# The Fast Bilateral Solver 

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ECCV 2016

## Bilateral Filter



Blur the input while respected edges in the reference image.

## Bilateral Filter



Input y

## 

Bilateral
Filter

Referenc
R
Output $\mathbf{x}$
e

$$
W_{i, j}=\exp \left(-\sum_{d} \frac{\left(R_{i, d}-R_{j, d}\right)^{2}}{2 \sigma_{d}^{2}}\right)
$$

## Bilateral Solver



Find the image that is as smooth as possible with respect to the reference image, and as close as possible to the input.

## Bilateral Solver



Input y


Referenc

## R

Output x
$\mathbf{x} \leftarrow \underset{\mathbf{x}}{\arg \min } \frac{\lambda}{2} \sum_{i, j} W_{i, j}\left(x_{i}-x_{j}\right)^{2}+\sum_{i}\left(x_{i}-y_{i}\right)^{2}$

## Bilateral Solver

The bilateral filter is one gradient descent step in the bilateral solver.

(assuming: step size $=1, \lambda=1$, initial state $=$

## Bilateral Solver

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Fast - Can be reformulated to be roughly as fast as a fast bilateral filter.

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General - Can be easily generalized into weighted, robust, low rank, and differentiable variants.

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Fast - Can be reformulated to be roughly as fast as a fast bilateral filter.

General - Can be easily generalized into weighted, robust, low rank, and differentiable variants.

Effective - Performs well on a variety of tasks:
depth superresolution
colorization
stereo
semantic segmentation, etc.

## Review: Bilateral-Space Optimization



Splat
resample from pixels into "bilateral-space"


Blur
apply a series of blurs in bilateral-space

Output


Slice
resample back into pixel-space

Adams et al, Eurographics 2010

## Review: Bilateral-Space Optimization

## Input




Splat
resample from pixels into "bilateral-space"


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apply a series of blurs in bilateral-space

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Adams et al, Eurographics 2010
We can solve optimization problems based on bilateral kernels in this "bilateral space"

Barron et al, CVPR 2015
Märki et al, CVPR 2016

## Bilateral Solver

If we know that :

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We can show that:

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\mathbf{W}=\mathbf{S}^{\mathrm{T}} \mathbf{D}_{\mathrm{m}}^{-1} \mathbf{D}_{\mathrm{n}} \mathbf{B D}_{\mathrm{n}} \mathbf{D}_{\mathrm{m}}^{-1} \mathbf{S}
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\mathbf{x} \leftarrow \underset{\mathbf{x}}{\arg \min } \frac{\lambda}{2} \sum_{i, j} W_{i, j}\left(x_{i}-x_{j}\right)^{2}+\sum_{i}\left(x_{i}-y_{i}\right)^{2}
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$$

$$
\begin{aligned}
& \mathbf{A}=\lambda\left(\mathbf{D}_{\mathrm{m}}-\mathbf{D}_{\mathrm{n}} \mathbf{B D}_{\mathrm{n}}\right)+\mathbf{S} \overrightarrow{1} \\
& \mathbf{x} \leftarrow \mathbf{S}^{\mathrm{T}}\left(\mathrm{~A}^{-1}(\mathbf{S y})\right)
\end{aligned}
$$

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\end{aligned}
$$

(see paper for details)

## Depth Superresolution

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Input y

task from Ferstl et al ICCV 2013, data from the Middlebury stereo dataset

## Depth Superresolution


task from Ferstl et al ICCV 2013, data from the Middlebury stereo dataset

## Depth Superresolution


task from Ferstl et al ICCV 2013, data from the Middlebury stereo dataset


Reference Image


Input Depth


True Depth


Chan et al.


Park et al.


FGF



GF


DT


Yang 2015


Ferstl et al.


Min et al.


Ma et al.


Yang 2007

$\dagger \mathrm{Lu}$


Zhang et al.


WLS






## Bilateral Solver

$$
\mathbf{x} \leftarrow \underset{\mathbf{x}}{\arg \min } \frac{\lambda}{2} \sum_{i, j} W_{i, j}\left(x_{i}-x_{j}\right)^{2}+\sum_{i}\left(x_{i}-y_{i}\right)^{2}
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\end{aligned}
$$

## Weighted Bilateral Solver

$\mathbf{x} \leftarrow \underset{\mathbf{x}}{\arg \min } \frac{\lambda}{2} \sum_{i, j} W_{i, j}\left(x_{i}-x_{j}\right)^{2}+\sum_{i} c_{i}\left(x_{i}-y_{i}\right)^{2}$

$$
\begin{aligned}
& \mathbf{A}=\lambda\left(\mathbf{D}_{\mathrm{m}}-\mathbf{D}_{\mathrm{n}} \mathbf{B D} \mathbf{D}_{\mathrm{n}}\right)+\mathbf{S c} \\
& \mathbf{x} \leftarrow \mathbf{S}^{\mathrm{T}}\left(\mathrm{~A}^{-1}(\mathbf{S} \mathbf{y})\right)
\end{aligned}
$$

## Weighted Bilateral Solver


nevian $R$



Input y



Confidence c

## Weighted Bilateral Solver



Bilateral Solver
$0.85 \mathrm{sec} /$ megapixel


Levin et al $81.0 \mathrm{sec} / \mathrm{meg}$ apixel

## $95 \times$ speedup

Levin et al, Colorization using optimization. SIGGRAPH 2004

## Weighted Bilateral Solver

$\mathbf{x} \leftarrow \underset{\mathbf{x}}{\arg \min } \frac{\lambda}{2} \sum_{i, j} W_{i, j}\left(x_{i}-x_{j}\right)^{2}+\sum_{i} c_{i}\left(x_{i}-y_{i}\right)^{2}$

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& \mathbf{x} \leftarrow \mathbf{S}^{\mathrm{T}}\left(\mathrm{~A}^{-1}(\mathbf{S} \mathbf{y})\right)
\end{aligned}
$$

## Robust Bilateral Solver

$$
\begin{gathered}
\mathbf{x} \leftarrow \underset{\mathbf{x}}{\arg \min } \frac{\lambda}{2} \sum_{i, j} W_{i, j}\left(x_{i}-x_{j}\right)^{2}+\sum_{i} \rho\left(x_{i}-y_{i}\right) \\
\equiv
\end{gathered}
$$

while not converged :

$$
\begin{aligned}
& \mathbf{c} \leftarrow \frac{\rho^{\prime}(\mathbf{x}-\mathbf{y})}{\mathbf{x}-\mathbf{y}} \\
& \mathbf{A}=\lambda\left(\mathbf{D}_{\mathrm{m}}-\mathbf{D}_{\mathrm{n}} \mathbf{B D}_{\mathrm{n}}\right)+\mathbf{S c} \\
& \mathbf{x} \leftarrow \mathbf{S}^{\mathrm{T}}\left(\mathrm{~A}^{-1}(\mathbf{S} \mathbf{y})\right)
\end{aligned}
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& \mathbf{x} \leftarrow \mathbf{S}^{\mathrm{T}}\left(\mathrm{~A}^{-1}(\mathbf{S} \mathbf{y})\right)
\end{aligned}
$$

can be done in
bilateral space

## Stereo

## MC-CNN

## Stereo

MC-CNN + Robust Bilateral Solver

## Stereo

MC-CNN

## Stereo

MC-CNN + Robust Bilateral Solver

Zbontar \& LeCun, CVPR 2015

## Stereo

| Middlebury Test Set V3 |  |  |
| :--- | :---: | :---: |
| Method | MAE | RMSE |
| MC-CNN | 17.9 | 55.0 |
| MC-CNN + RBS | 8.19 | 29.9 |

(Lowest test-set MAE and RMSE at submission time)

## Stereo

## Middlebury Test Set V3

| Method | MAE RMSE |  |
| :--- | :---: | :---: |
| MC-CNN | 17.9 | 55.0 |
| MC-CNN + RBS | 8.19 | 29.9 |

## Middlebury Training Set V3

| Method | MAE RMSE |  |
| :--- | :---: | :---: |
| MC-CNN | 5.93 | 18.36 |
| MC-CNN + TF | 5.67 | 16.18 |
| MC-CNN + FGF | 5.91 | 16.32 |
| MC-CNN + WMF | 5.30 | 15.62 |
| MC-CNN + DT | 5.69 | 16.53 |
| MC-CNN + RBS | 2.81 | 8.44 |

## Stereo

## Middlebury Test Set V3

| Method | MAE RMSE |  |
| :--- | :---: | :---: |
| MC-CNN | 17.9 | 55.0 |
| MC-CNN + RBS | 8.19 | 29.9 |

## Middlebury Training Set V3

| Method | MAE | RMSE |
| :--- | :---: | :---: |
| MeshStereo | 3.83 | 10.75 |
| MeshStereo + TF | 3.81 | 9.91 |
| MeshStereo + FGF | 3.96 | 10.03 |
| MeshStereo + WMF | 3.87 | 10.10 |
| MeshStereo + DT | 3.77 | 10.12 |
| MeshStereo + RBS | 3.22 | 8.72 |

## Stereo

## Middlebury Test Set V3

| Method | MAE RMSE |  |
| :--- | :---: | :---: |
| MC-CNN | 17.9 | 55.0 |
| MC-CNN + RBS | 8.19 | 29.9 |

## Middlebury Training Set V3

| Method | MAE RMSE |  |
| :--- | :---: | :--- |
| TMAP | 3.98 | 11.55 |
| TMAP + TF | 3.94 | 10.90 |
| TMAP + FGF | 4.17 | 10.79 |
| TMAP + WMF | 4.11 | 11.01 |
| TMAP + DT | 3.86 | 10.92 |
| TMAP + RBS | 3.31 | 9.44 |

## Stereo

## Middlebury Test Set V3

| Method | MAE RMSE |  |
| :--- | :---: | :---: |
| MC-CNN | 17.9 | 55.0 |
| MC-CNN + RBS | 8.19 | 29.9 |

## Middlebury Training Set V3

| Method | MAE RMSE |  |
| :--- | :---: | :---: |
| SGM | 3.85 | 10.68 |
| SGM + TF | 3.82 | 9.55 |
| SGM + FGF | 4.05 | 9.66 |
| SGM + WMF | 3.97 | 9.99 |
| SGM + DT | 3.85 | 9.90 |
| SGM + RBS | 3.44 | 9.21 |

## Bilateral Solver as a "Layer"



## Bilateral Solver as a "Layer"



Reference

## Bilateral Solver as a "Layer"

Forward: $\quad \mathbf{x} \leftarrow \mathbf{S}^{\mathrm{T}}\left(\mathrm{A}^{-1}(\mathbf{S y})\right)$

## Bilateral Solver as a "Layer"

$$
\text { Forward: } \quad \mathbf{x} \leftarrow \mathbf{S}^{\mathrm{T}}\left(\mathrm{~A}^{-1}(\mathbf{S y})\right)
$$

$$
\text { Backward: } \quad \nabla_{\mathbf{y}} \leftarrow \mathbf{S}^{\mathrm{T}}\left(\mathrm{~A}^{-1}\left(\mathbf{S} \nabla_{\mathbf{x}}\right)\right)
$$

## Bilateral Solver as a "Layer"

$$
\text { Forward: } \quad \mathbf{x} \leftarrow \mathbf{S}^{\mathrm{T}}\left(\mathrm{~A}^{-1}(\mathbf{S y})\right)
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$$

No annoying bookkeeping, "unrolling", etc.

## Semantic Segmentation



Reference

## Semantic Segmentation



## Semantic Segmentation



Accuracy Runtime (IOU) (ms)

Deeplab
62.3\% | 58

Deeplab + DenseCRF $67.6 \% \mid 58+918$

Deeplab + DenseCRF

## Semantic Segmentation



Deeplab + Bilateral Solver


Deeplab + Bilateral Solver Ь৮.U\% | Ђర + ૪Ь

70\% as accurate
11x faster

## Preview: Optical Flow for VR

Anderson et al, Jump: Virtual Reality Video, SIGGRAPH Asia 2016


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Works well on a variety of tasks: depth superresolution, colorization, stereo, semantic segmentation, etc.

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The bilateral solver is a simple, fast, and effective tool.

Nearly as fast as a bilateral filter, but significantly more accurate.

Works well on a variety of tasks: depth superresolution, colorization, stereo, semantic segmentation, etc.

Can be easily integrated into deep learning pipelines.

## Code Available*

*not the exact same code that was used for the paper

## github.com/poolio/bilateral solver

Thanks!

