Reinforcement Learning: Basic concepts

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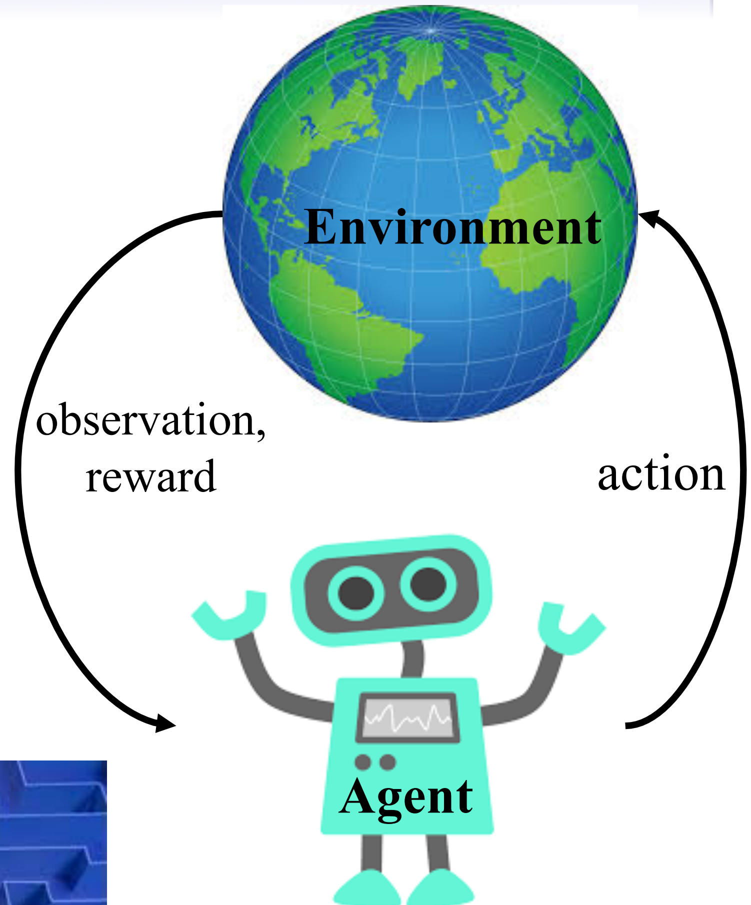
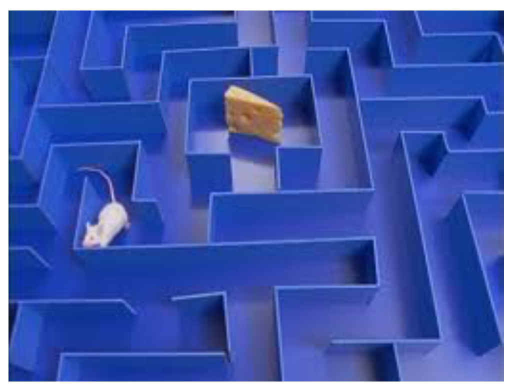
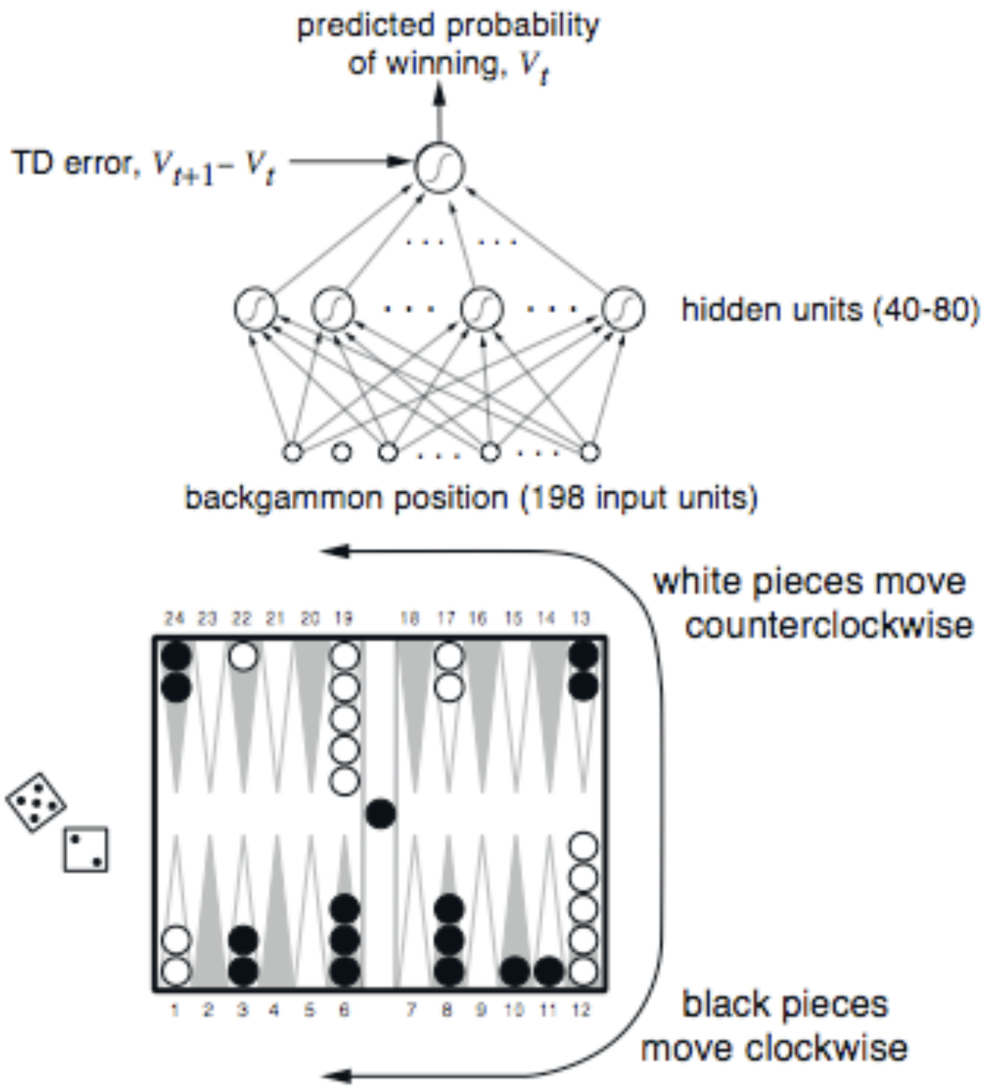
Reinforcement learning

- Learning by trial-and-error, in real-time.
- Improves with experience
- Inspired by psychology
  - Agent + Environment
  - Agent selects actions to maximize utility function.





RL system circa 1990's: TD-Gammon



Reward function:

+100 if win

- 100 if lose

0 for all other states

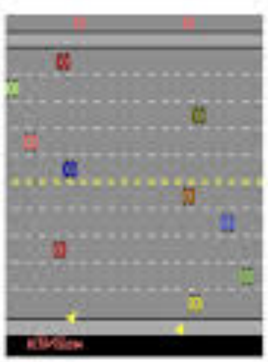
Trained by playing 1.5x10^6 million games against itself.

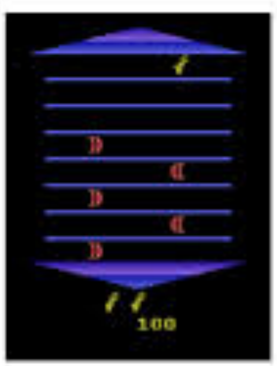
Enough to beat the best human player.

2016: World Go Champion Beaten by Deep Learning

RL applications at RLDM 2017

- Robotics
- Video games
- Conversational systems
- Medical intervention
- Algorithm improvement
- Improvisational theatre
- Autonomous driving
- Prosthetic arm control
- Financial trading
- Query completion





When to use RL?

- Data in the form of trajectories.
- Need to make a sequence of (related) decisions.
- Observe (partial, noisy) feedback to choice of actions.
- Tasks that require both learning and planning.

RL vs supervised learning

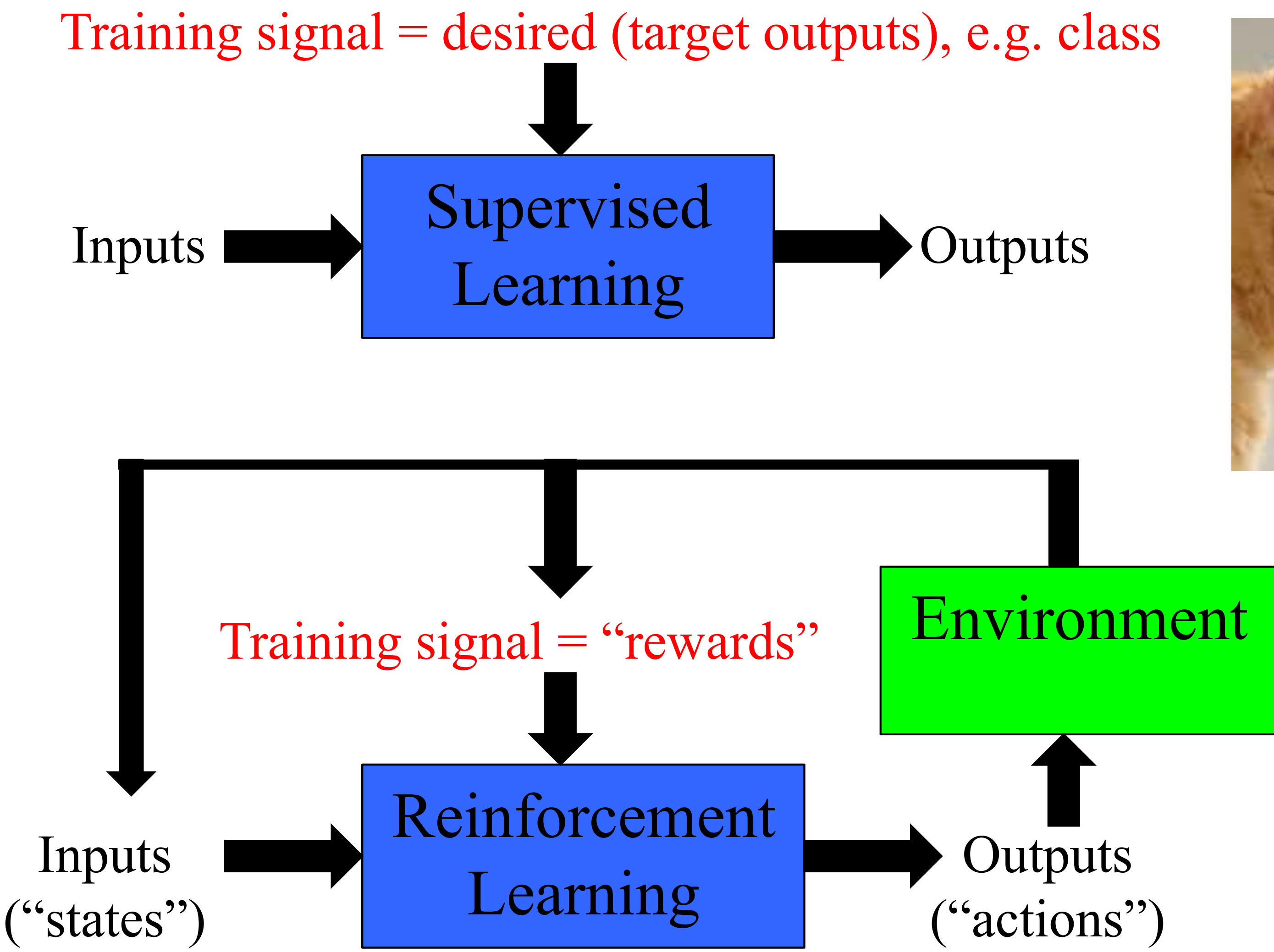
Training signal = desired (target outputs), e.g. class

Inputs → Supervised Learning → Outputs

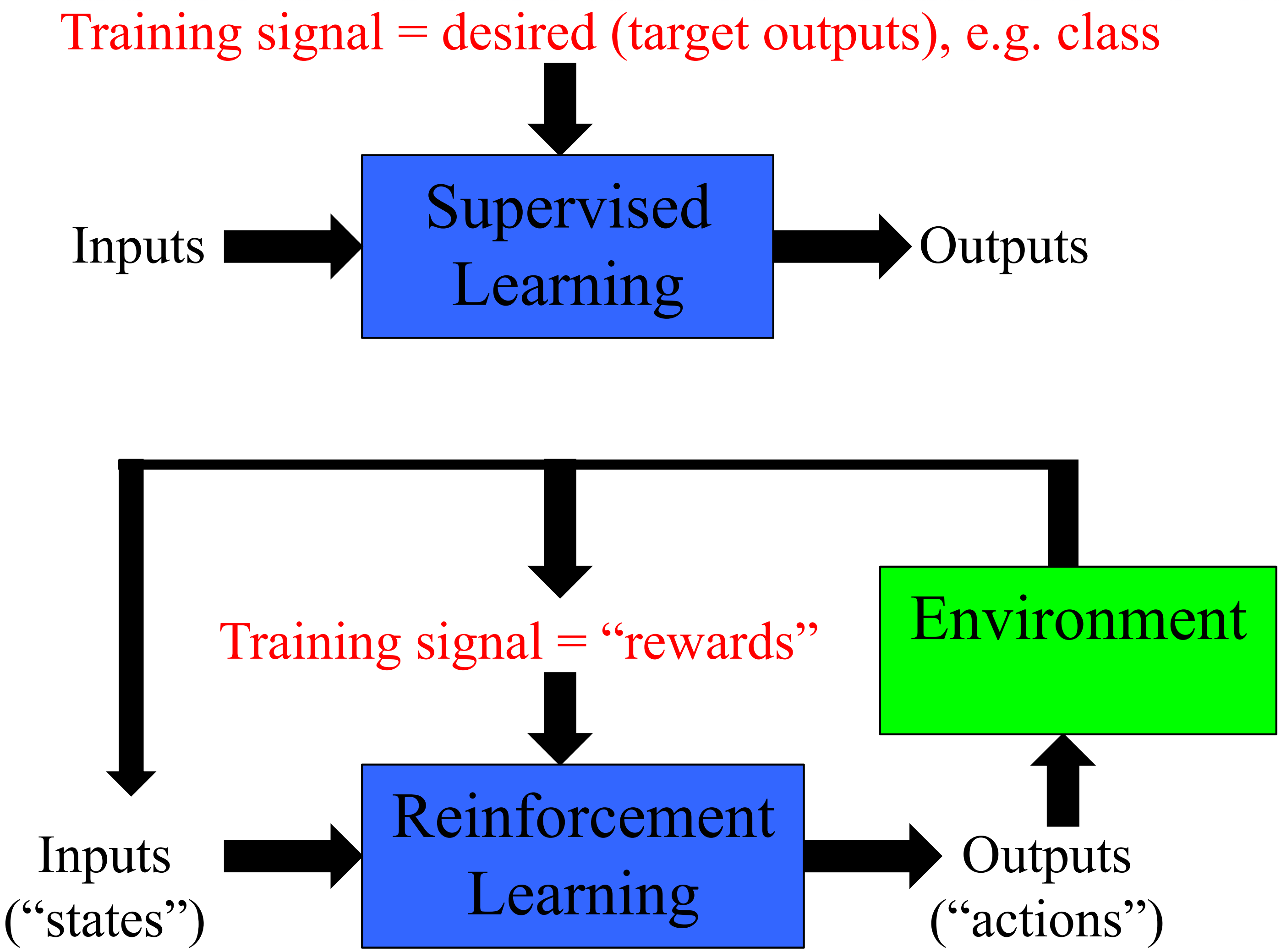
Training signal = “rewards”

Inputs (“states”) → Reinforcement Learning → Outputs (“actions”)

RL vs supervised learning



RL vs supervised learning



Practical & technical challenges:

1. Need access to the environment.

2. Jointly learning AND planning from **correlated** samples.

3. Data distribution changes with action choice.

Markov Decision Process (MDP)

Defined by:

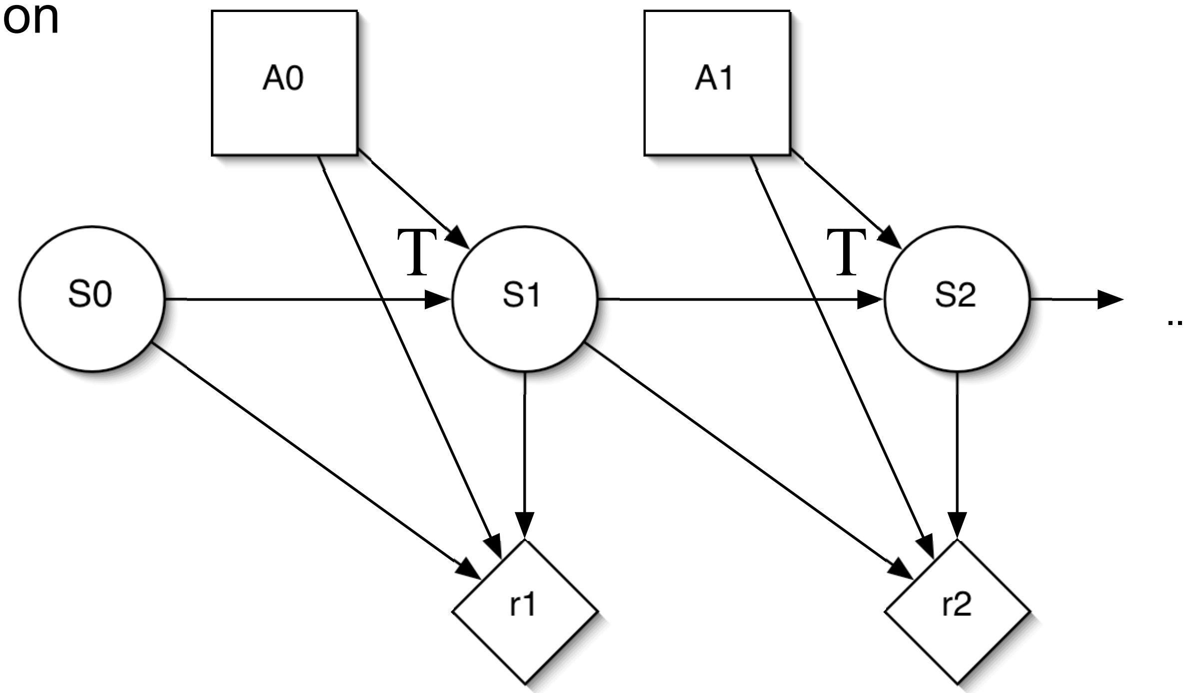
***S*: = {s_1, s_2, …, s_n}**, the set of states *(can be infinite/continuous)*

***A*: = {a_1, a_2, …, a_m}**, the set of actions *(can be infinite/continuous)*

T(s,a,s') := Pr(s'|s,a), the dynamics of the environment

R(s,a): Reward function

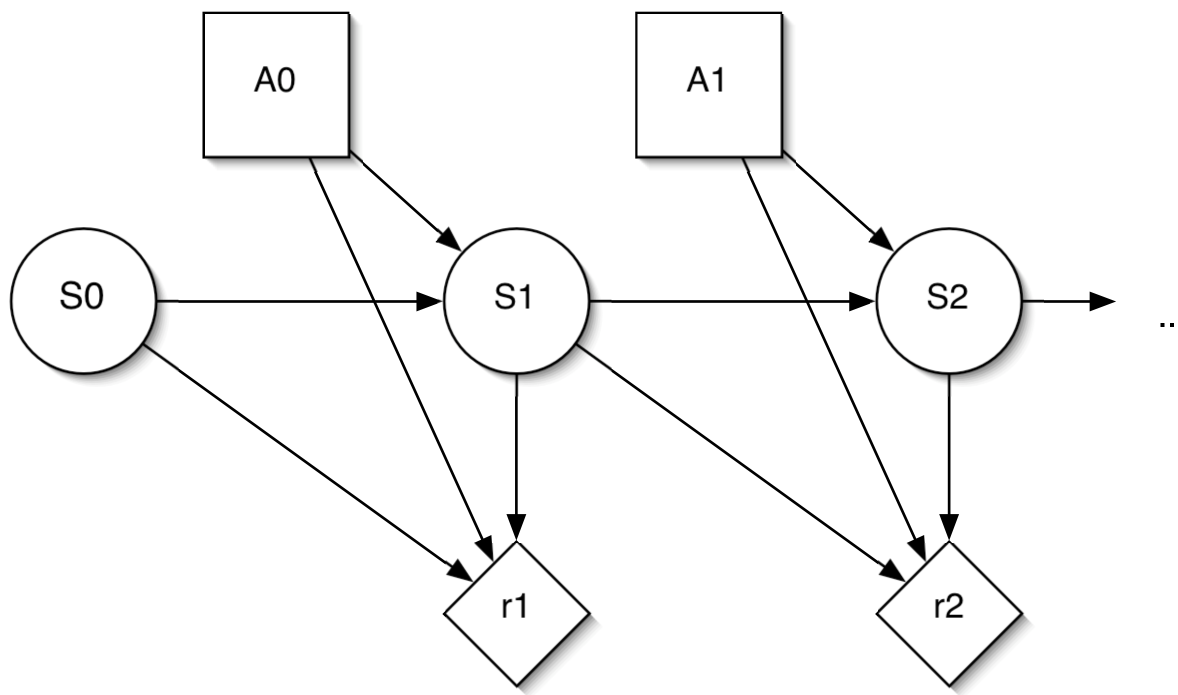
μ(s) : Initial state distribution



The **Markov** property

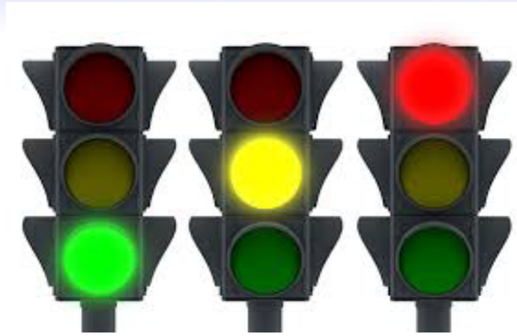
The distribution over future states **depends only on the present state and action**, not on any other previous event.

$$Pr(s_{t+1} \mid s_0, \ldots, s_t, a_0, \ldots a_t) = Pr(s_{t+1} \mid s_t, a_t)$$



The **Markov** property

- Traffic lights?



- Chess?

The **Markov** property

- Traffic lights?

- Chess?

- Poker?

Tip: Incorporate past observations in the state to have sufficient information to predict next state.

The goal of RL? Maximize return!

- Return, U_t of a trajectory, is the sum of rewards starting from step t.

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- Return, U_t of a trajectory, is the sum of rewards starting from step t.

- **Episodic task:** consider return over finite horizon (e.g. games, maze).

$$U_t = r_t + r_{t+1} + r_{t+2} + \ldots + r_T$$

- **Continuing task**: consider return over infinite horizon (e.g. juggling, balancing).

$$U_t = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \gamma^3 r_{t+3} \ldots = \sum_{k=0:\infty} \gamma^k r_{t+k}$$

The discount factor, γ

- Discount factor, $\gamma \in [0, 1)$ (usually close to 1).

- Intuition:

  – Receiving \$80 today is worth the same as \$100 tomorrow (assuming a discount factor of factor of $\gamma = 0.8$).

  – At each time step, there is a $1- \gamma$ chance that the agent dies, and does not receive rewards afterwards.

Defining behavior: The policy

- Policy, π defines the action-selection strategy at every state:

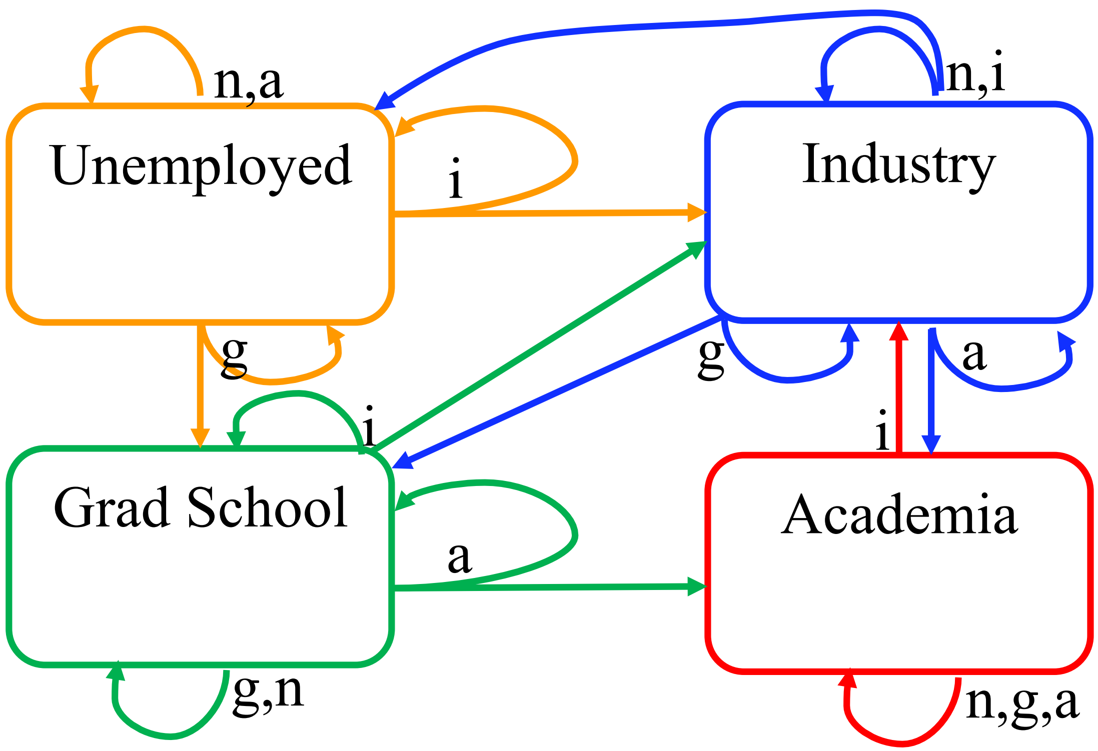
$$\pi(s,a) = P(a_t = a \mid s_t = s)$$

$$\pi : S \rightarrow A$$

Goal: **Find the policy that maximizes expected total reward.**
(But there are many policies!)

$$\mathbf{argmax}_{\pi}\, E_{\pi}\, [\, r_0 + r_1 + \ldots + r_T \mid s_0\,]$$

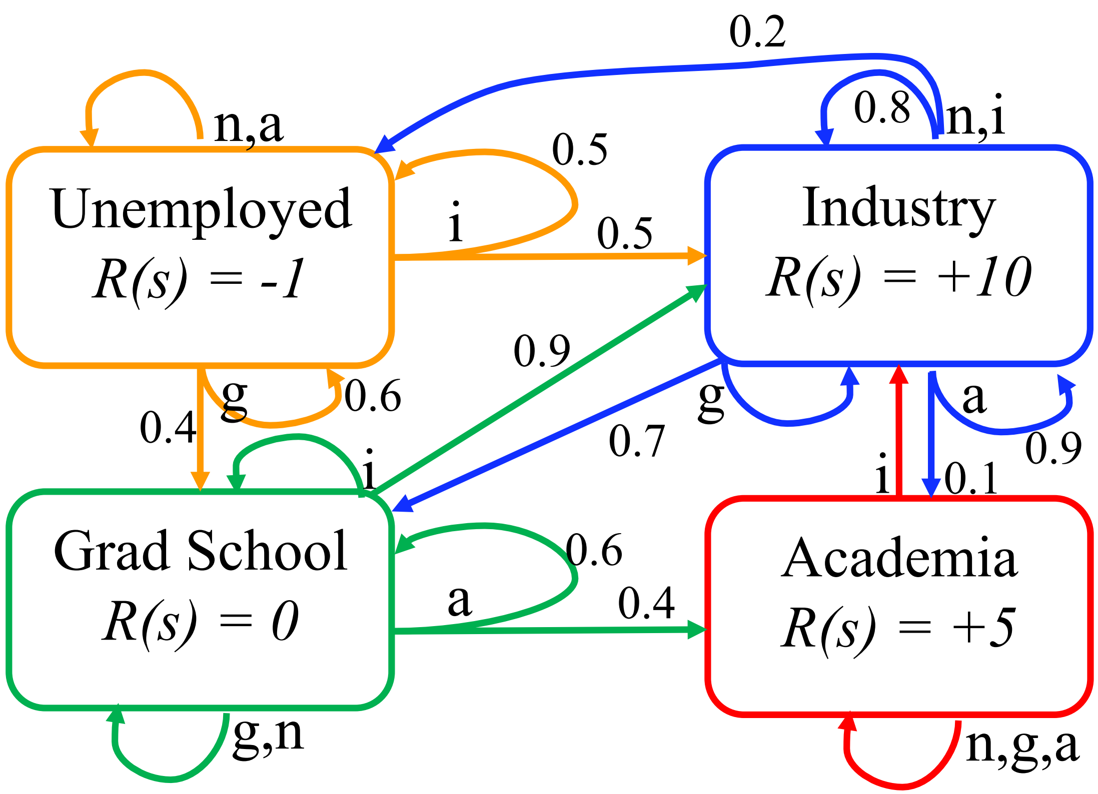
Example: Career Options



n=Do Nothing
i = Apply to industry
g = Apply to grad school
a = Apply to academia

What is the best policy?

Example: Career Options



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What is the best policy?

Value functions

The **expected return of a policy** (for every state) is called the **value function**: $V^{\pi}(s) = \boldsymbol{E}_{\pi}\,[\boldsymbol{r_t + r_{t+t} + \ldots + r_T \mid s_t = s}\,]$

<u>Simple strategy to find the best policy</u>:

1. Enumerate the space of all possible policies.
2. Estimate the expected return of each one.
3. Keep the policy that has maximum expected return.

Getting confused with terminology?

- **Reward**?
- **Return**?
- **Value**?
- **Utility**?

Getting confused with terminology?

- **Reward**: 1 step numerical feedback
- **Return**: Sum of rewards over the agent's trajectory.
- **Value**: Expected sum of rewards over the agent's trajector.
- **Utility**: Numerical function representing preferences.
- In RL, we assume **Utility** = **Return**.

The value of a policy

$$V^{\pi}(s) = E_{\pi}[r_t + r_{t+1} + \ldots + r_T \mid s_t = s]$$

$$V^{\pi}(s) = E_{\pi}[r_t] + E_{\pi}[r_{t+1} + \ldots + r_T \mid s_t = s]$$

$$V^{\pi}(s) = \underbrace{\textstyle\sum_{a \in A} \pi(s,a) R(s,a)}_{\textit{Immediate reward}} + \underbrace{E_{\pi}[r_{t+1} + \ldots + r_T \mid s_t = s]}_{\textit{Future expected sum of rewards}}$$

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$V^{\pi}(s) = \sum_{a \in A} \pi(s,a)R(s,a) + \underbrace{\sum_{a \in A} \pi(s,a)\sum_{s' \in S} T(s,a,s')}_{\text{Expectation over 1-step transition}} E_{\pi}\,[r_{t+1} + \ldots + r_T \mid s_{t+1} = s'\,]$

The value of a policy

$$V^\pi(s) = E_\pi [r_t + r_{t+1} + \ldots + r_T \mid s_t = s]$$

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$$V^\pi(s) = \sum_{a \in A} \pi(s,a)R(s,a) + \sum_{a \in A} \pi(s,a)\sum_{s' \in S} T(s,a,s') \underbrace{V^\pi(s')}_{\text{By definition}}$$

This is a **dynamic programming** algorithm.

The value of a policy

State value function (for a **fixed** policy):

$$V^{\pi}(s) = \sum_{a \in A} \pi(s,a) \, [\, \underbrace{R(s,a)}_{\textit{Immediate}} + \gamma \underbrace{\sum_{s' \in S} T(s,a,s') V^{\pi}(s')}_{\textit{Future expected sum of rewards}} \,]$$

State-action value function:

$$Q^{\pi}(s,a) = R(s,a) + \gamma \sum_{s'} T(s,a,s') [\sum_{a' \in A} \pi(s',a') Q^{\pi}(s',a')]$$

These are two forms of **Bellman's equation**.

The value of a policy

State value function:

$$V^{\pi}(s) = \sum_{a \in A} \pi(s,a) \left(R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') V^{\pi}(s') \right)$$

When S is a **finite set of states**, this is a **system of linear equations** (one per state) with a unique solution V^{π}.

Bellman's equation in matrix form: $V^{\pi} = R^{\pi} + \gamma\, T^{\pi} V^{\pi}$

Which can solved exactly: $V^{\pi} = (I - \gamma\, T^{\pi})^{-1} R^{\pi}$

Iterative Policy Evaluation: Fixed policy

Main idea: turn Bellman equations into update rules.

1. Start with some initial guess $V_0(s)$, $\forall s$. (Can be 0, or $r(s,\cdot)$.)

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2. During every iteration k, update the value function for all states:

$$V_{k+1}(s) \leftarrow \Big(R(s, \pi(s)) + \gamma \sum_{s' \in S} T(s, \pi(s), s') V_k(s') \Big)$$

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3. Stop when the maximum changes between two iterations is smaller than a desired threshold (the values stop changing.)

This is a dynamic programming algorithm. Guaranteed to converge!

Convergence of Iterative Policy Evaluation

- Consider the absolute error in our estimate $V_{k+1}(s)$:

$$\begin{aligned}
|V_{k+1}(s) - V^{\pi}(s)| &= \Big| \sum_{a} \pi(s,a)(R(s,a) + \gamma \sum_{s'} T(s,a,s') V_k(s')) \\
&\quad - \sum_{a} \pi(s,a)(R(s,a) + \gamma \sum_{s'} T(s,a,s') V^{\pi}(s')) \Big| \\
&= \gamma \Big| \sum_{a} \pi(s,a) \sum_{s'} T(s,a,s')(V_k(s') - V^{\pi}(s')) \Big| \\
&\leq \gamma \sum_{a} \pi(s,a) \sum_{s'} T(s,a,s') |V_k(s') - V^{\pi}(s')|
\end{aligned}$$

- As long as $\gamma<1$, the error **contracts** and eventually goes to 0.

Optimal policies and optimal value functions

- **Optimal value function,** V^* is the highest value that can be achieved for each state:

$$V^*(s) = max_{\pi} V^{\pi}(s)$$

- Any policy that achieves V^* is called an **optimal policy**, π^*.

Optimal policies and optimal value functions

- **Optimal value function,** V^* is the highest value that can be achieved for each state:

  $$V^*(s) = max_{\pi}\, V^{\pi}(s)$$

- Any policy that achieves V^* is called an **optimal policy**, π^*.

- For each MDP there is a **unique** **optimal value function** *(Bellman, 1957)*.

- The optimal policy is not necessarily unique.

Optimal policies and optimal value functions

- If we know V^* (and R, T, γ), then we can compute π^* easily.

$$\pi^*(s) = \text{argmax}_{a \in A} \left(R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') V^*(s') \right)$$

- If we know π^* (and R, T, γ), then we can compute V^* easily.

$$V^*(s) = \sum_{a \in A} \pi^*(s,a) \left(R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') V^*(s') \right)$$

$$V^*(s) = R(s, \pi(s)) + \gamma \sum_{s' \in S} T(s, \pi(s), s') V^*(s')$$

Take-home: Both V^* and π^* are "solutions" to the MDP.

Finding a good policy: **Policy Iteration**

- Start with an initial policy π_0 (e.g. random)

- Repeat:
  - Compute V^{π}, using iterative policy evaluation.
  - Compute a new policy π' that is greedy with respect to V^{π}

- Terminate when $\pi = \pi'$

Finding a good policy: **Value iteration**

Main idea: Turn the Bellman optimality equation into an iterative update rule (same as done in policy evaluation):

1. Start with an arbitrary initial approximation $V_0(s)$

2. On each iteration, update the value function estimate:

   $V_k(s) = \max_{a \in A} (R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') V_{k-1}(s'))$

3. Stop when max value change between iterations is below threshold.

The algorithm converges (in the limit) to the true V^*.

Three related algorithms

1. **Policy evaluation**: Fix the policy, estimate its value.

2. **Policy iteration**: Find the best policy at each state.
   » Policy evaluation + greedy improvement.

3. **Value iteration**: Find the optimal value function.

Three related algorithms

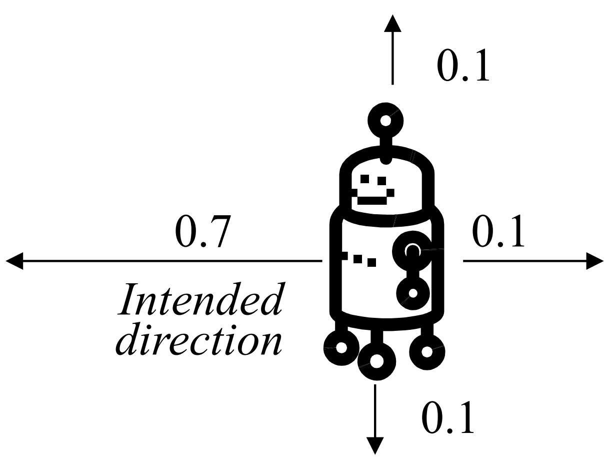
1. **Policy evaluation**: Fix the policy, estimate its value.
   – $O(S^3)$

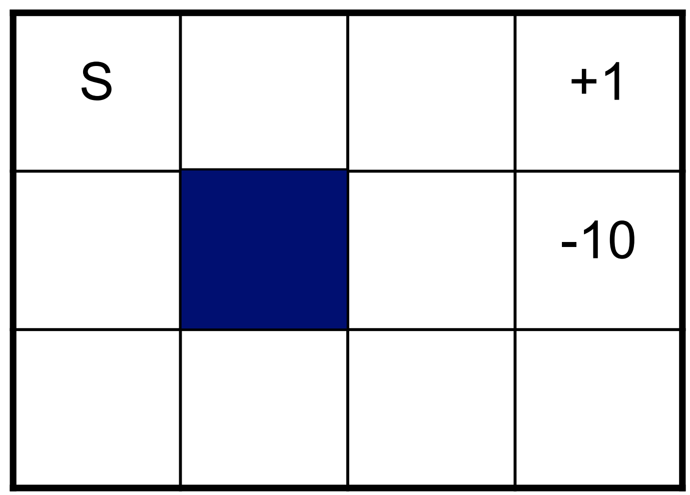
2. **Policy iteration**: Find the best policy at each state.
   » Policy evaluation + greedy improvement.
   – $O(S^3+S^2A)$ per iteration

3. **Value iteration**: Find the optimal value function.
   – $O(S^2A)$ per iteration

A 4x3 gridworld example

- 11 discrete states, 4 motion actions (N, S, E, W) in each state.
- Transitions are mildly **stochastic**.
- Reward is +1 in top right state, -10 in state directly below, -0 elsewhere.
- Episode terminates when the agent reaches +1 or -10 state.
- Discount factor γ = *0.99*.





Value Iteration (1)

0	0	0	+1
0		0	-10
0	0	0	0

Value Iteration (2)

0	0	0.69	+1
0		-0.99	-10
0	0	0	-0.99

Bellman residual: $|V_2(s) - V_1(s)| = 0.99$

Value Iteration (5)

0.48	0.70	0.76	+1
0.23		-0.55	-10
0	-0.20	-0.23	-1.40

Bellman residual: $|V_5(s) - V_4(s)| = 0.23$

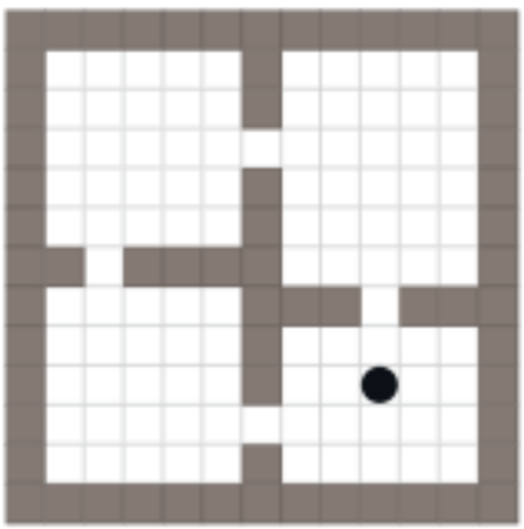
Value Iteration (20)

0.78	0.80	0.81	+1
0.77		-0.44	-10
0.75	0.69	0.37	-0.92

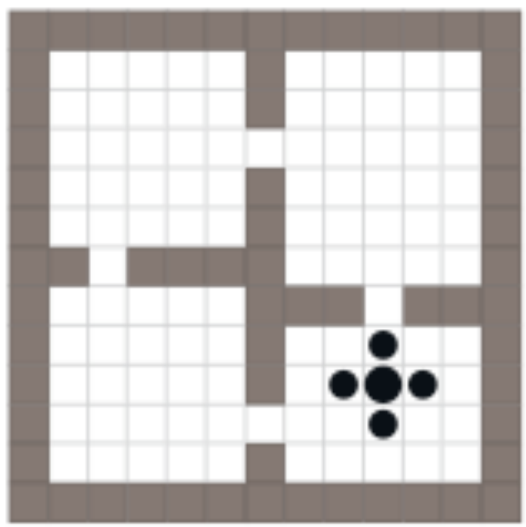
Bellman residual: $|V_5(s) - V_4(s)| = 0.008$

Another example: Four Rooms

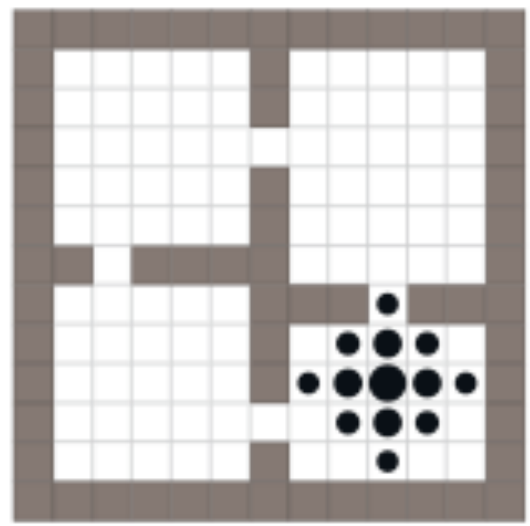
- Four actions, fail 30% of the time.
- No rewards until the goal is reached, $\gamma = 0.9$.
- Values propagate backwards from the goal.



Iteration #1



Iteration #2



Iteration #3

Asynchronous value iteration

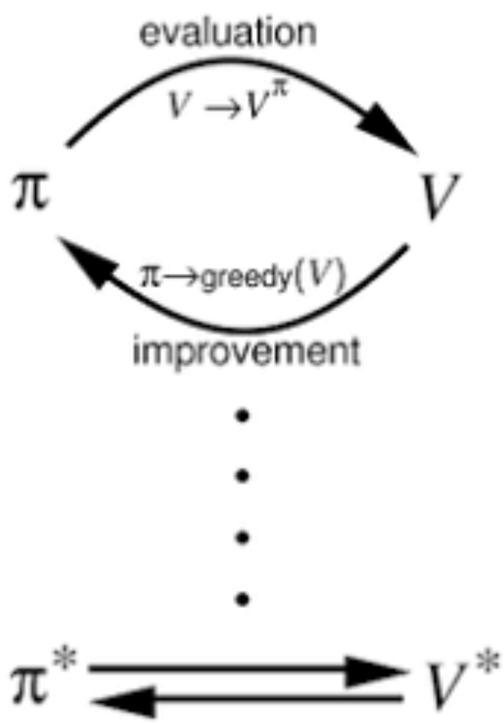
- Instead of updating all states on every iteration, focus on *important states*.
  - E.g., board positions that occur on every game, rather than just once in 100 games.

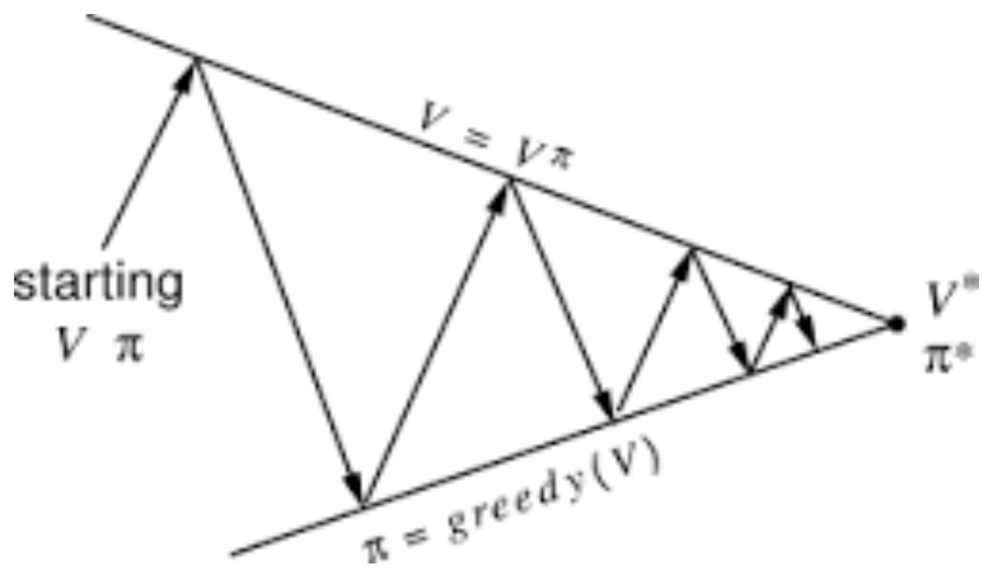
- Asynchronous dynamic programming algorithm:
  - Generate trajectories through the MDP.
  - Update states whenever they appear on such a trajectory.

- Focuses the updates on states that are actually possible.

Generalized Policy Iteration

- Any combination of policy evaluation and policy improvement steps.
  e.g. only update value of one state and improve policy at that state.

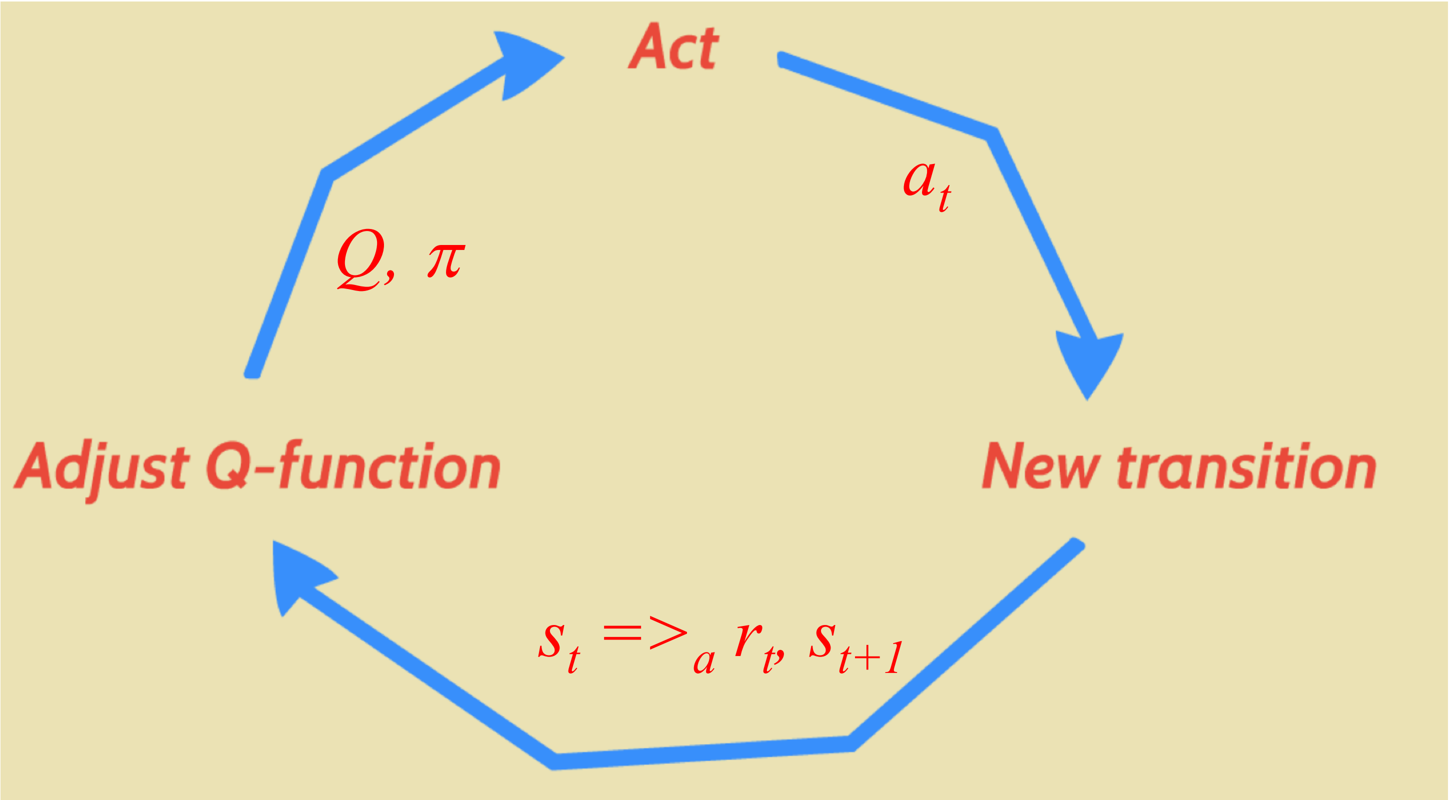




Key challenges in RL

- Designing the problem domain
  - State representation
  - Action choice
  - Cost/reward signal
- Acquiring data for training
  - Exploration / exploitation
  - High cost actions
  - Time-delayed cost/reward signal
- Function approximation
- Validation / confidence measures

Learning online from trial & error



Online reinforcement learning

- **Monte-Carlo** value estimate: Use the empirical return, $U(s_t)$ as a target estimate for the actual value function:

$$V(s_t) \leftarrow V(s_t) + \alpha\big(U(s_t) - V(s_t)\big)$$

** Not a Bellman equation. More like a gradient equation.*

  – Here α is the learning rate (a parameter).

  – Need to wait until the end of the trajectory to compute $U(s_t)$.

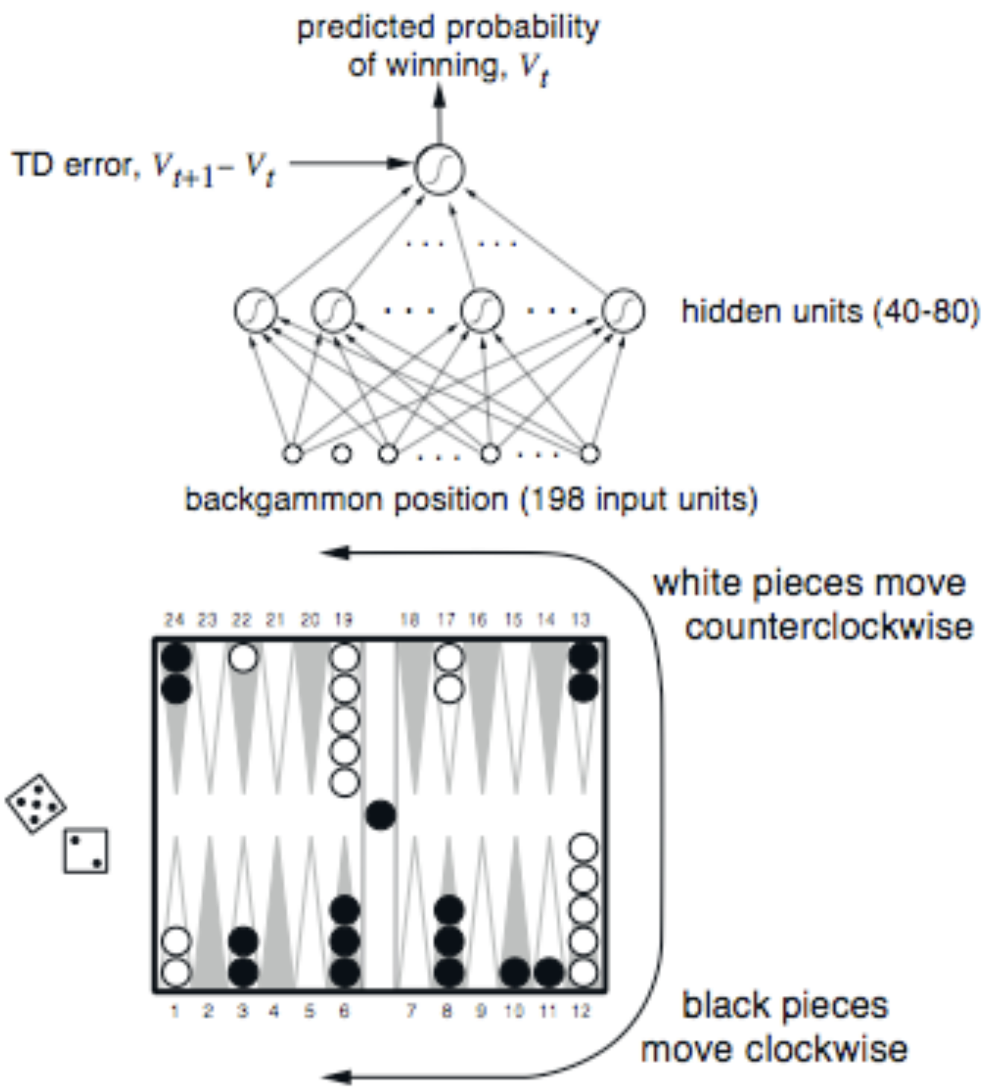
Temporal-Difference (TD) learning

- Monte-Carlo learning: $V(s_t) \leftarrow V(s_t) + \alpha\big(U(s_t) - V(s_t)\big)$

- TD-learning:

$$V(s_t) \leftarrow V(s_t) + \underset{\text{learning rate}}{\alpha}\,\big(\underbrace{r_{t+1} + \gamma V(s_{t+1}) - V(s_t)}_{\textbf{TD-error}}\big)\ \forall t = 0, 1, 2, \ldots$$

TD-Gammon (Tesauro, 1992)



Reward function:

+100 if win

- 100 if lose

0 for all other states

Trained by playing 1.5x10^6 million games against itself.

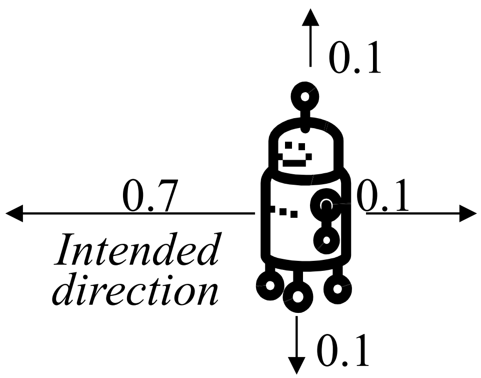
Enough to beat the best human player.

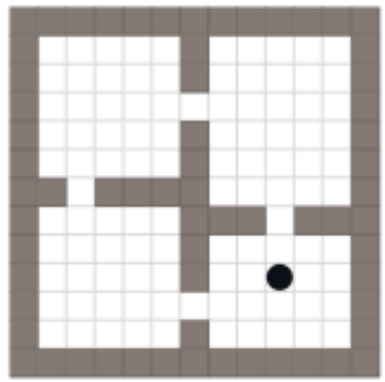
Several challenges in RL

- Designing the problem domain
  - State representation
  - Action choice
  - Cost/reward signal
- Acquiring data for training
  - Exploration / exploitation
  - High cost actions
- Time-delayed cost/reward signal
- **Function approximation**
- Validation / confidence measures

Tabular / Function approximation

- **Tabular**: Can store in memory a <u>list of the states</u> and their value.





** Can prove many more **theoretical properties** in this case, about convergence, sample complexity.*

- **Function approximation**: Too many states, continuous state spaces.

In large state spaces: Need approximation

Challenge: finding good features

$$\hat{Q}^{\pi}(s,a) = \sum_{i=1}^{d} \theta_i \underbrace{\phi_i(s,a)}_{\text{feature vector}}$$

Learning representations for RL

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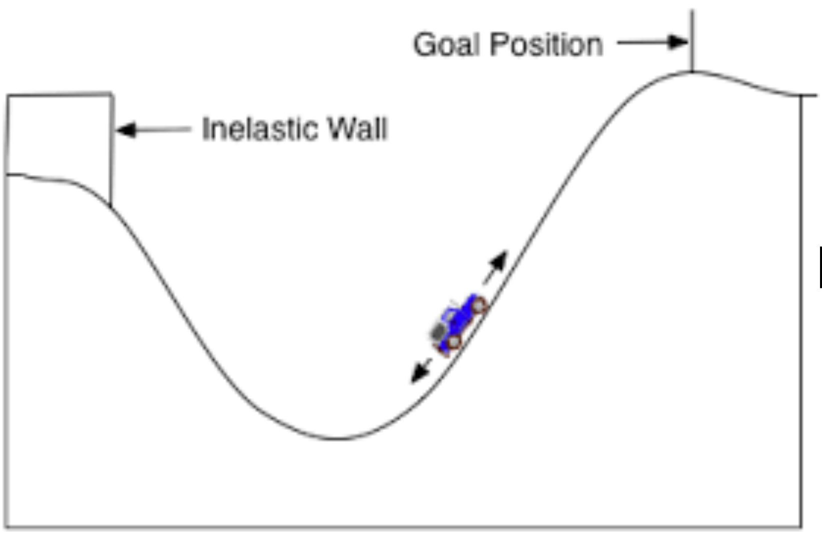
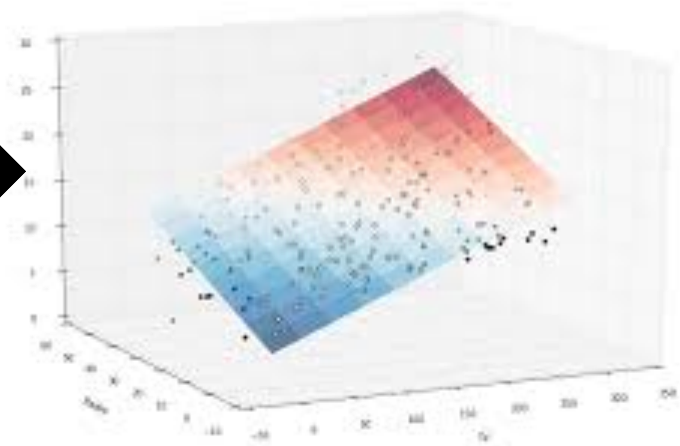


Original state



Linear function

$Q_\theta(s,a)$

Deep Reinforcement Learning

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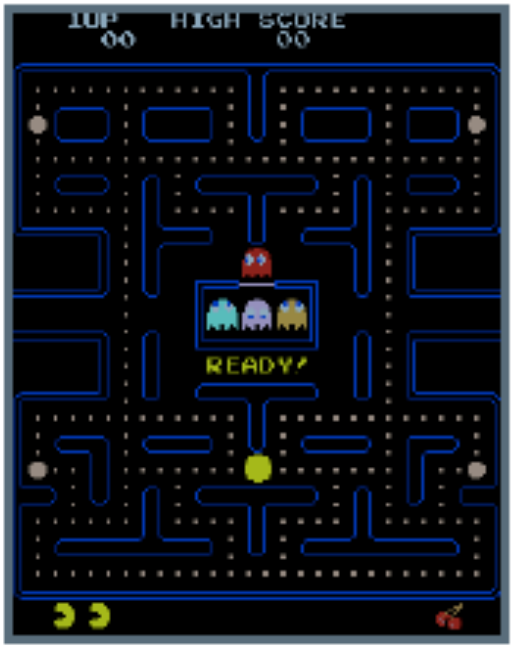

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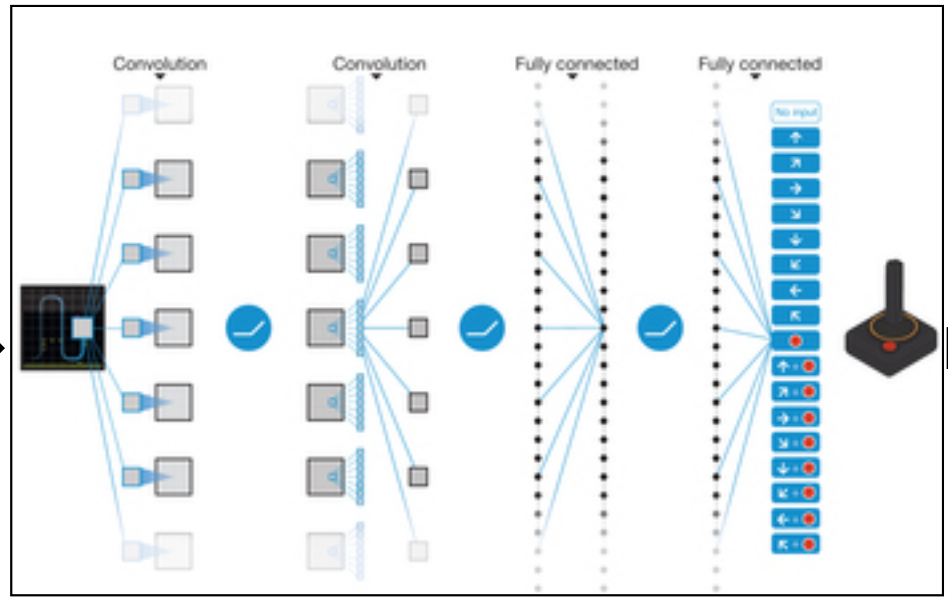


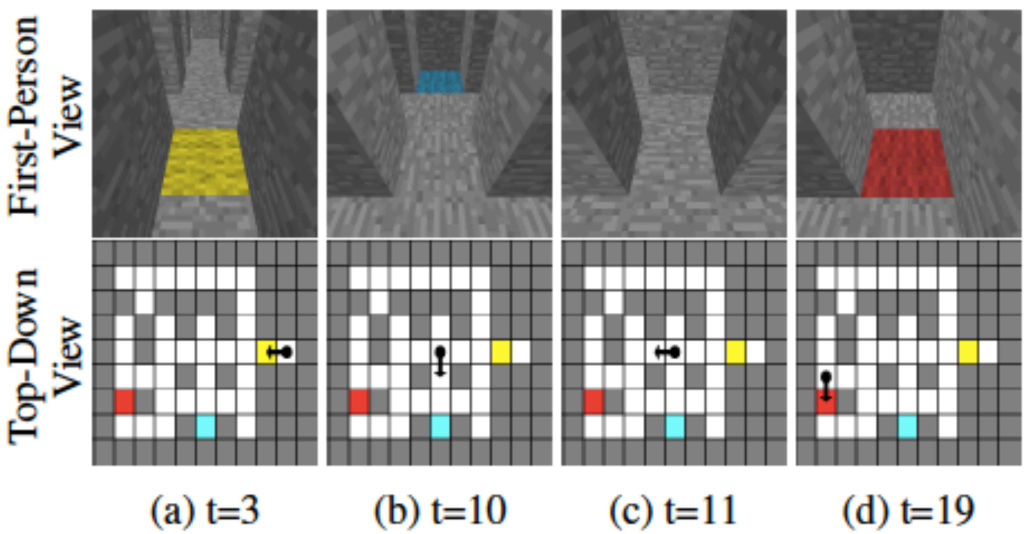
Convolutional Neural Net

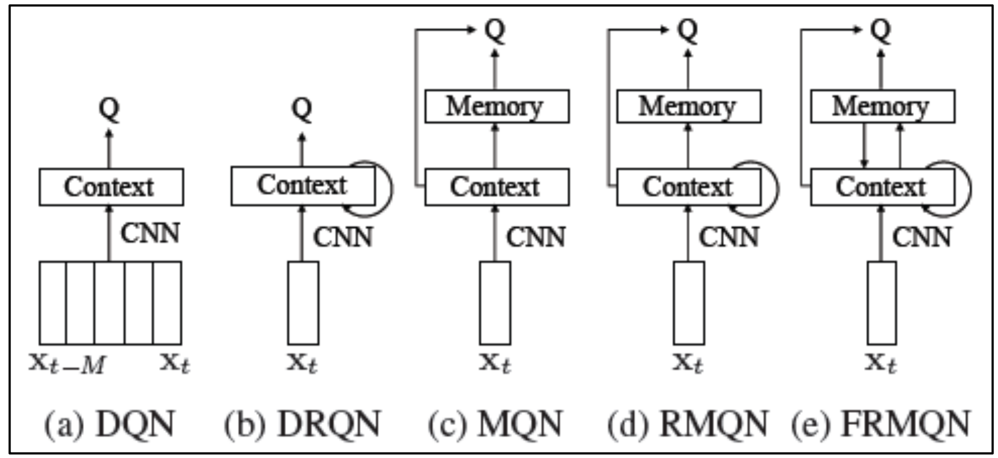
$Q_\theta(s,a)$

Deep Q-Network trained with stochastic gradient descent.

[DeepMind: Mnih et al., 2015].

Deep RL in Minecraft





Many possible architectures, incl. **memory** and **context**

Online videos: *https://sites.google.com/a/umich.edu/junhyuk-oh/icml2016-minecraft*

[U.Michigan: Oh et al., 2016].

The RL lingo

- Episodic / Continuing task
- Batch / Online
- **On-policy / Off-policy**
- Exploration / Exploitation
- Model-based / Model-free
- Policy optimization / Value function methods

On-policy / Off-policy

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  - Data distribution **changes** every time you change the policy!

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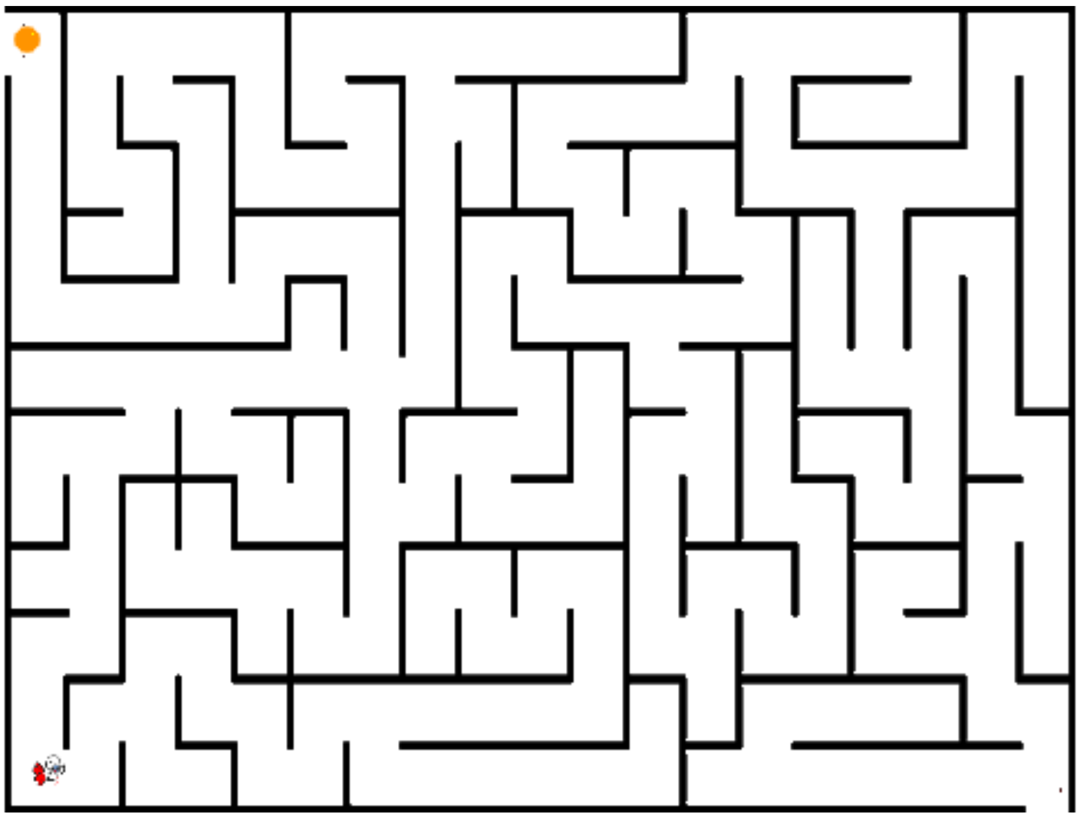
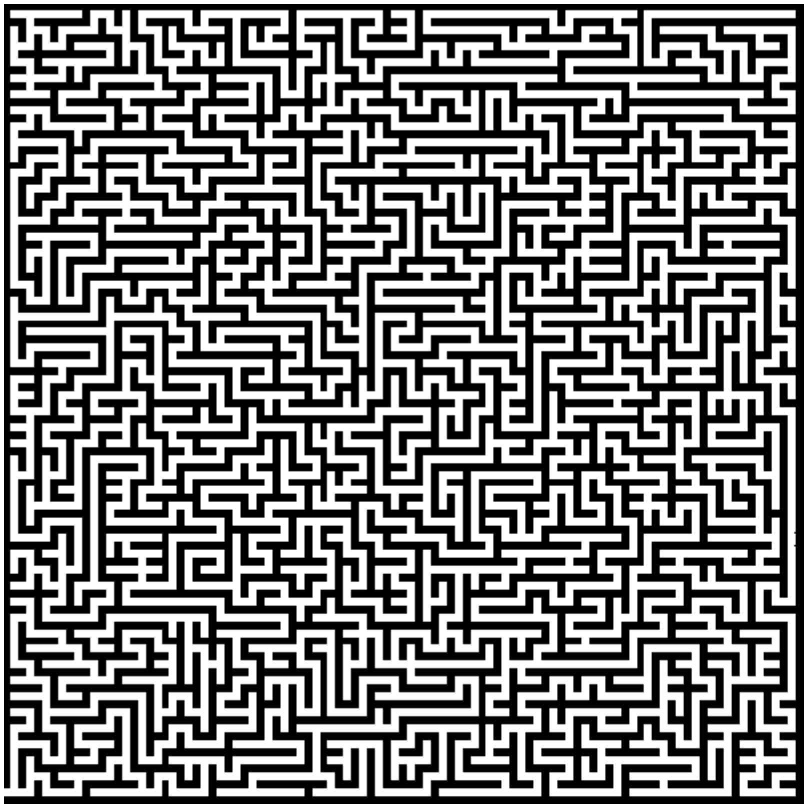
- Use importance sampling to reweigh data samples to compute unbiased estimates of a new policy.

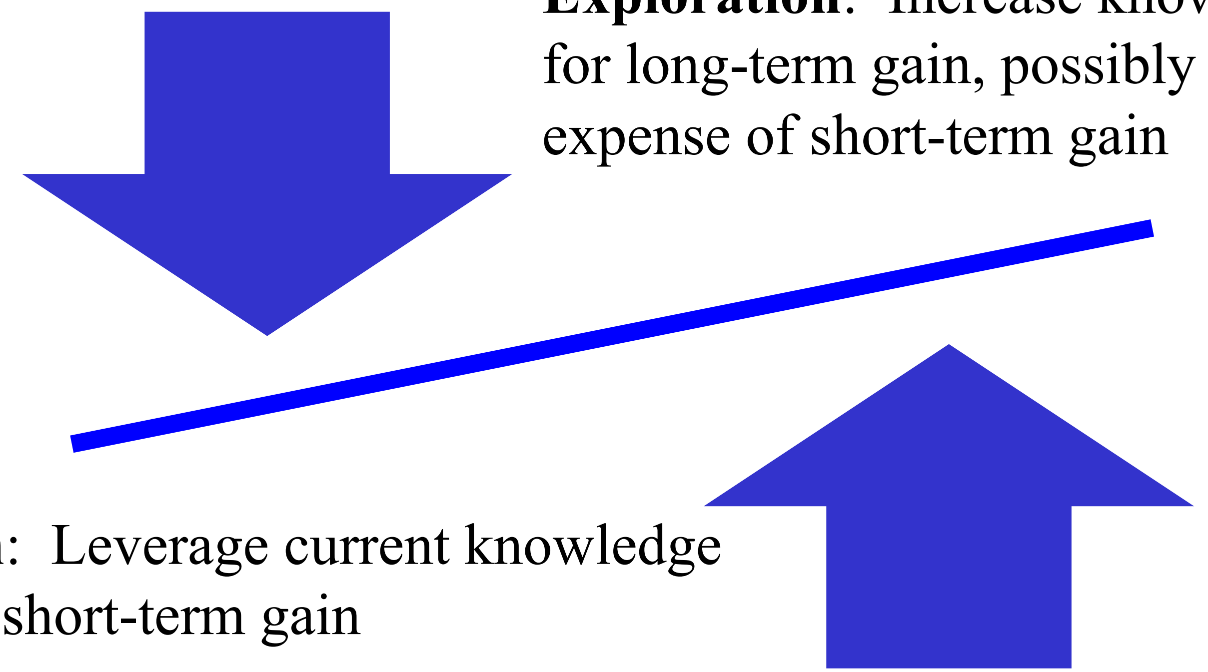
$$\rho_t = \frac{\pi(s_t, a_t)}{b(s_t, a_t)}$$

Exploration / Exploitation





Exploration / Exploitation



Exploration: Increase knowledge for long-term gain, possibly at the expense of short-term gain

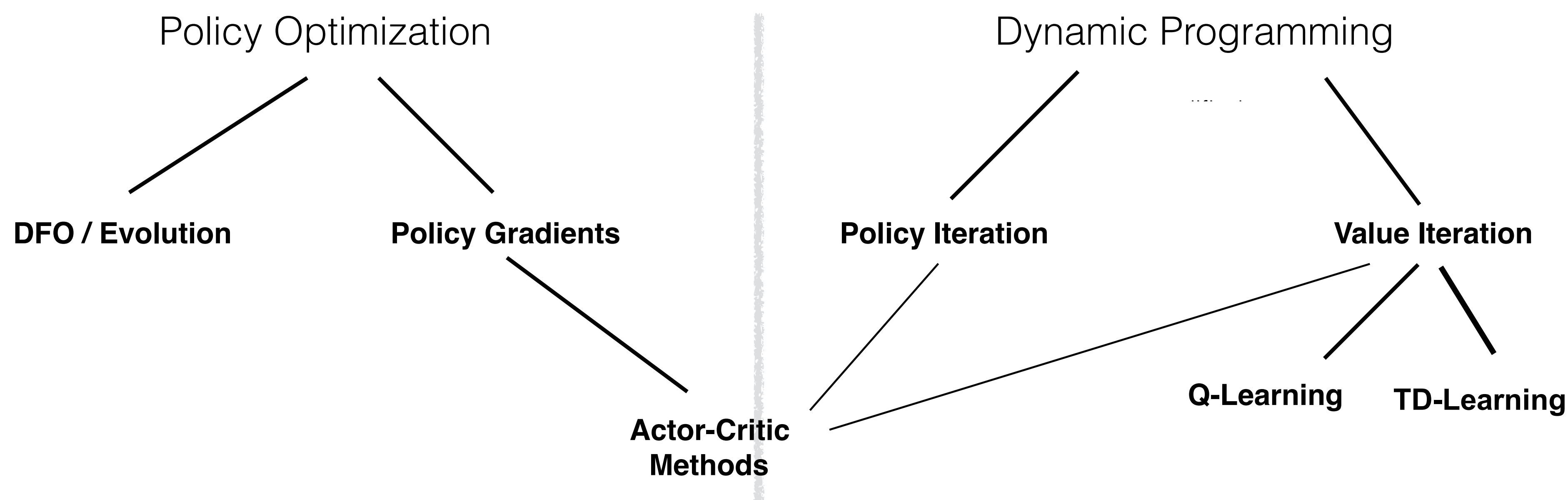
Exploitation: Leverage current knowledge to maximize short-term gain

Model-based vs Model-free RL

- **Option #1**: Collect large amounts of observed trajectories. Learn an approximate model of the dynamics (e.g. with supervised learning). Pretend the model is correct and apply value iteration.

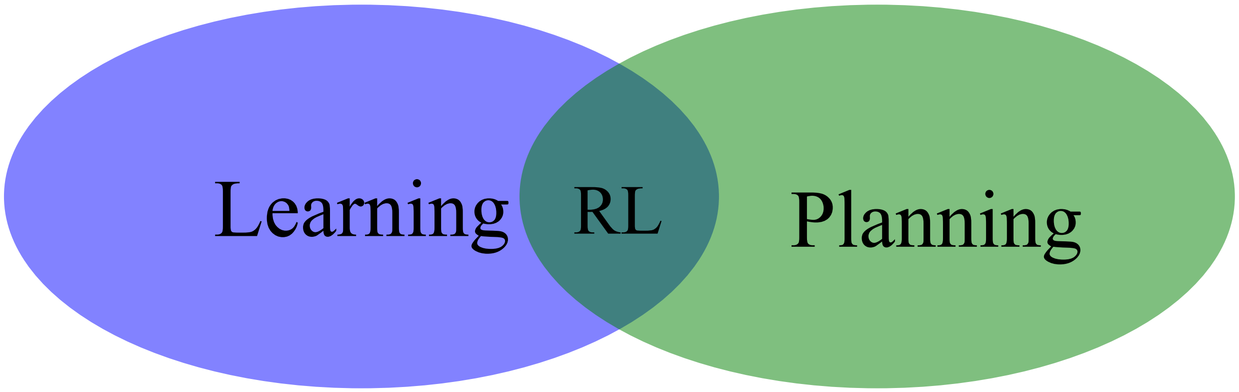
- **Option #2**: Use data to directly learn the value function or optimal policy.

Policy Optimization / Value Function



Quick summary

- RL problems are everywhere!
  - Games, text, robotics, medicine, …
- Need access to the “environment” to generate samples.
  - Most recent results make extensive use of a simulator.
- Feasible methods for large, complex tasks.
- Intuition about what is “easy”, “hard” is different than supervised learning.



RL resources

Comprehensive list of resources:

- https://github.com/aikorea/awesome-rl

Environments & algorithms:

- http://glue.rl-community.org/wiki/Main_Page
- https://gym.openai.com
- https://github.com/deepmind/lab