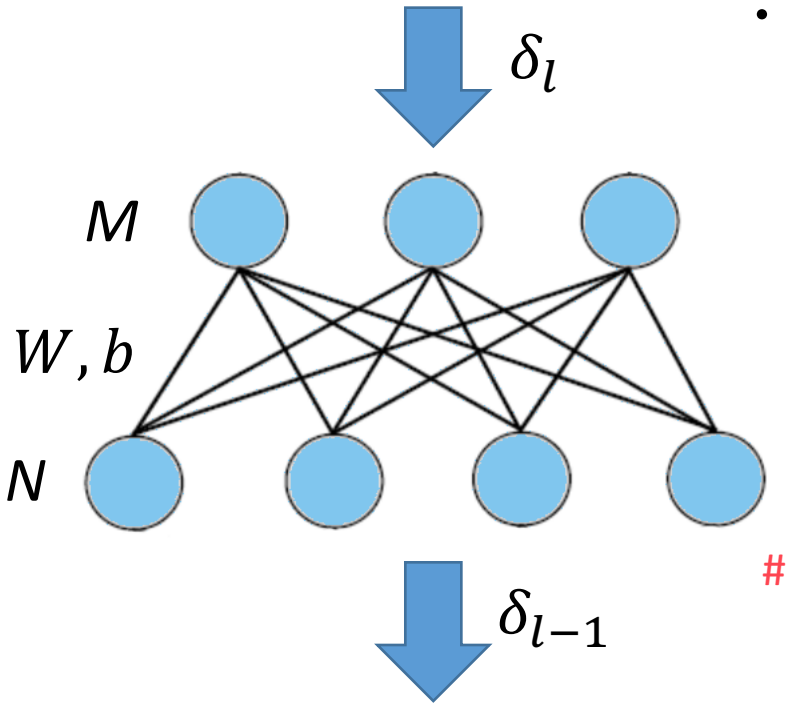
  - It is hard to bound the values of h', thus make it hard to decide how many bits it will need.
  - We choose to quantize x.

Quantized Backprop



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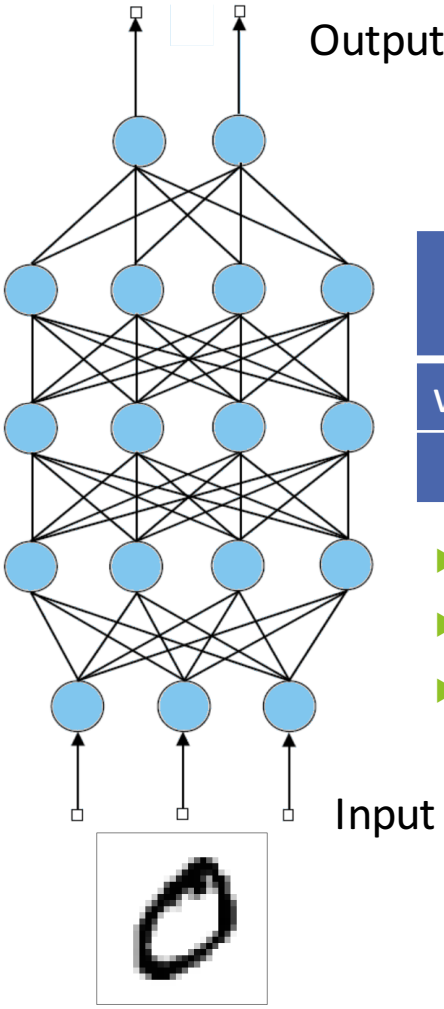
multiplications: 2M M

- **3M** multiplications in total
- A standard backprop would have to compute all the multiplications, requiring **2MN + 3M** multiplications.

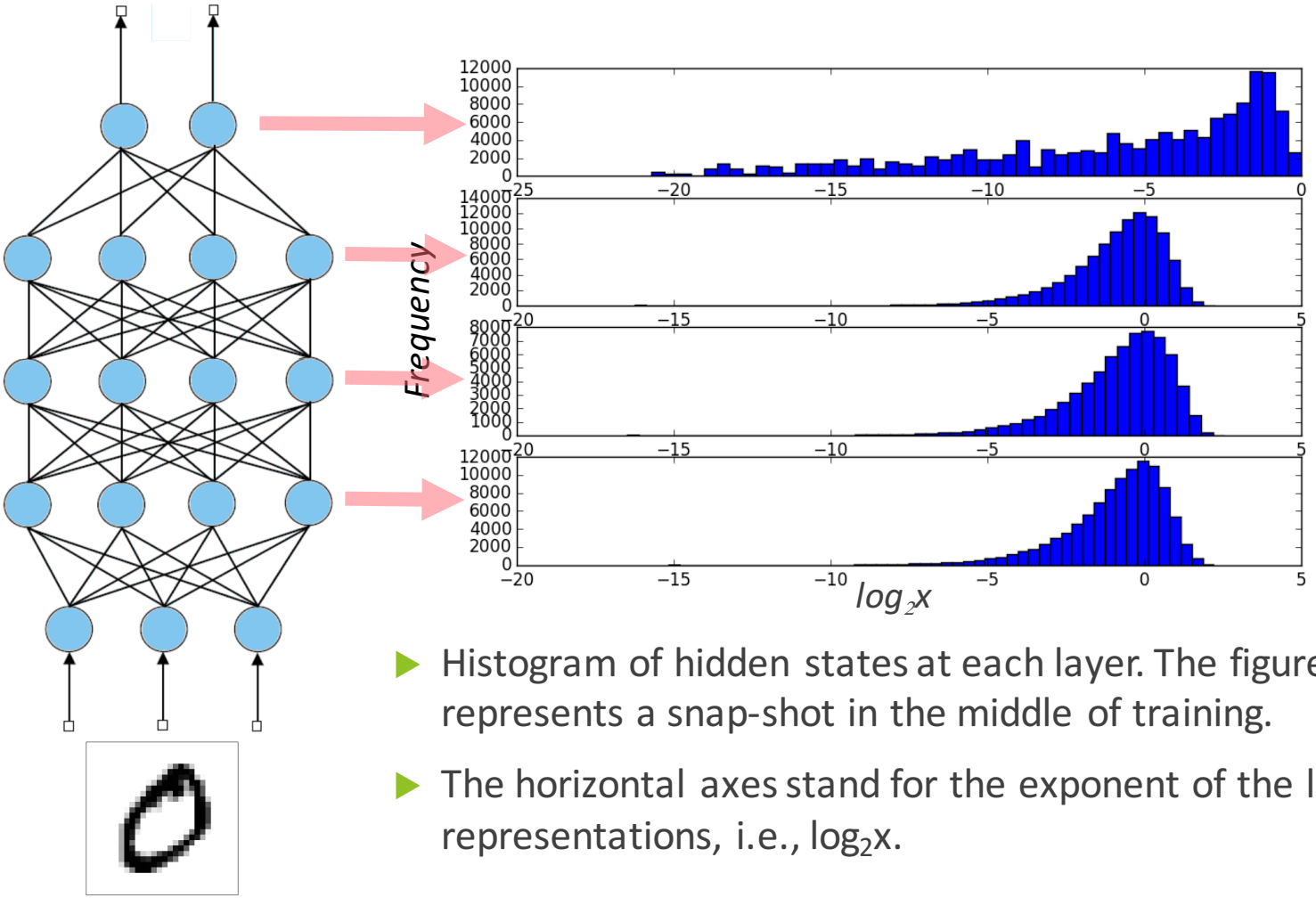
How many multiplications saved?



	Full precision	Ternary connect + Quantized backprop	ratio
without BN	1.7480×10^9	1.8492×10^6	0.001058
with BN	1.7535×10^9	7.4245×10^6	0.004234

- MLP with ReLU, 4 layers (784-1024-1024-1024-10)
- Assume that standard SGD are used as the optimization algorithm.
- BN stands for Batch Normalization

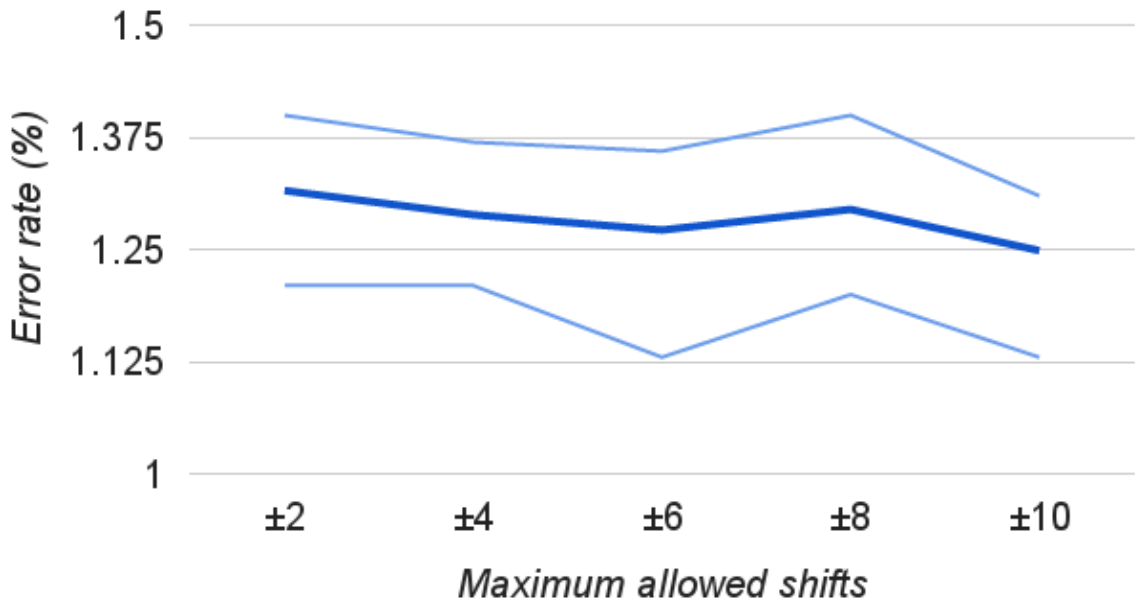
Range of Hidden Representations



- Histogram of hidden states at each layer. The figure represents a snap-shot in the middle of training.
- The horizontal axes stand for the exponent of the layers' representations, i.e., $\log_2 x$.

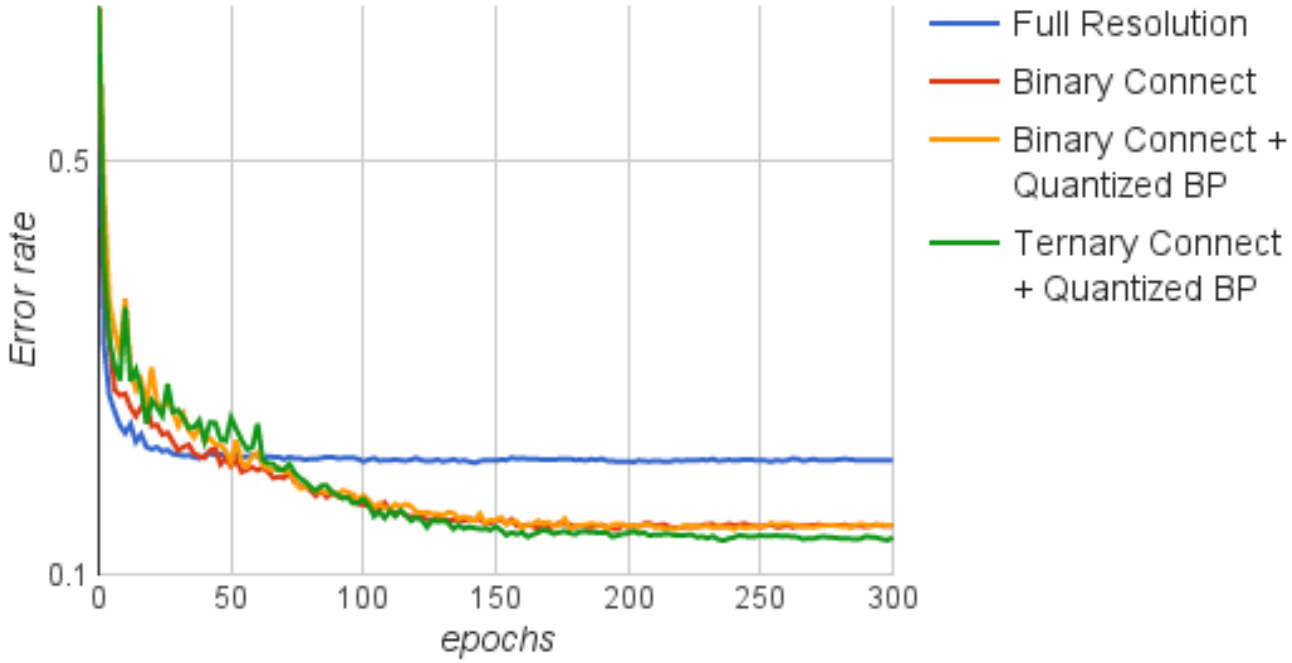
The Effect of Limiting the Range of Exponent

- Constraining the maximum allowed amount of bit shifts in quantized backprop.



General Performance

	Full precision	Binary connect	Binary connect + Quantized backprop	Ternary connect + Quantized backprop
MNIST	1.33%	1.23%	1.29%	1.15%
CIFAR10	15.64%	12.04%	12.08%	12.01%
SVHN	2.85%	2.47%	2.48%	2.42%



Related Works & Recent Advances

- Binarize both weights and activations [Courbariaux, et al., 2016]
- Exponential quantization over the forward pass.
- Larger, more serious datasets.
- Actual dedicated hardware realization.

Any questions?

References, Code & More: