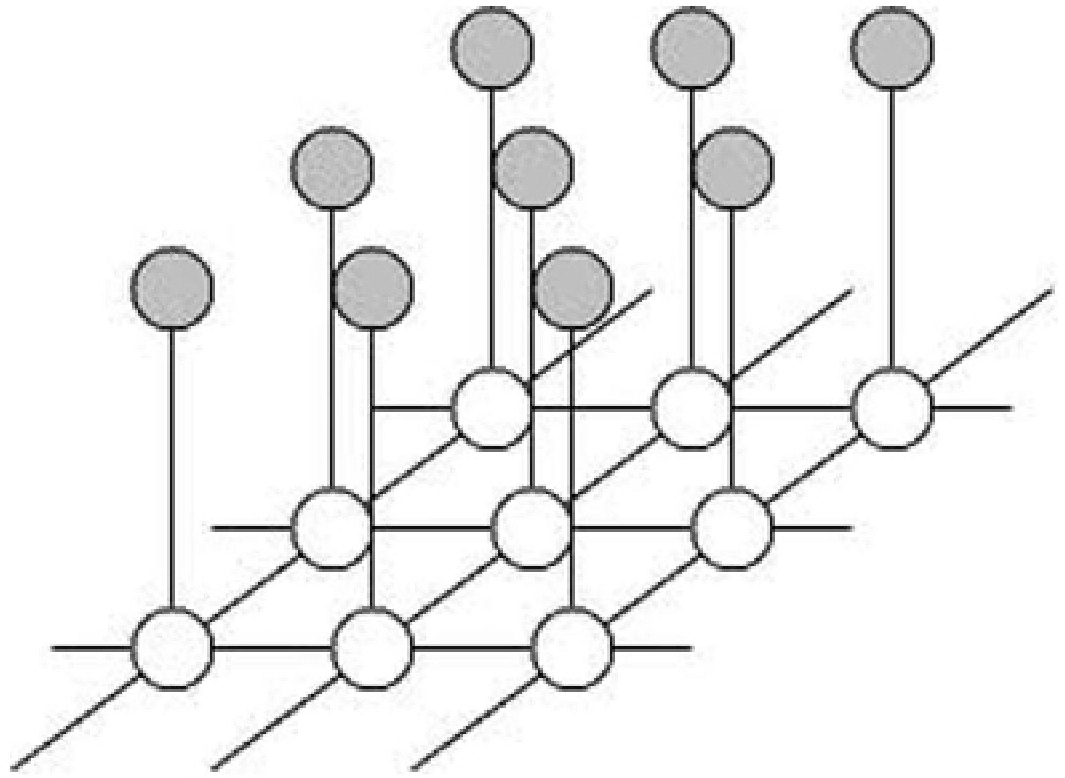
Efficient Continuous Relaxations for Dense CRF

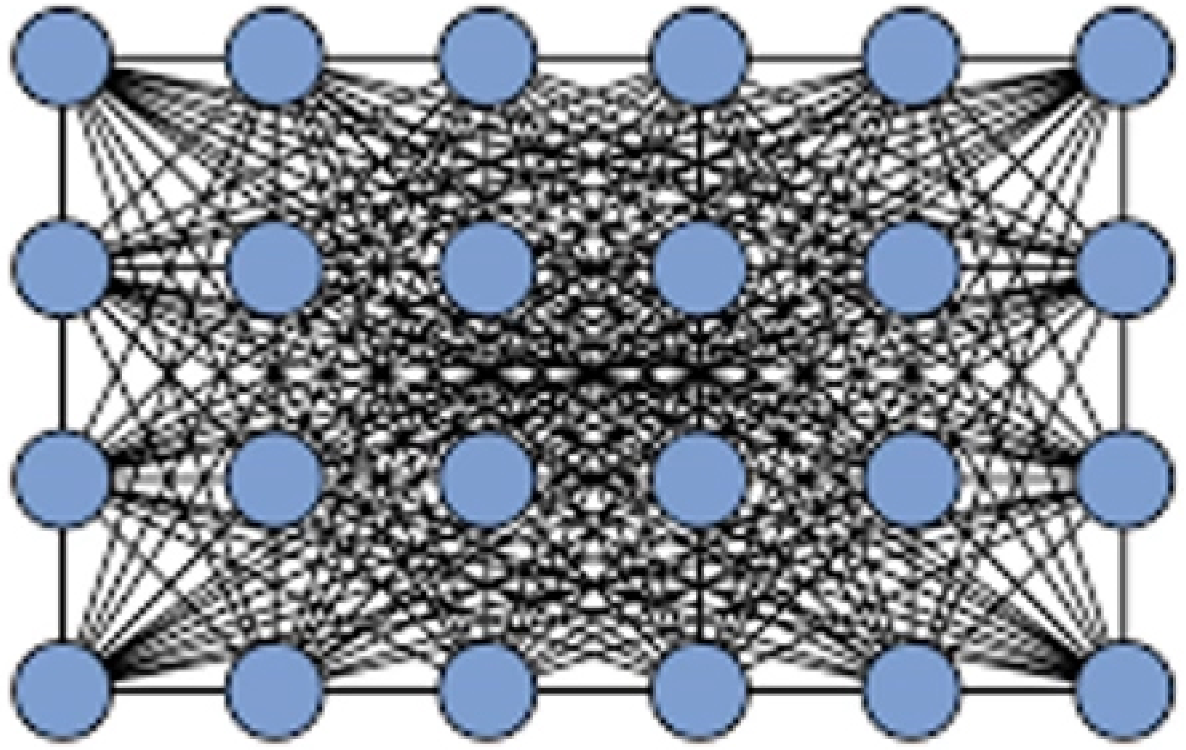
Alban Desmaison, Rudy Bunel,

Pushmeet Kohli, Philip H.S. Torr, M. Pawan Kumar

Dense CRF model

- CRF are widely used in vision
- Dense-CRF when all nodes are connected to each other
- Solved via Mean-Field inference





Solving problem using CRFs

- The problem is formulated as an energy function:

$$E(\mathbf{x}) = \sum_{a=1}^{N} \phi_a(x_a) + \sum_{a=1}^{N} \sum_{\substack{b=1 \\ b \neq a}}^{N} \psi_{a,b}(x_a, x_b).$$

- The goal is then to perform MAP estimation.

Content of the presentation

1. Continuous relaxations:
   - Convex QP
   - Difference of convex QP
   - LP

2. Experiments:
   - Segmentation
   - Stereo matching

QP relaxation

$$\min \quad \sum_{a=1}^{N} \sum_{i \in \mathcal{L}} \phi_a(i) y_a(i) + \sum_{a=1}^{N} \sum_{\substack{b=1 \\ b \neq a}}^{N} \sum_{i,j \in \mathcal{L}} \psi_{a,b}(i,j) y_a(i) y_b(j),$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{L}} y_a(i) = 1 \quad \forall a \in [1, N],$$

$$y_a(i) \in \{0, 1\} \quad \forall a \in [1, N] \quad \forall i \in \mathcal{L}.$$

QP relaxation

$$\begin{aligned} \min \quad & \sum_{a=1}^{N} \sum_{i \in \mathcal{L}} \phi_a(i) y_a(i) + \sum_{a=1}^{N} \sum_{\substack{b=1 \\ b \neq a}}^{N} \sum_{i,j \in \mathcal{L}} \psi_{a,b}(i,j) y_a(i) y_b(j), \\ \text{s.t.} \quad & \sum_{i \in \mathcal{L}} y_a(i) = 1 \quad \forall a \in [1, N], \\ & y_a(i) \geq 0 \quad \forall a \in [1, N] \quad \forall i \in \mathcal{L}. \end{aligned}$$

Convex QP

- Convex approximation

- A different problem
- But a convex problem
- Easy and fast to solve (Frank-Wolfe)

- Converges to single global minimum
- But no good guarantee on the quality of the solution

Difference of Convex relaxation

- Rewrite original problem as difference of convex functions

- Iteratively solving simple convex QPs (CCCP algorithms)
- Each subproblem is solved using the above algorithm

- Converges to zero gradient point of the true objective

LP relaxation

$$\min \quad S_{LP}(\mathbf{y}) = \underbrace{\sum_a \sum_i \phi_a(i) y_a(i)}_{unary} + \underbrace{\sum_a \sum_{b \neq a} \sum_i \psi_{a,b} \frac{|y_a(i) - y_b(i)|}{2}}_{pairwise},$$

$$\text{s.t.} \quad \mathbf{y} \in \mathcal{M}.$$

Solving the LP

- Take care of the absolute value
- Sort + divide and conquer algorithm
- Complete algorithm has O(n log(n)) complexity

- LP always converges to its global minimum
- Best theorethical bound: tight for 2 labels problem and 2-approximation for more labels

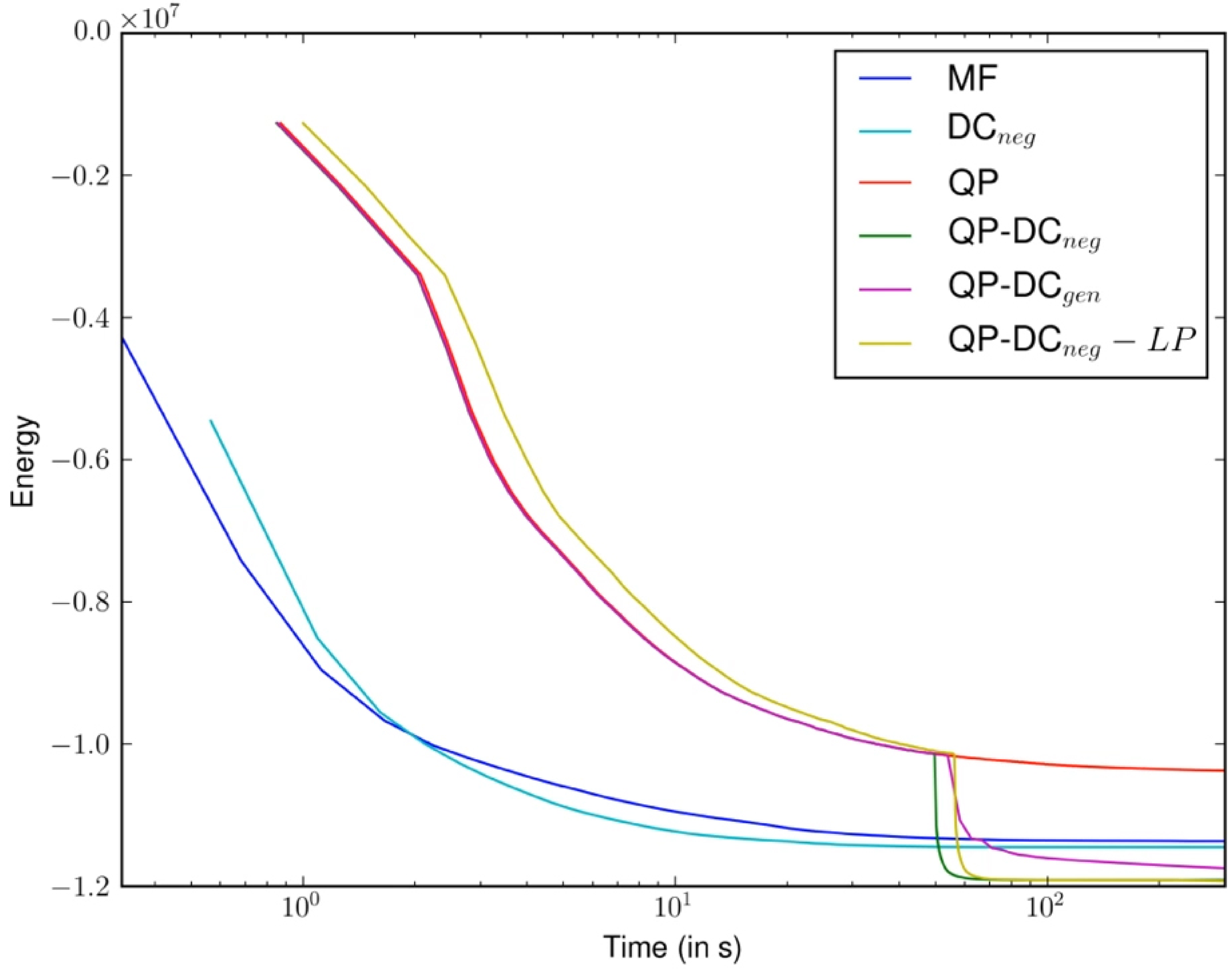
Experimental protocol

- Segmentation on Pascal VOC 2010 (in the paper)
  - Unaries from Krähenbühl and Koltun [1]
  - Pairwise cross validated
- Stereo matching
  - Unaries: absolute difference matching function
  - Pairwise (parameters from segmentation part)

[1] Krähenbühl and Koltun: Efficient inference in fully connected CRFs with gaussian edge potentials. NIPS 2011

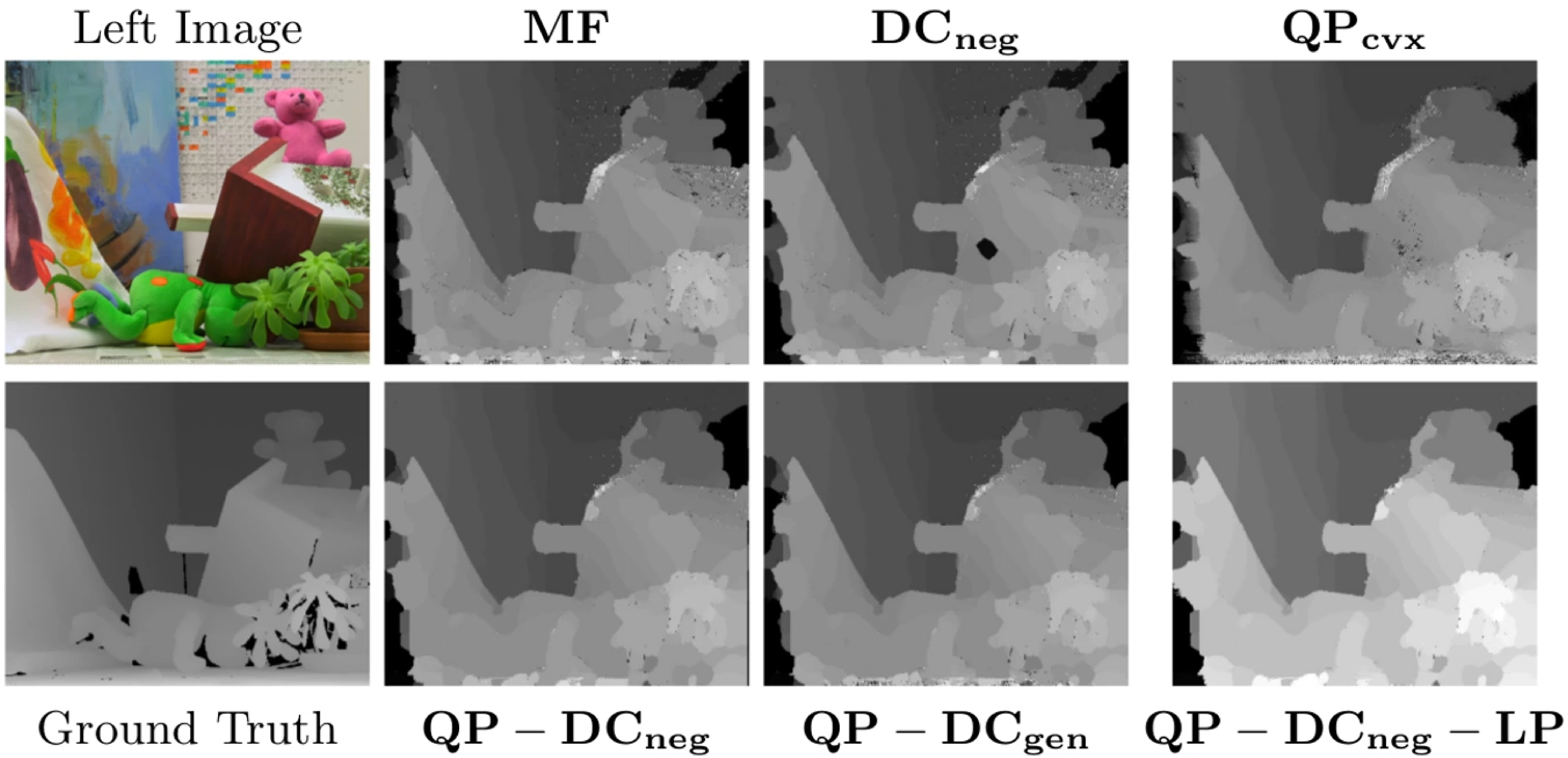
Stereo matching

- Energy results (Teddy)



Stereo matching

- Visual results



Conclusion

- Continuous relaxations for MAP estimation lead to better energies than MF
- Considering different level of relaxation allow to do a trade-off between speed and precision
- But what you minimize is important to get good results
- From this basis, many extensions are possible
  - More complex pixel compatibility functions
  - More efficient QP/LP solving strategies