Binary Hashing with Semidefinite Relaxation and Augmented Lagrangian

Thanh-Toan Do, Anh-Dzung Doan, Duc-Thanh Nguyen, Ngai-Man Cheung

Singapore University of Technology and Design

Overview and Contribution

- Two-step approach for hashing
  - inference binary codes
  - learn hash function, given inferenced binary codes

  → reduce the complexity; flexible using of different hash functions
- Contribution
  - Unified formulation for supervised/unsupervised hashing
  - **Two approaches for inferencing binary codes**
    - Semidefinite Programming
    - Augmented Lagrangian

Unified formulation for supervised/unsupervised hashing

- **Input:** $\mathbf{S}$: similarity matrix between samples, i.e., pairwise distance/pairwise label matrix for unsupervised/supervised; L: code length; n: number of training samples
- **Target:** learning binary codes $\mathbf{Z}$ s.t. $\mathbf{S}$ is preserved in Hamming space, i.e., solving

$$\min_{\mathbf{Z} \in \{-1,1\}^{L \times n}} \left\| \mathbf{Z}^T \mathbf{Z} - \mathbf{Y} \right\|^2 \quad (1)$$

  - unsupervised: $\mathbf{Y} = \mathbf{L} - \frac{L\mathbf{S}}{2}$; supervised: $\mathbf{Y} = \mathbf{L} - 2L\mathbf{S}$
- Using coordinate descent approach for solving above NP-hard, i.e., solving one row of $\mathbf{Z}$ at a time.
  Let $\mathbf{x} = [x_1, ..., x_n]^T = \mathbf{z}^{(k)}$, we solve BQP

$$\begin{aligned} &\min_{\mathbf{x}} \mathbf{x}^T \mathbf{A} \mathbf{x} \\ &s.t. \; {x_i}^2 = 1, \forall i = 1, ..., n. \end{aligned} \quad (2)$$

  where $\mathbf{A} = \{a_{ij}\} \in \mathbb{R}^{n \times n}$; $a_{ij} = \bar{\mathbf{z}}_i^T \bar{\mathbf{z}}_j - y_{ij}$.

Semidefinite Relaxation (SDR) approach

Original BQP:

$$\min_{\mathbf{x}} \mathbf{x}^T\mathbf{A}\mathbf{x}$$
$$s.t.\ x_i^2 = 1, \forall i = 1, ..., n.$$

Let $\mathbf{B} = \mathbf{A} - \lambda_1\mathbf{I}$, where λ_1 is the largest eigenvalue of $\mathbf{A}$; $\mathbf{X} = \mathbf{x}\mathbf{x}^T$
$\rightarrow$ solve **equivalent problem**

$$\min_{\mathbf{X}}\ trace(\mathbf{BX})$$
$$s.t.\ diag(\mathbf{X}) = \mathbf{1}; \mathbf{X} \succeq 0; rank(\mathbf{X}) = 1 \tag{3}$$

Two steps solution:

- Drop rank one constraint $\rightarrow$ solving Semidefinite Program

$$\min_{\mathbf{X}}\ trace(\mathbf{BX})$$
$$s.t.\ diag(\mathbf{X}) = \mathbf{1}; \mathbf{X} \succeq 0 \tag{4}$$

  Solving: Using Convex OPT packages: SeDuMi, SDPT3 $\rightarrow$ achieving the global optimal solution $\mathbf{X}^*$

- Recover binary solution $\hat{\mathbf{x}}$ from $\mathbf{X}^*$

  Solving: using randomized rounding process
  - generate ξ by $\xi \sim \mathcal{N}(0, \mathbf{X}^*)$
  - get feasible point: $\hat{\mathbf{x}} = sgn(\xi)$

Semidefinite Relaxation (SDR) approach (cont'd)

Problem need to be solved:

$$\begin{aligned} &\min_{\mathbf{X}} \ trace(\mathbf{BX}) \\ &s.t.\ diag(\mathbf{X}) = \mathbf{1}; \mathbf{X} \succeq 0; rank(\mathbf{X}) = 1 \end{aligned} \tag{5}$$

Solution provided SDR-rounding: $\hat{\mathbf{x}}$

Bound on objective value at $\hat{\mathbf{x}}$: Let f_{opt} be global optimum objective value of (5) and $f_{SDR-round} = \hat{\mathbf{x}}^T\mathbf{B}\hat{\mathbf{x}}$, we have

$$f_{opt} \leq E[f_{SDR-round}] \leq \frac{2}{\pi} f_{opt} \tag{6}$$

Augmented Lagrangian (AL) approach

Original problem:

$$\min_{\mathbf{x}} \mathbf{x}^T \mathbf{A} \mathbf{x}$$
$$s.t.\ {x_i}^2 = 1, \forall i = 1, ..., n.$$

Let $\Phi(\mathbf{x}) = \left[(x_1)^2 - 1, ..., (x_n)^2 - 1\right]^T$; $\Lambda = [\lambda_1, ..., \lambda_n]^T$: Lagrange multipliers. Minimizing the unconstrained augmented Lagrangian function

$$\mathcal{L}(\mathbf{x}, \Lambda; \mu) = \mathbf{x}^T \mathbf{A} \mathbf{x} - \Lambda^T \Phi(\mathbf{x}) + \frac{\mu}{2} \|\Phi(\mathbf{x})\|^2 \qquad (7)$$

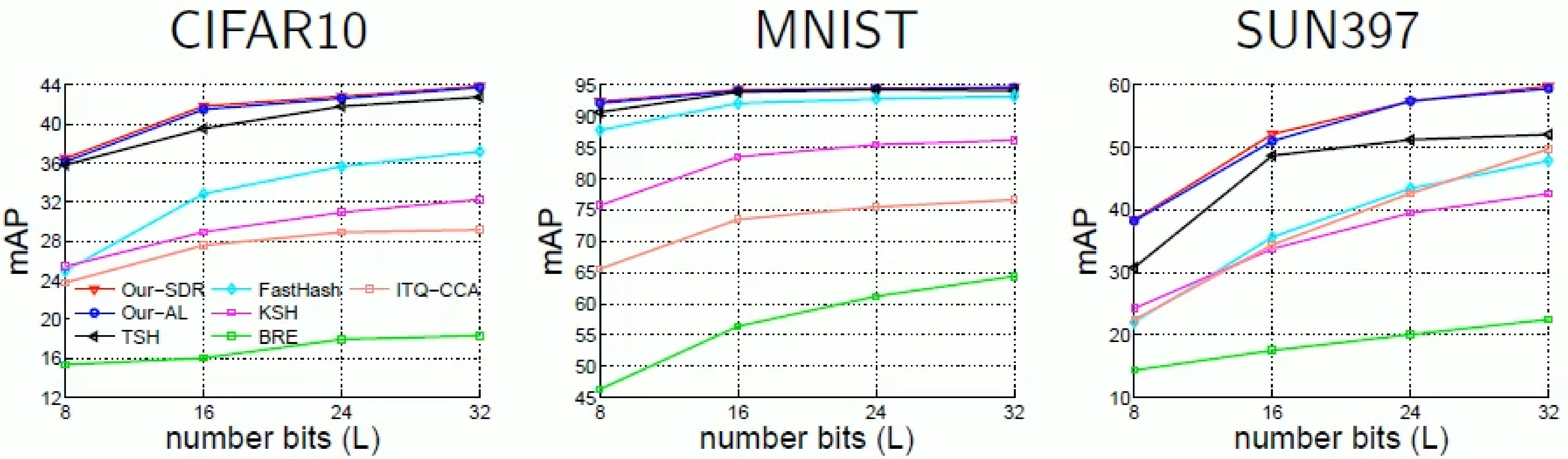
- When μ is large $\rightarrow$ penalize the binary constraint violation severely $\rightarrow$ force the minimizer of the AL function (7) closer to the feasible region of the original problem
- Theoretically, not necessary to take $\mu \rightarrow \infty$ in order to achieve a local optimum of original problem

Dataset

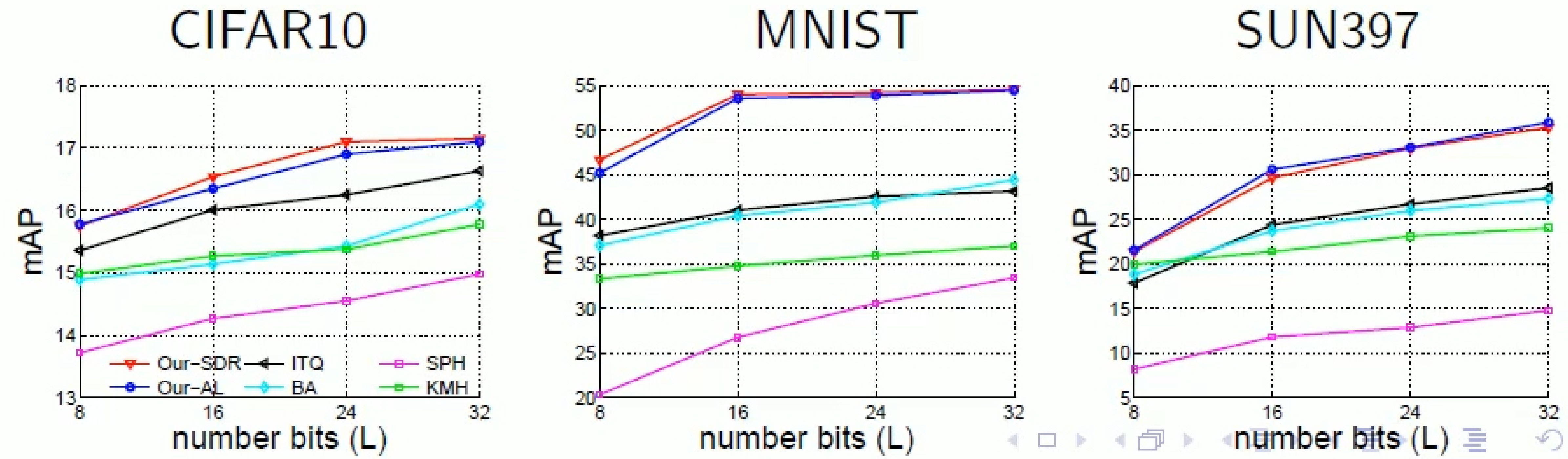
- CIFAR10: 60k images, 10 categories; GIST features.
- MNIST: 70k images, 10 categories; raw (intensity) features.
- SUN397: 108K images. We only use 42 most frequent categories; AlexNet features.

Evaluation of Supervised/Unsupervised Hashing

Supervised hashing



Unsupervised hashing



Thank you!