# The Fast Bilateral Solver

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## **Bilateral Filter**



Blur the input while respected edges in the reference image.

## **Bilateral Filter**





Find the image that is as smooth as possible with respect to the reference image, and as close as possible to the input.



The bilateral filter is one gradient descent step in the bilateral solver.

(assuming: step size = 
$$1_{\lambda} = 1$$
, initial state =  $1_{\lambda}$ 

Fast - Can be reformulated to be roughly as fast as a fast bilateral filter.

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**General** - Can be easily generalized into weighted, robust, low rank, and differentiable variants.

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**General** - Can be easily generalized into weighted, robust, low rank, and differentiable variants.

Effective - Performs well on a variety of tasks: depth superresolution colorization stereo semantic segmentation, etc.

## **Review: Bilateral-Space Optimization**







**Splat** resample from pixels into "bilateral-space" Blur apply a series of blurs in bilateral-space

Slice resample back into pixel-space

Adams et al, Eurographics 2010

# **Review: Bilateral-Space Optimization**



resample from pixels into "bilateral-space" Blur apply a series of blurs in bilateral-space

Slice resample back into pixel-space

Adams et al, Eurographics 2010

We can solve optimization problems based on bilateral kernels in this "bilateral space"

Barron et al, CVPR 2015 Märki et al, CVPR 2016

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$$W_{i,j} = \exp\left(-\sum_{d} \frac{(R_{i,d} - R_{j,d})^2}{2\sigma^2}\right)$$

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$$\begin{split} \mathbf{A} &= \lambda \left( \mathbf{D}_{m} - \mathbf{D}_{n} \mathbf{B} \mathbf{D}_{n} \right) + \mathbf{S} \vec{1} \\ \mathbf{x} &\leftarrow \mathbf{S}^{T} \left( \mathbf{A}^{-1} \left( \mathbf{S} \mathbf{y} \right) \right) \end{split}$$

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We can show that:

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(see paper for details)



Input y











Reference Image



Park et al.



 $\mathbf{FGF}$ 





 $\operatorname{GF}$ 

 ${\rm Input\,Depth}$ 

Min *et al*.

Ma et al.

Yang 2007

† Liet al.



Ferstl *et al*.



 ${\rm True}\,{\rm Depth}$ 





Zhang *et al*.







Method	Err	Time (sec
Nearest Neighbor	7.26	0.003
Bicubic	5.91	0.007
Kiechle <i>et al</i> .	5.86	450
Bilinear	5.16	0.004
Liu <i>et al</i> .	5.10	16.60
Shen <i>et al</i> .	4.24	31.48
Diebel & Thrun	3.98	
Chan <i>et al</i> .	3.83	3.02
Guided Filter	3.76	23.89
Min <i>et al</i> .	3.74	0.383
Lu & Forsyth	3.69	20
Park <i>et al</i> .	3.61	24.05
Domain Transform	3.56	0.021
Ma et al.	3.49	18
GuidedFilter (Matlab)	3.47	0.434
Zhang <i>et al</i> .	3.45	1.346
Fast Guided Filter	3.41	0.225
Yang 2015	3.41	0.304
Yang <i>et al.</i> 2007	3.25	
Farbman <i>et al</i> .	3.19	6.11
Joint Bilateral Upsample	3.14	1.98
Ferstl <i>et al</i> .	2.93	140
Li et al.	2.56	700
Kwon <i>et al</i> .	1.21	300
Bilateral Solver	2.70	0.234



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 $_{\mathrm{JB}}$ 

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Yang 2007

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BS

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#### Bilateral Solver 0.85 sec/megapixel

#### Levin *et al* 81.0 sec/megapixel

#### 95× speedup

Levin et al, Colorization using optimization. SIGGRAPH 2004

$$\mathbf{x} \leftarrow \underset{\mathbf{x}}{\operatorname{arg\,min}} \ \frac{\lambda}{2} \sum_{i,j} W_{i,j} \left( x_i - x_j \right)^2 + \sum_i c_i (x_i - y_i)^2$$

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## **Robust Bilateral Solver**

$$\mathbf{x} \leftarrow \underset{\mathbf{x}}{\operatorname{arg\,min}} \frac{\lambda}{2} \sum_{i,j} W_{i,j} \left( x_i - x_j \right)^2 + \sum_i \rho(x_i - y_i) = =$$

 $\mathbf{while} \operatorname{not} \operatorname{converged}$ :

$$\begin{split} \mathbf{c} &\leftarrow \frac{\rho'(\mathbf{x} - \mathbf{y})}{\mathbf{x} - \mathbf{y}} \\ \mathbf{A} &= \lambda \left( \mathbf{D}_{m} - \mathbf{D}_{n} \mathbf{B} \mathbf{D}_{n} \right) + \mathbf{S} \mathbf{c} \\ \mathbf{x} &\leftarrow \mathbf{S}^{T} \left( \mathbf{A}^{-1} \left( \mathbf{S} \mathbf{y} \right) \right) \end{split}$$

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can be done in bilateral space



## MC-CNN



## MC-CNN + Robust Bilateral Solver



## MC-CNN



### MC-CNN + Robust Bilateral Solver

# Method MAE RMSE MC-CNN 17.9 55.0 MC-CNN + RBS 8.19 29.9

(Lowest test-set MAE and RMSE at submission time)

# Method MAE RMSE MC-CNN 17.9 55.0 MC-CNN + RBS 8.19 29.9

#### Middlebury Training Set V3

Method	MAE	RMSE
MC-CNN	5.93	18.36
MC-CNN + TF	5.67	16.18
MC-CNN + FGF	5.91	16.32
MC-CNN + WMF	5.30	15.62
MC-CNN + DT	5.69	16.53
MC-CNN + RBS	2.81	8.44

# Method MAE RMSE MC-CNN 17.9 55.0 MC-CNN + RBS 8.19 29.9

#### Middlebury Training Set V3

Method	MAE	RMSE
MeshStereo	3.83	10.75
MeshStereo + TF	3.81	9.91
MeshStereo + FGF	3.96	10.03
MeshStereo + WMF	3.87	10.10
MeshStereo + DT	3.77	10.12
MeshStereo + RBS	3.22	8.72

# Method MAE RMSE MC-CNN 17.9 55.0 MC-CNN + RBS 8.19 29.9

Middlebury Training Set V3		
Method	MAE	RMSE
TMAP	3.98	11.55
TMAP + TF	3.94	10.90
TMAP + FGF	4.17	10.79
TMAP + WMF	4.11	11.01
TMAP + DT	3.86	10.92
TMAP + RBS	3.31	9.44

# Method MAE RMSE MC-CNN 17.9 55.0 MC-CNN + RBS 8.19 29.9

#### Middlebury Training Set V3

Method	MAE	RMSE
SGM	3.85	10.68
SGM + TF	3.82	9.55
SGM + FGF	4.05	9.66
SGM + WMF	3.97	9.99
SGM + DT	3.85	9.90
SGM + RBS	3.44	9.21





Reference

Forward: 
$$\mathbf{x} \leftarrow \mathbf{S}^{\mathrm{T}} \left( \mathrm{A}^{-1} \left( \mathbf{S} \mathbf{y} \right) \right)$$

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$$\mathbf{x} \leftarrow \mathbf{S}^{\mathrm{T}} \left( \mathrm{A}^{-1} \left( \mathbf{S} \mathbf{y} \right) \right)$$

Backward: 
$$\nabla_{\mathbf{y}} \leftarrow \mathbf{S}^{\mathrm{T}} \left( \mathrm{A}^{-1} \left( \mathbf{S} \nabla_{\mathbf{x}} \right) \right)$$

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$$\nabla_{\mathbf{y}} \leftarrow \mathbf{S}^{\mathrm{T}} \left( \mathrm{A}^{-1} \left( \mathbf{S} \nabla_{\mathbf{x}} \right) \right)$$

#### No annoying bookkeeping, "unrolling", etc.



#### Reference



#### Deeplab

Chen et al, ICLR 2015



#### Deeplab + DenseCRF

Chen et al, ICLR 2015

(ms)

58



#### Deeplab + Bilateral Solver

Accuracy Runtime (IOU) (ms) Deeplab 62.3% 58 Deeplab + DenseCRF 67.6% 58 +918 Deeplab + Bilateral Solver 66.0% 58 + 85

70% as accurate 11x faster

## Preview: Optical Flow for VR

Anderson et al, Jump: Virtual Reality Video, SIGGRAPH Asia 2016



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Anderson et al, Jump: Virtual Reality Video, SIGGRAPH Asia 2016



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#### Anderson et al, Jump: Virtual Reality Video, SIGGRAPH Asia 2016



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Works well on a variety of tasks: depth superresolution, colorization, stereo, semantic segmentation, etc.

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Works well on a variety of tasks: depth superresolution, colorization, stereo, semantic segmentation, etc.

Can be easily integrated into deep learning pipelines.

## Code Available\*

#### \*not the exact same code that was used for the paper

github.com/poolio/bilateral\_solver

# Thanks!