



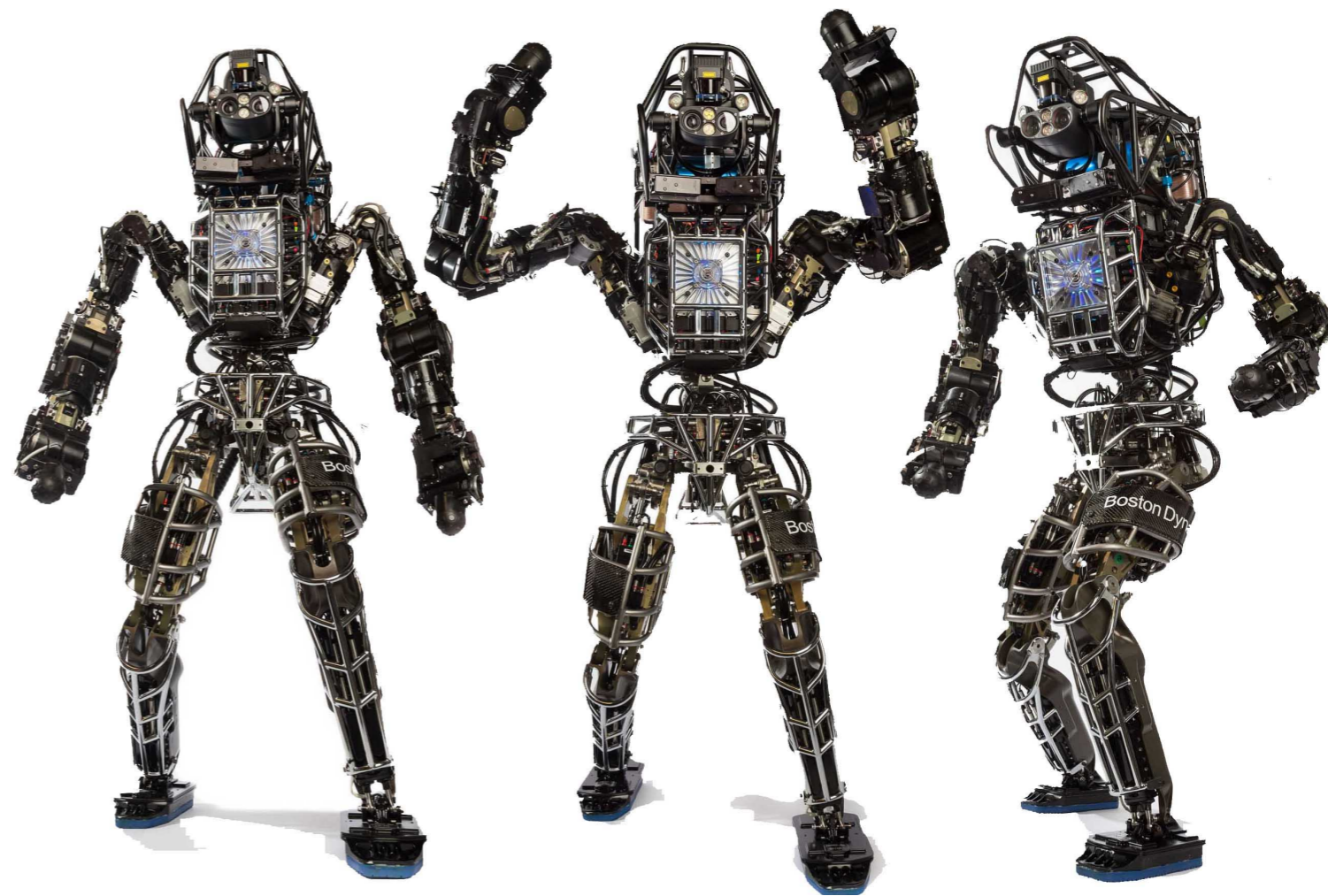
# WHAT WOULD SHANNON DO? BAYESIAN COMPRESSION FOR DL

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DEEP LEARNING AND REINFORCEMENT LEARNING  
SUMMER SCHOOL MONTREAL 2017

# Motivation



# Motivation

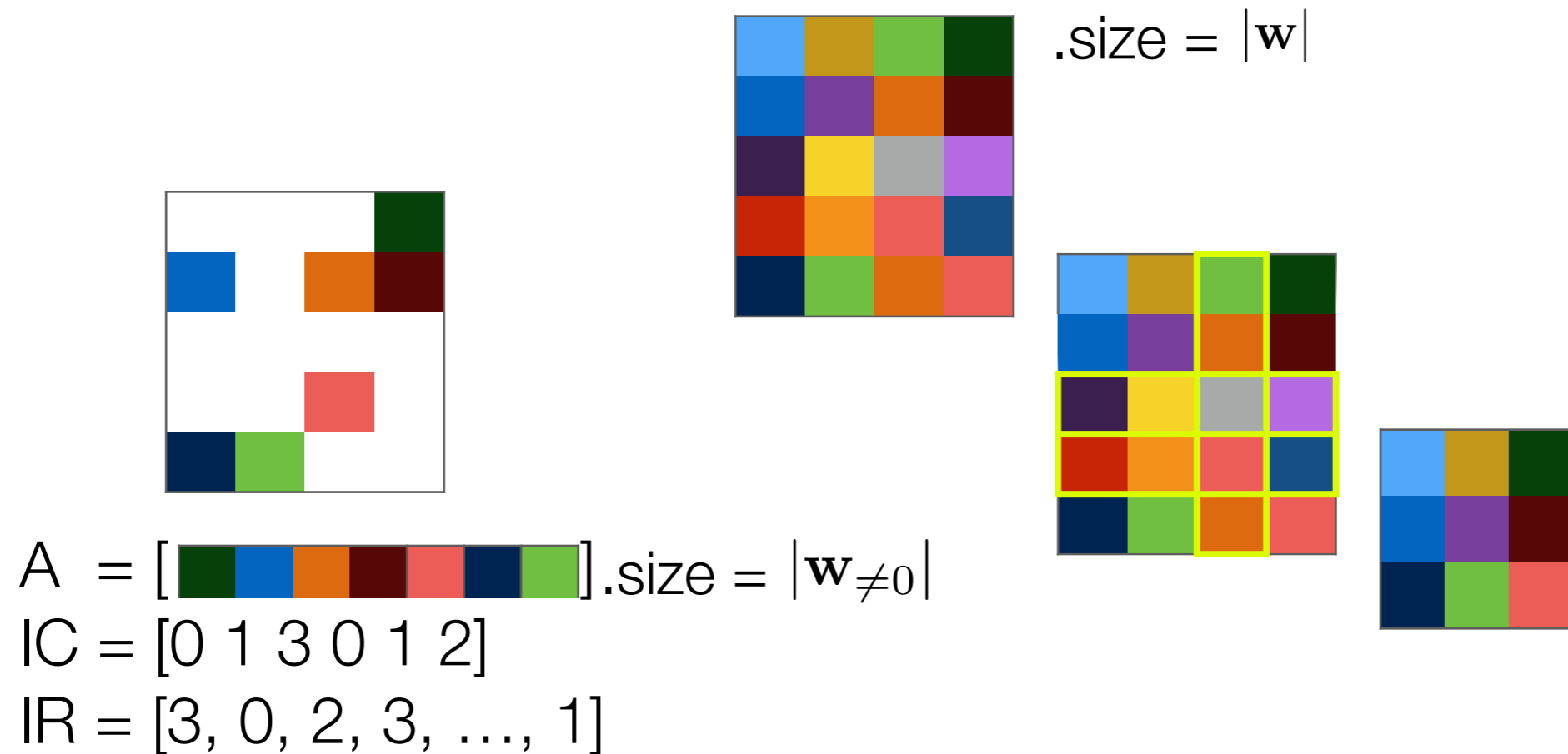
- 1 Wh costs 0.0225 cent
- running a Titan X for 1h: 5.625 cent
- facebook has 1.86 billion active users
- VGG takes ~147ms/16 predictions
- making one prediction for all users costs 20 k€

# Motivation - Summary

- mobile devices have **limited hardware**
- **energy costs** for predictions
- bandwidth **transmitting** models
- speeding up inference for **real time processing**
- relation to **privacy**

# Practical view on compression

A : Sparsity learning



- (Unstructured) Pruning

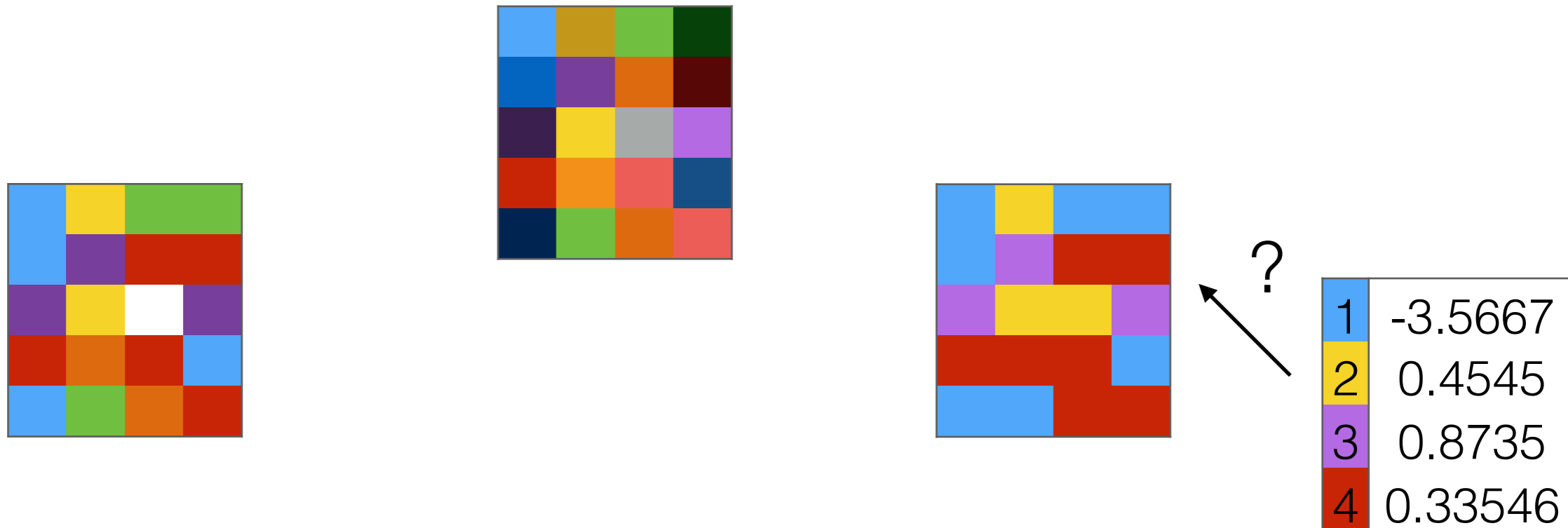
- CR:  $\approx \frac{|\mathbf{w}|}{2|\mathbf{w}_{\neq 0}|}$

- Structured Pruning:

- CR:  $\frac{|\mathbf{w}|}{|\mathbf{w}_{\neq 0}|}$

# Practical view on compression

B : Bit per weight reduction



- precision quantisation
- CR:  $32/10 = 3$
- PRO: fast inference
- CON: savings is not too big

- Set quantisation by clustering
- CR:  $32/4 = 8$
- PRO: extreme compressible with e.g. further Huffman encoding
- CON: inference?

# Practical view on compression

## Summary - Properties

	Set quantisation	Bit quantisation
Unstructured pruning	<ul style="list-style-type: none"><li>- highest compression</li><li>- flop and energy savings moderate</li></ul>	
Structured pruning		<ul style="list-style-type: none"><li>- lowest expected compression</li><li>- BUT will save considerable amount of flops and thus energy</li></ul>

# Practical view on compression

## Summary - Applications

	Set quantisation	Bit quantisation
Unstructured pruning	<ul style="list-style-type: none"><li>- “ZIP”-format for NN</li><li>- transmitting via limited channels</li><li>- save millions of nets efficiently</li></ul>	
Structured pruning		<ul style="list-style-type: none"><li>- inference at scale</li><li>- real time predictions</li><li>- hardware limited devices</li></ul>



# Variational lower bound

$$\begin{aligned}\log p(\mathcal{D}) \geq \mathcal{L}(q(\mathbf{w}), \mathbf{w}) &= \mathbf{E}_{q(\mathbf{w})} \left[ \log \frac{p(\mathcal{D}, \mathbf{w})}{q(\mathbf{w})} \right] \\ &= \mathbf{E}_{q(\mathbf{w})} [\log p(\mathcal{D} | \mathbf{w})] - KL(q(\mathbf{w}) || p(\mathbf{w}))\end{aligned}$$

Hinton, Geoffrey E., and Drew Van Camp. "Keeping the neural networks simple by minimizing the description length of the weights." *Proceedings of the sixth annual conference on Computational learning theory*. ACM, 1993.

# MDL principle and Variational Learning

The best model is the one that compresses the data best. There are two costs, one for **transmitting a model** and one for reporting the **data misfit**.

Jorma Rissanen, 1978

# Variational lower bound

$$\begin{aligned}\log p(\mathcal{D}) \geq \mathcal{L}(q(\mathbf{w}), \mathbf{w}) &= \mathbf{E}_{q(\mathbf{w})} \left[ \log \frac{p(\mathcal{D}, \mathbf{w})}{q(\mathbf{w})} \right] \\ &= \mathbf{E}_{q(\mathbf{w})} [\log p(\mathcal{D} | \mathbf{w})] - KL(q(\mathbf{w}) || p(\mathbf{w}))\end{aligned}$$

transmitting  
data misfit

transmitting  
the model

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# Variational lower bound

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$$p(\mathcal{D}|\mathbf{w}) = p(\mathbf{T}|\mathbf{X}, \mathbf{w}) = \prod_{n=1}^N \mathcal{N}(\mathbf{t}_n|\mathbf{x}_n, \mathbf{w}) \quad KL(q(\mathbf{w})||p(\mathbf{w})) = \mathbb{E}_{q(\mathbf{w})} [-\log p(\mathbf{w})] - H(q(\mathbf{w}))$$

$$H(q(\mathbf{w})) = - \int_{\Omega} q(\mathbf{w}) \log q(\mathbf{w}) \, d\mathbf{w} = - \int_{\mathbb{R}^I} \mathcal{N}(\mathbf{w}|\mathbf{0}, \sigma\mathbf{I}) \log \mathcal{N}(\mathbf{w}|\mathbf{0}, \sigma\mathbf{I}) \, d\mathbf{w} = [I \log(2\pi e\sigma^2)]$$

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# Practical view on compression

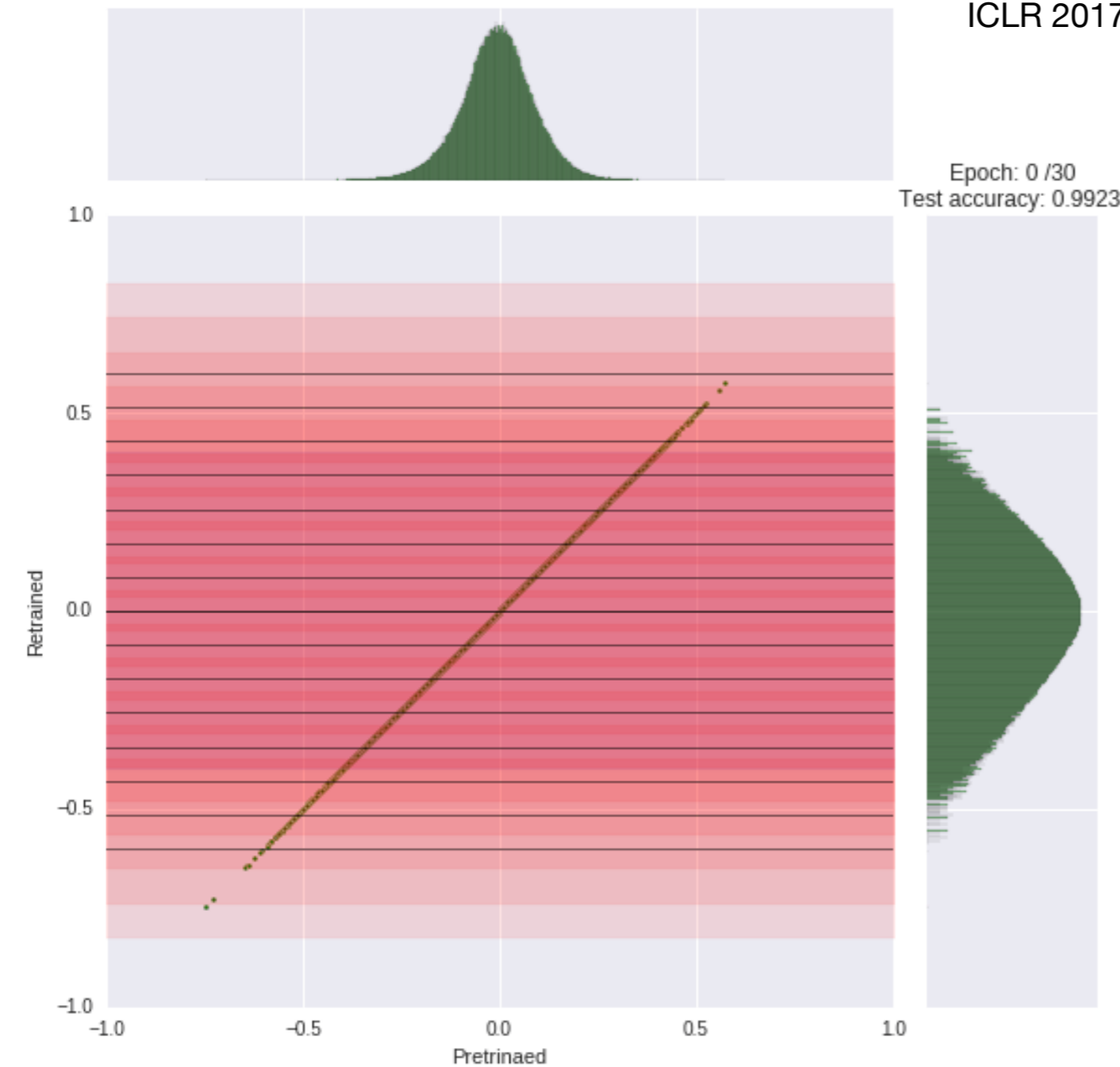
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# Soft weight-sharing for NN compression

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ICLR 2017



- **Solution:** train a neural network with **gaussian mixture model prior**

$$q(\mathbf{w}) = \prod q(w_i) = \delta(w_i | \mu_i)$$

$$p(\mathbf{w}) = \prod_{i=1}^I \sum_{j=0}^J \pi_j \mathcal{N}(w_i | \mu_j, \sigma_j^2).$$

- Pruning by setting one component to zero with high mixing proportion

# Soft weight-sharing for NN compression

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ICLR 2017

Model	Method	Top-1 Error[%]	$\Delta$ [%]	$ \mathbf{W} $ [ $10^6$ ]	$\frac{ \mathbf{W}_{\neq 0} }{ \mathbf{W} }$ [%]	CR
LeNet-300-100	Han et al. (2015a)	1.64 $\rightarrow$ 1.58	0.06	0.2	8.0	40
	Guo et al. (2016)	2.28 $\rightarrow$ 1.99	-0.29		1.8	56
	Ours	1.89 $\rightarrow$ 1.94	-0.05		4.3	<b>64</b>
LeNet-5-Caffe	Han et al. (2015a)	0.80 $\rightarrow$ 0.74	-0.06	0.4	8.0	39
	Guo et al. (2016)	0.91 $\rightarrow$ 0.91	0.00		0.9	108
	Ours	0.88 $\rightarrow$ 0.97	0.09		0.5	<b>162</b>
ResNet (light)	Ours	6.48 $\rightarrow$ 8.50	2.02	2.7	6.6	45

# Practical view on compression

## Summary - Properties

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# Bayesian Compression for Deep Learning

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UNDER SUBMISSION NIPS 2017

- ***Idea: use dropout to learn architecture***
- the variational version of dropout learns the dropout rate
- ***Solution: Learn dropout rate for each weight structure, when weights have a high dropout rate we can safely ignore them***
- uncertainty in left over weights to compute bit precision

Kingma, Diederik P., Tim Salimans, and Max Welling. "Variational dropout and the local reparameterization trick." *NIPS*. 2015.

Molchanov, Dmitry, Arsenii Ashukha, and Dmitry Vetrov. "Variational Dropout Sparsifies Deep Neural Networks." *arXiv preprint arXiv:1701.05369* (2017).

# Bayesian Compression for Deep Learning

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$$q(\mathbf{z}) = \prod q(z_i) = \mathcal{N}(z_i | \mu_i^z, \alpha_i)$$

$$q(\mathbf{w} | \mathbf{z}) = \prod q(w_i | z_i) = \mathcal{N}(w_i | z_i \mu_i, z_i^2 \sigma_i^2)$$



force high dropout rates

push to zero for high dropout rates

# Bayesian Compression for Deep Learning

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$$q(z) = \prod q(z_i) = \mathcal{N}(z_i | \mu_i^z, \alpha_i)$$

$$q(\mathbf{w}|z) = \prod q(w_i|z_i) = \mathcal{N}(w_i | z_i \mu_i, z_i^2 \sigma_i^2)$$

$$p(w) = \int p(z) p(w|z) dz$$

$$p(w) \propto \int \frac{1}{|z|} \mathcal{N}(w|0, z^2) dz = \frac{1}{|w|}$$

# Bayesian Compression for Deep Learning

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Network & size	Method	Pruned architecture	Bit-precision
LeNet-300-100	Sparse VD	512-114-72	8-11-14
784-300-100	BC-GNJ	278-98-13	8-9-14
	BC-GHS	311-86-14	13-11-10
LeNet-5-Caffe	Sparse VD	14-19-242-131	13-10-8-12
20-50-800-500	GD	7-13-208-16	-
	GL	3-12-192-500	-
	BC-GNJ	8-13-88-13	18-10-7-9
	BC-GHS	5-10-76-16	10-10-14-13
VGG	BC-GNJ	63-64-128-128-245-155-63- -26-24-20-14-12-11-11-15	10-10-10-10-8-8-8- -5-5-5-5-5-6-7-11
(2× 64)-(2× 128)- -(3× 256)-(8× 512)	BC-GHS	51-62-125-128-228-129-38- -13-9-6-5-6-6-6-20	11-12-9-14-10-8-5- -5-6-6-6-8-11-17-10

# Bayesian Compression for Deep Learning

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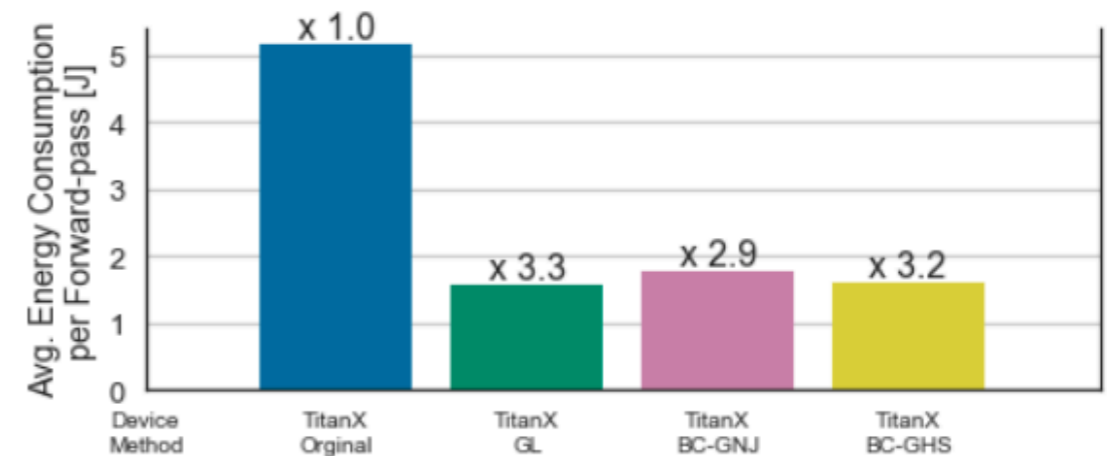
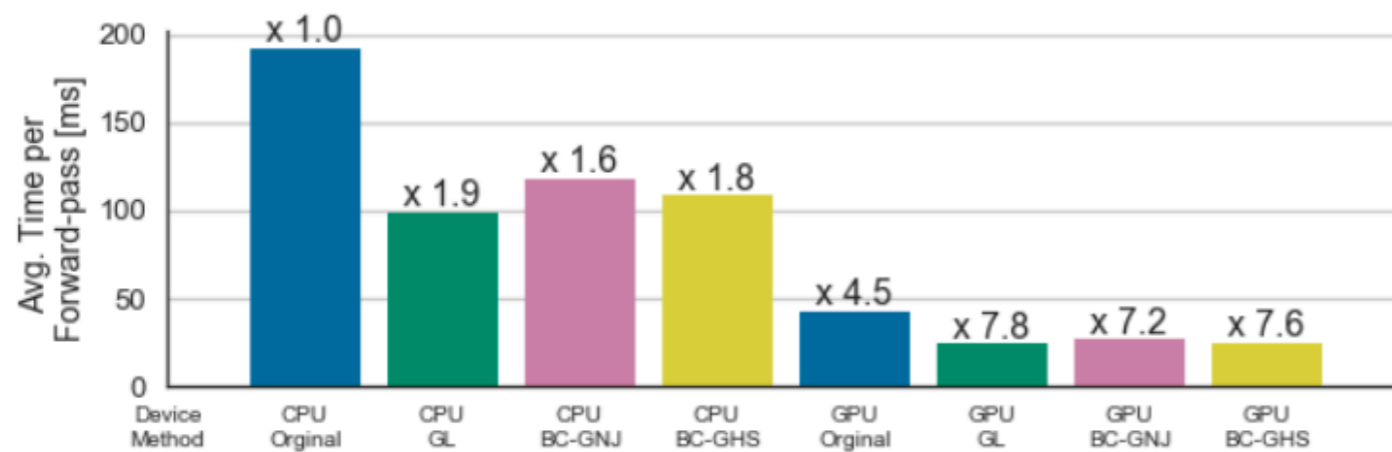
UNDER SUBMISSION NIPS 2017

Model		Compression Rates (Error %)			
Original Error %	Method	$\frac{ \mathbf{w} \neq 0 }{ \mathbf{w} } \%$	Pruning	Fast Prediction	Maximum Compression
LeNet-300-100 1.6	DC	8.0	6 (1.6)	-	40 (1.6)
	DNS	1.8	28* (2.0)	-	-
	SWS	4.3	12* (1.9)	-	64(1.9)
	Sparse VD	2.2	21(1.8)	84(1.8)	113 (1.8)
	BC-GNJ	10.8	9(1.8)	36(1.8)	58(1.8)
	BC-GHS	10.6	9(1.8)	23(1.9)	59(2.0)
	LeNet-5-Caffe 0.9	DC	8.0	6*(0.7)	-
DNS		0.9	55*(0.9)	-	108(0.9)
SWS		0.5	100*(1.0)	-	162(1.0)
Sparse VD		0.7	63(1.0)	228(1.0)	365(1.0)
BC-GNJ		0.9	108(1.0)	361(1.0)	573(1.0)
BC-GHS		0.6	156(1.0)	419(1.0)	771(1.0)
VGG 8.4		BC-GNJ	6.7	14(8.6)	56(8.8)
	BC-GHS	5.5	18(9.0)	59(9.0)	116(9.2)

# Bayesian Compression for Deep Learning





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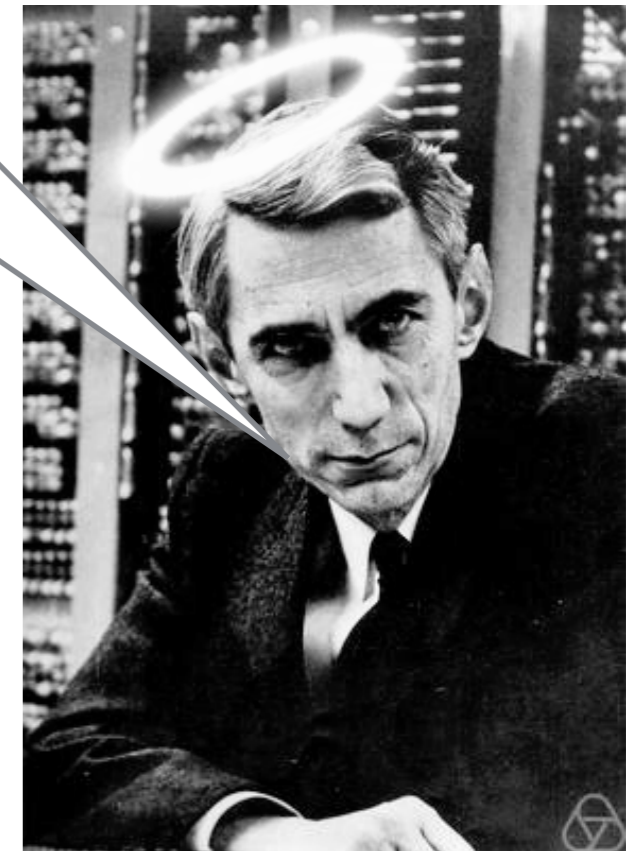


# Warning: Don't be too enthusiastic!

These algorithms are merely proposals, little can be realised by common frameworks today.

- Architecture pruning 
- Sparse matrix support  (partially in big frameworks)
- Reduced bit precision  (NVIDIA is starting)
- Clustering 

Thank you for your attention.  
Any questions?



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