A Convex Solution to Spatially-Regularized Correspondence Problems

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Correspondence Problem

$$f : \Omega \quad \rightarrow \quad \Omega$$

Goal: find optimal f

$$\min_{f \in \mathrm{Diff}^+(\Omega,\Omega)} \int_{\Gamma(f)} \mathbf{c}(\mathbf{p}) d\mathbf{p}$$

where $\Gamma(f) = \{(p, f(p)) | p \in \Omega\} \subset \Omega \times \Omega$

Minimal Surface Problem

from **Correspondence Problems**

$$\min_{f \in \mathrm{Diff}^+(\Omega,\Omega)} \int_{\Gamma(f)} \mathbf{c}(\mathbf{p}) d\mathbf{p}$$

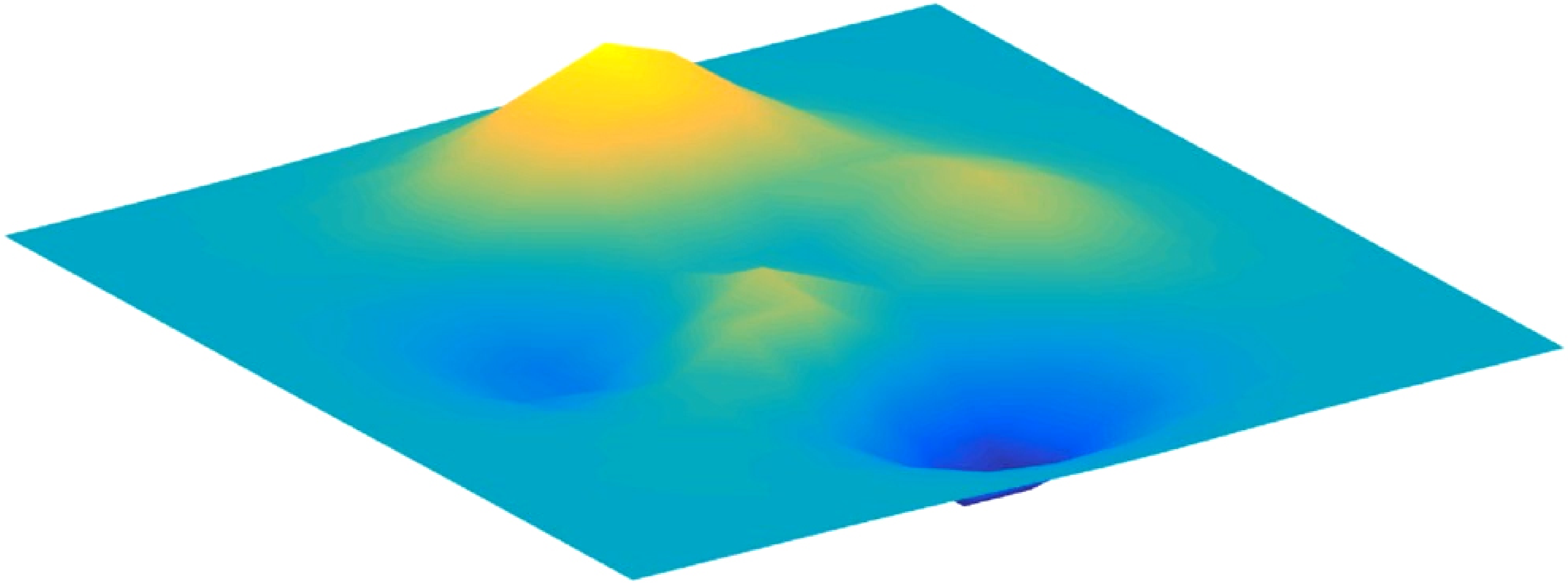
to **Minimal Surface Problems**

$$\min_{\Gamma \subset \Omega \times \Omega} \int_{\Gamma} \mathbf{c}(\mathbf{p}) d\mathbf{p}$$

$$\text{s. t. } \Gamma \text{ closed surface}$$

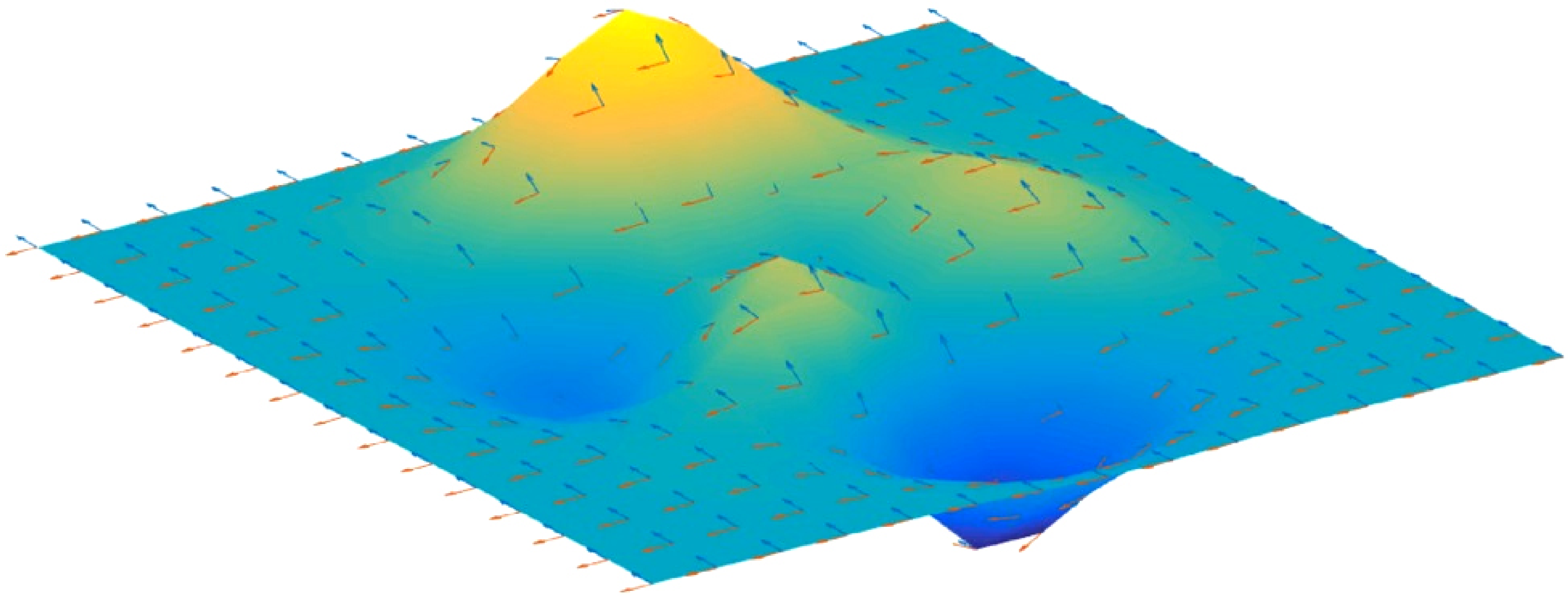
$$\pi_1(\Gamma) = \pi_2(\Gamma) = \Omega$$

Main Challenge



How do we find a minimal 2-dimensional surface of codimension 2?

Surface Representation

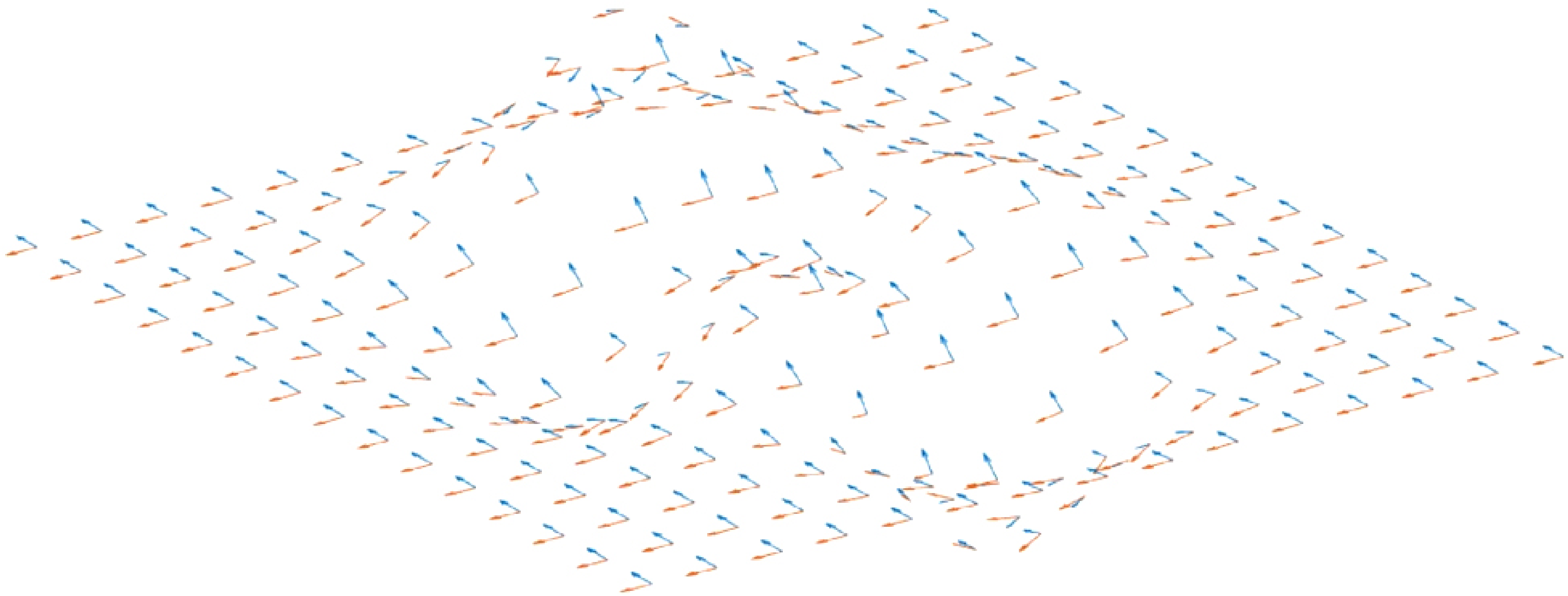


$$\min_{\Gamma \subset \Omega \times \Omega} \int_\Gamma \mathbf{c}(\mathbf{p}) d\mathbf{p} = \int_\Omega \mathbf{c}(x(p)) \sqrt{\det dx^\top dx}\, dp$$

$$\text{s. t. } \Gamma \text{ closed surface}$$

$$\pi_1(\Gamma) = \pi_2(\Gamma) = \Omega$$

Minimal 2-Vector Fields



$$\min_{\omega:\Omega^2\to\Lambda_2\mathcal{T}\mathbb{R}^4} \int_{\Omega^2} \mathbf{c}(\mathbf{p})\|\omega(\mathbf{p})\| d\mathbf{p}$$

$$\text{s. t.} \quad \operatorname{div}\omega = 0$$

$$\pi_1(\omega) = \pi_2(\omega) = \Omega$$