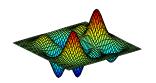
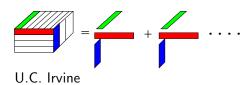
Guaranteed Non-convex Machine Learning Using Tensor Methods

Anima Anandkumar





Regime of Modern Machine Learning

Massive datasets, growth in computation power, challenging tasks

Success of Supervised Learning

- Learn p(y|x) from labeled samples $\{(x_i, y_i)\}$.
- Extract relevant features from large amounts of labeled data.



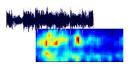




Image classification

Speech recognition

Text processing

Regime of Modern Machine Learning

Massive datasets, growth in computation power, challenging tasks

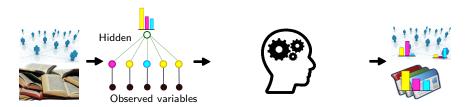
Missing Link in AI: Unsupervised Learning

- Learn p(x) from unlabeled samples $\{x_i\}$.
- Discover latent variables related to observed variable x.
- Human vs. Machine Learning: Make discoveries automatically.



Unsupervised Learning via Probabilistic Models

 $\mathsf{Data} \to \mathsf{Model} \to \mathsf{Learning} \ \mathsf{Algorithm} \to \mathsf{Predictions}$



Challenges in High dimensional Learning

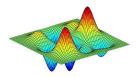
- Dimension of $x \gg \dim$ of latent variable h.
- Learning is like finding needle in a haystack.
- Computationally & statistically challenging.



Overview of Unsupervised Learning Methods

Goal: learn model parameters θ from observations x.

- Maximum likelihood: $\max_{\theta} p(x; \theta)$.
- Non-convex: stuck in local optima.
- Curse of dimensionality: Exponential no. of critical points.



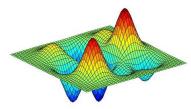
- Heuristics: Expectation Maximization, Variational Inference
- Other mechanisms such as Generative Adversarial Nets also non-convex.

Guaranteed Learning through Tensor Methods



Replace the objective function

Max Likelihood vs. Best Tensor decomp.



Preserves Global Optimum (infinite samples)

$$\arg \max_{\theta} p(x; \theta) = \arg \min_{\theta} \|\widehat{T}(x) - T(\theta)\|_{\mathbb{F}}^{2}$$

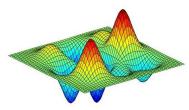
 $\widehat{T}(x)$: empirical tensor, $T(\theta)$: low rank tensor based on θ .

Guaranteed Learning through Tensor Methods



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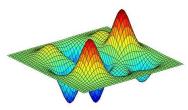
Simple algorithms succeed under mild and natural conditions for many learning problems.

Guaranteed Learning through Tensor Methods



Replace the objective function

Max Likelihood vs. Best Tensor decomp.



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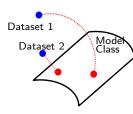
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 $\widehat{T}(x)$: empirical tensor, $T(\theta)$: low rank tensor based on θ .



Finding globally opt tensor decomposition

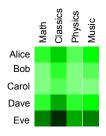
Simple algorithms succeed under mild and natural conditions for many learning problems.



Outline

- Introduction
- 2 Tensor Decomposition Algorithms
- Tensors for Probabilistic Models
- Tensors in Deep Learning
- 5 Steps Forward

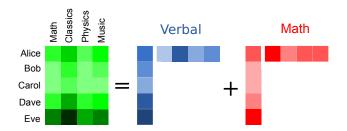
Matrix Decomposition: Discovering Latent Factors



- List of scores for students in different tests
- Learn hidden factors for Verbal and Mathematical Intelligence [C. Spearman 1904]

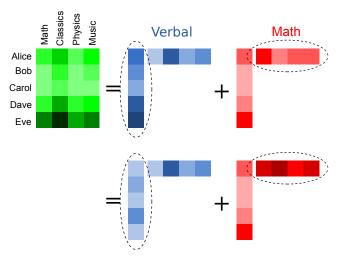
$$Score (student,test) = student_{verbal-intlg} \times test_{verbal} \\ + student_{math-intlg} \times test_{math}$$

Matrix Decomposition: Discovering Latent Factors



- Identifying hidden factors influencing the observations
- Characterized as matrix decomposition

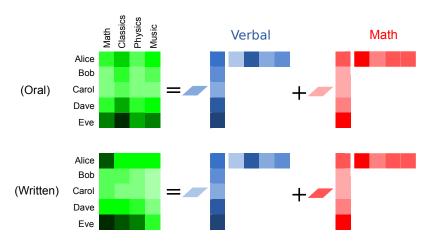
Matrix Decomposition: Discovering Latent Factors



- Decomposition is **not** necessarily **unique**.
- Decomposition cannot be overcomplete.



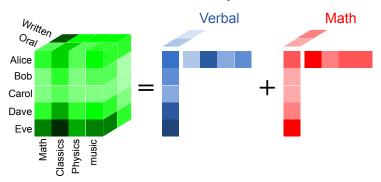
Tensor: Shared Matrix Decomposition



- Shared decomposition with different scaling factors
- Combine matrix slices as a tensor



Tensor Decomposition



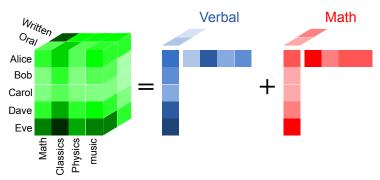
Outer product notation:

$$T = u \otimes v \otimes w + \tilde{\mathbf{u}} \otimes \tilde{\mathbf{v}} \otimes \tilde{\mathbf{w}}$$

$$\updownarrow$$

$$T_{i_1, i_2, i_3} = u_{i_1} \cdot v_{i_2} \cdot w_{i_3} + \tilde{\mathbf{u}}_{i_1} \cdot \tilde{v}_{i_2} \cdot \tilde{\mathbf{w}}_{i_3}$$

Tensor Decomposition



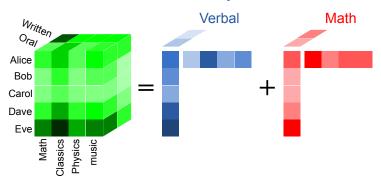
Uniqueness of Tensor Decomposition [J. Kruskal 1977]

- Above tensor decomposition: unique when rank one pairs are linearly independent
- Matrix case: when rank one pairs are orthogonal





Tensor Decomposition

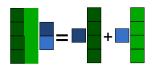


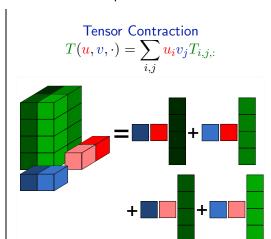
Finding Best Tensor Decomposition? Overcome Non-convexity?

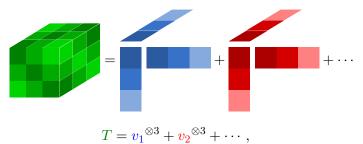
Notion of Tensor Contraction

Extends the notion of matrix product

$Mv = \sum_{i} v_{j} M_{j}$

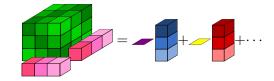






Tensor Power Method

$$v \mapsto \frac{T(v, v, \cdot)}{\|T(v, v, \cdot)\|}.$$

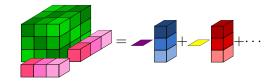


$$T(v, v, \cdot) = \langle v, v_1 \rangle^2 v_1 + \langle v, v_2 \rangle^2 v_2$$

A., R. Ge, D. Hsu, S. Kakade, M. Telgarsky, "Tensor Decompositions for Learning Latent

Tensor Power Method

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Orthogonal Tensors

- \bullet $\vec{v}_1 \perp \vec{v}_2$.
- $v_1 \perp v_2$. $T(v_1, v_1, \cdot) = \lambda_1 v_1$.



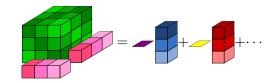
A., R. Ge, D. Hsu, S. Kakade, M. Telgarsky, "Tensor Decompositions for Learning Latent

Variable Models," JMLR 2014.



Tensor Power Method

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Orthogonal Tensors

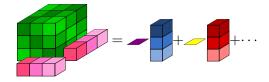
- \bullet $\vec{v}_1 \perp \vec{v}_2$.
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A., R. Ge, D. Hsu, S. Kakade, M. Telgarsky, "Tensor Decompositions for Learning Latent

Tensor Power Method

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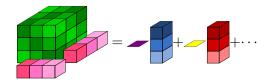
Exponential no. of stationary points for power method:

$$T(v, v, \cdot) = \lambda v$$

A., R. Ge, D. Hsu, S. Kakade, M. Telgarsky, "Tensor Decompositions for Learning Latent Variable Models," JMLR 2014.

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Exponential no. of stationary points for power method:

$$T(v, v, \cdot) = \lambda v$$
 Stable





Unstable

Other statitionary points



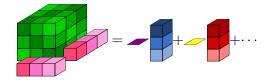
A., R. Ge, D. Hsu, S. Kakade, M. Telgarsky, "Tensor Decompositions for Learning Latent

Variable Models," JMLR 2014.



Tensor Power Method

$$v \mapsto \frac{T(v, v, \cdot)}{\|T(v, v, \cdot)\|}.$$



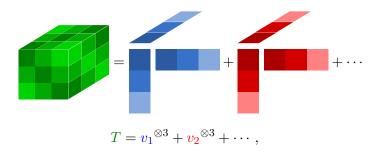
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Exponential no. of stationary points for power method:

$$T(v, v, \cdot) = \lambda v$$

For power method on orthogonal tensor, no spurious stable points

A., R. Ge, D. Hsu, S. Kakade, M. Telgarsky, "Tensor Decompositions for Learning Latent Variable Models," JMLR 2014.



A., R. Ge, D. Hsu, S. Kakade, M. Telgarsky, "Tensor Decompositions for Learning Latent Variable Models," JMLR 2014.

Orthogonalization

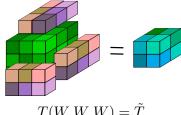


Input tensor T

A., R. Ge, D. Hsu, S. Kakade, M. Telgarsky, "Tensor Decompositions for Learning Latent Variable Models." JMLR 2014.



Orthogonalization

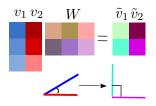


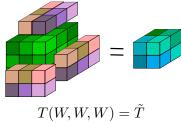
 $T(W, W, W) = \tilde{T}$

A., R. Ge, D. Hsu, S. Kakade, M. Telgarsky, "Tensor Decompositions for Learning Latent Variable Models." JMLR 2014.



Orthogonalization

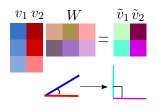


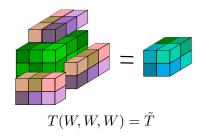


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Orthogonalization



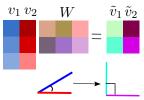


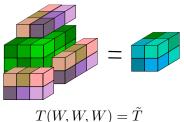
$$\tilde{T} = T(W, W, W) = \tilde{v_1}^{\otimes 3} + \tilde{v_2}^{\otimes 3} + \cdots,$$



A., R. Ge, D. Hsu, S. Kakade, M. Telgarsky, "Tensor Decompositions for Learning Latent Variable Models." JMLR 2014.

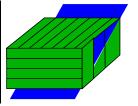






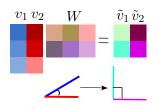
Find W using SVD of Matrix Slice

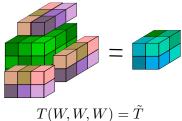




A., R. Ge, D. Hsu, S. Kakade, M. Telgarsky, "Tensor Decompositions for Learning Latent Variable Models," JMLR 2014.

Orthogonalization





Orthogonalization: invertible when v_i 's linearly independent.

Guaranteed tensor decomposition: when v_i 's linearly independent.

A., R. Ge, D. Hsu, S. Kakade, M. Telgarsky, "Tensor Decompositions for Learning Latent Variable Models." JMLR 2014.



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Extracting Topics from Documents



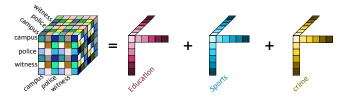
A., D. P. Foster, D. Hsu, S.M. Kakade, Y.K. Liu. "Two SVDs Suffice: Spectral decompositions for probabilistic topic modeling and latent Dirichlet allocation," NIPS 2012.

Tensor Methods for Topic Modeling

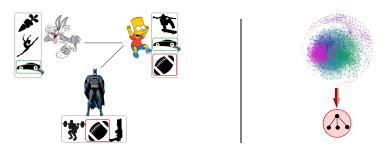


- Topic-word matrix $\mathbb{P}[\mathsf{word} = i | \mathsf{topic} = j]$
- Linearly independent columns

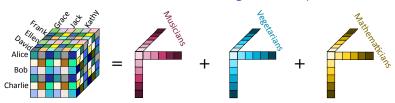
Moment Tensor: Co-occurrence of Word Triplets



Extracting Communities in Social Networks



Moment Tensor: Common Friends among Node Triplets



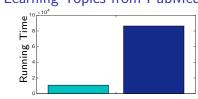
A., R. Ge, D. Hsu, S.M. Kakade. "A Tensor Spectral Approach to Learning Mixed Membership Community Models" COLT 2013.

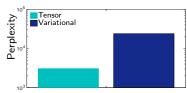


Tensors vs. Variational Inference

Criterion: Perplexity = $\exp[-likelihood]$.

Learning Topics from PubMed on Spark, 8mil articles



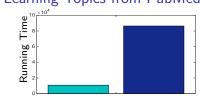


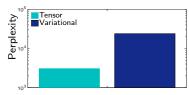


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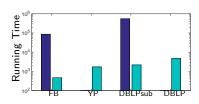
Learning Topics from PubMed on Spark, 8mil articles

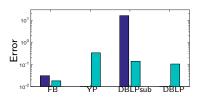




Learning network communities on single workstation

Facebook $n\sim 20k$, Yelp $n\sim 40k$, DBLP-sub $n\sim 1e5$, DBLP $n\sim 1e6$.





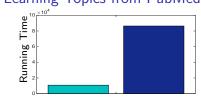
F. Huang, U.N. Niranjan, M. Hakeem, A, "Online tensor methods for training latent variable models," JMLR 2014.

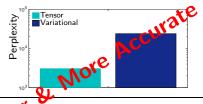


Tensors vs. Variational Inference

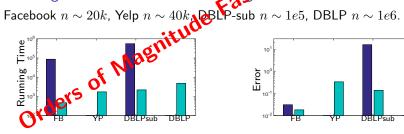
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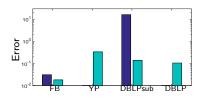
Learning Topics from PubMed on Spark, 8mil articles





Learning network communities on significant workstation





F. Huang, U.N. Niranjan, M. Hakeem, A, "Online tensor methods for training latent variable models," JMLR 2014.



Sparse coding prevalent in neural signaling.

Neural sparse coding [Papadopoulou11]

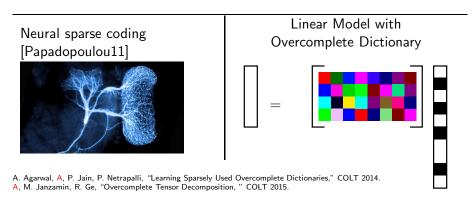


A. Agarwal, A, P. Jain, P. Netrapalli, "Learning Sparsely Used Overcomplete Dictionaries," COLT 2014.

A, M. Janzamin, R. Ge, "Overcomplete Tensor Decomposition," COLT 2015.



Sparse coding prevalent in neural signaling.



Contribution: learn overcomplete incoherent dictionaries

Neural sparse coding [Papadopoulou11] A. Agarwal, A. P. Jain, P. Netrapalli, "Learning Sparsely Used Overcomplete Dictionaries," COLT 2014. A. M. Janzamin, R. Ge, "Overcomplete Tensor Decomposition," COLT 2015.

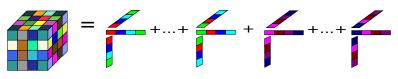
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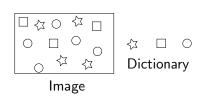
A. Agarwal, A, P. Jain, P. Netrapalli, "Learning Sparsely Used Overcomplete Dictionaries," COLT 2014.

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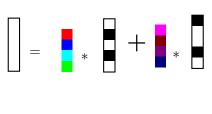
Efficient Tensor Decomposition with Shifted Components



Shift-invariant Dictionary

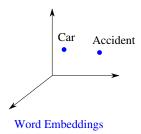


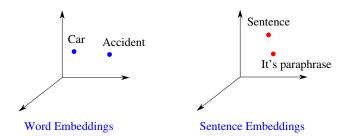
Convolutional Model



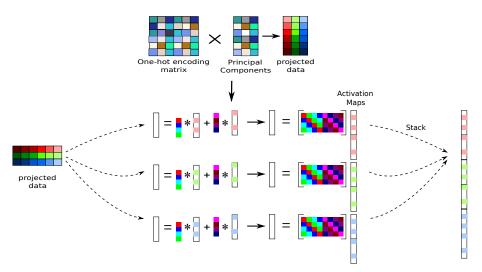
A. Agarwal, A, P. Jain, P. Netrapalli, "Learning Sparsely Used Overcomplete Dictionaries," COLT 2014.

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Paraphrase Detection on MSR corpus with ~ 5000 Sentences



Paraphrase Detection on MSR corpus with ~ 5000 Sentences

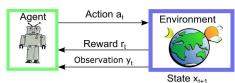
Method	F score	No. of samples
Vector Similarity (Baseline)	75%	$\sim 4k$
Tensor (Proposed)	81%	$\sim 4k$
Skipthought (RNN)	82%	~ 74 mil

- Unsupervised learning of embeddings.
- No outside info for tensor vs. large book corpus (74 million) for skipthought
- Similar story with holographic embeddings for knowledge bases by M.
 Nickel et al.

Reinforcement Learning of Partially Observable Markov Decision Process

Learning in Adaptive Environments

- Learner changes environment
- Hidden state estimation.



Reinforcement Learning of Partially Observable Markov Decision Process

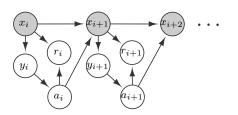
Learning in Adaptive Environments

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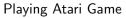
Agent Reward r_t Observation y_t State x_{t+1}

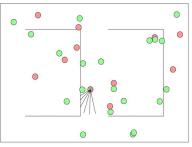
Partially Observable Markov Decision Process

- Design of tensor algorithms under memoryless policies
- Guaranteed regret bounds: comparable to fully observed environment.

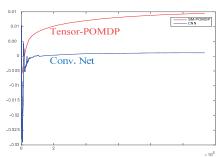


Reinforcement Learning of Partially Observable Markov Decision Process



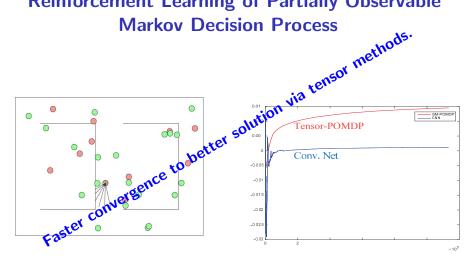


Average Reward vs. Time.



K. Azizzadenesheli, A. Lazaric, A, "Reinforcement Learning of POMDPs using Spectral Methods," 2016.

Reinforcement Learning of Partially Observable



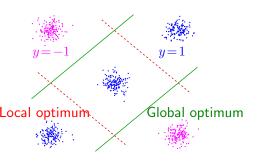
K. Azizzadenesheli, A. Lazaric, A, "Reinforcement Learning of POMDPs using Spectral Methods," 2016.

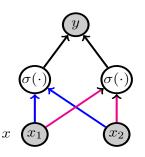
Outline

- Introduction
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Local Optima in Backpropagation

"..few researchers dare to train their models from scratch.. small miscalibration of initial weights leads to vanishing or exploding gradients.. poor convergence..*"



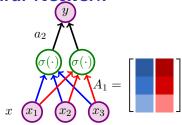


Exponential (in dimensions) no. of local optima for backpropagation

P. Krahenbhl, C. Doersch, J. Donahue, T. Darrell "Data-dependent Initializations of Convolutional Neural Networks", ICLR 2016.



$$\mathbb{E}[y|x] := f(x) = \langle a_2, \sigma(A_1^\top x) \rangle$$

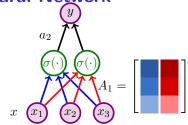


[&]quot;Score Function Features for Discriminative Learning: Matrix and Tensor Framework" by M. Janzamin, H. Sedghi, and A., Dec. 2014.



$$\mathbb{E}[y|x] := f(x) = \langle a_2, \sigma(A_1^\top x) \rangle$$

Moments using score functions $S(\cdot)$



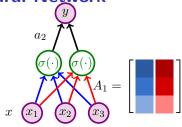
[&]quot;Score Function Features for Discriminative Learning: Matrix and Tensor Framework" by M. Janzamin, H. Sedghi, and A., Dec. 2014.



$$\boxed{\mathbb{E}[y|x] := f(x) = \langle a_2, \sigma(A_1^\top x) \rangle}$$

Moments using score functions $\mathcal{S}(\cdot)$

$$\mathbb{E}\left[y\cdot\mathcal{S}_1(x)\right] = +$$



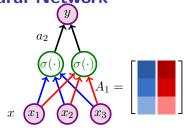
[&]quot;Score Function Features for Discriminative Learning: Matrix and Tensor Framework" by M. Janzamin, H. Sedghi, and A., Dec. 2014.



$$\boxed{\mathbb{E}[y|x] := f(x) = \langle a_2, \sigma(A_1^\top x) \rangle}$$

Moments using score functions $S(\cdot)$

$$\mathbb{E}\left[y\cdot\mathcal{S}_2(x)\right] = -$$



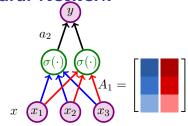
[&]quot;Score Function Features for Discriminative Learning: Matrix and Tensor Framework" by M. Janzamin, H. Sedghi, and A., Dec. 2014.



$$\mathbb{E}[y|x] := f(x) = \langle a_2, \sigma(A_1^\top x) \rangle$$

Moments using score functions $\mathcal{S}(\cdot)$

$$\mathbb{E}\left[y\cdot\mathcal{S}_3(x)\right] = +$$



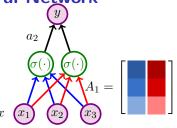
[&]quot;Score Function Features for Discriminative Learning: Matrix and Tensor Framework" by M. Janzamin, H. Sedghi, and A., Dec. 2014.



$$\mathbb{E}[y|x] := f(x) = \langle a_2, \sigma(A_1^\top x) \rangle$$

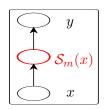
Moments using score functions $\mathcal{S}(\cdot)$

$$\mathbb{E}\left[y\cdot\mathcal{S}_3(x)\right] = +$$



Given input pdf
$$p(\cdot)$$
, $S_m(x) := (-1)^m \frac{\nabla^{(m)} p(x)}{p(x)}$.

Gaussian $x \Rightarrow$ Hermite polynomials.



[&]quot;Score Function Features for Discriminative Learning: Matrix and Tensor Framework" by M. Janzamin, H. Sedghi, and A., Dec. 2014.

Tensorizing Neural Networks

- Multi-linear representation of dense layers of CNNs.
 - ► Tensor train format for low rank approximation of weight matrix.
- Compact representation: solves memory problem.

$$Y(i_1, i_2 \dots) = \sum_{j_1, j_2 \dots} G(i_1, j_1) G(i_2, j_2) \dots X(j_1, j_2 \dots)$$



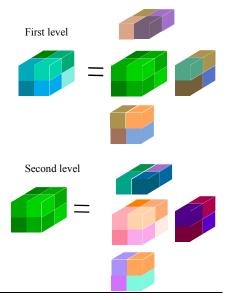
Results on ImageNet

- Compression rate 200,000!
- Negligible performance loss.

A. Novikov, D. Podoprikhin, A. Osokin, D. Vetrov, "Tensorizing Neural Networks", NIPS 2015.

Tensor Analysis for Expressive Power

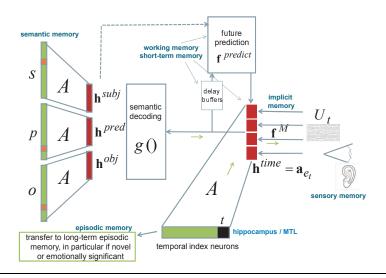
- Hierarchical Tucker tensors for representing arithmetic conv nets.
- Employs locality, sharing and pooling.
- Exponentially more parameters in shallow net vs. deep net.



N. Cohen, O. Sharir, A. Shashua, "On the Expressive Power of Deep Learning: A Tensor

Tensors in Memory Embeddings

Human Memory Model. Semantic decoding through Tensor Tucker.



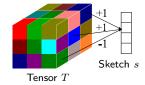
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Scaling up and Deploying Tensor Methods

Scaling up

- Dimensionality reduction through sketching.
- Communication efficient methods.



Deployment

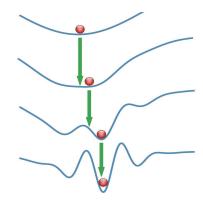
- Multi-platform support: CPU, GPU, Cloud, FPGA . . .
- Extended BLAS kernels: Beyond linear algebra.
- Many deep learning operations involve tensor contractions.

Wang, Tung, Smola, A. "Guaranteed Tensor Decomposition via Sketching", NIPS'15.

Cecka, Niranjan, Shi, A, "Tensor Contractions with Extended BLAS kernels on CPU and GPU", under preparation.

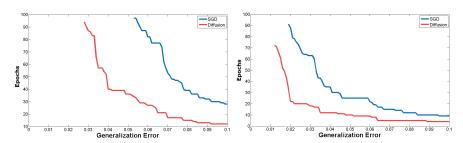
Smoothing and Continuation Methods

- Global approach vs. local search.
- Unified guarantees for non-convex problems?



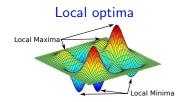
H. Mobahi, "Training RNNs by Diffusion" .

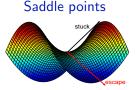
Learning to add using RNN



H. Mobahi, "Training RNNs by Diffusion" .

- Escaping saddle points in high dimensions?
- Can SGD escape in bounded time?
- Degeneracy of saddle points in various non-convex problems?



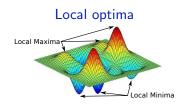


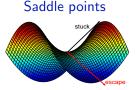
Efficient approaches for escaping higher order saddle points in non-convex optimization by A., R. Ge, COLT 2016.



Contribution: First method to escape third order saddle

- Escaping saddle points in high dimensions?
- Can SGD escape in bounded time?
- Degeneracy of saddle points in various non-convex problems?





Efficient approaches for escaping higher order saddle points in non-convex optimization by A., R. Ge, COLT 2016.



Research Connections and Resources

Collaborators

Jennifer Chayes, Christian Borgs, Prateek Jain, Alekh Agarwal & Praneeth Netrapalli (MSR), Srinivas Turaga (Janelia), Michael Hawrylycz & Ed Lein (Allen Brain), Allesandro Lazaric (Inria), Alex Smola (CMU), Rong Ge (Duke), Daniel Hsu (Columbia), Sham Kakade (UW), Hossein Mobahi (MIT).



 Podcast/lectures/papers/software available at http://newport.eecs.uci.edu/anandkumar/