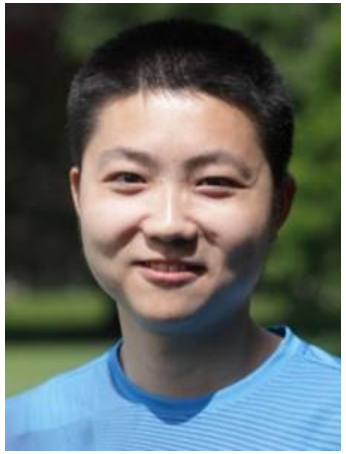
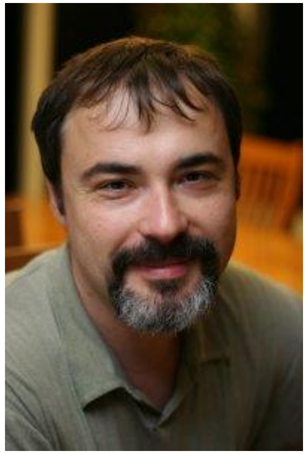
Western UNIVERSITY · CANADA

ÉTS Le génie pour l'industrie

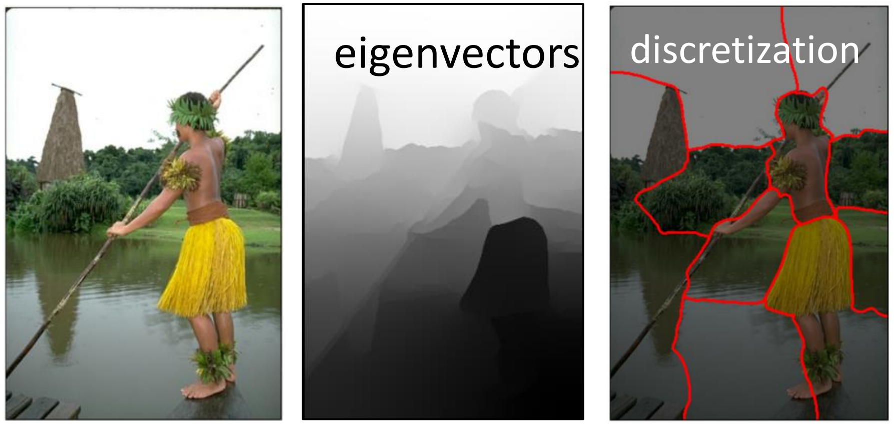
# Normalized Cut Meets MRF

Meng Tang, Dmitrii Marin, Ismail Ben Ayed, Yuri Boykov

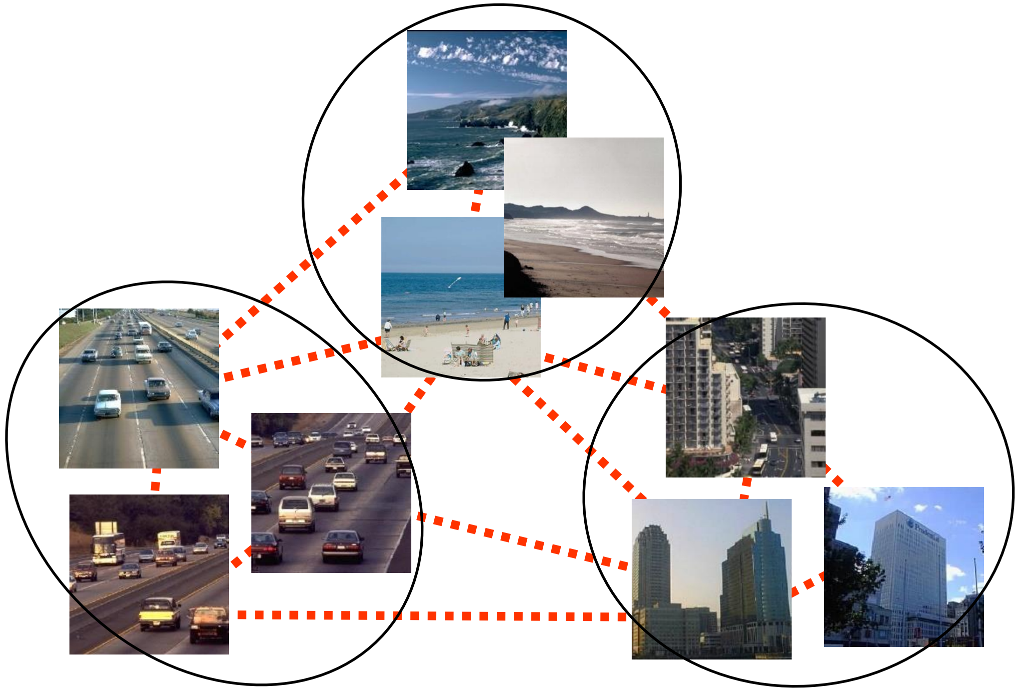
   

# Normalized Cut (**NC**)

$$NC(S) = -\sum_k \frac{assoc(S^k, S^k)}{assoc(S^k, \Omega)}$$




*[Arbelaez, Maire, Fowlkes & Malik, 2010]*



*[Shi & Malik, 2000] [Ng, Jordan & Weiss, 2002] [Belkin & Niyogi, 2003] [Luxburg, 2007]*

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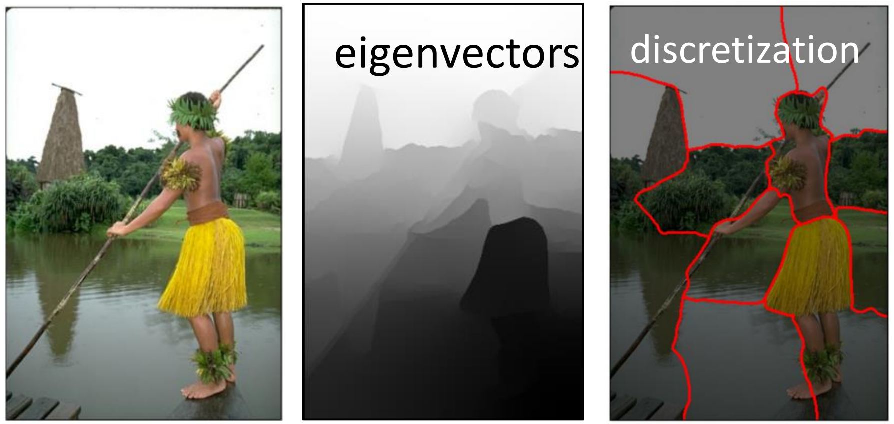
$$NC(S) = -\sum_k \frac{S^{k\prime} A S^k}{d' S^k}$$

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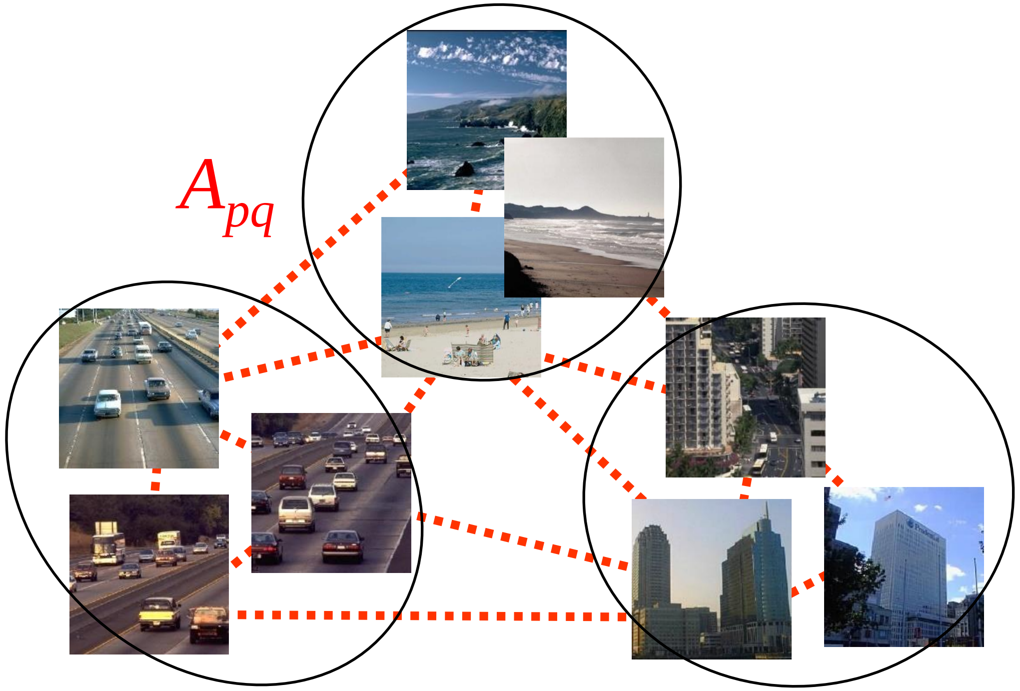


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# Markov Random Field (**MRF**)

(graphical models)

$$\sum_{c \in \mathcal{F}} E_c(S_c)$$

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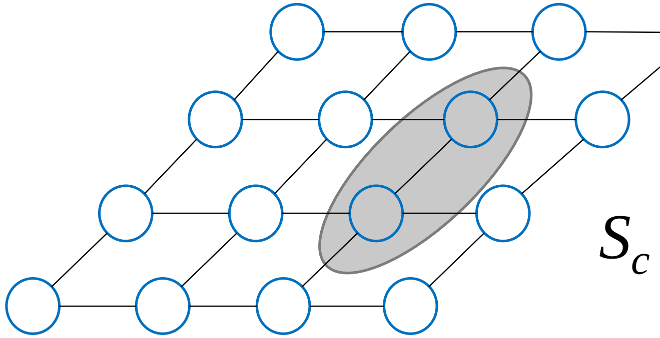


*Potts Model*: edge alignment

interactive segmentation *[Boykov & Jolly , 2001]*

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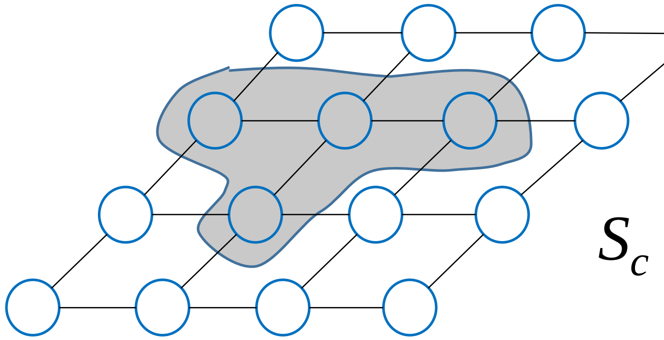
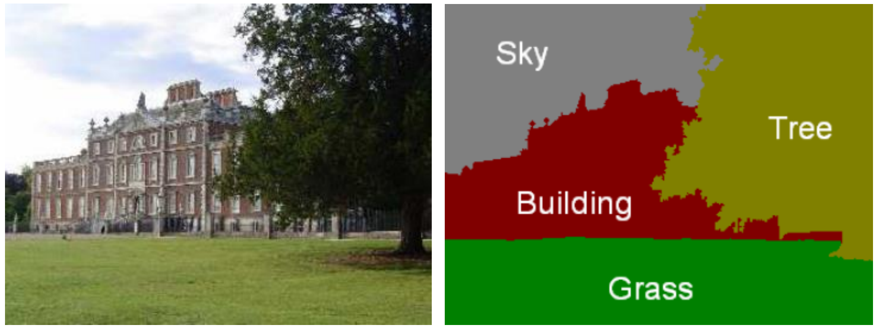



*Robust $P^n$ Potts*: bin consistency




semantic segmentation *[Kohli & Torr 2009] [Gould 2014]*

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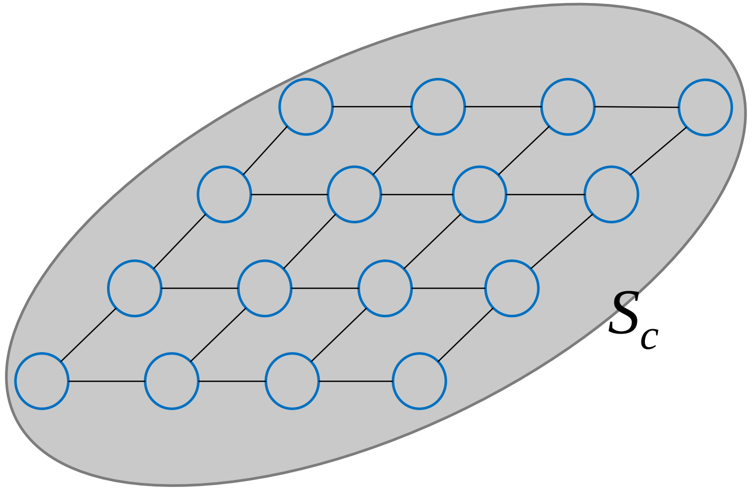
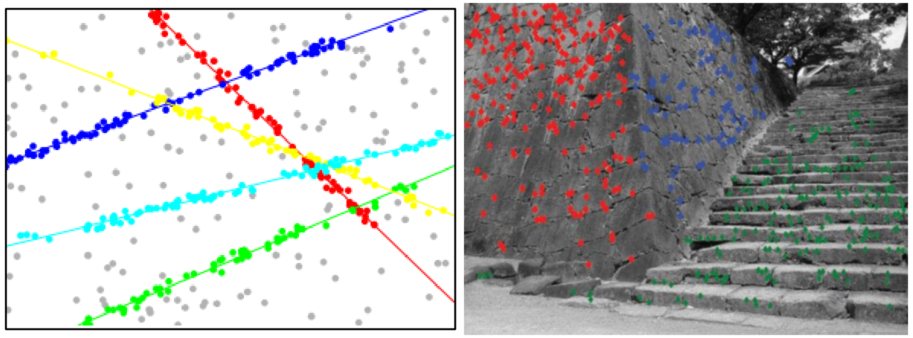



*Label Cost*: sparsity



geometric model fitting [Delong et al., 2012]

# Our proposal: Normalized Cut + MRF

$$E(S) = \boxed{\sum_k - \frac{S^{k'} A S^k}{d' S^k}} + \gamma \boxed{\sum_{c \in \mathcal{F}} E_c(S_c)}$$

balanced clustering

regularization constraints

# Why *MRF* for Normalized Cut?

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previous NC approach:

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weak edge alignment

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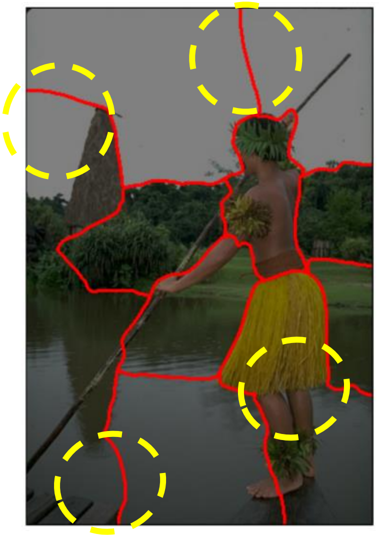

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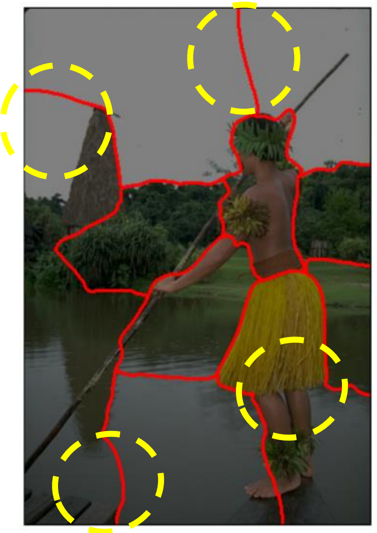

post-processing
(e.g. *[Arbelaez et al., 2011]*)

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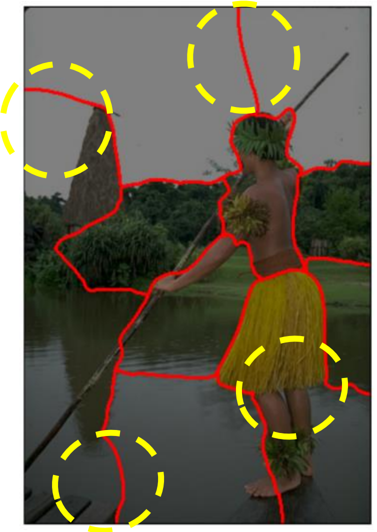

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**Our approach:**

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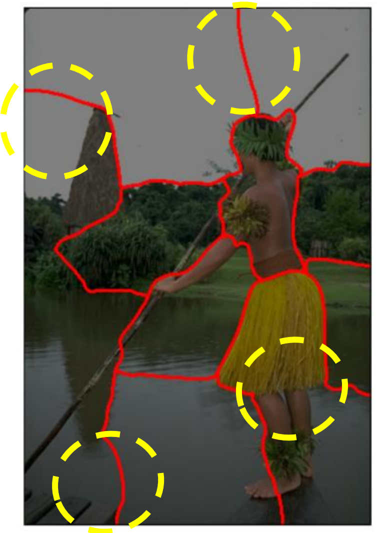

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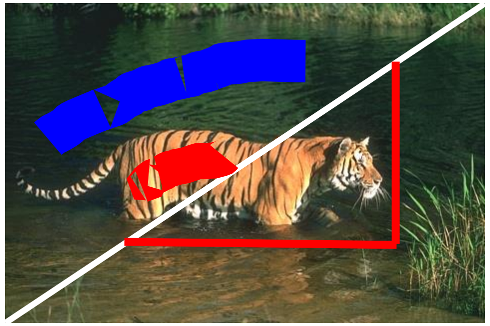

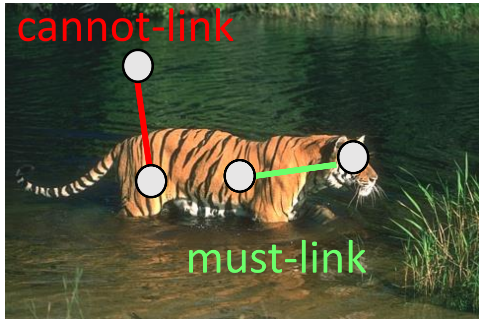


*reformulation* of NC
constrained eigen problem

*[Yu & Shi 2004]*
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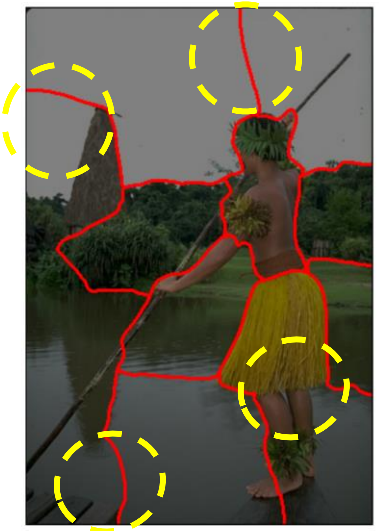

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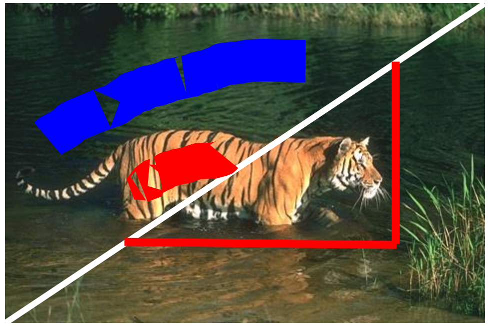

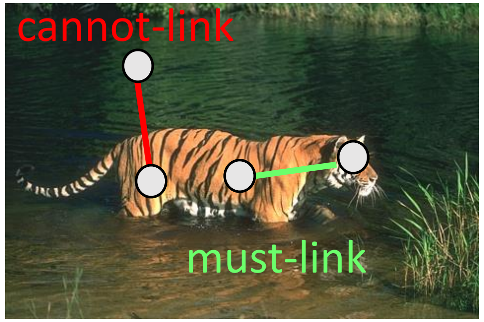


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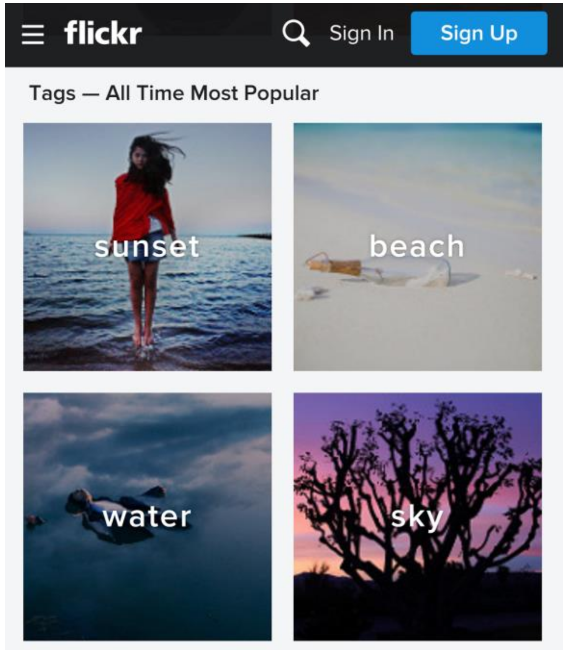
**Our approach:**

NC + Potts
MRF

NC + Potts + seeds
MRF

# Why *MRF* for Normalized Cut?

How to incorporate group priors?



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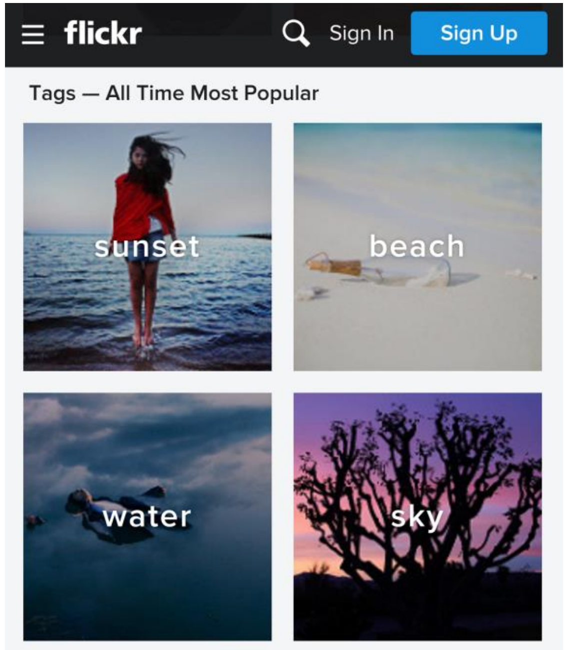


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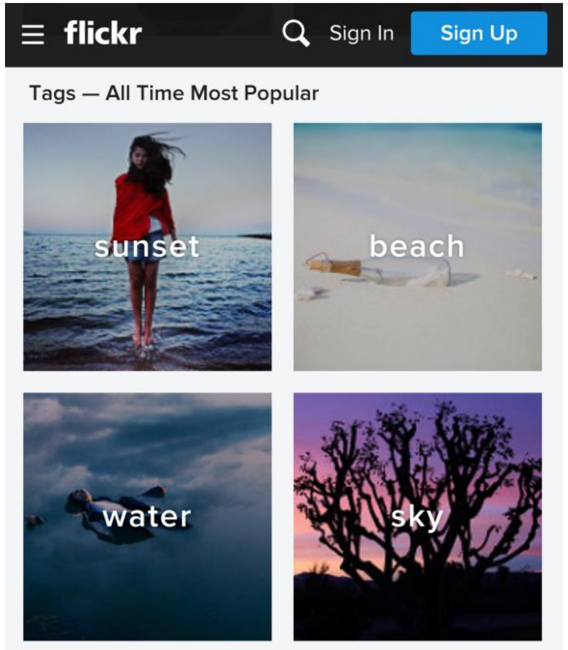


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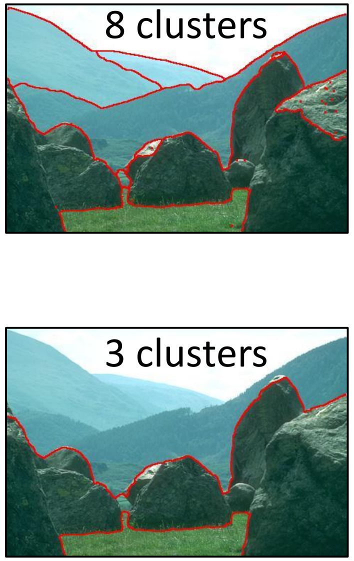


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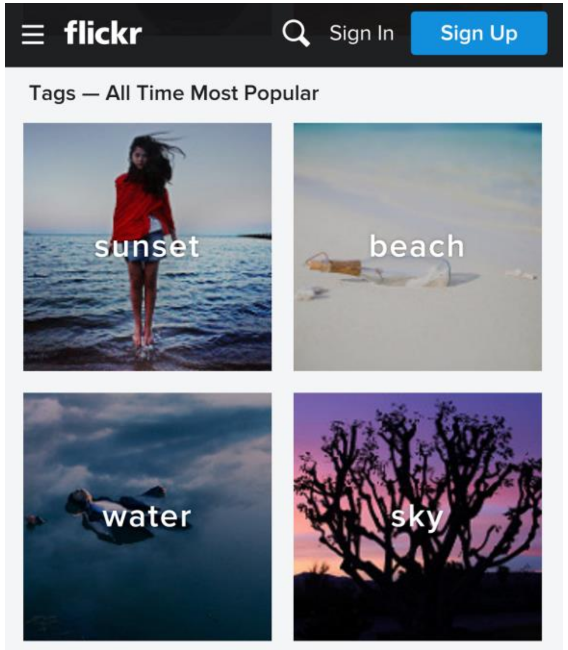


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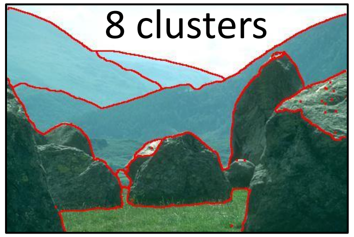


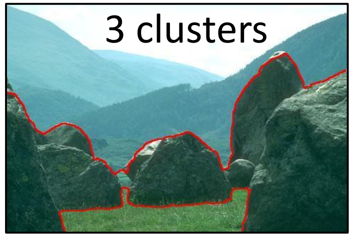


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**Our approach:**

NC + Robust $P^n$ Potts
(Robust $P^n$ Potts: MRF)

NC + label costs
(label costs: MRF)

# Why *Normalized* Cut for MRF?

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typical MRF for segmentation:

$$-\sum_k \sum_{p \in S^k} \ln P(I_p|\theta^k) + \sum_{pq \in \mathcal{N}} w \cdot [s_p \neq s_q]$$

ML term for $\theta^k$

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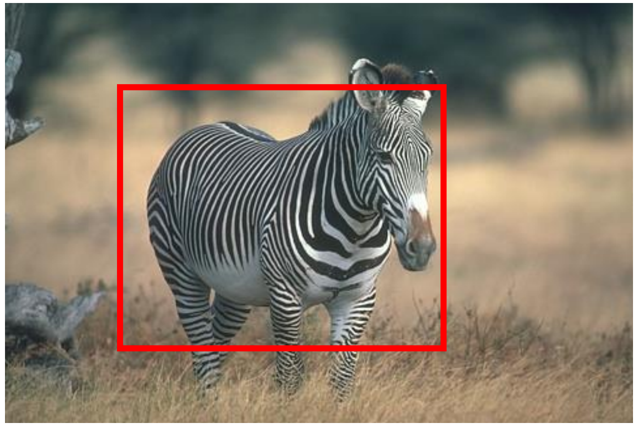

model fitting (*e.g.* GMM)

GrabCut *[Rother, Kolmogorov, Blake, 2004]*

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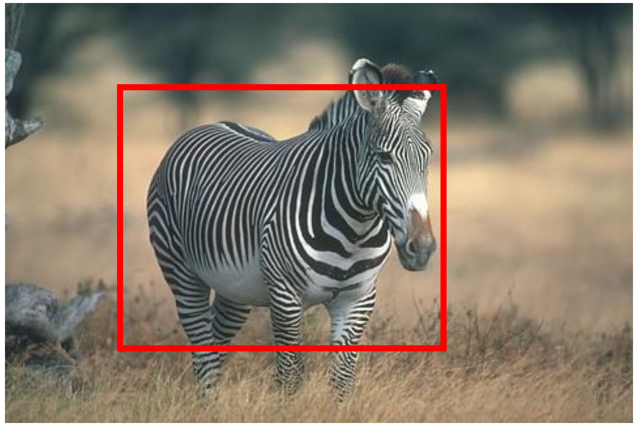

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color space **clustering**

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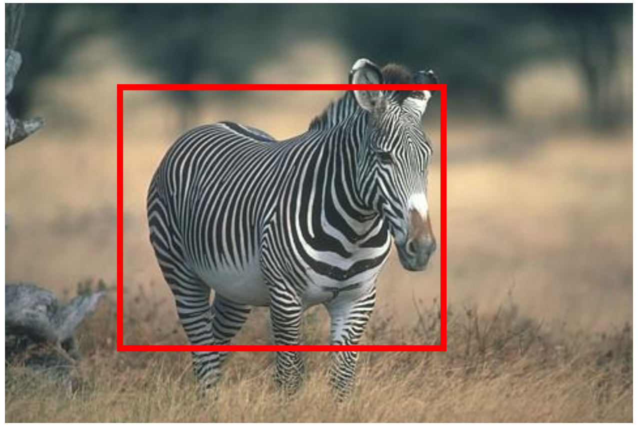
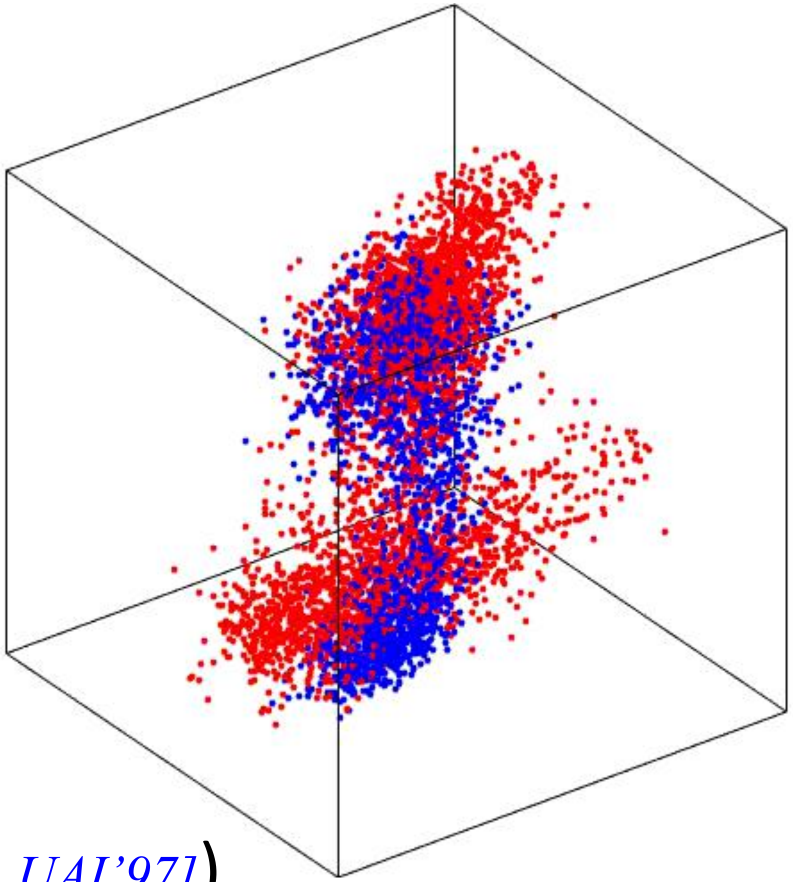
ML term for $\theta^k$

**poor clustering**
**(*overfitting* & *local minima*)**





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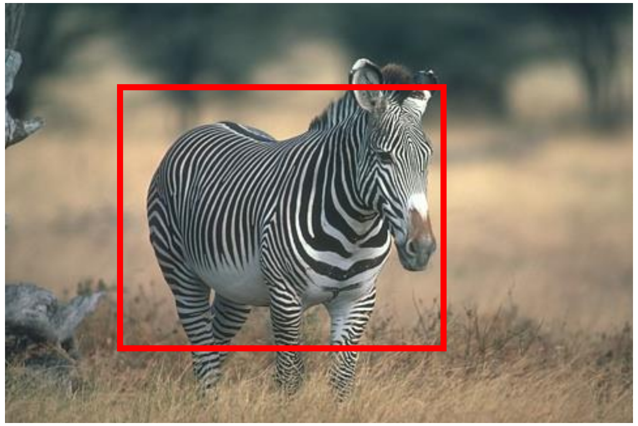
$$NC(S) + \sum_{pq \in \mathcal{N}} w \cdot [s_p \neq s_q]$$

NC for colors



Normalized Cut

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**good clustering**

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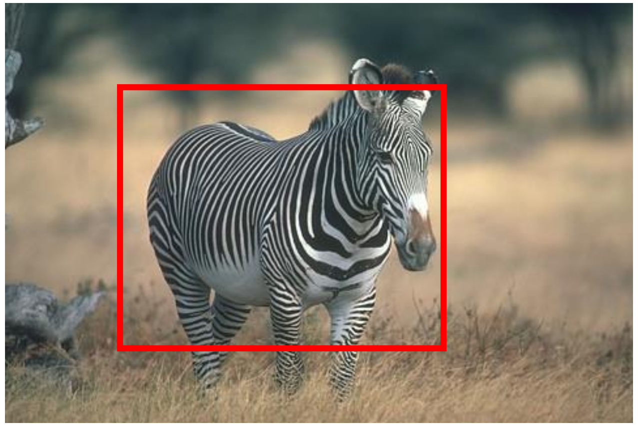

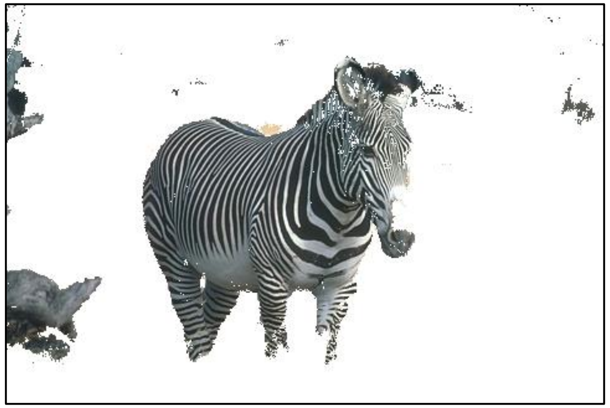

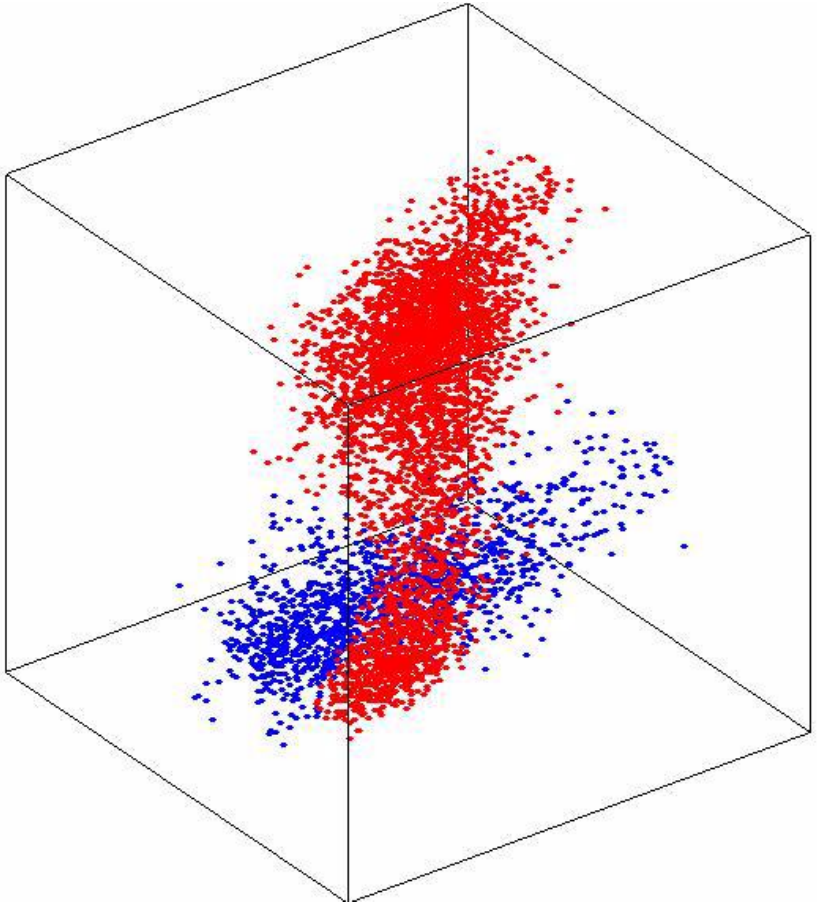

Normalized Cut

color space **clustering**

# Our proposal: Normalized Cut + MRF

$$E(S) = \boxed{\sum_k -\frac{S^{k'} A S^k}{d' S^k}} + \gamma \boxed{\sum_{c \in \mathcal{F}} E_c(S_c)}$$

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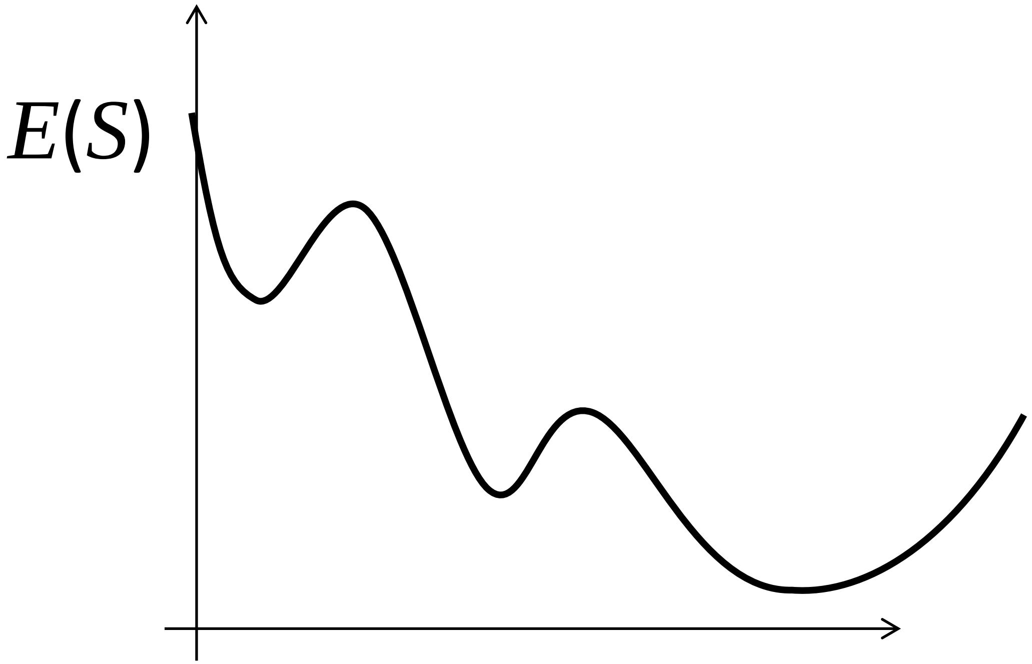
# Bound optimization, in general



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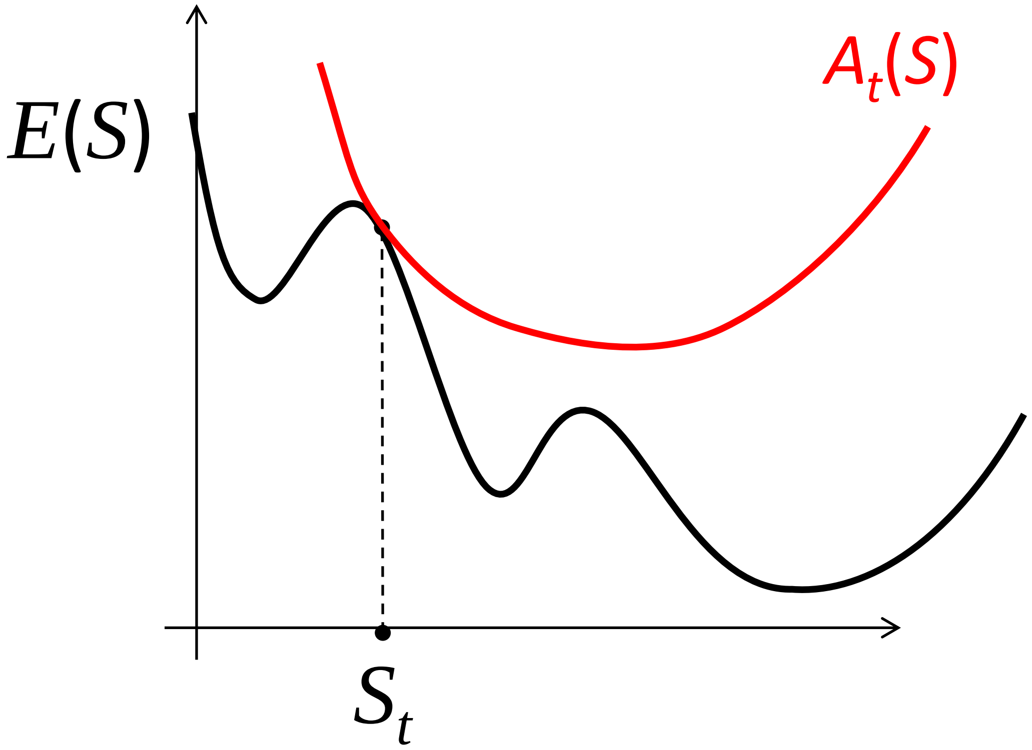

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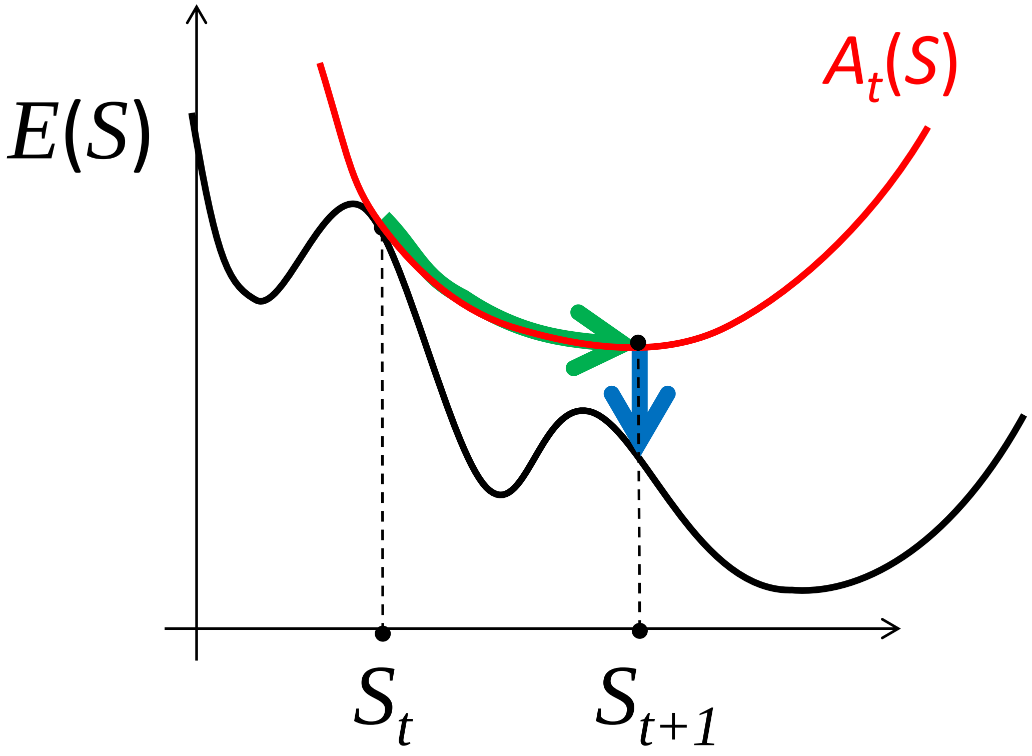

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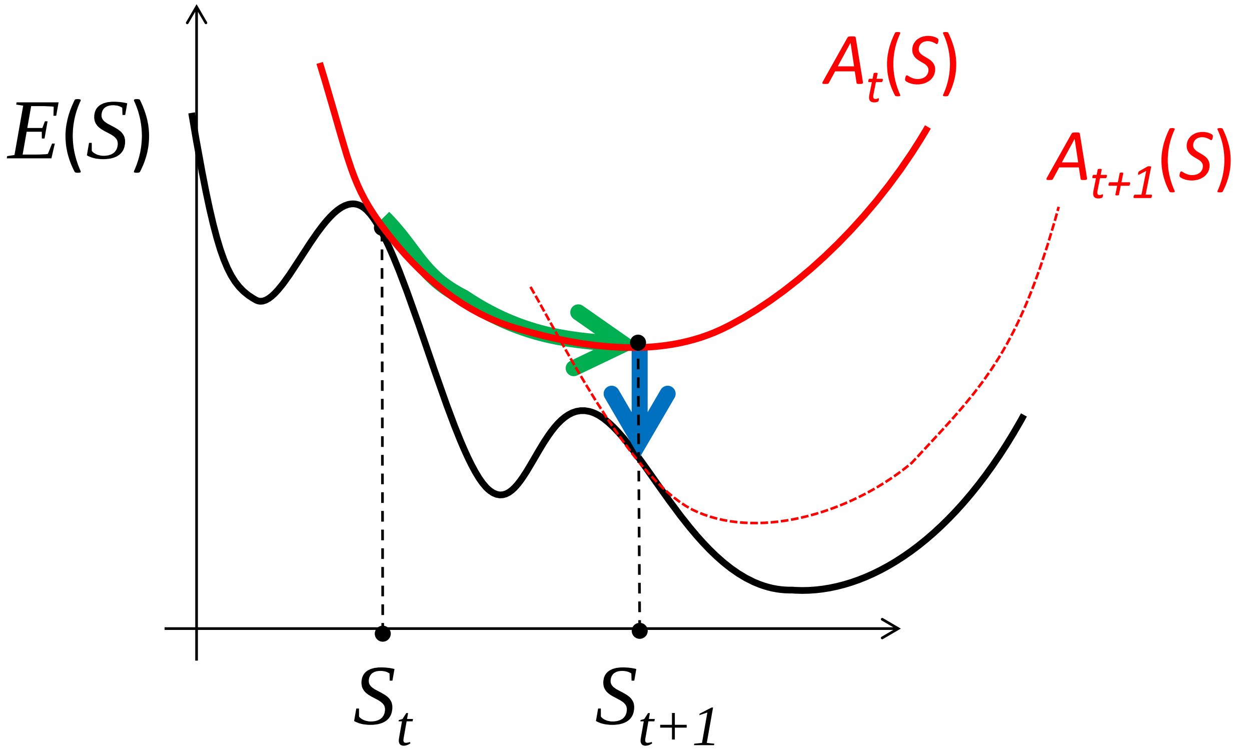

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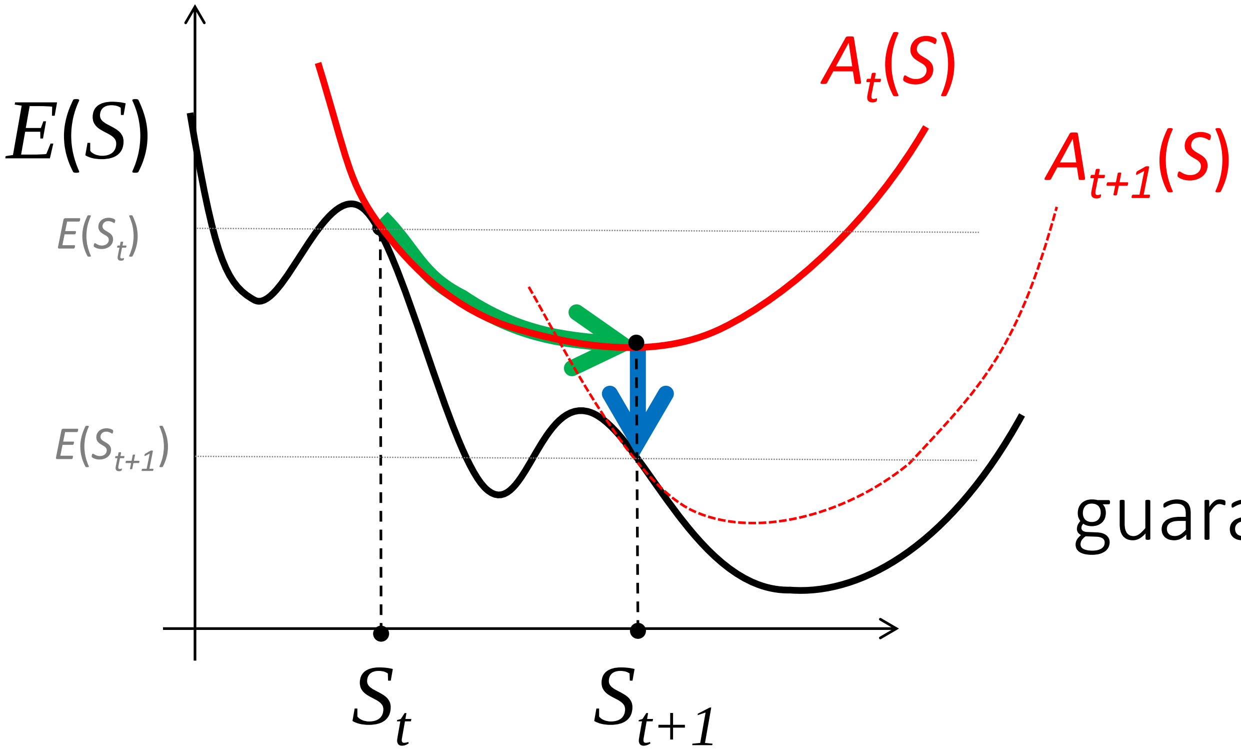


guaranteed energy decrease

# Bound for our joint energy

$$E(S) = \boxed{\sum_k -\frac{S^{k'} A S^k}{d' S^k}} + \boxed{\gamma \sum_{c \in \mathcal{F}} E_c(S_c)}$$

$//\bigwedge$ unary bound for NC

$$A_t(S) = \boxed{\sum_{p \in \Omega} U_p(S_p)} + \boxed{\gamma \sum_{c \in \mathcal{F}} E_c(S_c)}$$

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$$\Delta \quad \text{unary bound for NC}$$

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we propose ***kernel bound*** and ***spectral bound*** for NC

# Kernel bound for NC

**Lemma 1 (concavity)**

$$NC(S) = \sum_k -\frac{{S^k}' A S^k}{d' S^k}$$

# Kernel bound for NC

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*Function* $e : \mathbf{R}^{|\Omega|} \rightarrow \mathbf{R}$ *is concave over region* $S^k > 0$ *given p.s.d. affinity matrix* $A := [A_{pq}]$.

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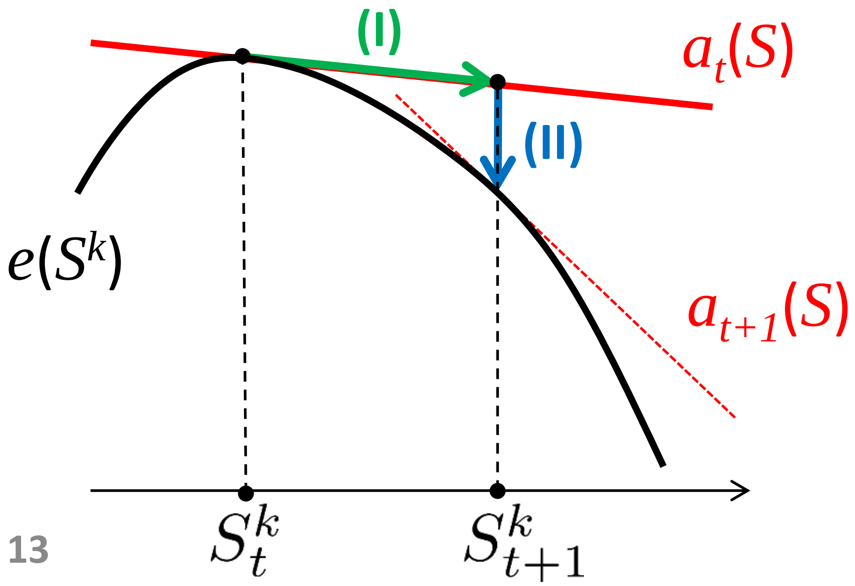

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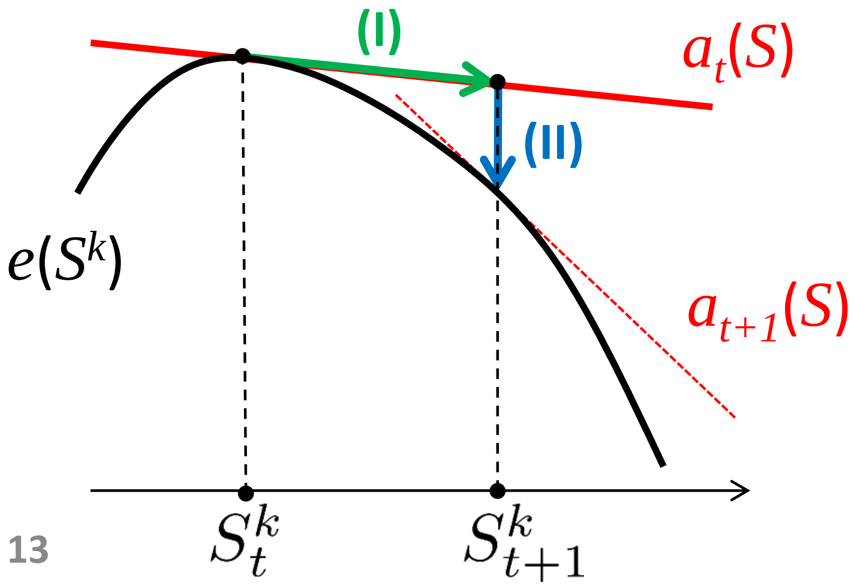


first-order Taylor expansion:

$$e(S_t^k) + \nabla e(S_t^k) \cdot (S^k - S_t^k)$$

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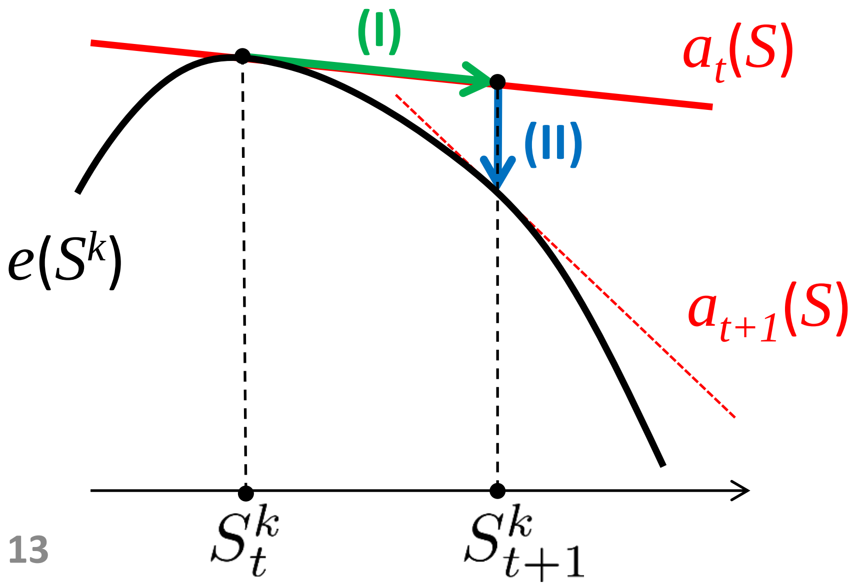


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equivalently kernel k-means for NC *[Dhillon et al., 2004]*

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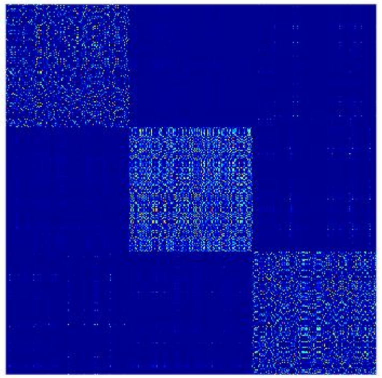
Consider rank $m$ approximation $\widetilde{A} \approx A$:

$$\min_{\widetilde{A}} \|A - \widetilde{A}\|_F$$



$A$

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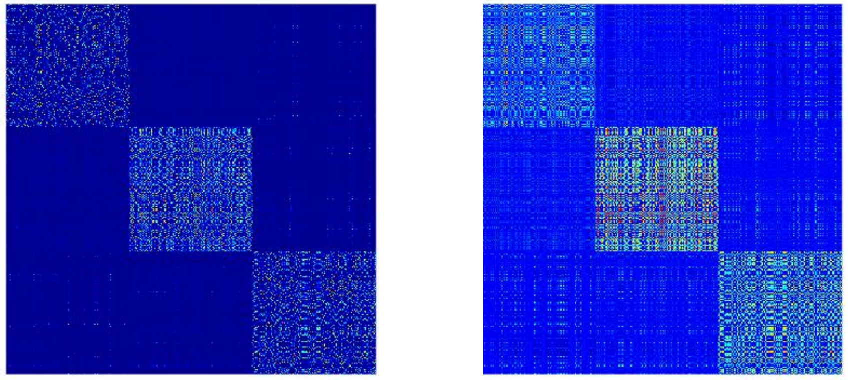

$A$ $\widetilde{A}$

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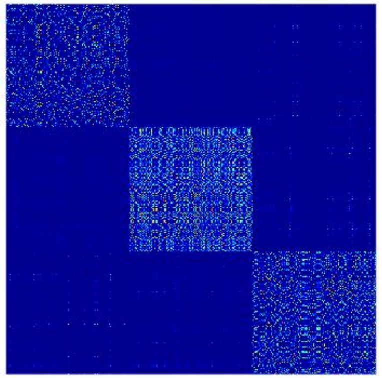
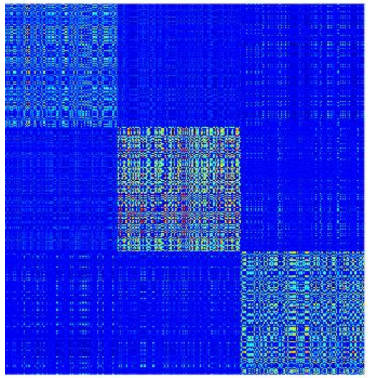
**low-dimensional** points:

$$\widetilde{\phi}_p := \sqrt{\Lambda^m} V_p^m \in \mathcal{R}^m$$



$A$

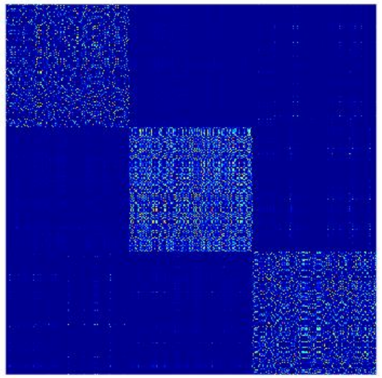


$\widetilde{A}$

# Low-rank Approximation

Consider rank $m$ approximation $\widetilde{A} \approx A$:

$$\min_{\widetilde{A}} \|A - \widetilde{A}\|_F$$



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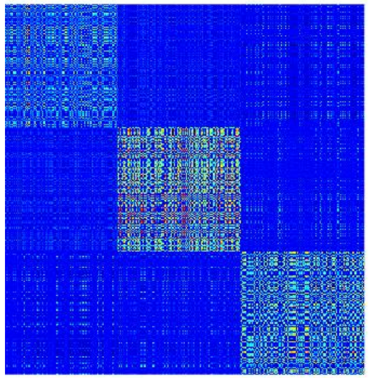

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using top $m$ eigenvectors and eigenvalues of $A$ :

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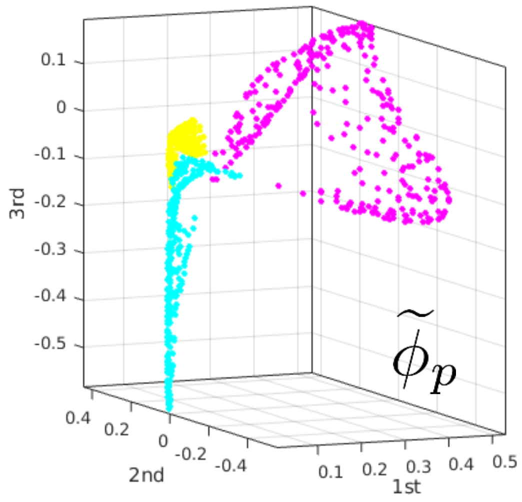

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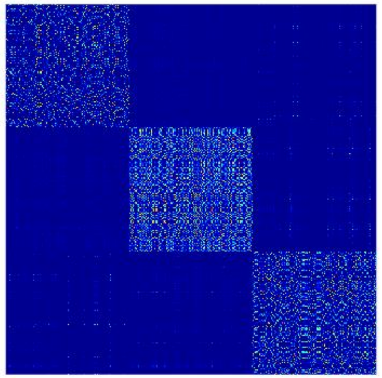

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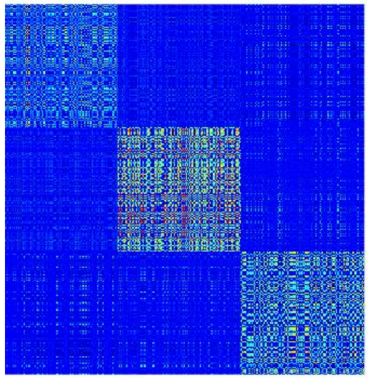
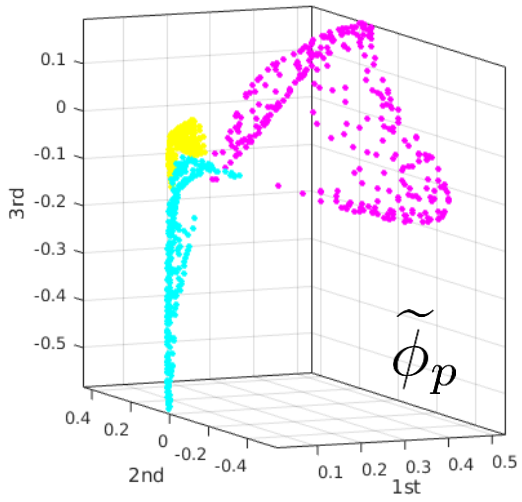


$\widetilde{A}$




$\widetilde{\phi}_p$

**isometric Euclidean Embedding**

$$A_{pq} \approx \widetilde{A}_{pq} = \langle \widetilde{\phi}_p, \widetilde{\phi}_q \rangle$$

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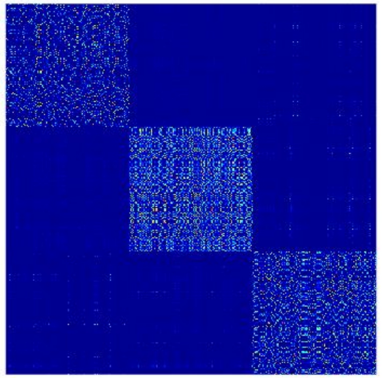

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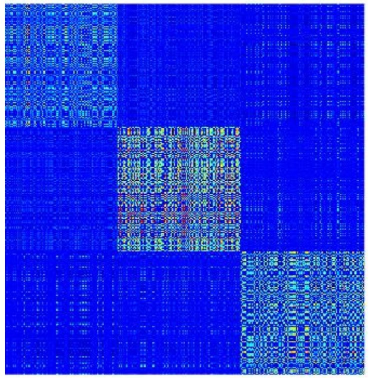

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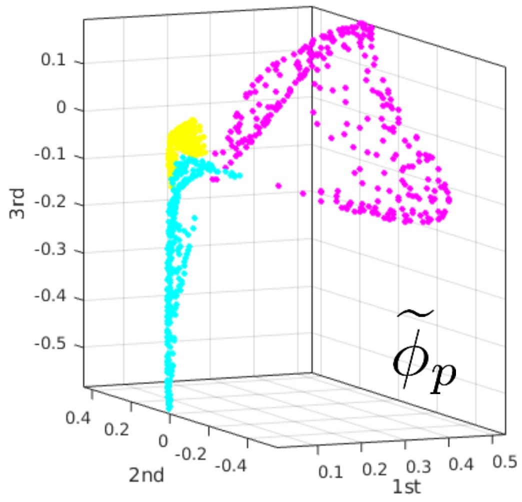


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$A_{pq} \approx \widetilde{A}_{pq} = \langle \widetilde{\phi}_p, \widetilde{\phi}_q \rangle$

MDS *[Cox & Cox., 2000]*

Kernel PCA *[Schölkopf, Smola, Müller, 1998]*

**NC**

$$\widetilde{A} \approx A$$

$$\sum_k -\frac{S^{k'} A S^k}{d' S^k} \approx \sum_k -\frac{S^{k'} \widetilde{A} S^k}{d' S^k}$$

**NC** $\widetilde{A} \approx A$ **K-means**

$$\sum_k -\frac{S^{k'} A S^k}{d' S^k} \approx \sum_k -\frac{S^{k'} \widetilde{A} S^k}{d' S^k} \overset{c}{=} \sum_k \sum_{p \in S^k} d_p \|\widetilde{\phi}_p - \mu^k\|^2$$

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- **NC = K-means** for exact embedding $\phi_p \in \mathcal{R}^{|\Omega|}$ *[Bach & Jordan 2003, Dhillon et al., 2004]*

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- **NC = K-means** for exact embedding $\phi_p \in \mathcal{R}^{|\Omega|}$ *[Bach & Jordan 2003, Dhillon et al., 2004]*
- We propose **NC ≈ K-means** for low-dimensional embedding $\widetilde{\phi}_p \in \mathcal{R}^m$ for $m \ll |\Omega|$

**NC**

$$\sum_k - \frac{{S^k}' A S^k}{d' S^k}$$

**K-means**

$$\overset{c}{\approx} \sum_k \sum_{p \in S^k} d_p \|\widetilde{\phi}_p - \mu^k\|^2$$

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unary spectral bound for NC

$$\leq \sum_k \sum_{p \in \Omega} d_p \|\widetilde{\phi}_p - \mu_t^k\|^2 \cdot S_p^k$$

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- **K-means** on $\phi_p^* \in \mathcal{R}^K$ - discretization heuristic for a spectral relaxation of **NC**

*[Shi & Malik 2000, Yu & Shi 2003]*

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## NC K-means

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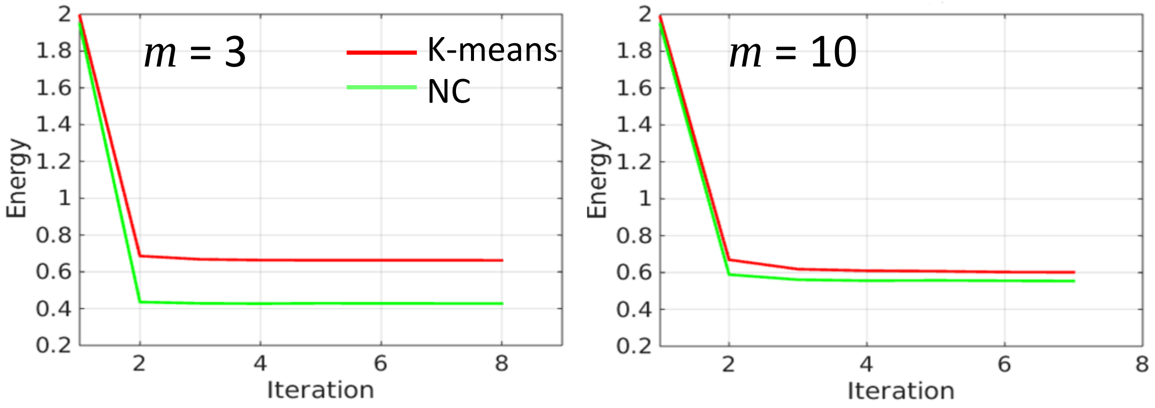


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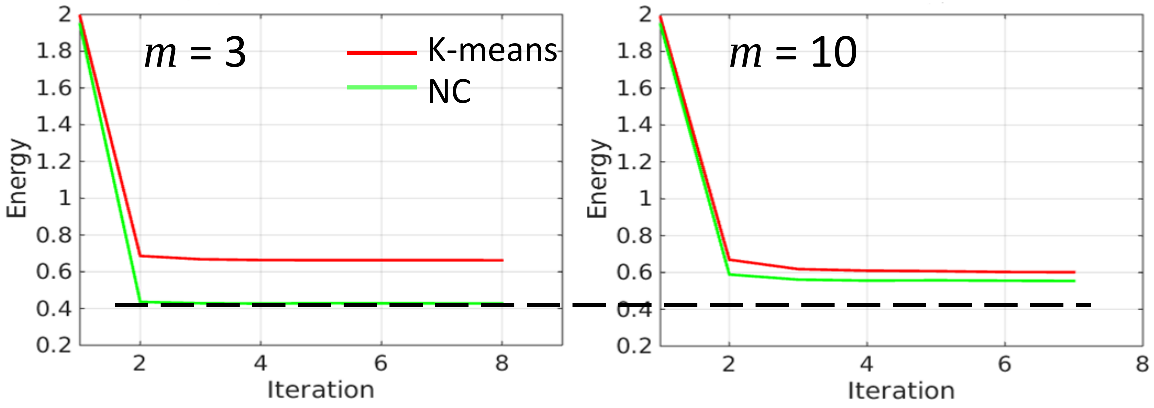


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$$E(S) = \boxed{\sum_k -\frac{S^{k\prime} A S^k}{d' S^k}} + \boxed{\gamma \sum_{c \in \mathcal{F}} E_c(S_c)}$$

$$/\!\!\bigwedge$$ unary bound for NC

$$A_t(S) = \boxed{\sum_{p \in \Omega} U_p(S_p)} + \boxed{\gamma \sum_{c \in \mathcal{F}} E_c(S_c)}$$

iterate

$$S_{t+1} = \arg\min_S A_t(S)$$ (move-making and graph cuts *[Boykov, Veksler, Zabih, 2001]*)

# Our **Kernel Cut** and **Spectral Cut**

$$E(S) = \sum_k -\frac{S^{k\prime} A S^k}{d' S^k} + \gamma \sum_{c \in \mathcal{F}} E_c(S_c)$$

$//\!\wedge$ unary bound for NC (**Kernel Bound** or **Spectral Bound**)

$$A_t(S) = \sum_{p \in \Omega} U_p(S_p) + \gamma \sum_{c \in \mathcal{F}} E_c(S_c)$$

iterate

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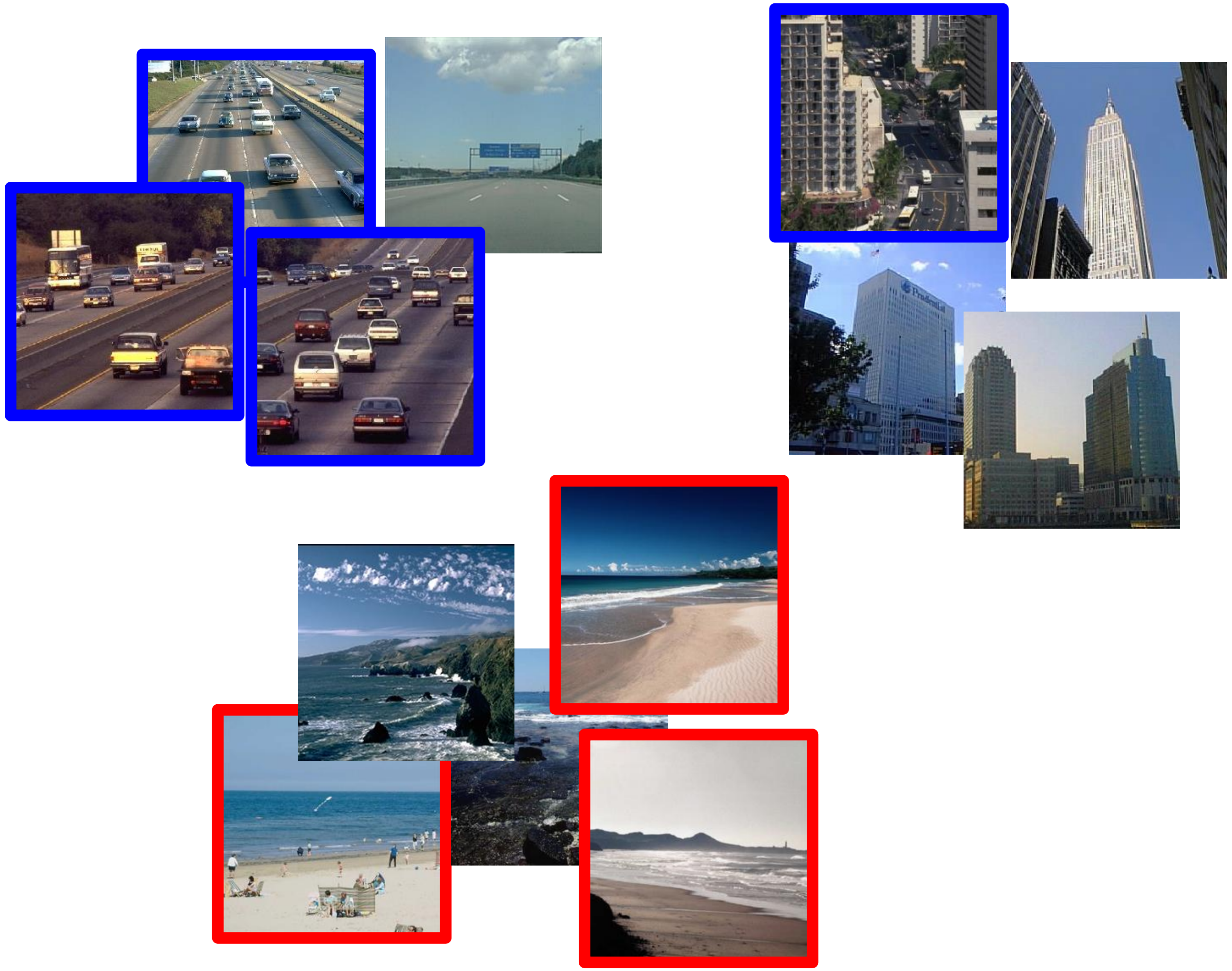
# Experiments: MRF helps Normalized Cut

using image tags (e.g. **beach**, **car**) to help image clustering



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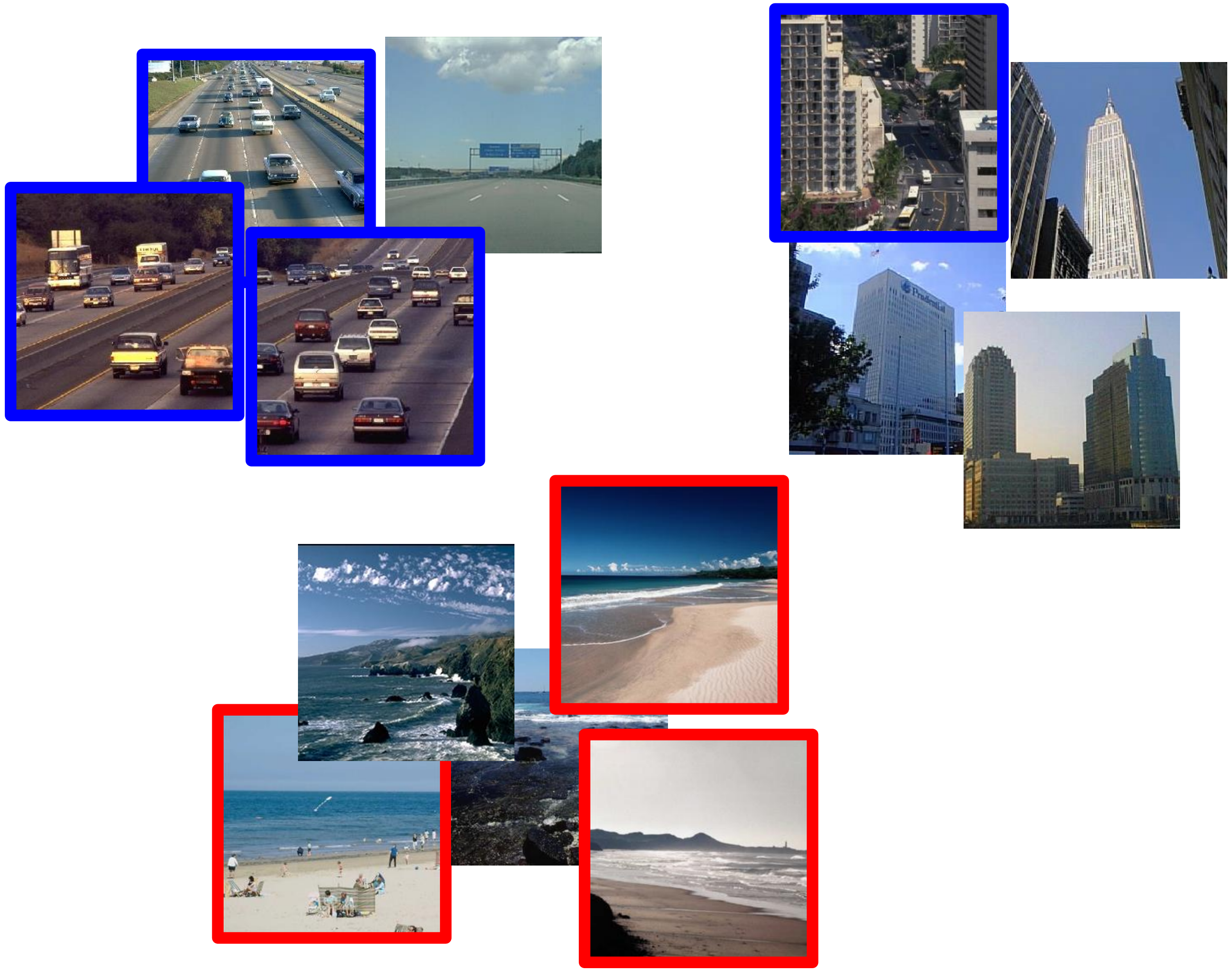
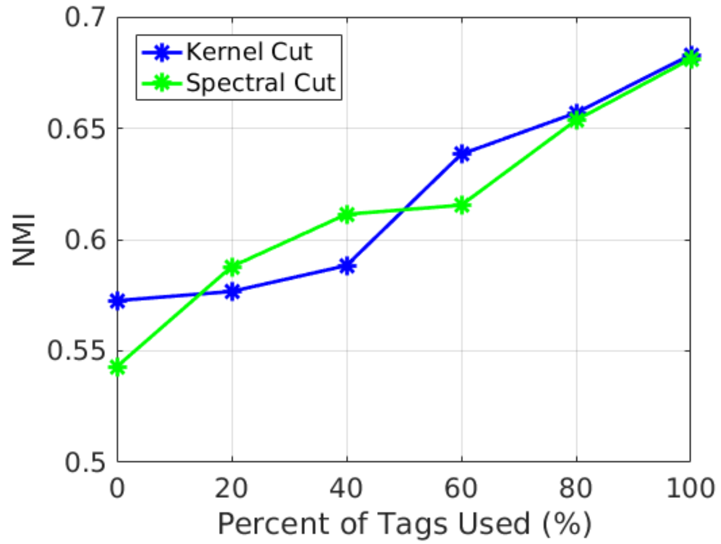


NC + robust $P^n$ Potts



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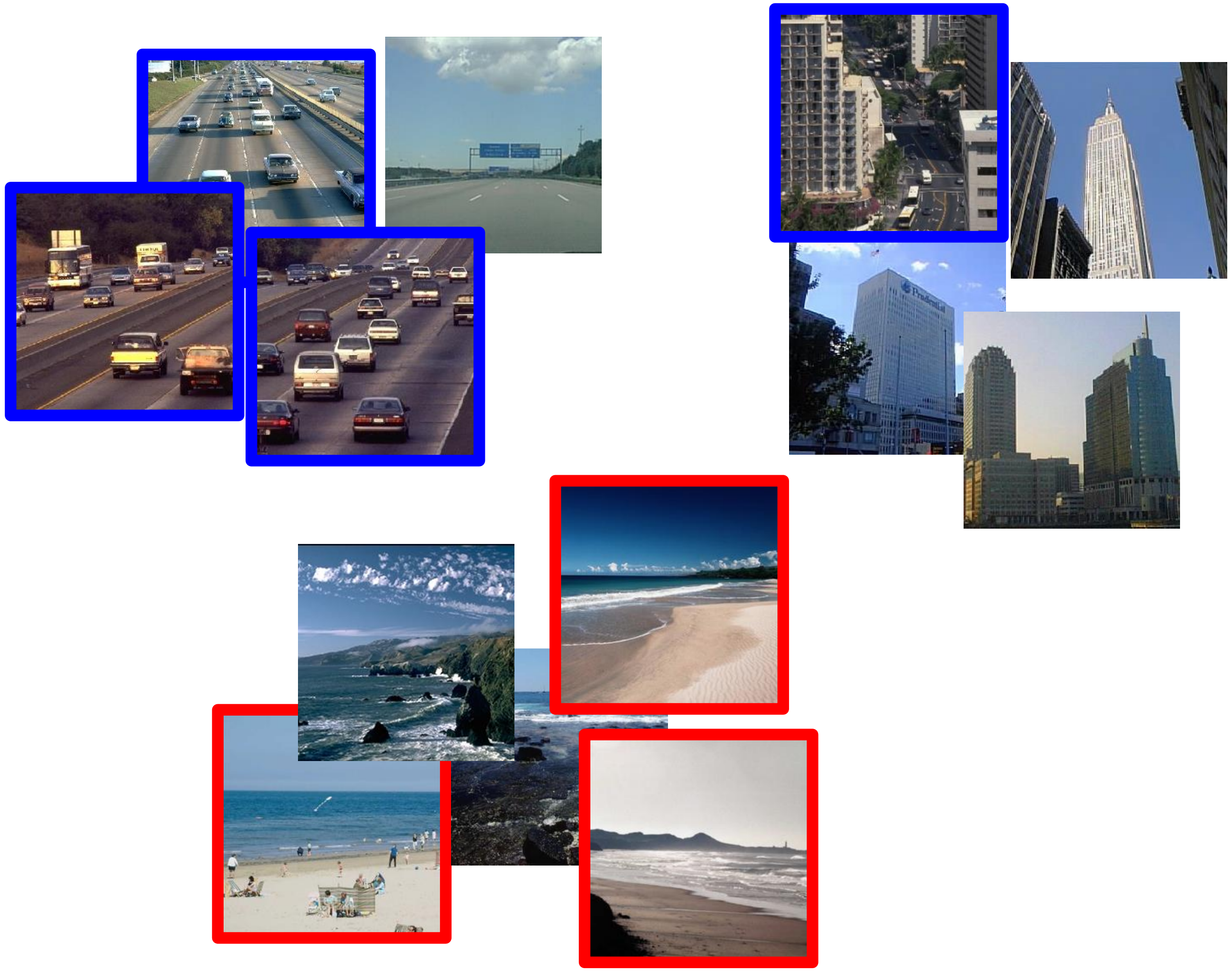

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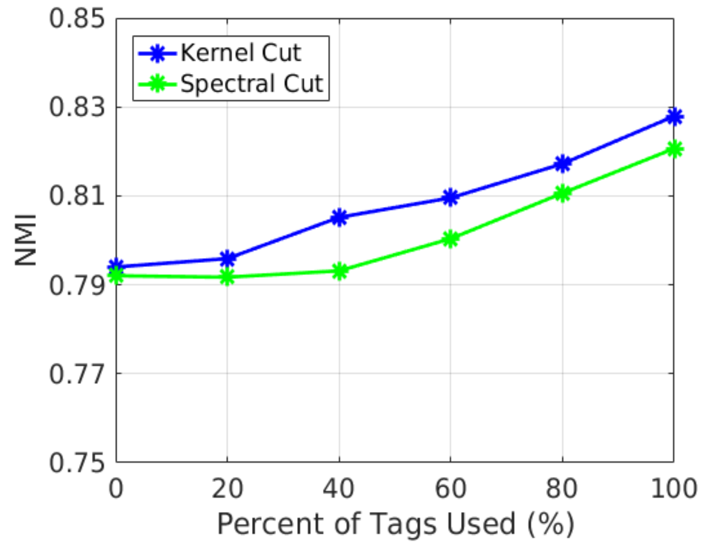


+with *knn* kernel on deep features

# Experiments: Normalized Cut helps MRF

(a) Video frames (b) Optical flow (c) Our Kernel Cut (NC + Potts)

Fig. motion segmentation using RGB, location (XY) and motion (M). “+xy” means with MRF

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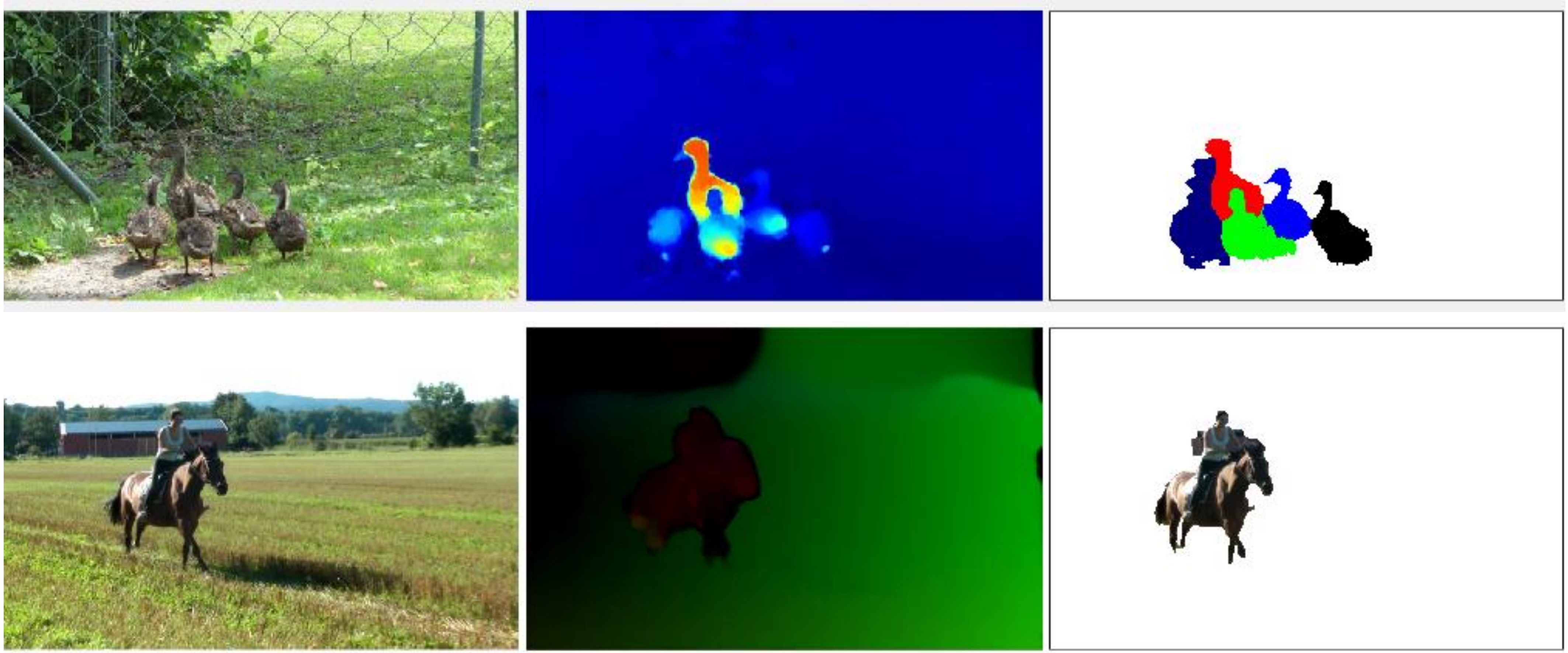
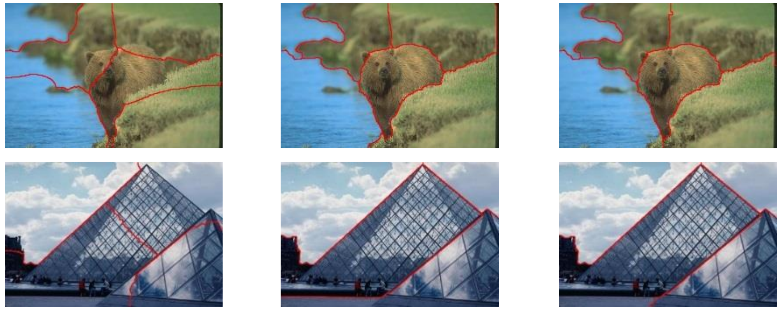


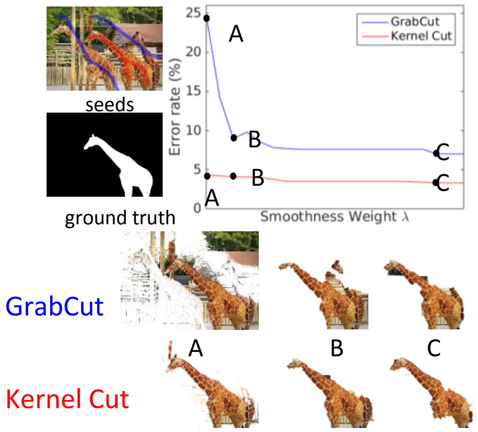
Fig. motion segmentation using RGB, location (XY) and motion (M). “+xy” means with MRF

# More Experiments

**Potts model** improves edge alignment



Spectral Clustering (no Potts) | Our Kernel Cut (NC with Potts) | Our Spectral Cut (NC with Potts)




separating similar objects

NC with increasing **label cost**

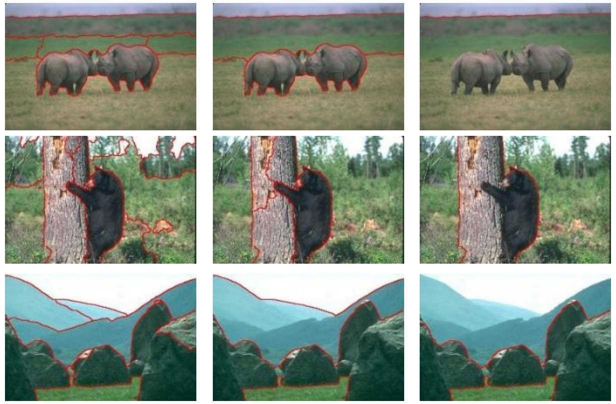
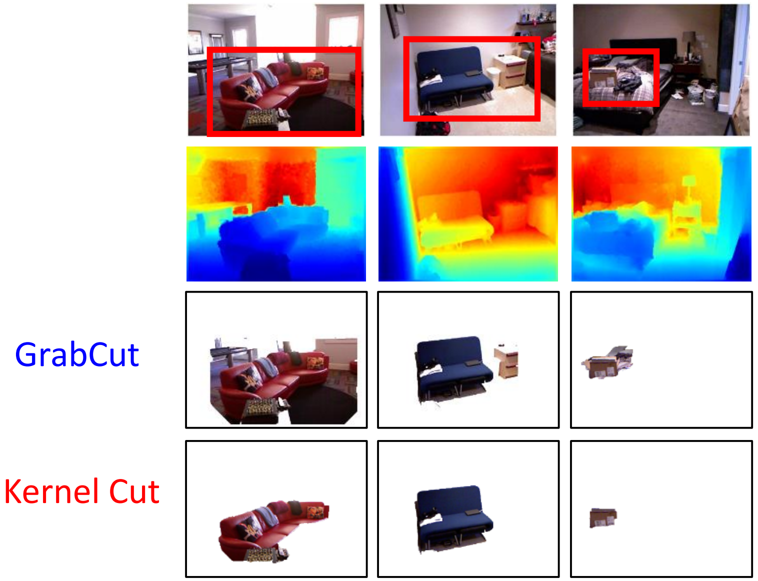
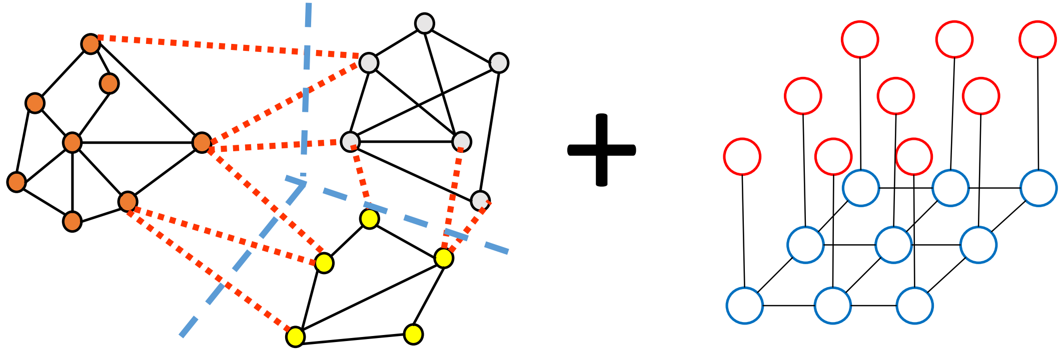


Fig. 1. RGBD segmentation



# Conclusion



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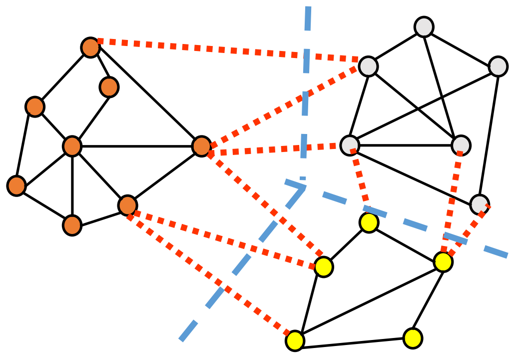

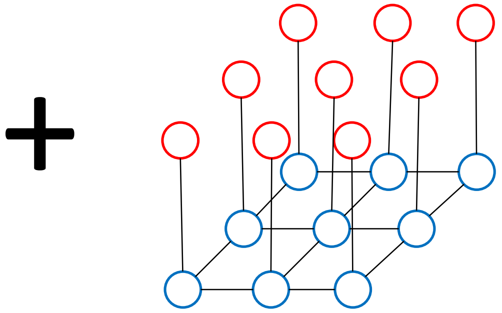

- ❑ new unary kernel and spectral bounds for NC

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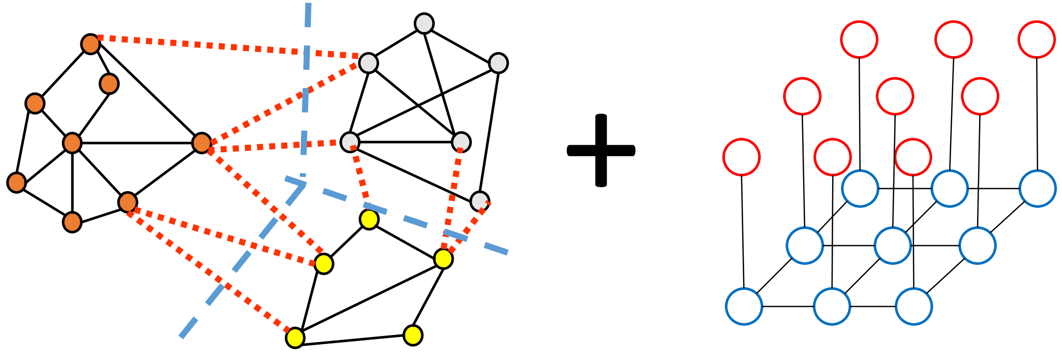

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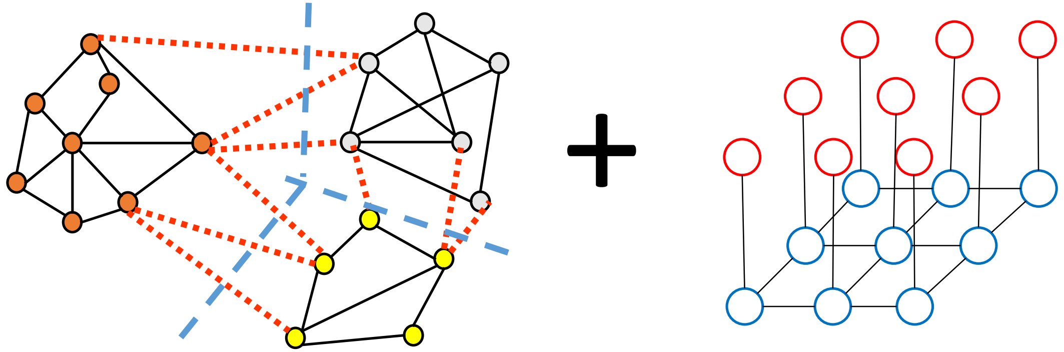

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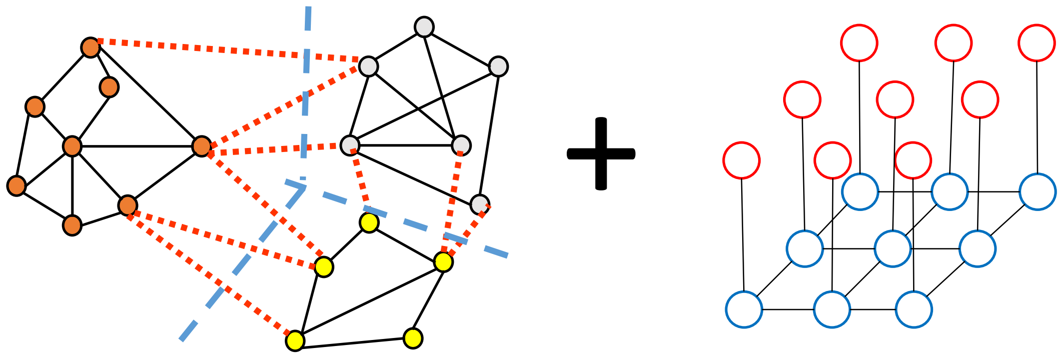

- ❑ new unary kernel and spectral bounds for NC
- ❑ can combine NC with any MRF constraints
- ❑ can combine MRF with balanced clustering
- ❑ MRF with features of any dimension (RGBD, RGBM, RGBXYM, deep,…)