Nonequilibrium quantum physics: Exact steps

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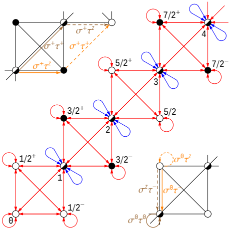

Kolokvij na IJS, 7.6.2016

How to DO theoretical physics?

Ludvig D. Faddeev vs. Phillip W. Anderson

`according to: L.D.Faddeev, 'After-dinner speech', Rome, 2010`

- 'Top-down approach'
  Principles, symmetries, conservation laws ⇒ models ⇒ phenomenology
- 'Bottom-up' approach
  Phenomenology, effects ⇒ models, simulations ⇒ principles, symmetries, conservation laws

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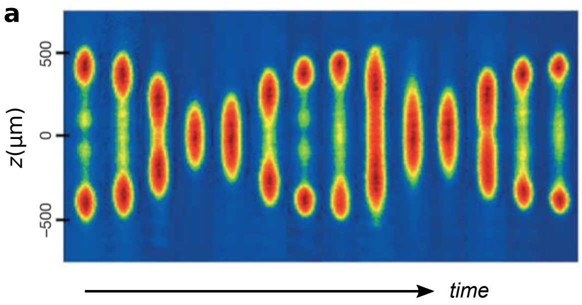
Time evolution

Newton's cradle with the ultracold bose-atom gas (^{87}Rb atoms)
(Konishita, Wegner and Weiss, Nature 440, 900 (2006))



$$|\Psi(t)\rangle\langle\Psi(t)| \longrightarrow Z^{-1} \exp\left(-\sum_j \beta_j F_j\right), \quad F_2 \equiv H$$

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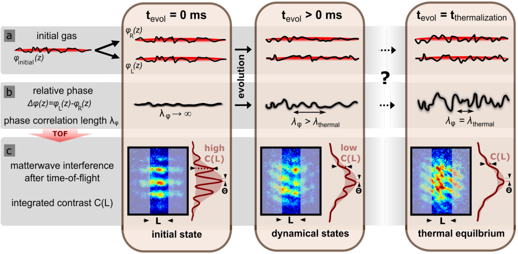
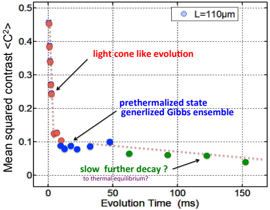
Observation of prethermalization and Generalized Gibbs ensemble

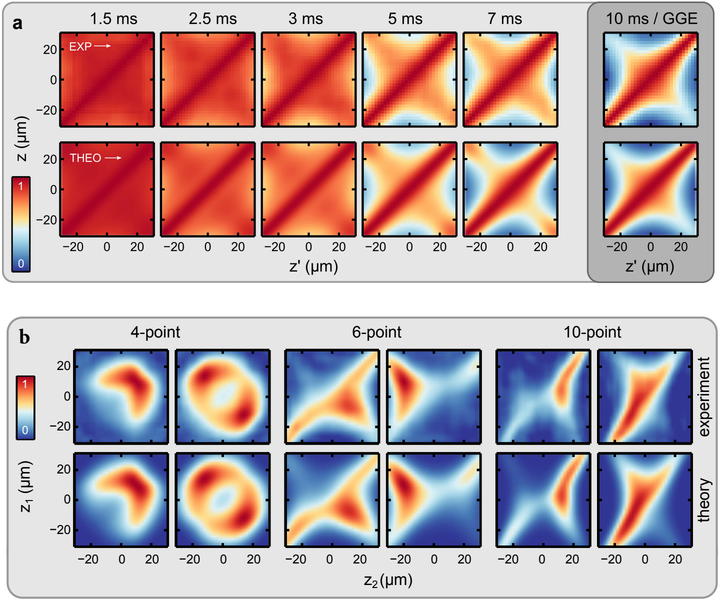
review: Langer, Gasenzer and Schmiedmayer, to appear in J. Stat. Mech. (arXiv:1603.09385)



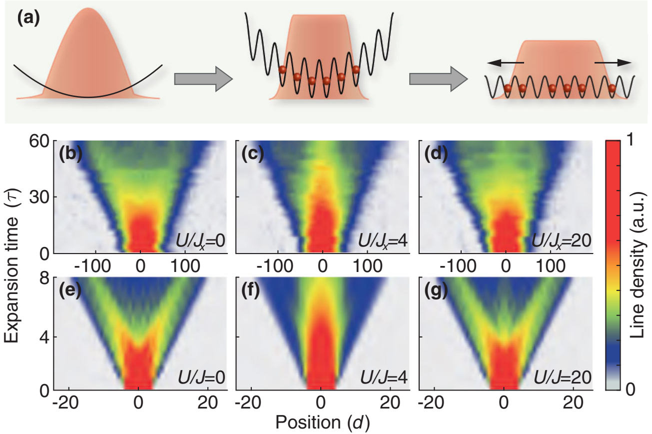


Gring et al. Science **337**, 1318 (2012)

Precise experimental measurement of correlation functions



Ronzheimer et al. Phys. Rev. Lett. **110**, 205301 (2013)
Bosonic ^{39}K atoms..



J.T. Barreiro et al. Nature **470**, 486 (2011)

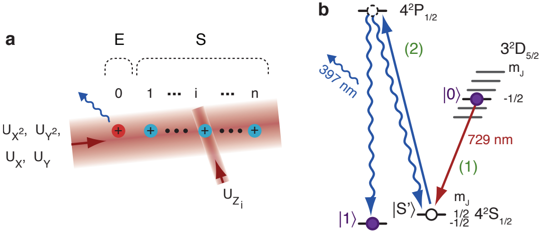
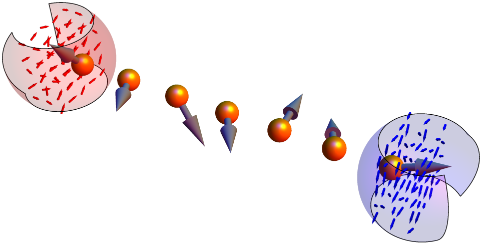


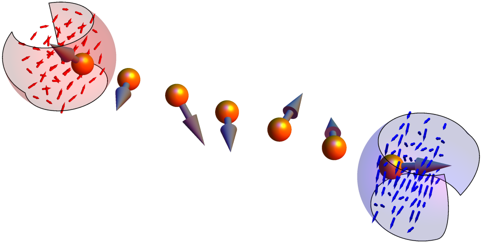
FIG. 1. **Experimental tools for the simulation of open quantum systems with ions. a,** The coherent component is realized by collective ($U_X, U_Y, U_{X^2}, U_{Y^2}$) and single-qubit operations (U_{Z_i}) on a string of $^{40}Ca^+$ ions which consists of the environment qubit (ion 0) and the system qubits (ions 1 through n). **b,** The dissipative mechanism on the ancilla qubit is realized in the two steps shown on the Zeeman-split $^{40}Ca^+$ levels by (1) a coherent transfer of the population from $|0\rangle$ to $|S'\rangle$ and (2) an optical pumping to $|1\rangle$ after a transfer to the $4^2P_{1/2}$ state by a circularly-polarised laser at 397 nm.



Open quantum system's approach:
Canonical markovian master equation for the many-body density matrix:

The Lindblad (L-GKS) equation:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \hat{\mathcal{L}}\rho := -\mathrm{i}[H,\rho] + \sum_{\mu}\left(2L_\mu \rho L_\mu^\dagger - \{L_\mu^\dagger L_\mu, \rho\}\right).$$



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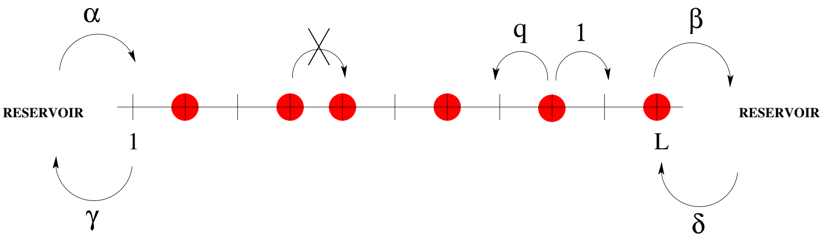
- *Bulk*: Fully **coherent**, local interactions,e.g. $H = \sum_{x=1}^{n-1} h_{x,x+1}$.
- *Boundaries:* Fully **incoherent**, ultra-local dissipation, jump operators L_μ supported near boundaries $x = 1$ or $x = n$.

Markovian model on a 2^L dimensional probability state vector $\underline{p}(t)$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\underline{p} = M\underline{p}$$



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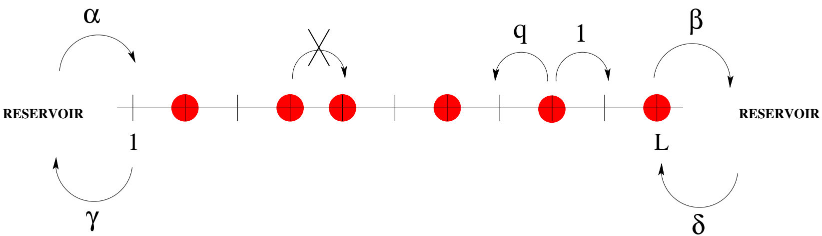

Nonequilibrium steady state (NESS): a fixed point probability state vector $\underline{p}_{\infty}$

$$M\underline{p}_{\infty} = 0$$

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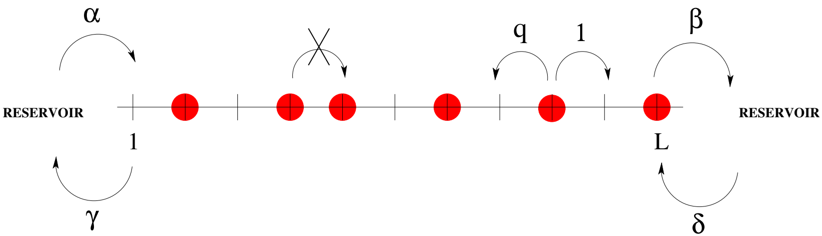

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Applications: driven diffusive systems, traffic flow, hopping conductivity in soild electrolytes, Motion of RNA templates, Brownian motors, etc.

Derrida, Evans, Hakim & Pasquier, J. Phys. A (1993): 

Let $\mathbf{A}_0, \mathbf{A}_1$ be a pair of matrices, and $\langle L|$, $|R\rangle$ a pair of left and right 'vacua'.

$$\mathrm{MPA}: \qquad p_{s_1,s_2,\ldots,s_L} = \langle L|\mathbf{A}_{s_1}\mathbf{A}_{s_2}\cdots\mathbf{A}_{s_L}|R\rangle, \quad s_j \in \{0,1\}$$

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Asking such MPA $\underline{p}$ to solve the Markov fixed point condition $M\underline{p} = 0$ results in a single algebraic relation in the bulk

$$\mathbf{A}_1\mathbf{A}_0 - q\mathbf{A}_0\mathbf{A}_1 = (1-q)(\mathbf{A}_0 + \mathbf{A}_1)$$

with two boundary conditions

$$\langle L|(\alpha\mathbf{A}_0 - \gamma\mathbf{A}_1) = \langle L|, \quad (\beta\mathbf{A}_1 - \delta\mathbf{A}_0)|R\rangle = |R\rangle$$

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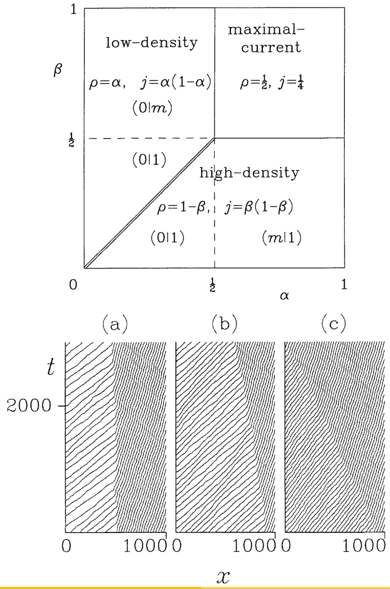
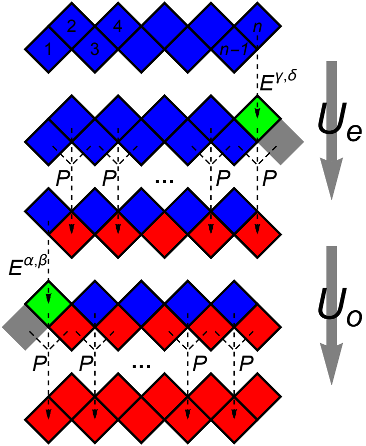
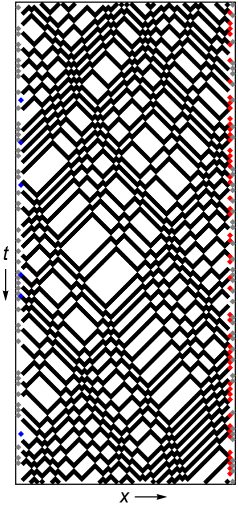
This algebraic structure is enough to yield all physical observables in NESS!

Rich non-equilibrium phase diagram of TASEP (q=0)

Kolomeisky et al. J. Phys. A: Math.& Gen. **31**, 6911 (1998)







TP, C. Mejia-Monasterio, J. Phys. A: Math. Theor. **49**, 185003 (2016)

Exact solution for NESS exist in terms of a specific matrix product ansatz!

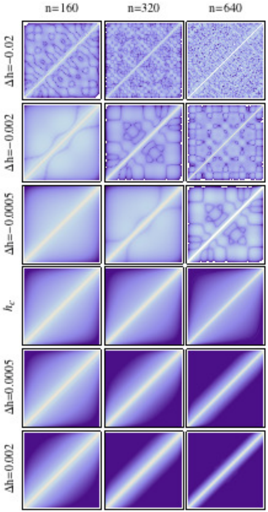
TP & I. Pižorn, PRL **101**, 105701 (2008)

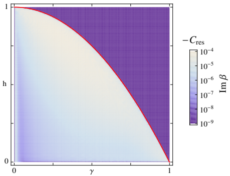
$$H = \sum_j^n \left(\frac{1+\gamma}{2} \sigma_j^{\mathrm{x}} \sigma_{j+1}^{\mathrm{x}} + \frac{1-\gamma}{2} \sigma_j^{\mathrm{y}} \sigma_{j+1}^{\mathrm{y}} + h \sigma_j^{\mathrm{z}} \right)$$

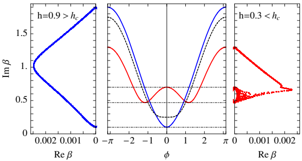
$$L_1 = \frac{1}{2}\sqrt{\Gamma_1^{\mathrm{L}}}\sigma_1^- \qquad L_3 = \frac{1}{2}\sqrt{\Gamma_1^{\mathrm{R}}}\sigma_n^-$$

$$L_2 = \frac{1}{2}\sqrt{\Gamma_2^{\mathrm{L}}}\sigma_1^+ \qquad L_4 = \frac{1}{2}\sqrt{\Gamma_2^{\mathrm{R}}}\sigma_n^+$$

$$C(j,k) = \langle \sigma_j^{\mathrm{z}} \sigma_k^{\mathrm{z}} \rangle - \langle \sigma_j^{\mathrm{z}} \rangle \langle \sigma_k^{\mathrm{z}} \rangle$$







Steady state Lindblad equation $\hat{\mathcal{L}}\rho_\infty = 0$:

$$\mathrm{i}[H, \rho_\infty] = \sum_\mu \left(2L_\mu \rho_\infty L_\mu^\dagger - \{L_\mu^\dagger L_\mu, \rho_\infty\}\right)$$

The *XXZ* Hamiltonian:

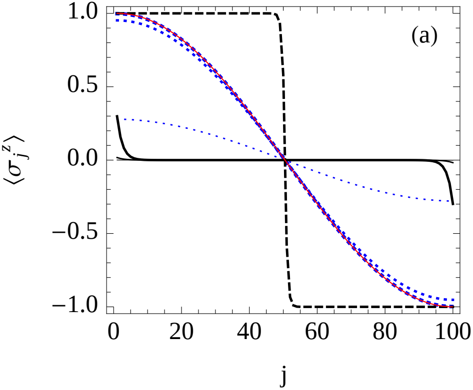
$$H = \sum_{x=1}^{n-1} (2\sigma_x^+ \sigma_{x+1}^- + 2\sigma_x^- \sigma_{x+1}^+ + \Delta \sigma_x^z \sigma_{x+1}^z)$$

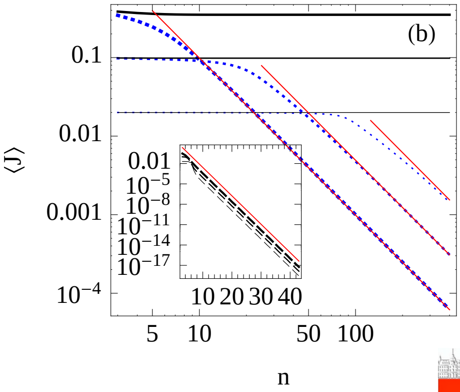
and in&out (source& sink) Lindblad jump operators:

$$L_1 = \sqrt{\varepsilon}\sigma_1^+, \quad L_2 = \sqrt{\varepsilon}\sigma_n^-.$$

Again, nonequilibrium phase transition in the steady state!

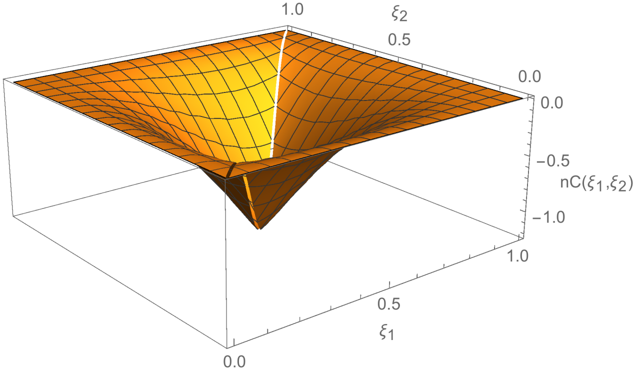
- For $|\Delta| < 1$, $\langle J \rangle \sim n^0$ (ballistic)
- For $|\Delta| > 1$, $\langle J \rangle \sim \exp(-\mathrm{const}\, n)$ (insulating)
- For $|\Delta| = 1$, $\langle J \rangle \sim n^{-2}$ (anomalous)





$$C\left(\frac{x}{n},\frac{y}{n}\right)=\langle\sigma^{\rm z}_x\sigma^{\rm z}_y\rangle-\langle\sigma^{\rm z}_x\rangle\langle\sigma^{\rm z}_y\rangle$$

for isotropic case $\Delta=1$ (XXX)



$$C(\xi_1,\xi_2)=-\frac{\pi^2}{2n}\xi_1(1-\xi_2)\sin(\pi\xi_1)\sin(\pi\xi_2),\quad \text{for}\quad \xi_1<\xi_2$$

Exact solution for the steady state: Matrix product ansatz

TP, PRL**106**(2011); PRL**107**(2011); Karevski, Popkov, Schütz, PRL**111**(2013)

$$\rho_\infty = (\operatorname{tr} R)^{-1} R, \quad R = \Omega\Omega^\dagger$$

$$\Omega = \sum_{(s_1,\ldots,s_n)\in\{+,-,0\}^n} \langle 0|\mathbf{A}_{s_1}\mathbf{A}_{s_2}\cdots\mathbf{A}_{s_n}|0\rangle \sigma^{s_1}\otimes\sigma^{s_2}\cdots\otimes\sigma^{s_n} = \langle 0| \begin{pmatrix} \mathbf{A}_0 & \mathbf{A}_+ \\ \mathbf{A}_- & \mathbf{A}_0 \end{pmatrix}^{\otimes n} |0\rangle$$

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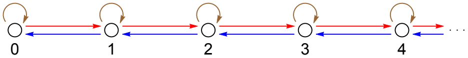
$$\begin{aligned}
\mathbf{A}_0 &= \sum_{k=0}^{\infty} a_k^0 |k\rangle\langle k|, \\
\mathbf{A}_+ &= \sum_{k=0}^{\infty} a_k^+ |k\rangle\langle k+1|, \\
\mathbf{A}_- &= \sum_{k=0}^{\infty} a_k^- |k+1\rangle\langle r|,
\end{aligned}$$



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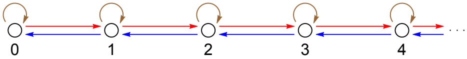

$$\mathbf{A}_- = \sum_{k=0}^{\infty} a_k^- |k+1\rangle\langle r|,$$

$$a_k^0 = \cos((s-k)\eta) \qquad \cos\eta := \Delta,$$

$$a_k^+ = \sin((k+1)\eta) \qquad \tan(\eta s) := \frac{\varepsilon}{2\mathrm{i}\sin\eta}$$

$$a_k^- = \cos((2s-k)\eta)$$

s is a q−deformed complex spin $q = e^{\mathrm{i}\eta}$

Example: Hamiltonian with local interactions

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Continuity equation — local conservation laws:

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$J_j \equiv$ energy current density

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$J_j \equiv$ energy current density

In general: F some general extensive quantity, $F = \sum_j f_j$, f_j local around site j. From $[H, F] = 0$ we have

$$\frac{\mathrm{d}}{\mathrm{d}t} f_j = \mathrm{i}[H, f_j] = g_j - g_{j+1}$$

$g_j \equiv$ corresponding density of current of F.

Near equilibrium transport and conservation laws

Green-Kubo formula:

$$\kappa(\omega) = \lim_{t\to\infty} \lim_{n\to\infty} \frac{\beta}{n} \int_0^t \mathrm{d}t' e^{\mathrm{i}\omega t} \langle J(t')J(0)\rangle_\beta$$

Divergence of d.c. conductivity defines the Drude weight D,

$$\kappa(\omega) = 2\pi D\delta(\omega) + \kappa_{\mathrm{reg}}(\omega)$$

which again can be expressed in terms of a *linear response* formula

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For integrable systems, Zotos et al. (1997) proposed to apply Mazur (1969) / Suzuki (1971) bound to estimate Drude weight in terms of conserved quantities F_j, $[H, F_j] = 0$:

$$D \geq \lim_{n\to\infty} \frac{\beta}{2n} \sum_j \frac{\langle J F_j \rangle_\beta^2}{\langle F_j^2 \rangle_\beta}$$

where operators F_j may be chosen mutually orthogonal $\langle F_j F_k \rangle = 0$, $j \neq k$.

$$D \geq \lim_{n\to\infty} \frac{\beta}{2n} \sum_j \frac{\langle J F_j \rangle_\beta^2}{\langle F_j^2 \rangle_\beta}$$

Conclusion: Integrable systems are ballistic conductors at any temperature, unless ...

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Conclusion: Integrable systems are ballistic conductors at any temperature, unless ...

... unless all conserved quantities F_j orthogonal to current operator J, $\langle J F_j \rangle = 0$, which may happen due to discrete symmetries

'Square root' NESS $\rho_\infty \propto \Omega(s)\Omega^\dagger(s)$, $s = s(\varepsilon)$, generates non-Hermitian commuting family

$$[\Omega(s), \Omega(s')] = 0, \qquad \forall s, s' \in \mathbb{C}$$

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Derivative of such non-equilibrium 'transfer matrix' defines a quasi-local conserved quantity

$$Z = \frac{\mathrm{d}}{\mathrm{d}s}\Omega(s)|_{s=0}, \quad \langle Z^\dagger Z\rangle \propto n.$$

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$$[H, Q] = 2\sigma_1^z - 2\sigma_n^z$$

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Q is in fact the first-order of NESS

$$\rho_\infty \propto \mathbb{1} + \varepsilon Q + \mathcal{O}(\varepsilon^2).$$

Corollary: Rigorous lower bound on spin Drude weight

$\exists$ spin flip symmetry $P = \prod_j \sigma_j^{\mathrm{x}}$: $[H, P] = 0$ ter $[F_j, P] = 0$ for all local conserved quantities F_j, but $PJ = -JP$, hence $\langle F_j J \rangle = 0$.

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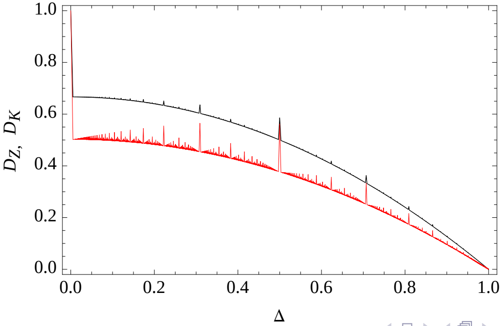
Fractal Mazur bound on Drude weight

$$\frac{D}{\beta} \geq D_Z := \frac{\sin^2(\pi l/m)}{\sin^2(\pi/m)} \left(1 - \frac{m}{2\pi} \sin\left(\frac{2\pi}{m}\right)\right), \quad \Delta = \cos\left(\frac{\pi l}{m}\right)$$

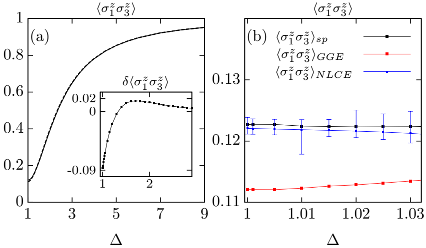
TP, Ilievski, PRL **111**, 057203 (2013); TP, NPB **886**, 1177 (2014)



Generalized Gibbs ensemble $\rho_{\mathrm{GGE}} = \exp(-\sum_{j=1}^{\infty} \beta_j F_j)$ for the steady state after a quantum quench of XXZ Hamiltonian gives **incorrect results**!

$$H = \sum_{x=1}^{n-1} (2\sigma_x^+ \sigma_{x+1}^- + 2\sigma_x^- \sigma_{x+1}^+ + \Delta \sigma_x^z \sigma_{x+1}^z)$$

B. Wouters *et al.*, Phys. Rev. Lett. **113**, 117202 (2014); M. Brockmann *et al.*, J. Stat. Mech. P12009 **(2014)**:



B. Pozsgay *et al.*, Phys. Rev. Lett. **113**, 117203 (2014); M. Mestyan *et al.*, J. Stat. Mech. P04001 **(2015)**:

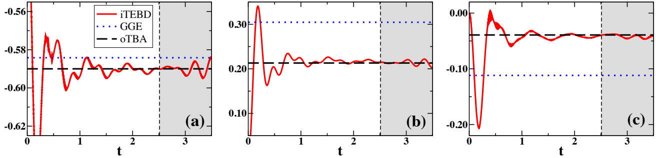
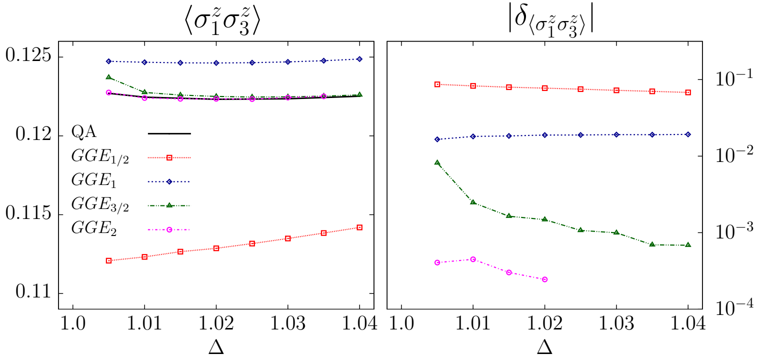


FIG. 1: Numerical simulation of the time evolution of correlation functions (a) $\langle\sigma_1^z\sigma_2^z\rangle$, (b) $\langle\sigma_1^z\sigma_3^z\rangle$, (c) $\langle\sigma_1^z\sigma_4^z\rangle$

New quasilocal conserved charges [E. Ilievski, M. Medenjak, TP, PRL **115**, 120601 (2015)] close the gap between GGE and QA:

E. Ilievski, J. De Nardis, B. Wouters, J.-S. Caux, F. H. L. Essler, TP, PRL **115**, 157201 (2015)



Consider $2s+1$ dimensional spin$-s$ auxiliary space $\mathcal{H}_{\rm a} = \mathcal{V}_s$ with SU(2) generators represented as

$$\mathbf{s}^{\rm z}|m\rangle = m|m\rangle,\ \mathbf{s}^{\pm}|m\rangle = \sqrt{(s+1\pm m)(s\mp m)}|m\pm 1\rangle$$

and define Lax operators acting over $\mathcal{H}_{\rm p}\otimes\mathcal{H}_{\rm a}$, $\mathcal{H}_{\rm p} = \mathcal{V}_{1/2}^{\otimes n}$,

$$\mathbf{L}_{x,{\rm a}}(\lambda) = \lambda\mathbb{1} + \sigma^{\rm z}_x\mathbf{s}^{\rm z}_{\rm a} + \sigma^{+}_x\mathbf{s}^{-}_{\rm a} + \sigma^{-}_x\mathbf{s}^{+}_{\rm a} = \lambda\mathbb{1} + \vec{\sigma}_x\cdot\vec{\mathbf{s}}_{\rm a},$$

in turn defining a commuting set of transfer matrices

$$T_s(\lambda) = {\rm tr}_{\rm a}\mathbf{L}_{0,{\rm a}}(\lambda)\mathbf{L}_{1,{\rm a}}(\lambda)\cdots\mathbf{L}_{n-1,{\rm a}}(\lambda),$$
$$[T_s(\lambda), T_{s'}(\lambda')] = 0,\quad \forall s,s',\lambda,\lambda'.$$

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The fundamenal TM $T_{1/2}(\lambda)$ generates all local charges as

$$F_k = -{\rm i}\partial_t^{k-1}\log T_{1/2}(\tfrac{1}{2}+{\rm i}t)|_{t=0},$$

with $H_{XXX} = Q_2$.

Theorem (PRL **115**, 120601 (2015)):

Traceless operators $X_s(t)$, $s \in \frac{1}{2}\mathbb{Z}^+, t \in \mathbb{R}$, defined as

$$\begin{aligned} X_s(t) &= [\tau_s(t)]^{-n} \left\{ T_s(-\tfrac{1}{2} + \mathrm{i}t) T_s'(\tfrac{1}{2} + \mathrm{i}t) \right\}, \\ \tau_s(t) &= -t^2 - \left(s + \tfrac{1}{2}\right)^2, \end{aligned}$$

where $T_s'(\lambda) \equiv \partial_\lambda T_s(\lambda)$ and $\{A\} \equiv A - (\operatorname{tr} A)\mathbb{1}/(\operatorname{tr} \mathbb{1})$, are quasilocal for all s, t and linearly independent from $\{F_k; k \geq 2\}$ for $s > \frac{1}{2}$.

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Inspiration: for $s = 1/2$, TM is asymptitically, $n \to \infty$, a unitary operator

$$
T_{1/2}(\tfrac{1}{2} + \mathrm{i}t) \simeq \exp\left(\mathrm{i} \sum_{k=1}^{\infty} F_{k+1} t^k / k! \right),
$$

[`Fagotti and Essler, JSTAT P07012 (2013)`] hence $X_{1/2}(t)$ is a logarithmic derivative, since $T_s^\dagger(\lambda) \equiv T_s^T(\bar{\lambda}) = (-1)^n T_s(-\bar{\lambda})$.

- Quasi-local charges exist in integrable lattice models and are as relevant for non-equilibrium physics and relaxation to equilibrium as the local ones!
- Stability of quasilocal charges and integrable nonequilibrium steady states against perturbations?
- Quantum field theories?
- Classical integrable lattice systems?
- Integrable bosonic lattices/field theories?

Acknowledgements:
My (former) students who directly or indirectly contributed to the work reported here: I. Pižorn, B. Žunkovič, E. Ilievski, B. Buča, M. Medenjak, L. Zadnik

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