Linear Depth Estimation from an Uncalibrated, Monocular Polarisation Image

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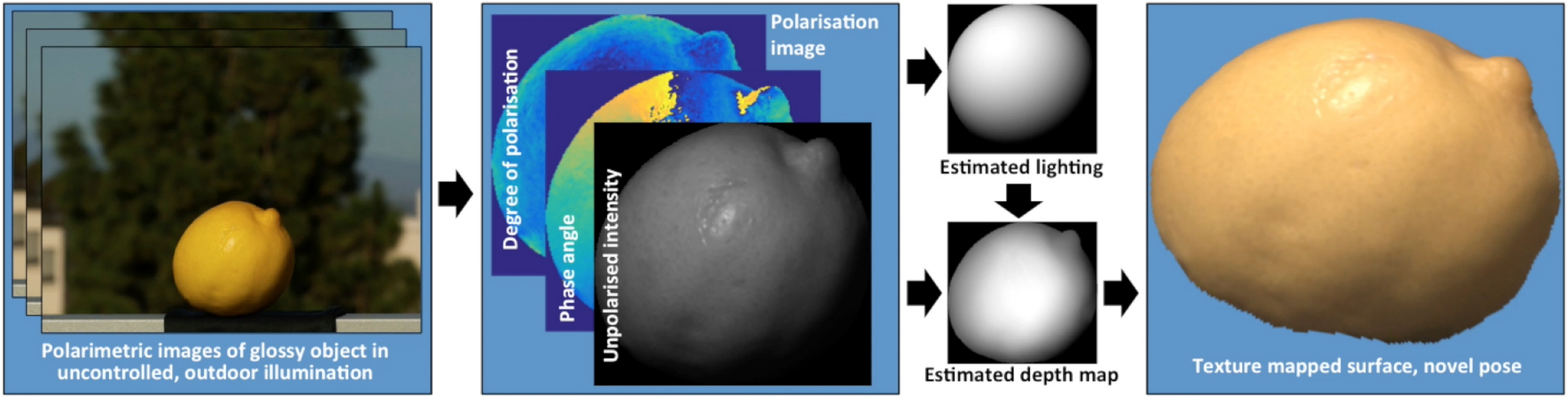
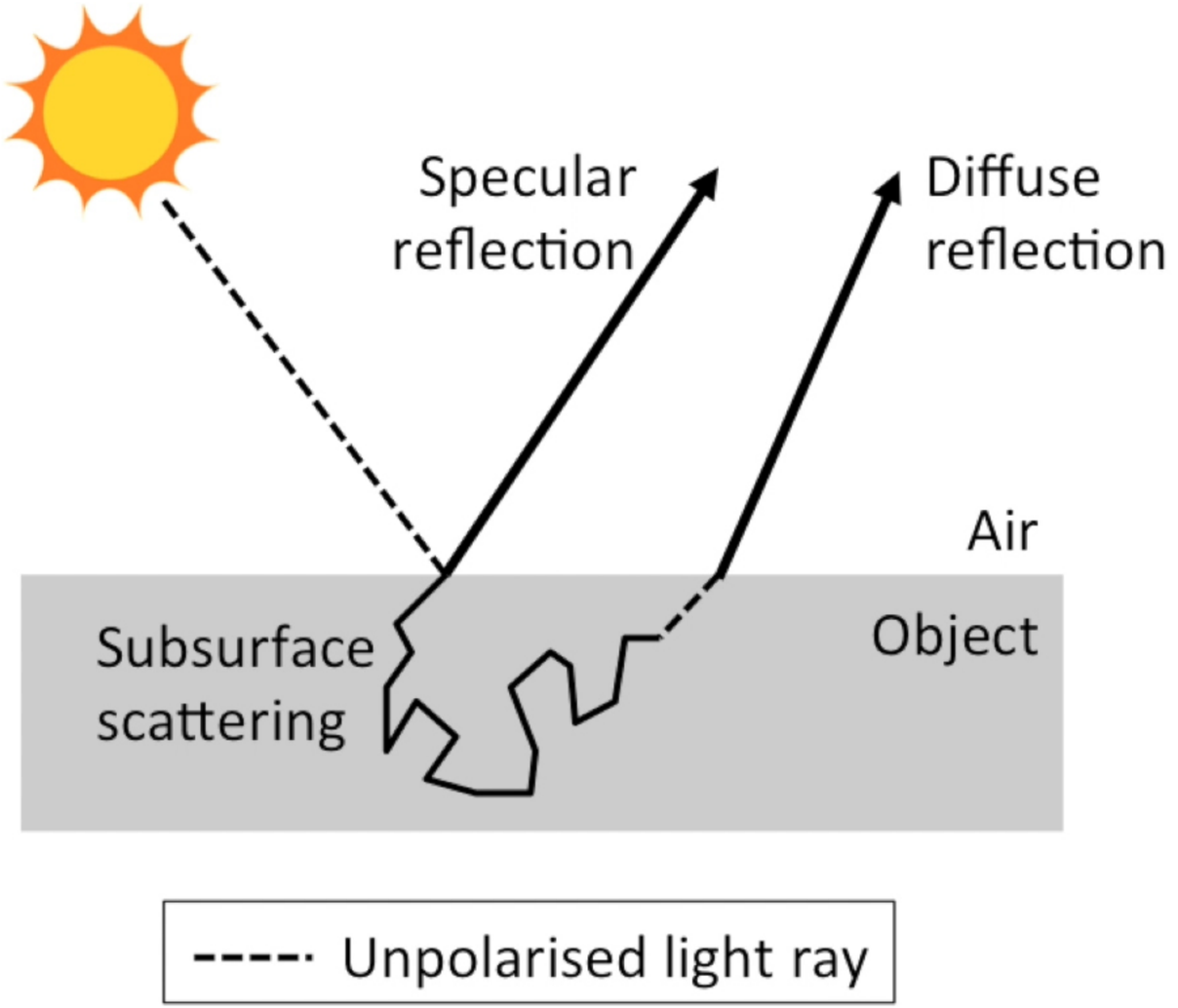
Overview



- Depth from a single polarisation image
- Only need to solve large, sparse linear system
- Monocular and passive
- Arbitrary uncalibrated illumination (estimated)

Shape-from-polarisation

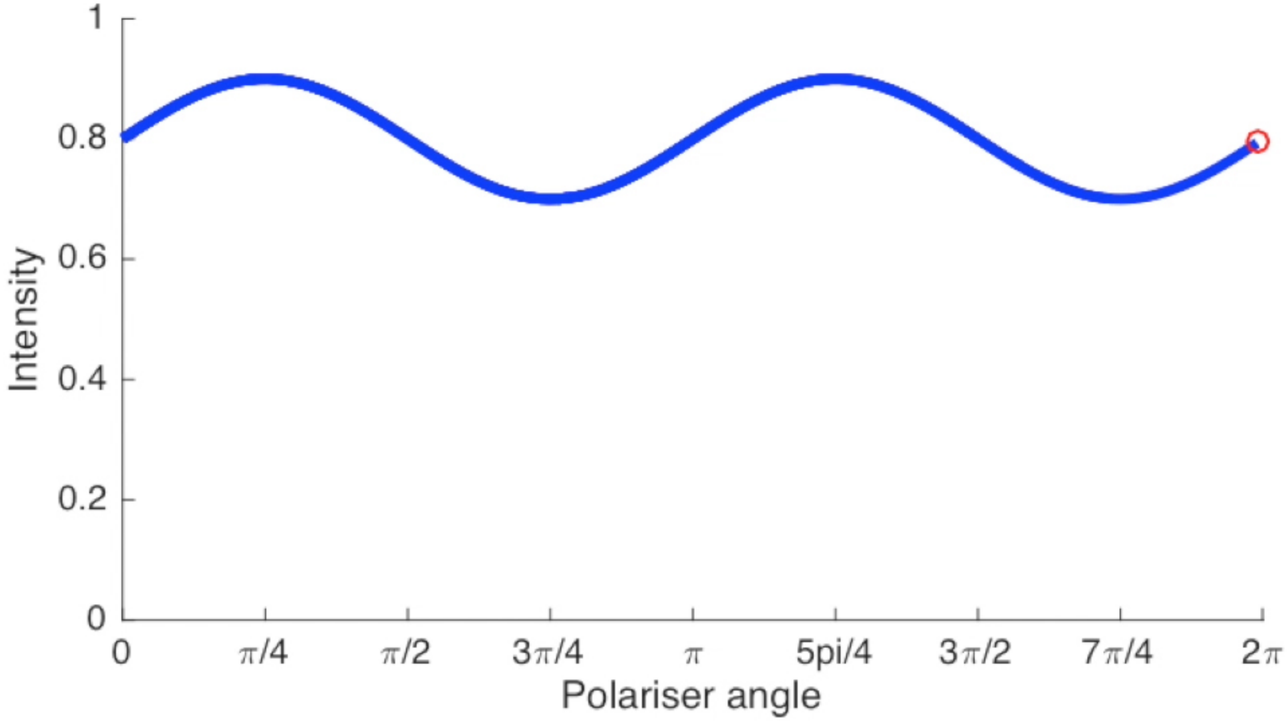


When light reflects from a surface it becomes partially polarised

Degree of polarisation and phase angle provide shape cue

Polarimetric Image Capture

Rotate linear polarising filter in front of camera

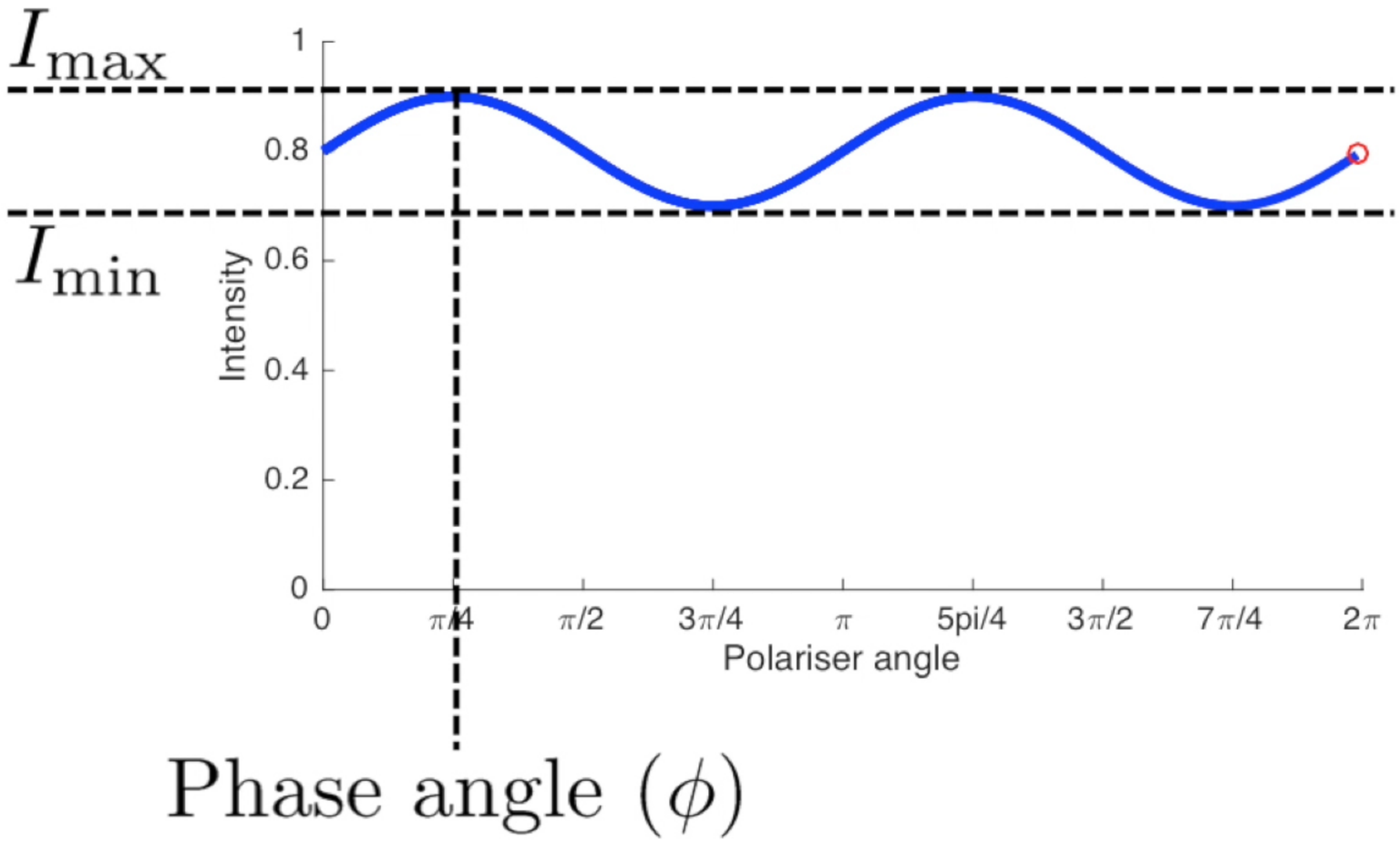


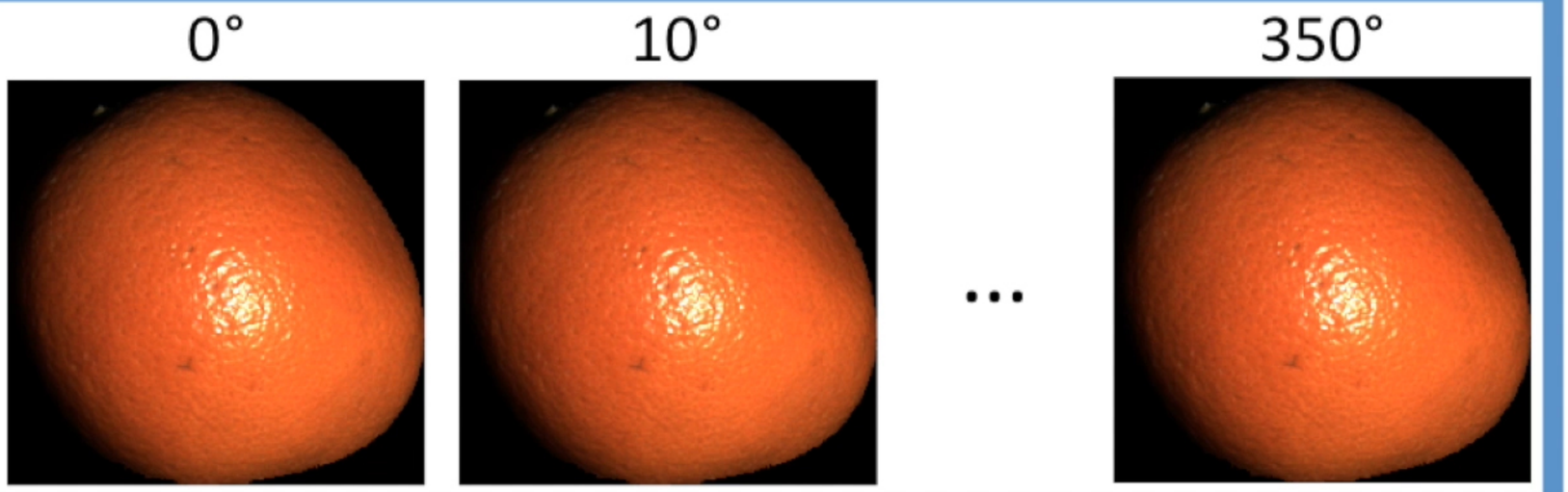
Intensity varies sinusoidally

Polarisation Image

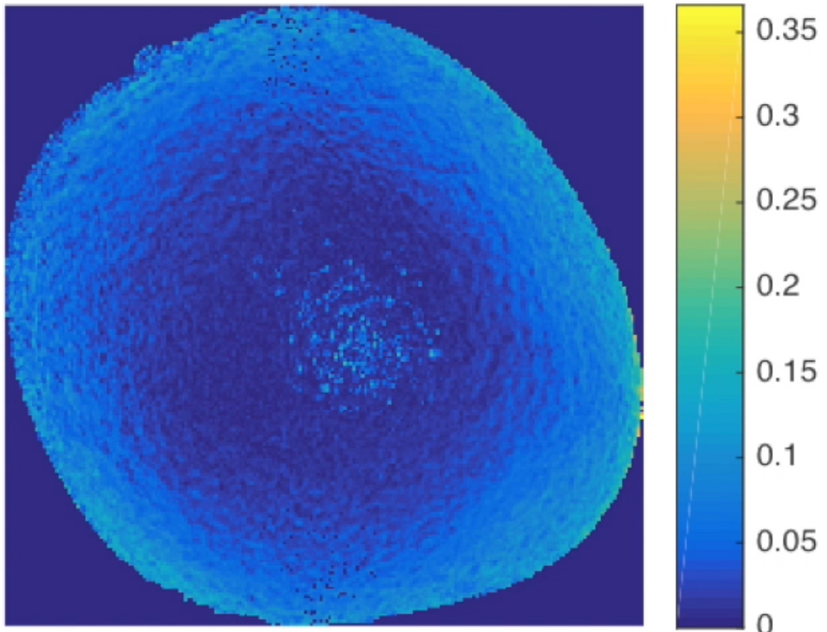
$$\mathrm{DOLP} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

$$I_{\mathrm{unpol}} = \frac{I_{\max} + I_{\min}}{2}$$

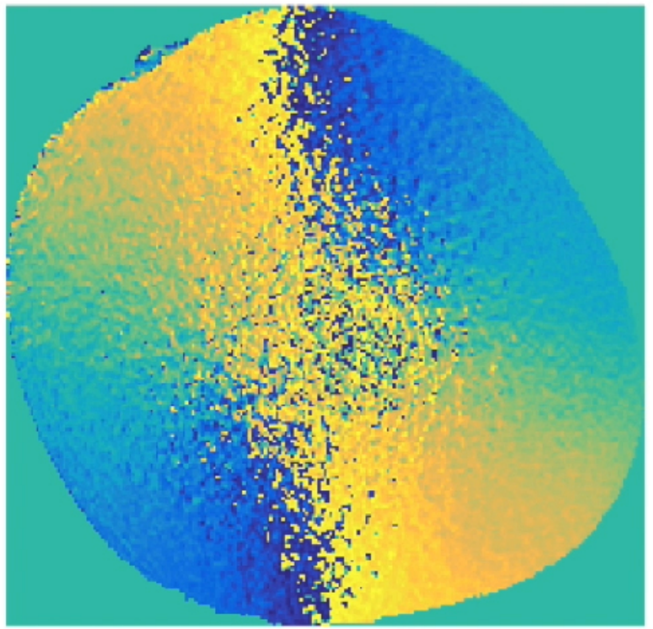


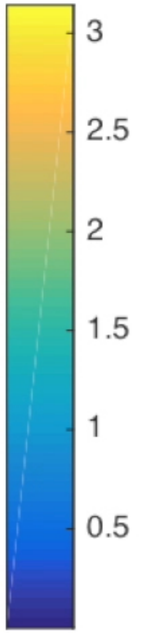


Polarisation Image

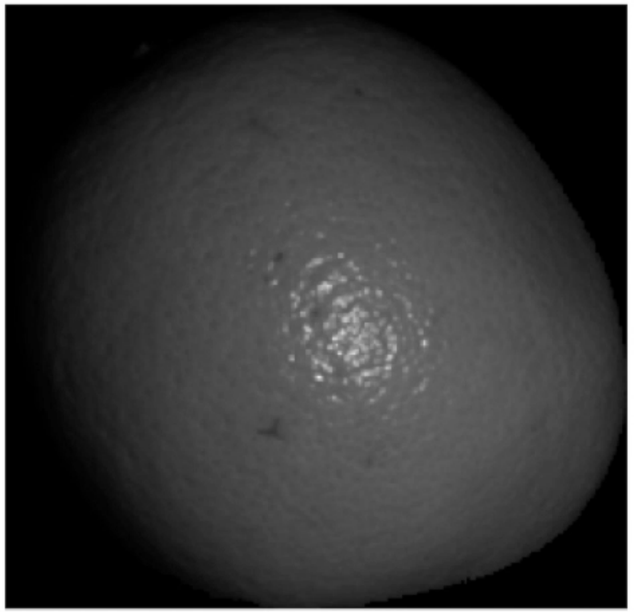


Degree of polarisation



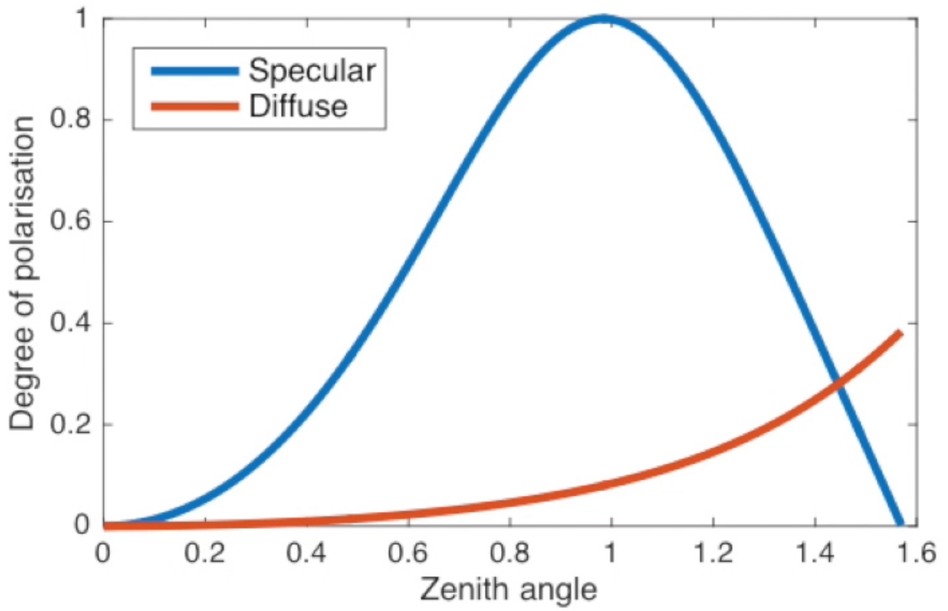


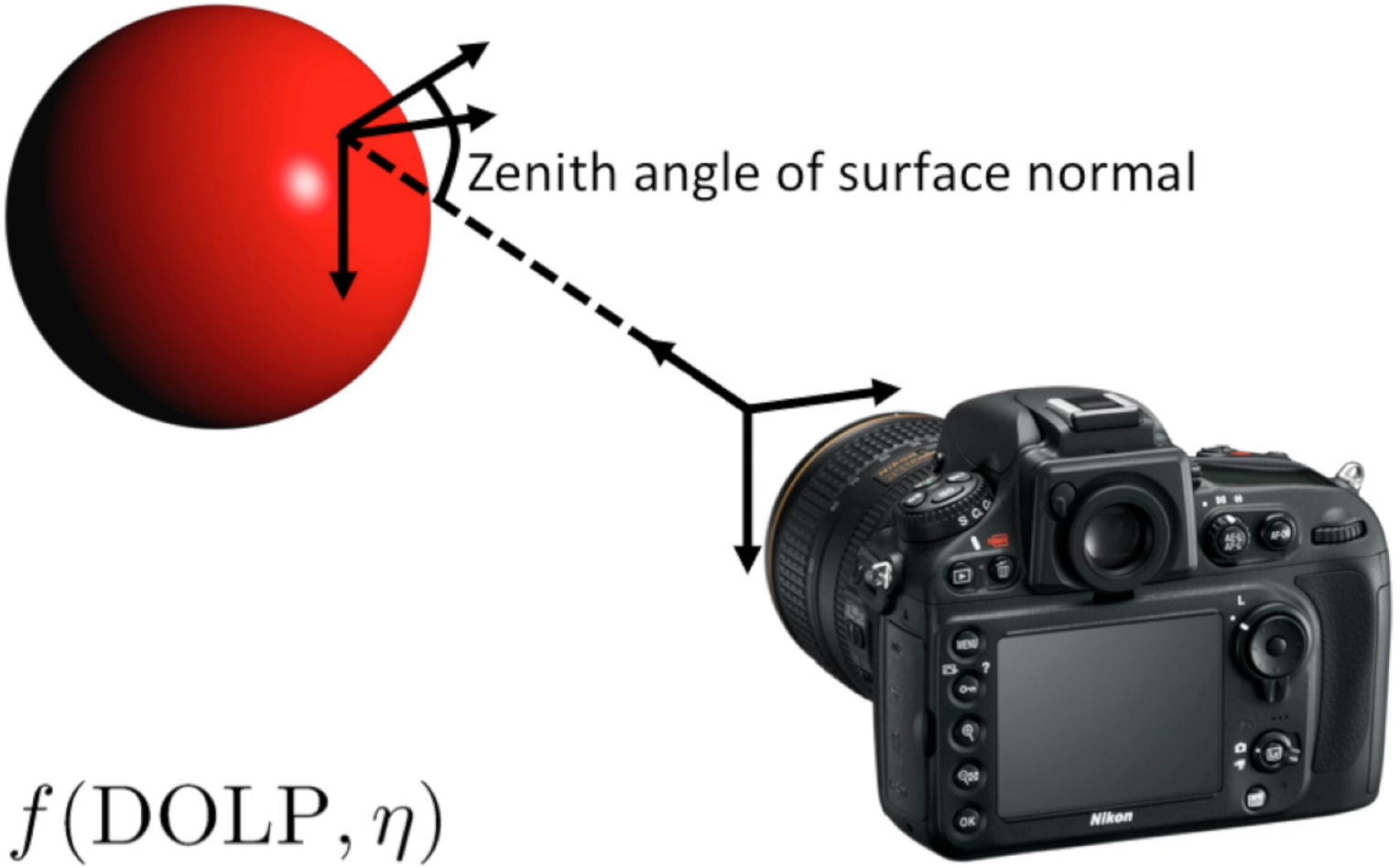
Phase Angle



Unpolarised Intensity

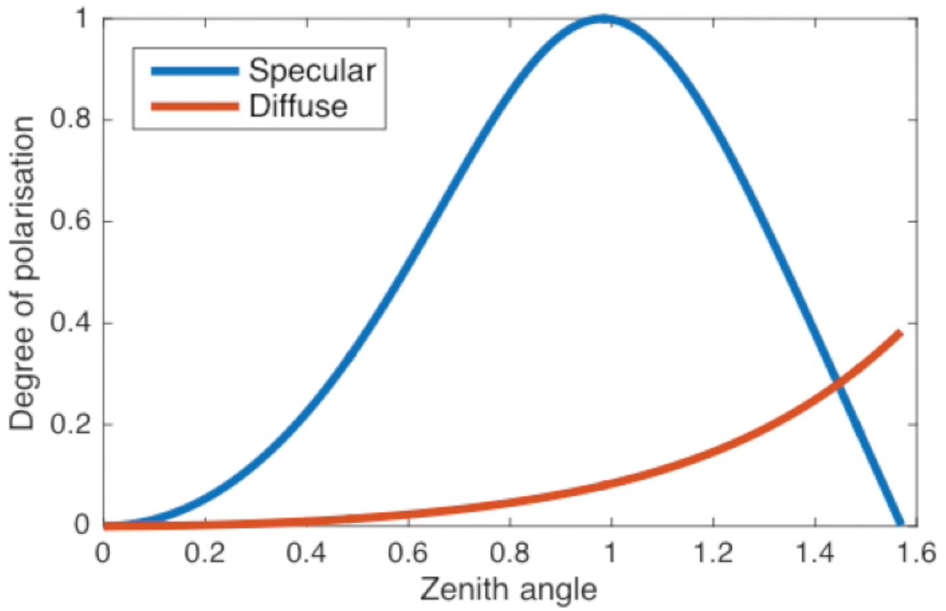
Shape-from-polarisation

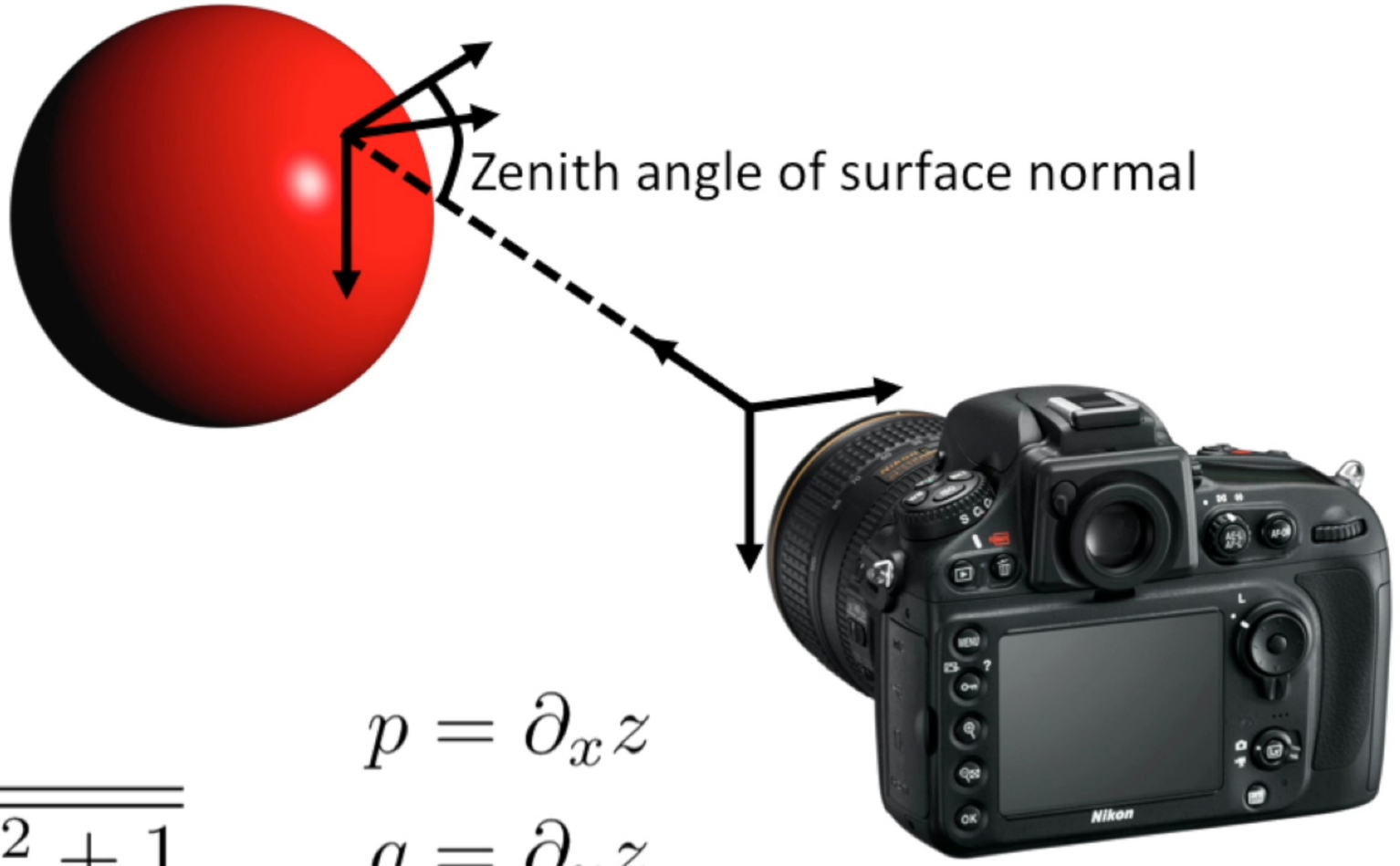




$$\cos(\theta) = \mathbf{n} \cdot \mathbf{v} = f(\mathrm{DOLP}, \eta)$$

Shape-from-polarisation





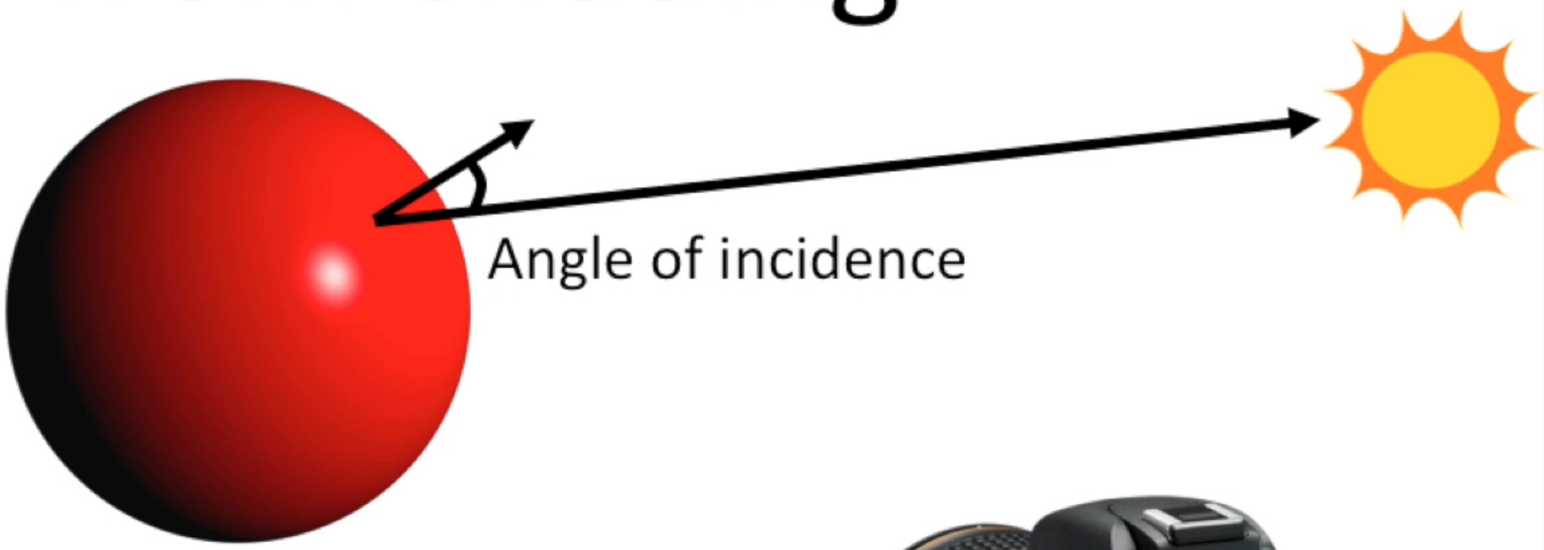
$$f(\mathrm{DOLP}, \eta) = \frac{1}{\sqrt{p^2 + q^2 + 1}}$$

$$p = \partial_x z$$

$$q = \partial_y z$$

Shape-from-shading

Unpolarised intensity provides shading cue



$$I_{\text{unpol}} = \mathbf{n} \cdot \mathbf{s}$$

$$= n_x s_x + n_y s_y + n_z s_z$$

$$= \frac{-p s_x - q s_y + s_z}{\sqrt{p^2 + q^2 + 1}}$$

$$p = \partial_x z$$

$$q = \partial_y z$$

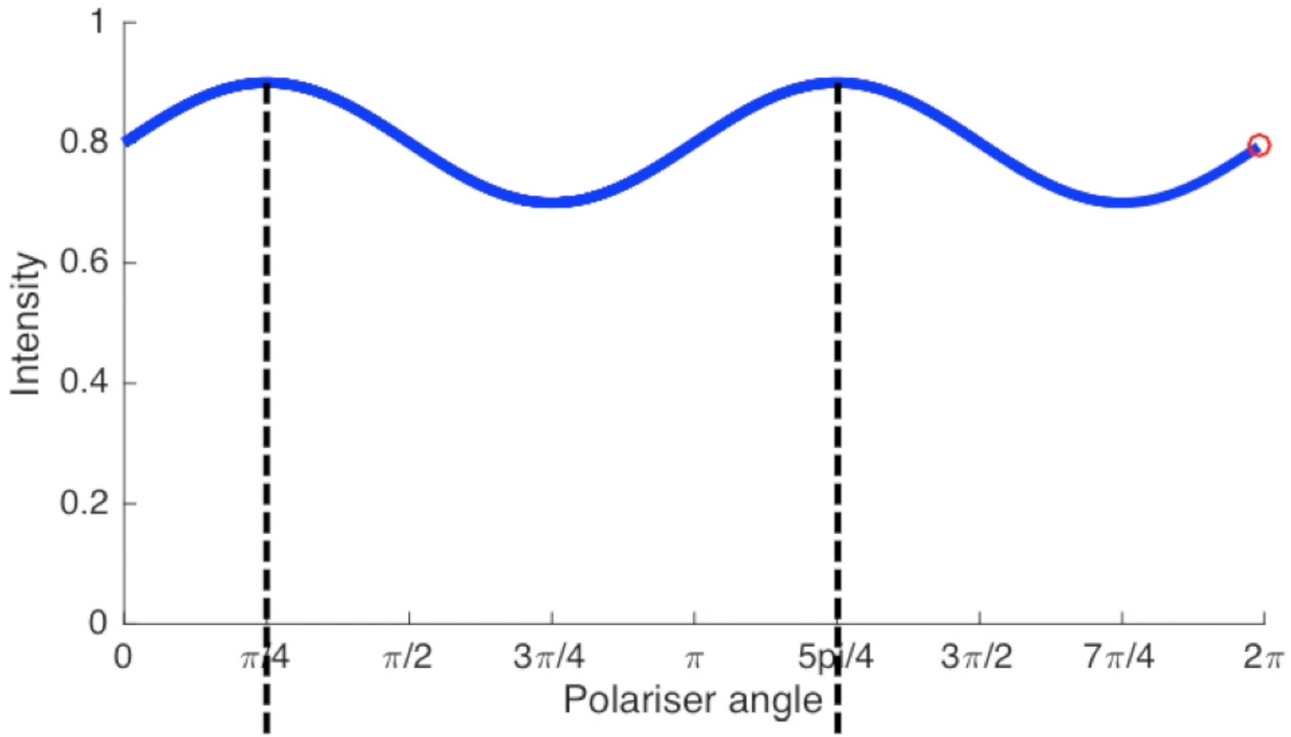
First linear expression

Ratio between degree of polarisation and unpolarised intensity:

First linear equation in surface gradient

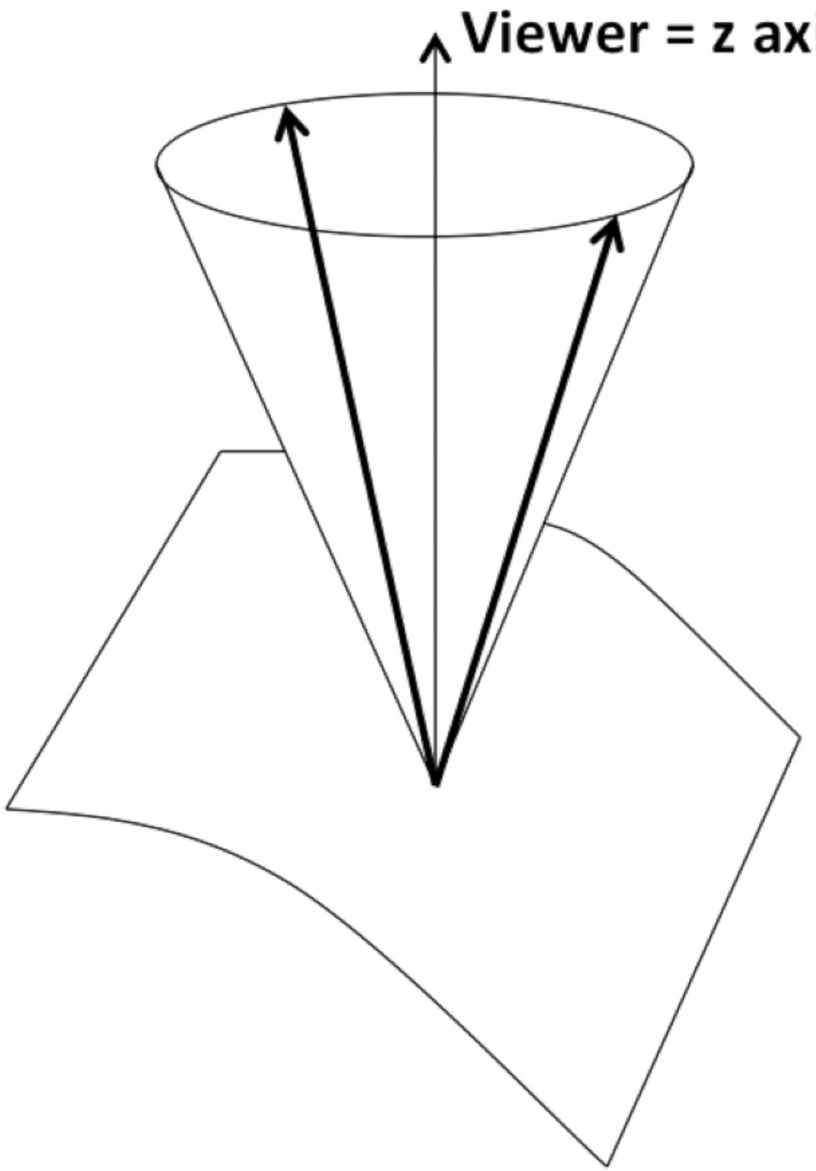
$$\frac{I_{\text{unpol}}}{f(\text{DOLP}, \eta)} = -ps_x - qs_y + s_z$$

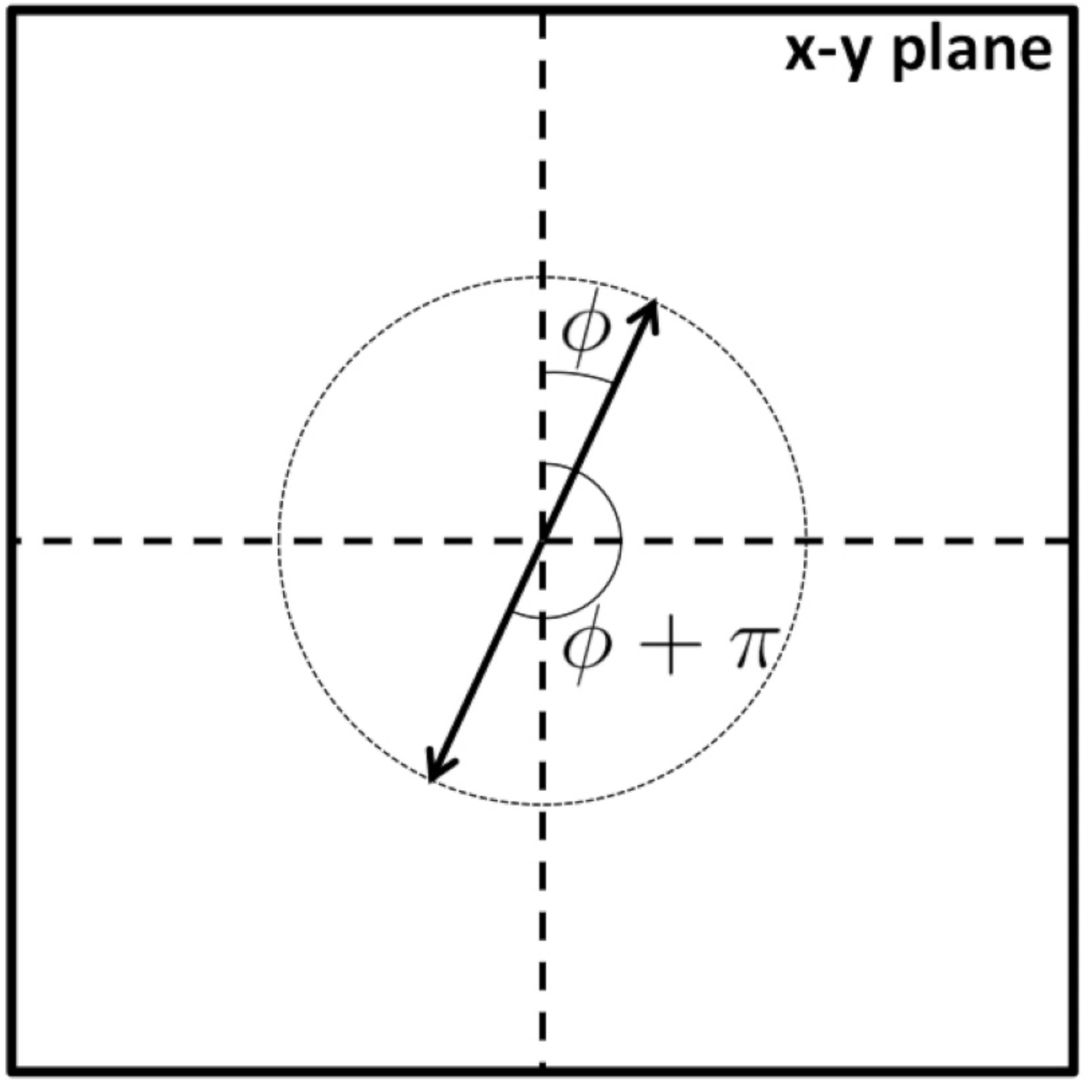
Azimuthal Ambiguity



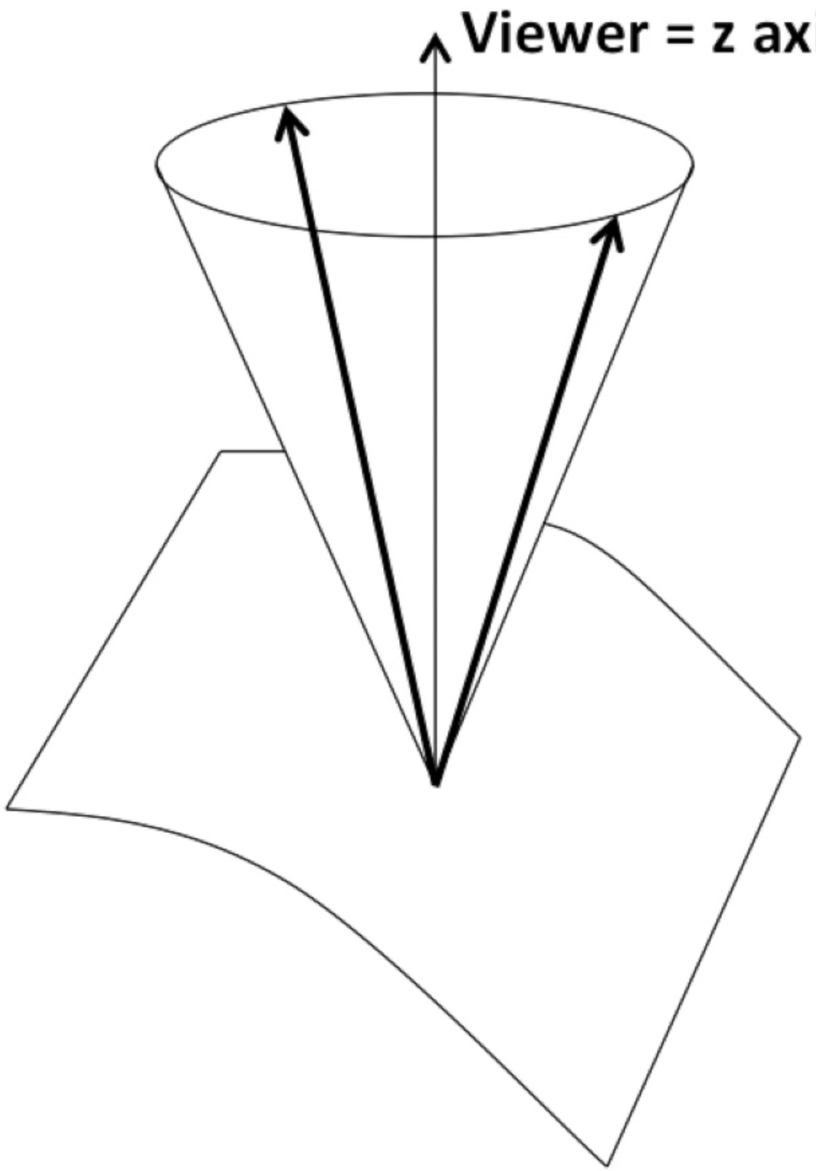
$$\alpha = \phi \quad \text{OR} \quad \phi + \pi$$

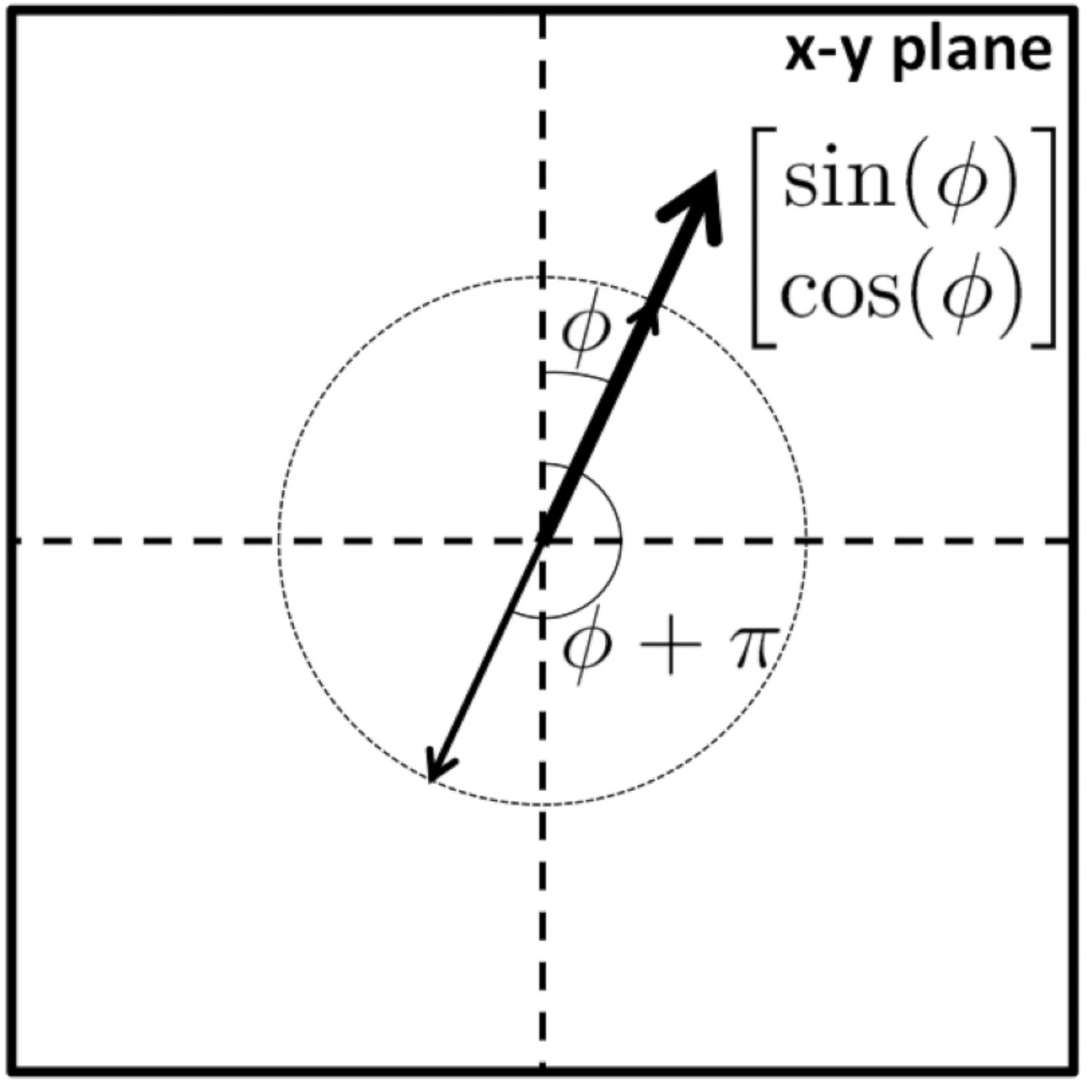
Azimuthal Ambiguity



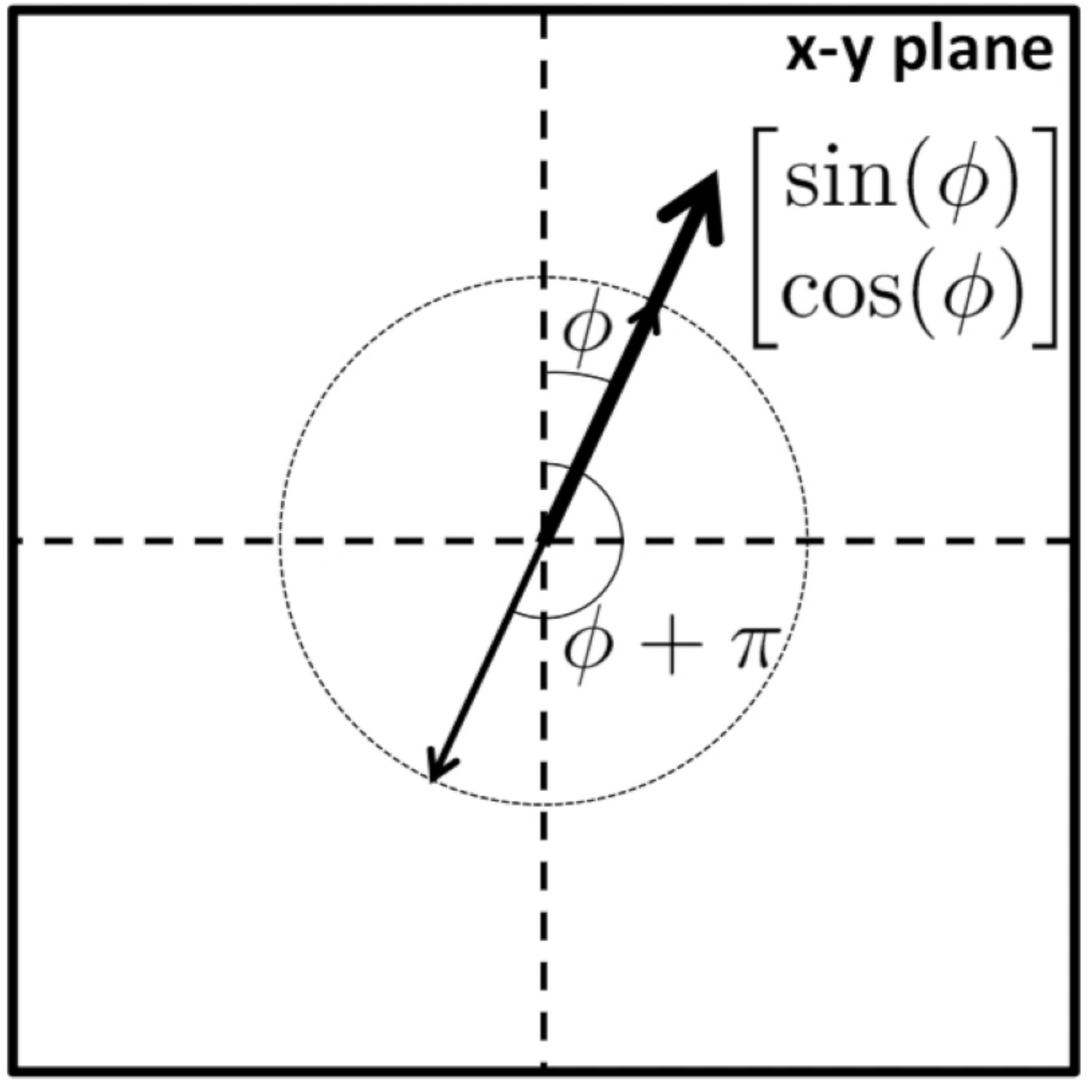


Azimuthal Ambiguity





Azimuthal Ambiguity



$$\mathbf{n} \cdot \begin{bmatrix}\cos(\phi)\\ -\sin(\phi)\\ 0\end{bmatrix} = 0$$

Second linear equation in surface gradient

$$-p\cos(\phi) + q\sin(\phi) = 0$$

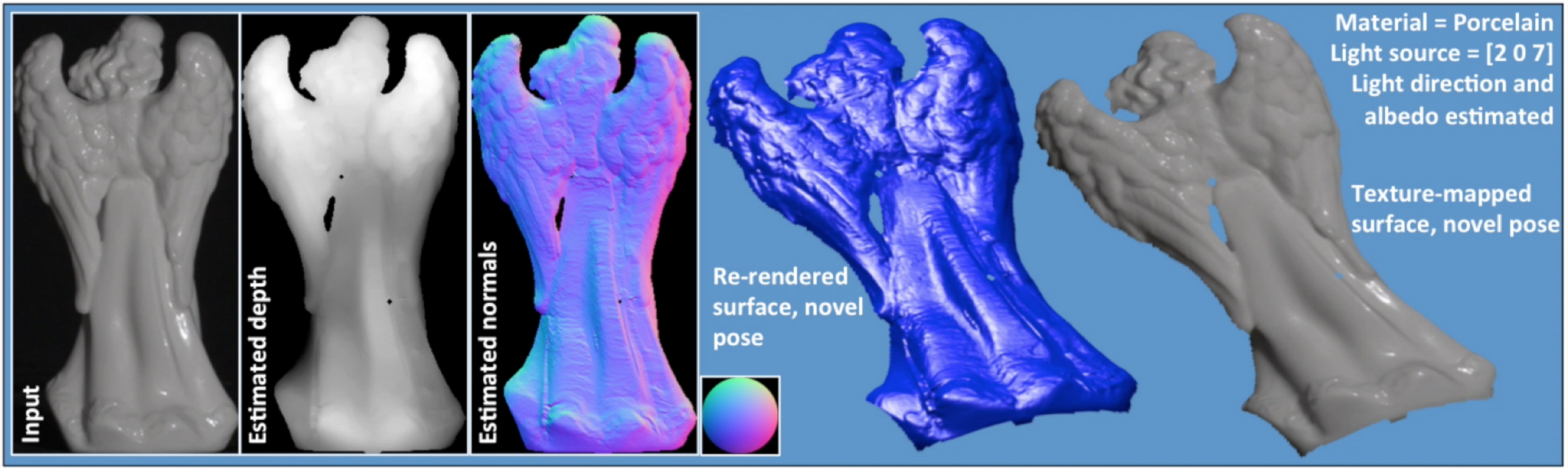
Solving for depth

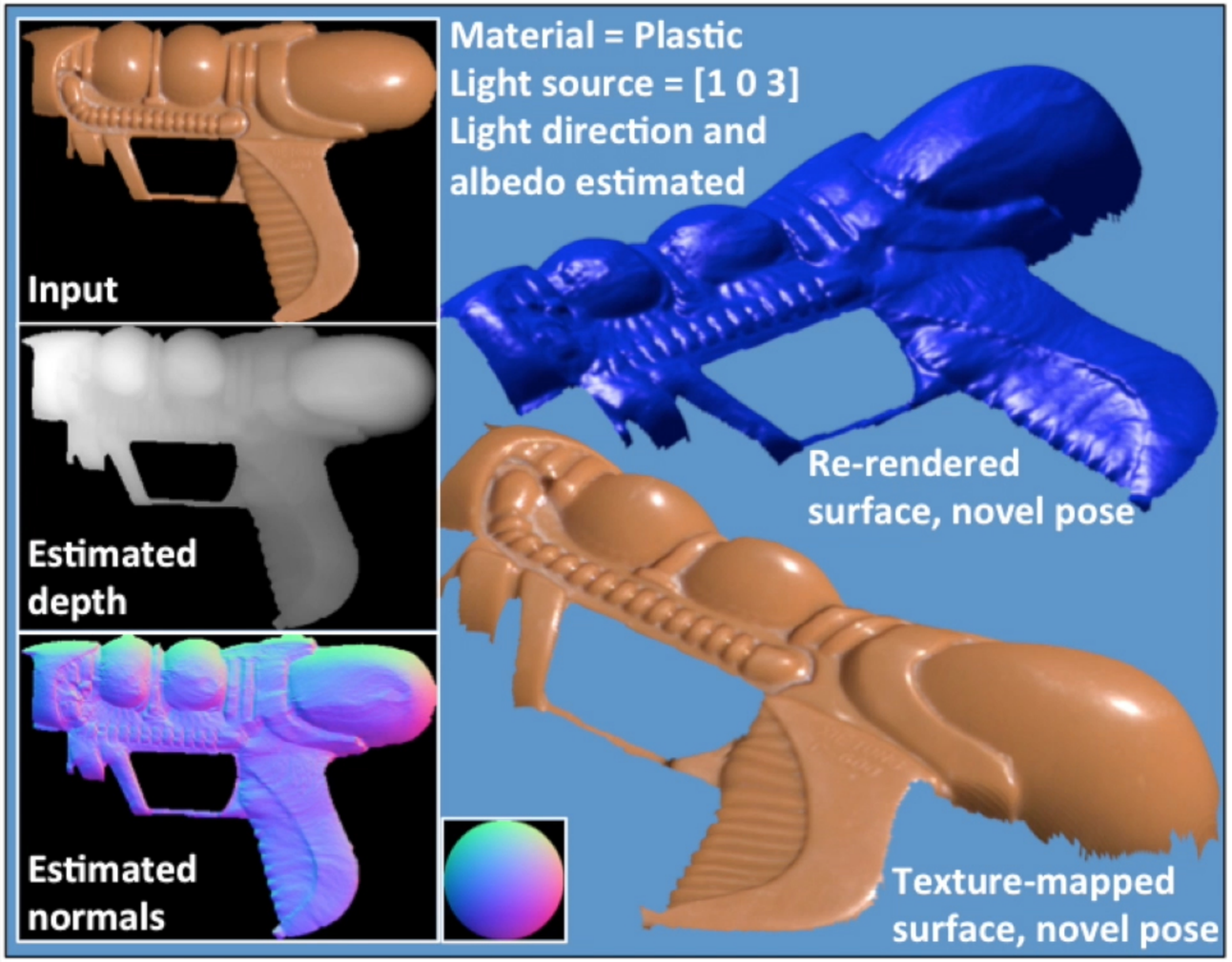
Finite difference approximations:

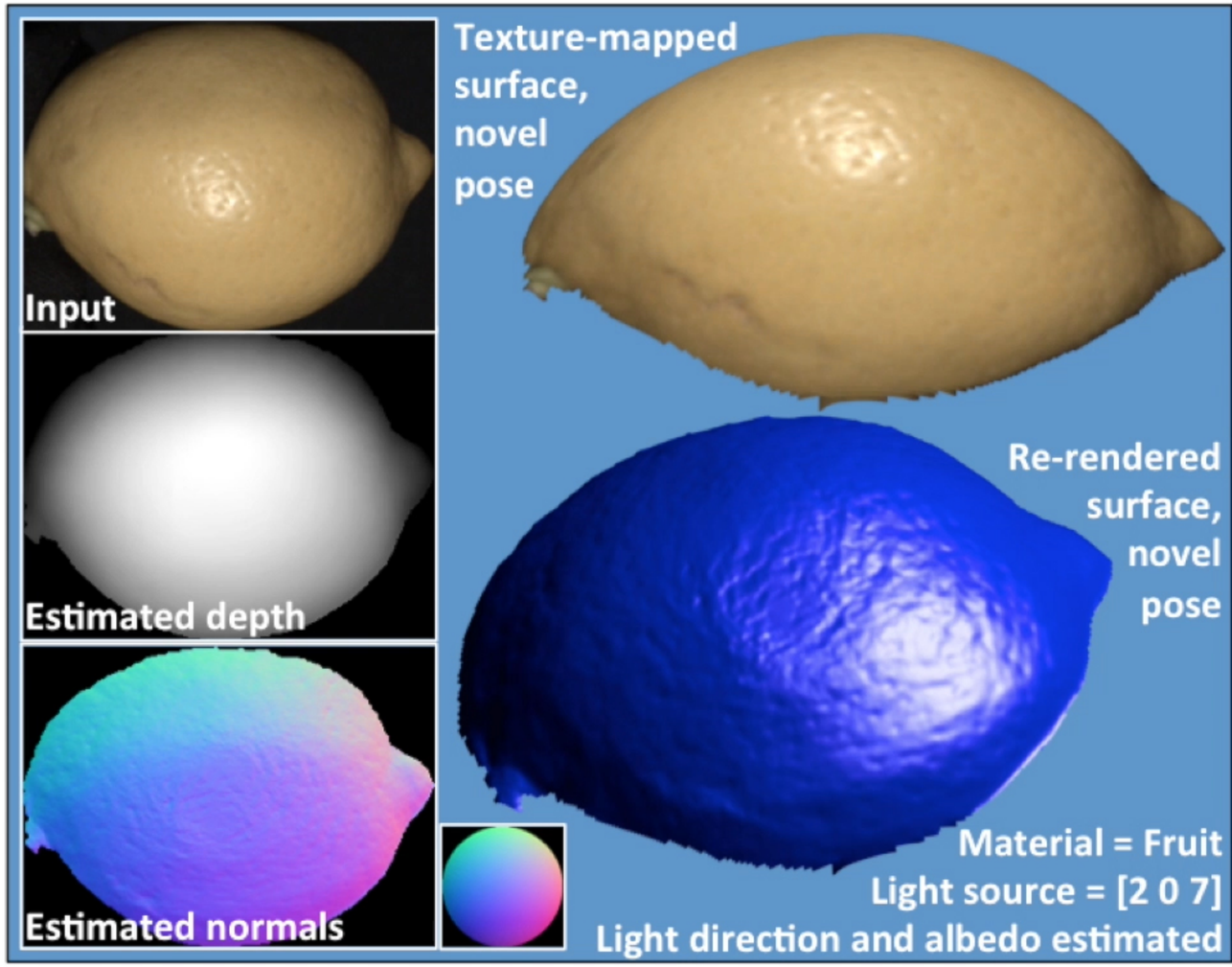
$$p(x,y) \approx z(x+1,y) - z(x,y)$$
$$q(x,y) \approx z(x,y+1) - z(x,y)$$

Form linear system from pairs of linear equations for each pixel:

$$\mathbf{z}^* = \arg\min_{\mathbf{z}} \|\mathbf{A}\mathbf{z} - \mathbf{b}\|^2$$


Input
Estimated depth
Estimated normals
Re-rendered surface, novel pose
Material = Porcelain
Light source = [2 0 7]
Light direction and albedo estimated
Texture-mapped surface, novel pose


Input
Estimated depth
Estimated normals
Material = Plastic
Light source = [1 0 3]
Light direction and albedo estimated
Re-rendered surface, novel pose
Texture-mapped surface, novel pose


Input
Estimated depth
Estimated normals
Texture-mapped surface, novel pose
Re-rendered surface, novel pose
Material = Fruit
Light source = [2 0 7]
Light direction and albedo estimated