ShapeFit and ShapeKick for Robust, Scalable Structure from Motion

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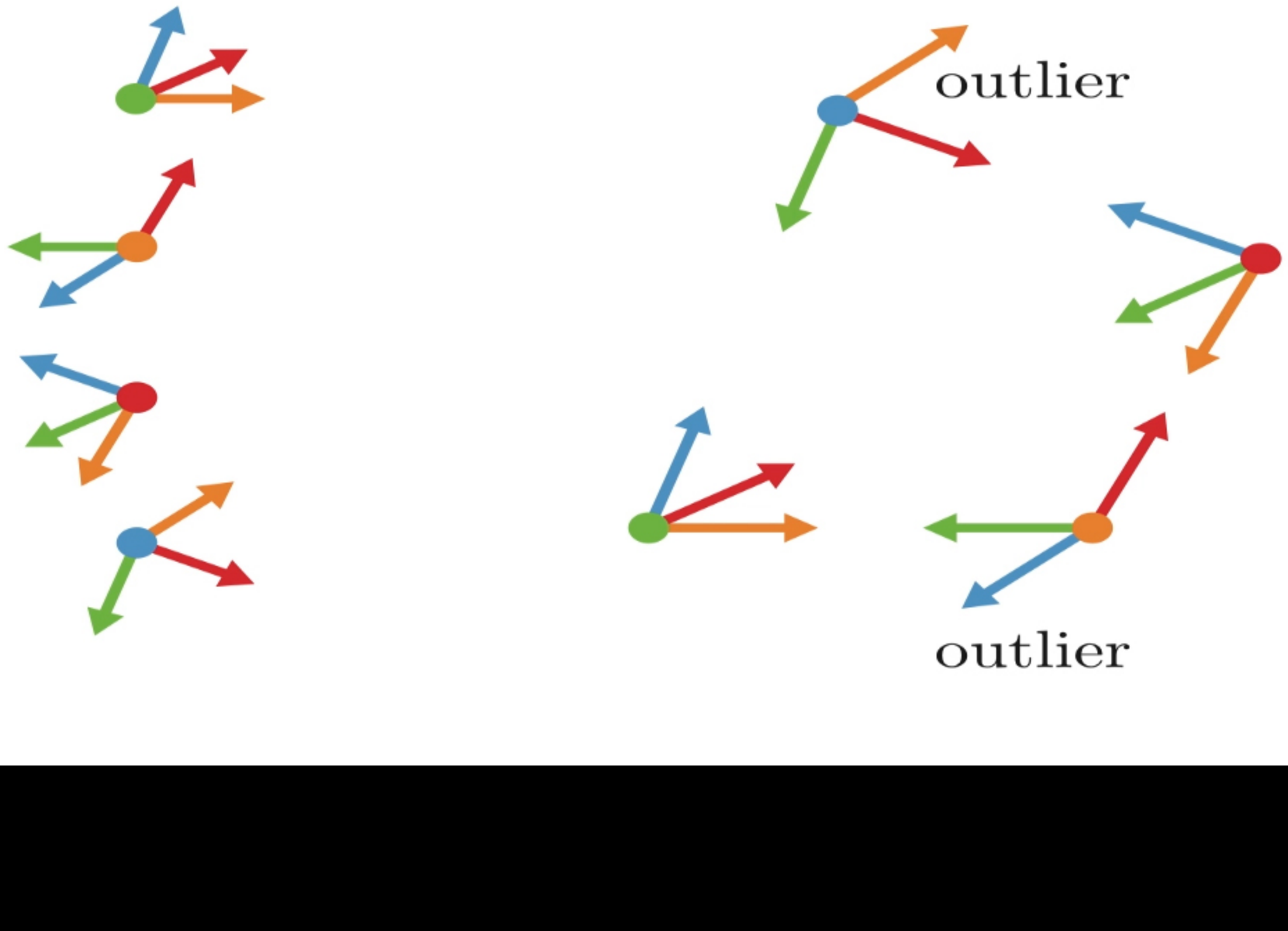
Location recovery from relative directions with outliers and known camera rotations



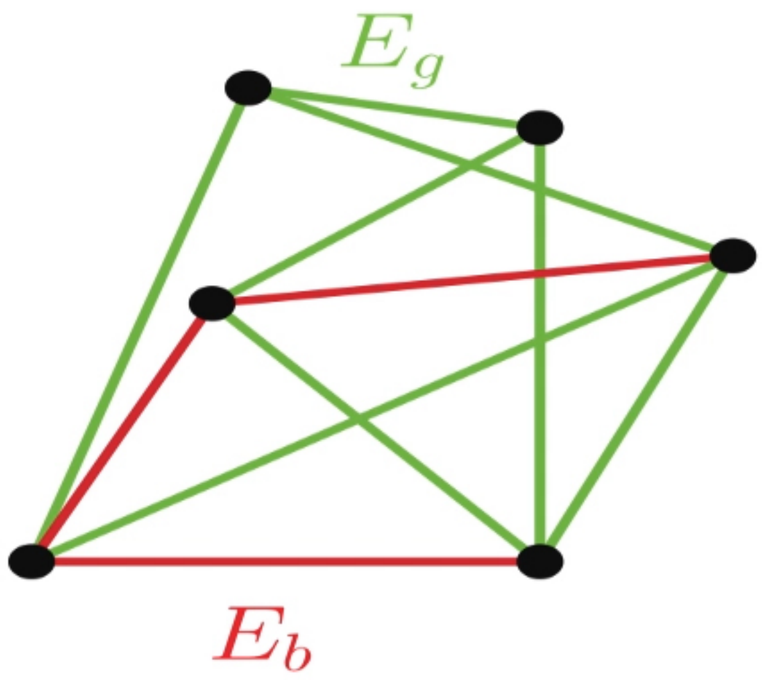
Precise formulation: location recovery from relative directions with outliers

Let: $t_1 \dots t_n \in \mathbb{R}^3$

$G = ([n], E = E_g \sqcup E_b)$

$v_{ij} = \dfrac{t_i - t_j}{\|t_i - t_j\|_2}$ for $ij \in E_g$

$v_{ij} \in \mathcal{S}^2$ for $ij \in E_b$



Given: $G, \{v_{ij}\}$

Find: $\{t_i\}$ up to translation and scale

ShapeFit:

A convex program for location recovery with outliers

$$\begin{aligned} \underset{\{t_i\}}{\text{minimize}} \quad & \sum_{ij\in E} \|P_{v_{ij}^{\perp}}(t_i - t_j)\|_2 \\ \text{subject to} \quad & \sum_{ij\in E} \langle t_i - t_j, v_{ij}\rangle = 1, \quad \sum_{i\in[n]} t_i = 0 \end{aligned}$$

ShapeKick:

A fast ADMM implementation of ShapeFit using kicking

ShapeFit is provably robust to adversarial outliers

Let: $t_1 \dots t_n \sim \mathcal{N}(0, I_{3\times 3})$ be i.i.d.

$ij \in E$ with prob. $p = \Omega(n^{-1/5} \log^{3/5} n)$

$E_b \subset E$ be an arbitrary subset

$v_{ij} \in \mathcal{S}^2$ be arbitrary for $ij \in E_b$

$\gamma = cp^5 / \log^3 n$ for some $c > 0$

Theorem

If n is large enough, and $\max \deg(E_b) \leq \gamma n$,
then with probability at least $1 - \frac{1}{n^4}$,
the minimizer of Shapefit is unique and
exactly equals $\{t_i\}$ up to translation and scale.

ShapeKick: comparable accuracy and 10x faster than the state of the art

Median recovery error (m)

	1d+Huber	1d + SK	1d+LUD
NYC Library	**2.2**	2.4	2.8
Piazza Pop.	3.2	**1.7**	2.0
Metropolis	4.0	**2.4**	3.7
Montreal ND	**0.9**	1.5	1.1
Tow. London	3.5	**3.3**	4.3
Notre Dame	0.5	0.5	**0.5**
Alamo	0.8	**0.8**	0.9
Union Sq.	7.9	**7.4**	7.9
Vienna Cath.	**4.3**	7.6	5.8
Roman For.	**6.4**	19	7.7
Piccadilly	**1.8**	2.1	2.1

Solution time (s)

1d+Huber	1d+SK	LUD
26	**2.2**	57
115	**1.9**	35
83	**2.4**	27
50	**3.5**	112
43	**2.8**	41
66	**7.1**	247
202	**11**	186
116	**3.7**	
462	**8.2**	255
130	**9.5**	
593	**40**	

Shown for Robust PCA translations formulation in Ozyesil and Singer (2015).

ShapeFit suggests a simpler global Structure from Motion pipeline:

Bipartite pipeline: