Multidataset Independent Subspace Analysis

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with

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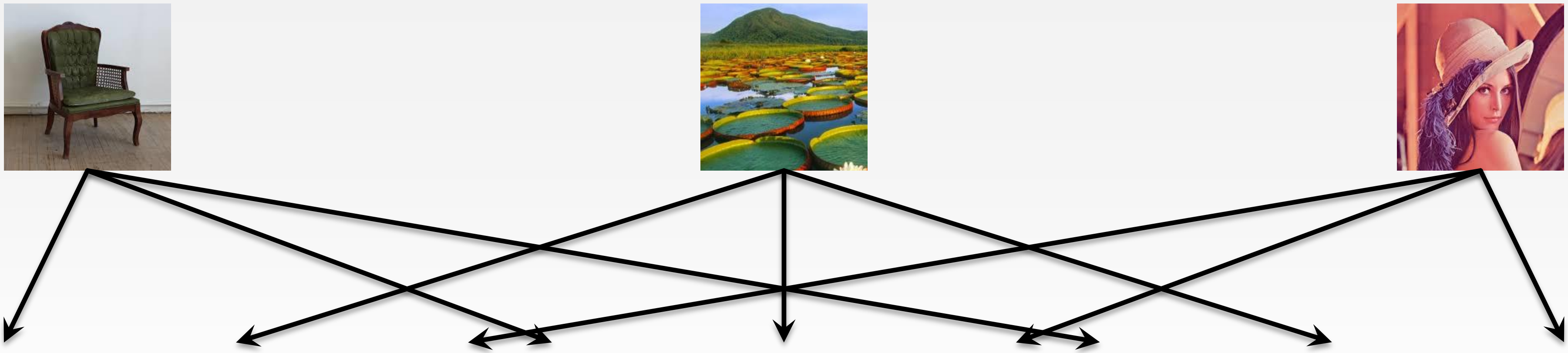
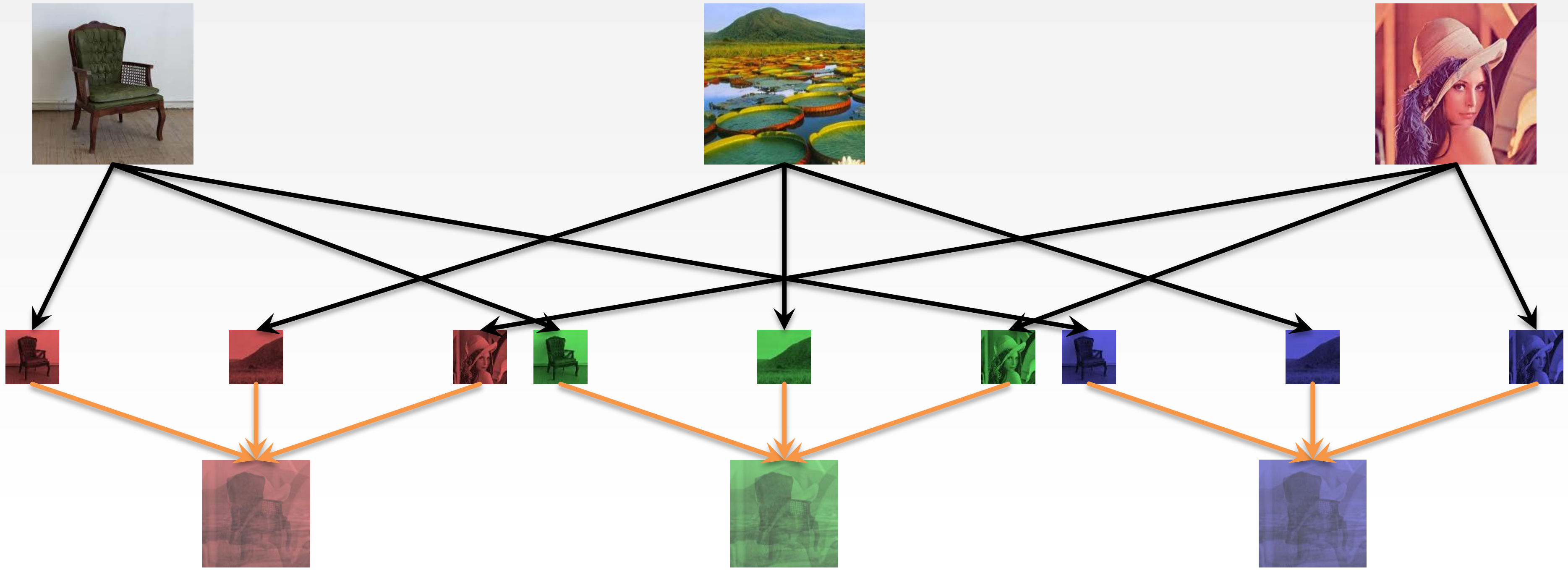
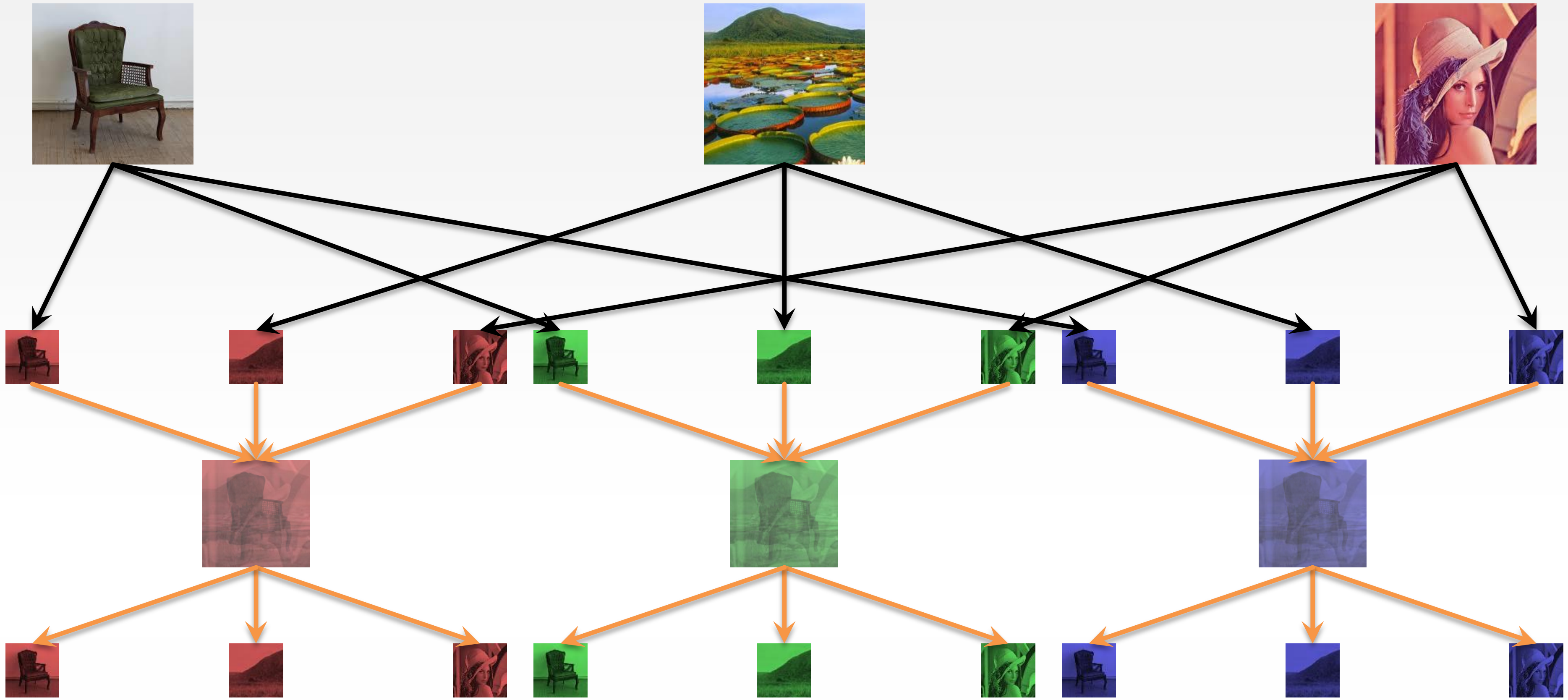
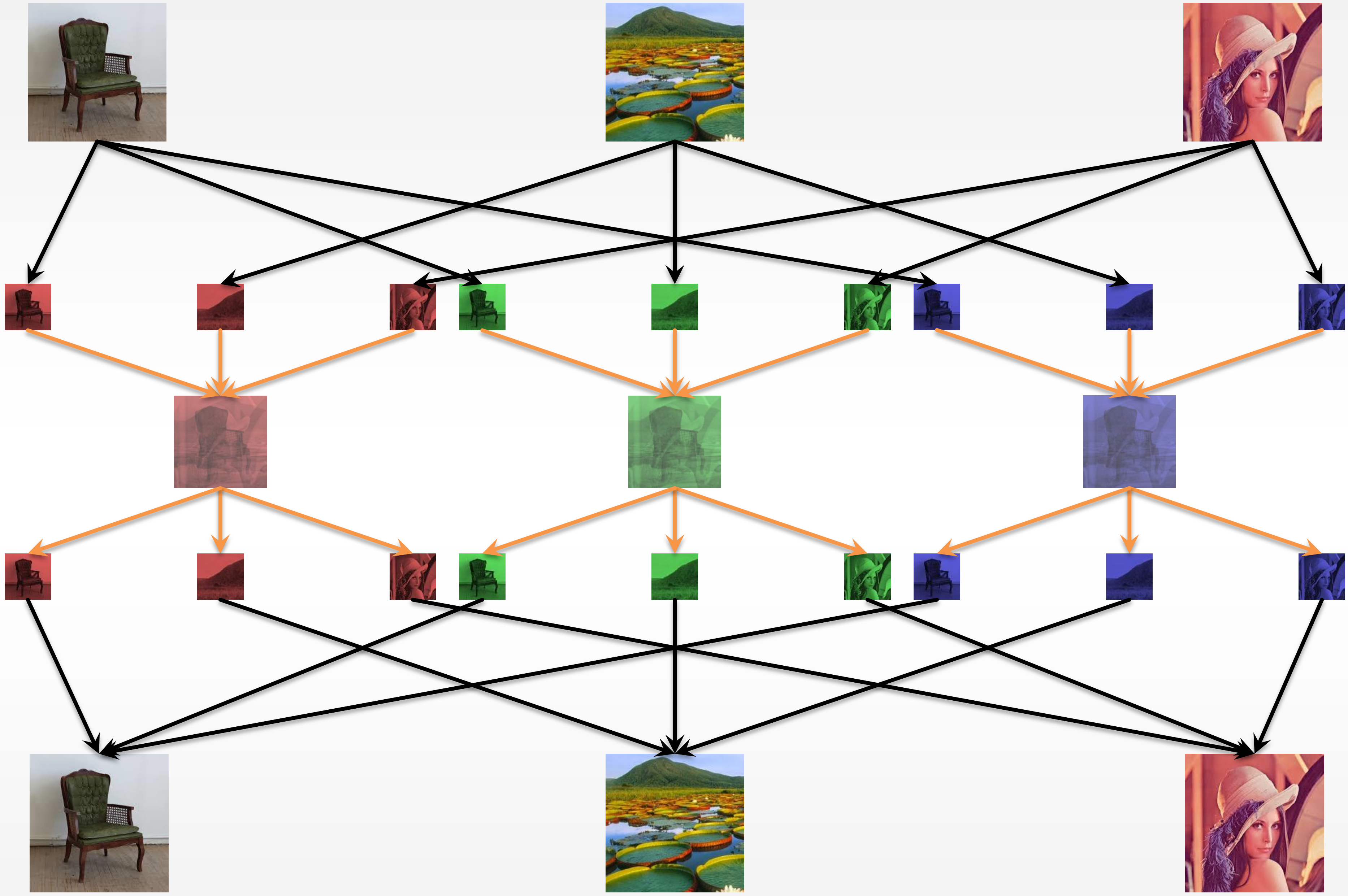
University of Montreal

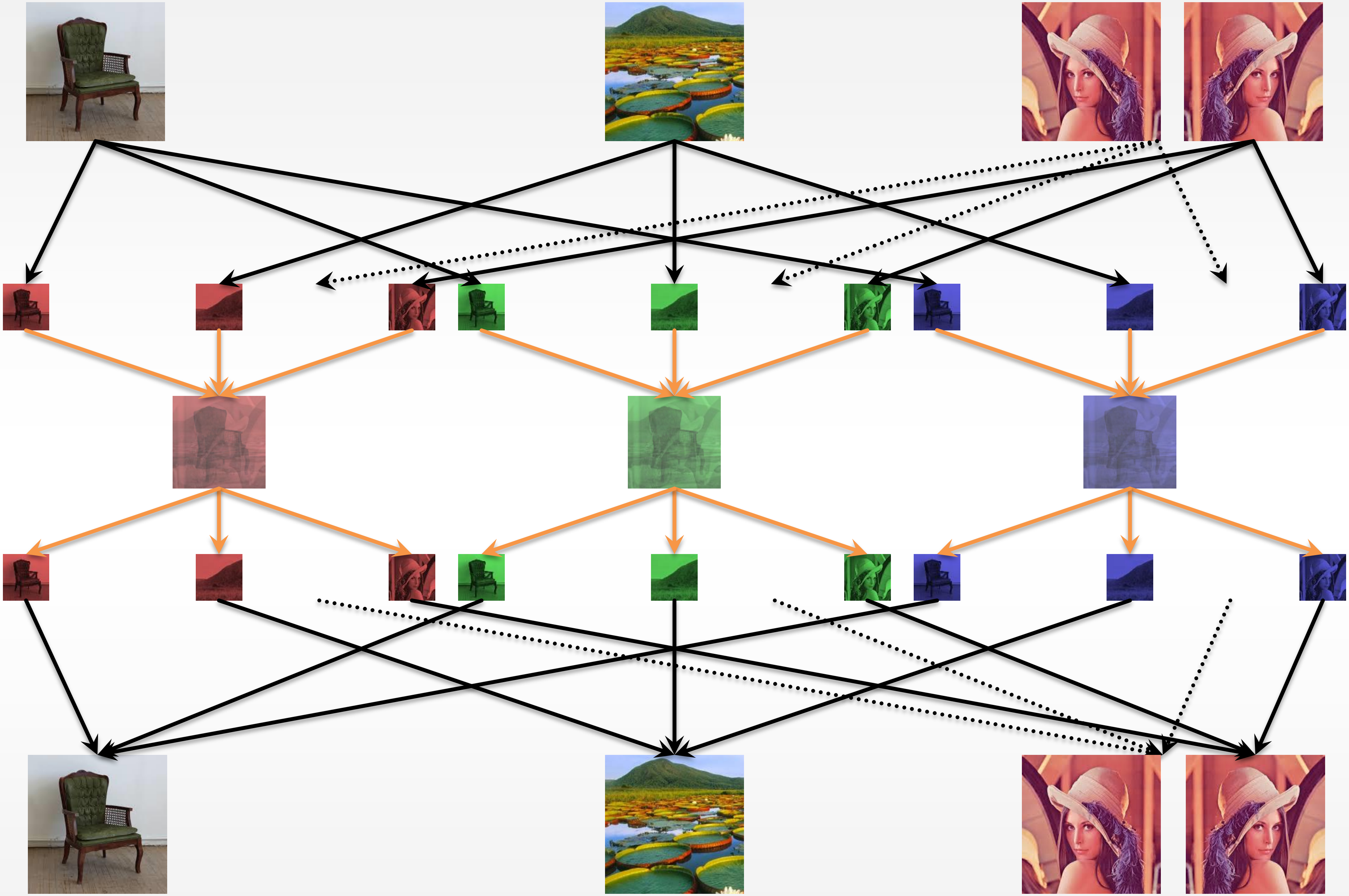












Multidataset Multidimensional Problems

- A Hierarchy of Blind Source Separation Models [*Silva et al.*, *2016*]

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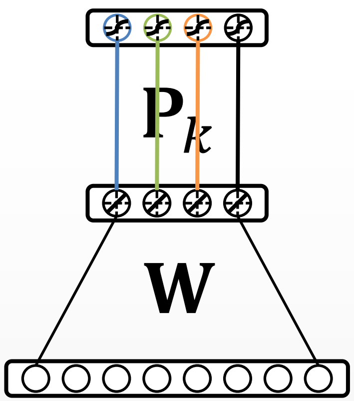
$$J(\mathbf{W}) = KL\langle p(\mathbf{y}), \prod_k^K p_k(\mathbf{y}_k)\rangle,$$

$$= -\sum_i^{C=K} \log|\sigma_i| - \sum_k^K \mathbb{E}[\mathrm{h}_k(\mathrm{y}_k)],$$



$$\mathrm{h}_k(\mathrm{y}_k) = -\sqrt{\frac{y_k^2}{\sigma_k^2}} - \frac{1}{2}\log\det\sigma_k^2$$

$$\mathbf{y} = \mathbf{W}\mathbf{x}$$

$\mathbf{x}$

SDU

Single Dataset (**SD**)

Multidataset Multidimensional Problems

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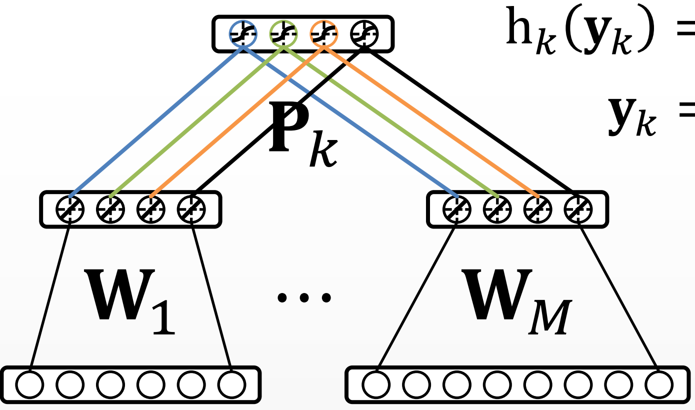
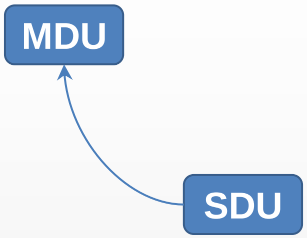


$$\mathrm{h}_k(\mathbf{y}_k) = -\sqrt{\mathbf{y}_k^\top \mathbf{\Sigma}_k^{\mathrm{y}^{-1}} \mathbf{y}_k} - \frac{1}{2}\log\det \mathbf{\Sigma}_k^{\mathbf{y}}$$

$$\mathbf{y}_k = \mathbf{P}_k \mathbf{W} \mathbf{x}$$

$$\mathbf{y}_m = \mathbf{W}_m \mathbf{x}_m$$



Single Dataset (**SD**) → Multidataset (**MD**)

Multidataset Multidimensional Problems

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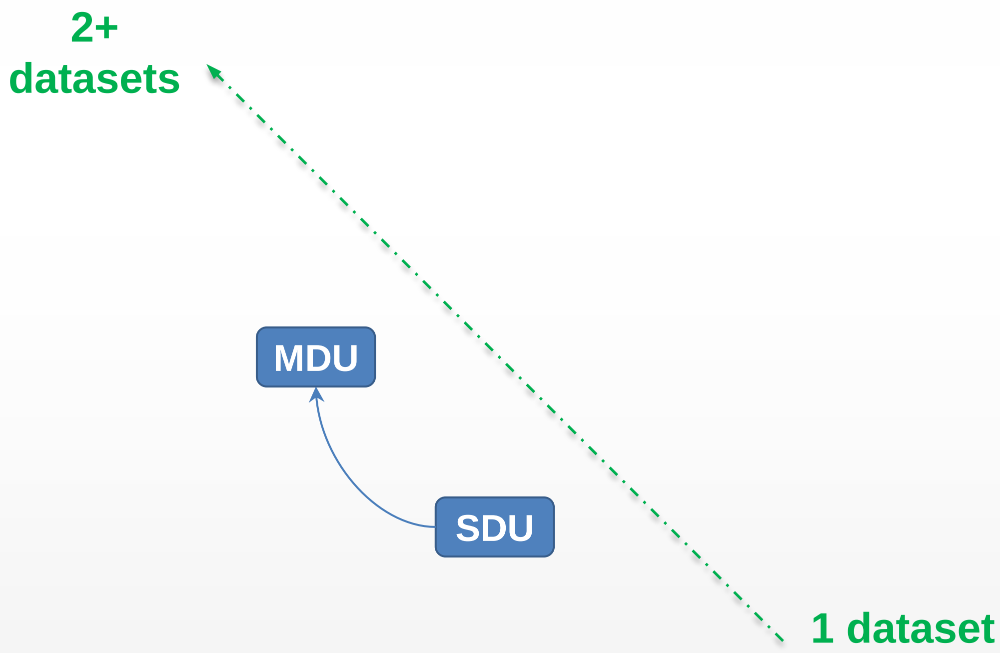

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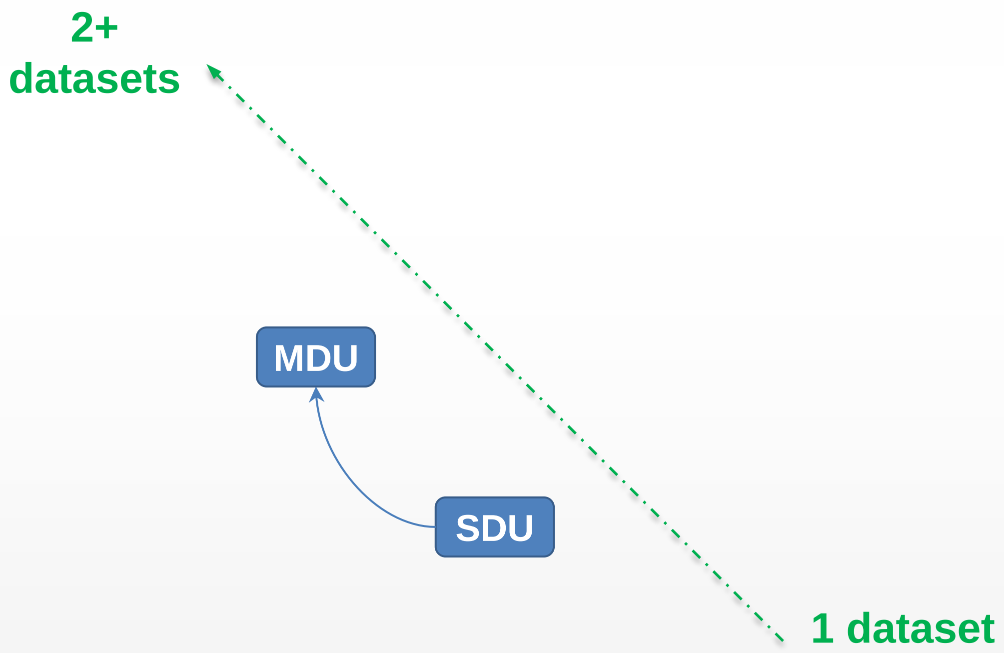

Single Dataset (**SD**) → Multidataset (**MD**)
Unidimensional (**U**)

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$$J(\mathbf{W}) = KL\langle p(\mathbf{y}), \prod_k^K p_k(\mathbf{y}_k)\rangle,$$
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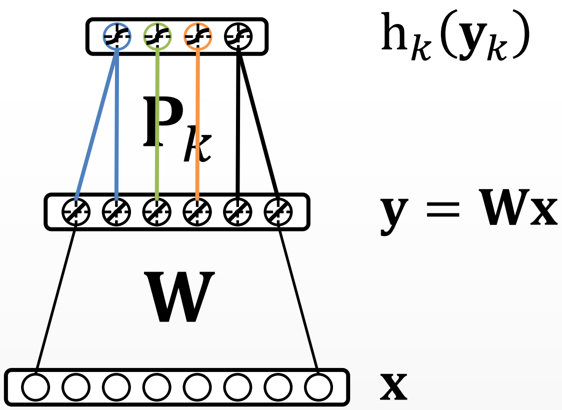

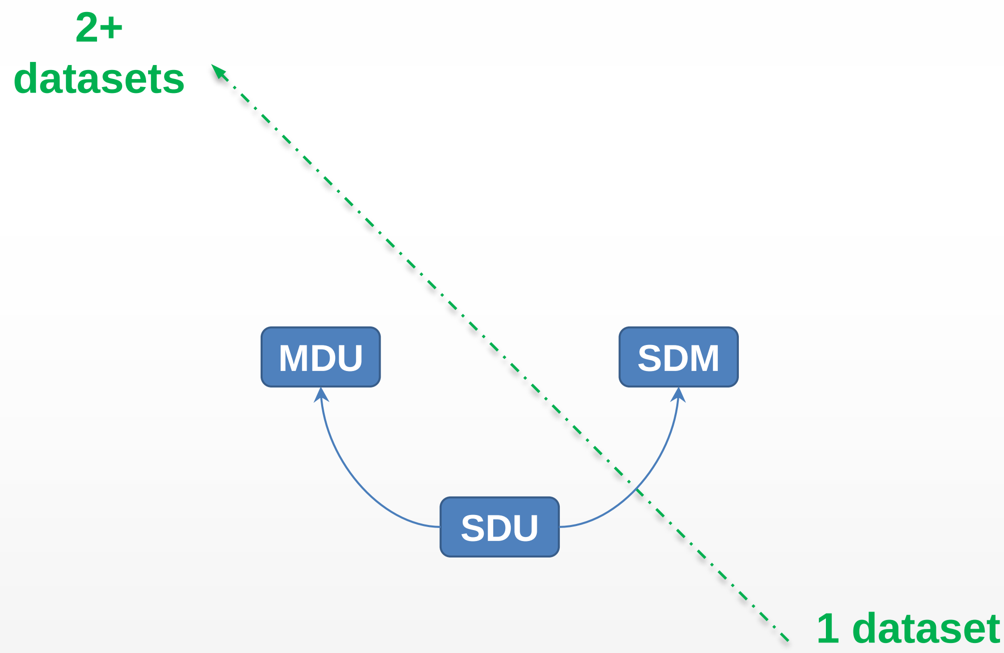

Single Dataset (**SD**) → Multidataset (**MD**)
Unidimensional (**U**) → Multidimensional (**M**)

Multidataset Multidimensional Problems

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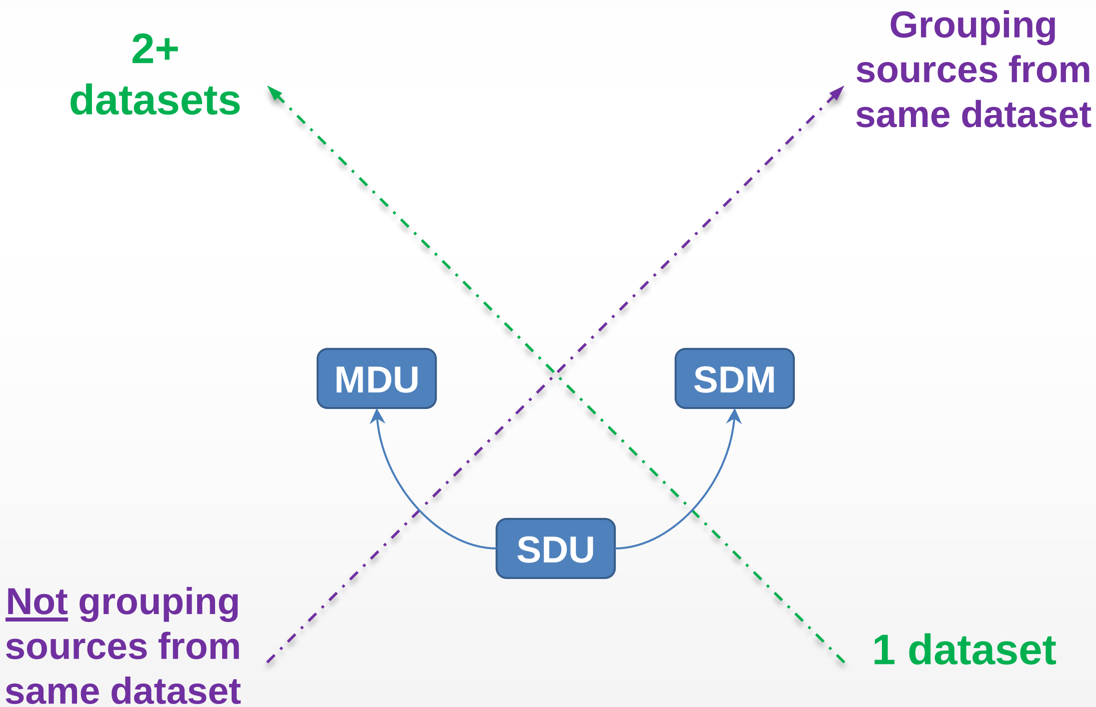

Single Dataset (**SD**) → Multidataset (**MD**)
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$$J(\mathbf{W}) = KL\langle p(\mathbf{y}), \prod_k^K p_k(\mathbf{y}_k)\rangle,$$
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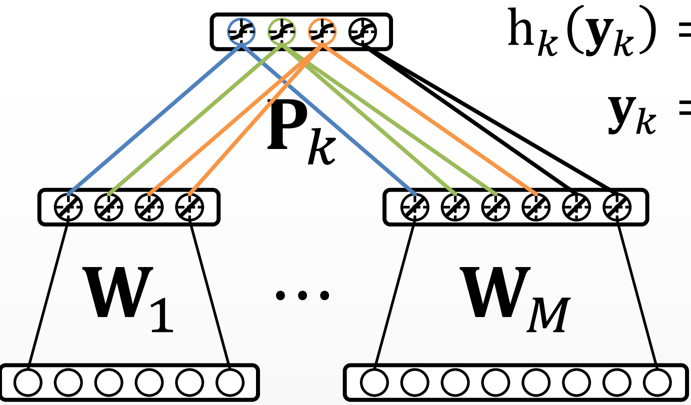
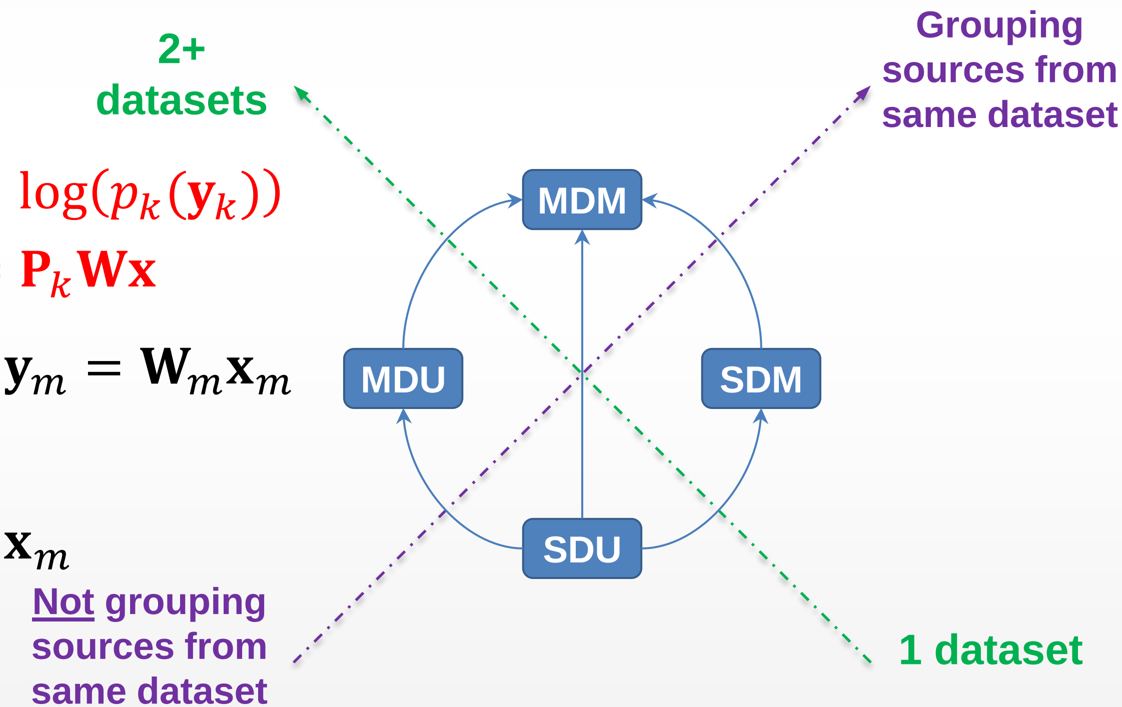


$$\mathrm{h}_k(\mathbf{y}_k) = \log(p_k(\mathbf{y}_k))$$
$$\mathbf{y}_k = \mathbf{P}_k\mathbf{W}\mathbf{x}$$
$$\mathbf{y}_m = \mathbf{W}_m\mathbf{x}_m$$
$$\mathbf{x}_m$$



Single Dataset (**SD**) → Multidataset (**MD**)
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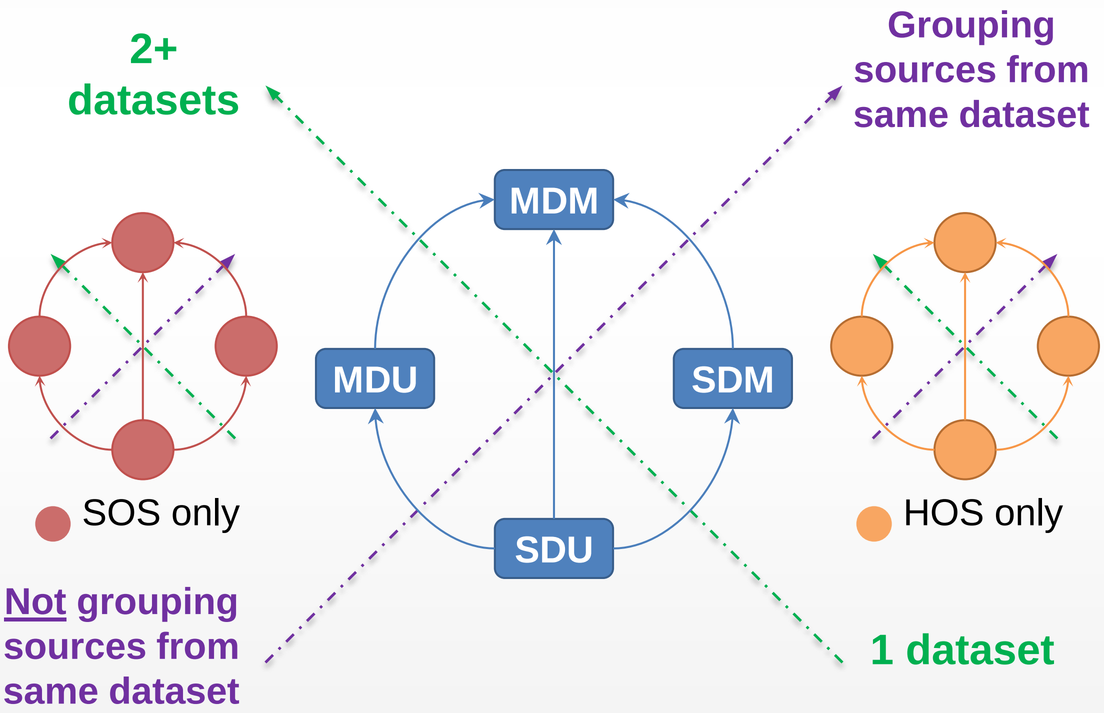

Single Dataset (**SD**) → Multidataset (**MD**)
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Multidataset Multidimensional Problems

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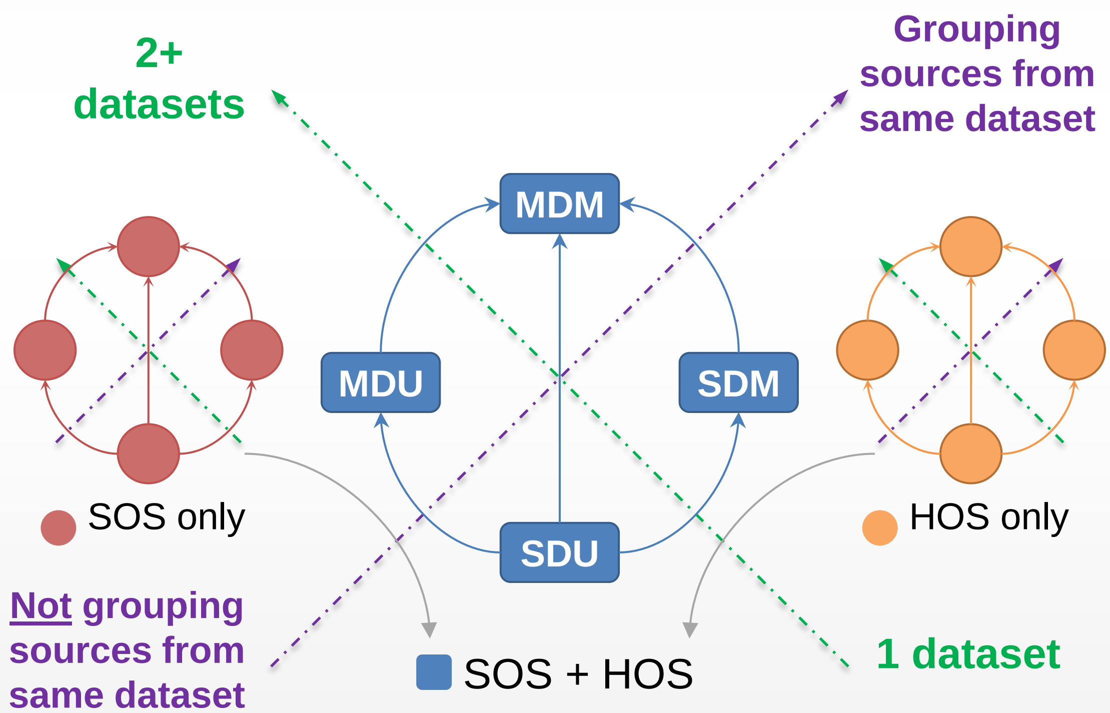
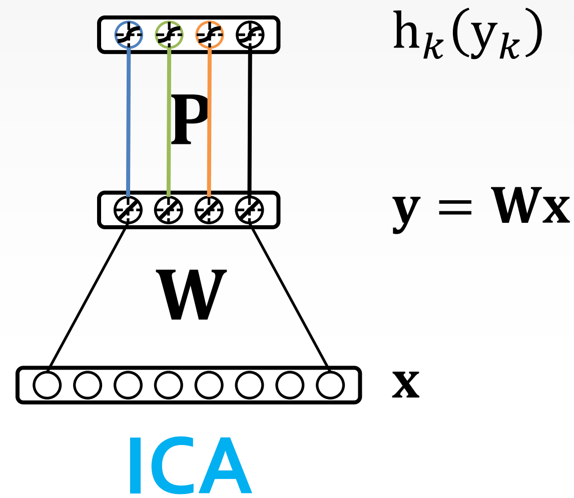


Single Dataset (**SD**) → Multidataset (**MD**)
Unidimensional (**U**) → Multidimensional (**M**)

ICA Regularized with a Linear Autoencoder

$$J(\mathbf{W}) = -\sum_{i}^{C=K} \log|\sigma_i| - \sum_{k}^{K} \mathbb{E}[\mathrm{h}_k(\mathrm{y}_k)]$$

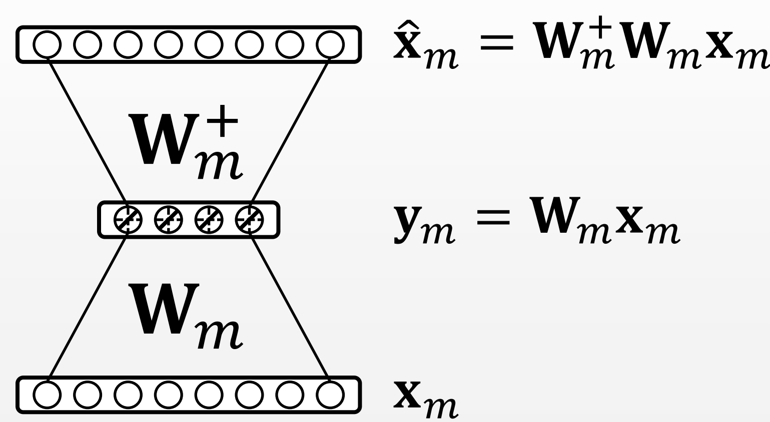


- Assuming white data $\mathbf{z}$, *Le et al., 2011* regularized ICA with a simple linear autoencoder $\hat{\mathbf{z}} = \mathbf{W}^{\top}\mathbf{W}\mathbf{z}$.

- Simultaneously estimate the reduced space while learning statistically independent hidden units.

- Modified linear autoencoder for non-white data: $\hat{\mathbf{x}}_m = \mathbf{W}_m^{+}\mathbf{W}_m\mathbf{x}_m$. Applied to <u>all</u> <u>modalities</u> as a **non-linear constraint**.

$$R(\mathbf{W}_m) = \|\mathbf{W}_m^{+}\mathbf{W}_m\mathbf{x}_m - \mathbf{x}_m\|_2^2$$

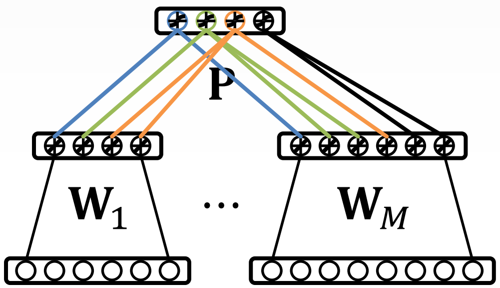


MISA: A Generalized Model for Independent Features

$$J(\mathbf{W}) = KL\langle p(\mathbf{y}), \textstyle\prod_k^K p_k(\mathbf{y}_k)\rangle,\ \mathbf{W} = \mathrm{blkdiag}(\mathbf{W}_m),\ \mathbf{x} = [\mathbf{x}_1^\top \ \cdots \ \mathbf{x}_M^\top]^\top$$

$$J(\mathbf{W}) = -\sum_m^M \sum_i^{C_m} \log|\sigma_{mi}| - \sum_k^K \mathbb{E}[\mathrm{h}_k(\mathbf{y}_k)]$$

$$J(\mathbf{W}) = -\sum_i^{C} \log|\sigma_{i}| - \sum_k^K \mathbb{E}[\mathrm{h}_k(\mathbf{y}_k)]$$



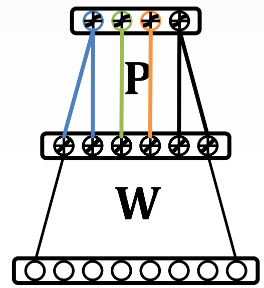
$\mathrm{h}_k(\mathbf{y}_k) = \log(p_k(\mathbf{y}_k|\mathbf{x}))$

$\mathbf{y}_k = \mathbf{P}_k\mathbf{W}\mathbf{x}$

$\mathbf{y}_m = \mathbf{W}_m\mathbf{x}_m$

$\mathbf{x}_m$

MISA



$\mathrm{h}_k(\mathbf{y}_k)$

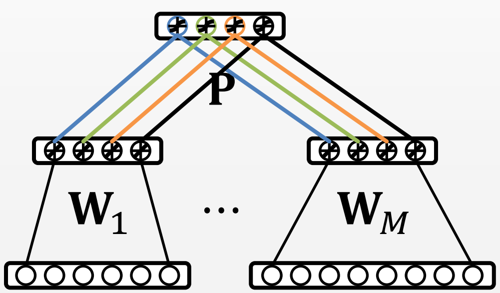
$\mathbf{y} = \mathbf{W}\mathbf{x}$

$\mathbf{x}$

ISA

$$J(\mathbf{W}) = -\sum_m^M \sum_i^{C_m=K} \log|\sigma_{mi}| - \sum_k^K \mathbb{E}[\mathrm{h}_k(\mathbf{y}_k)]$$

$$J(\mathbf{W}) = -\sum_i^{C=K} \log|\sigma_{i}| - \sum_k^K \mathbb{E}[\mathrm{h}_k(y_k)]$$

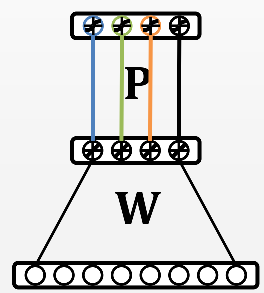


$\mathrm{h}_k(\mathbf{y}_k)$

$\mathbf{y}_m = \mathbf{W}_m\mathbf{x}_m$

$\mathbf{x}_m$

IVA



$\mathrm{h}_k(y_k)$

$\mathbf{y} = \mathbf{W}\mathbf{x}$

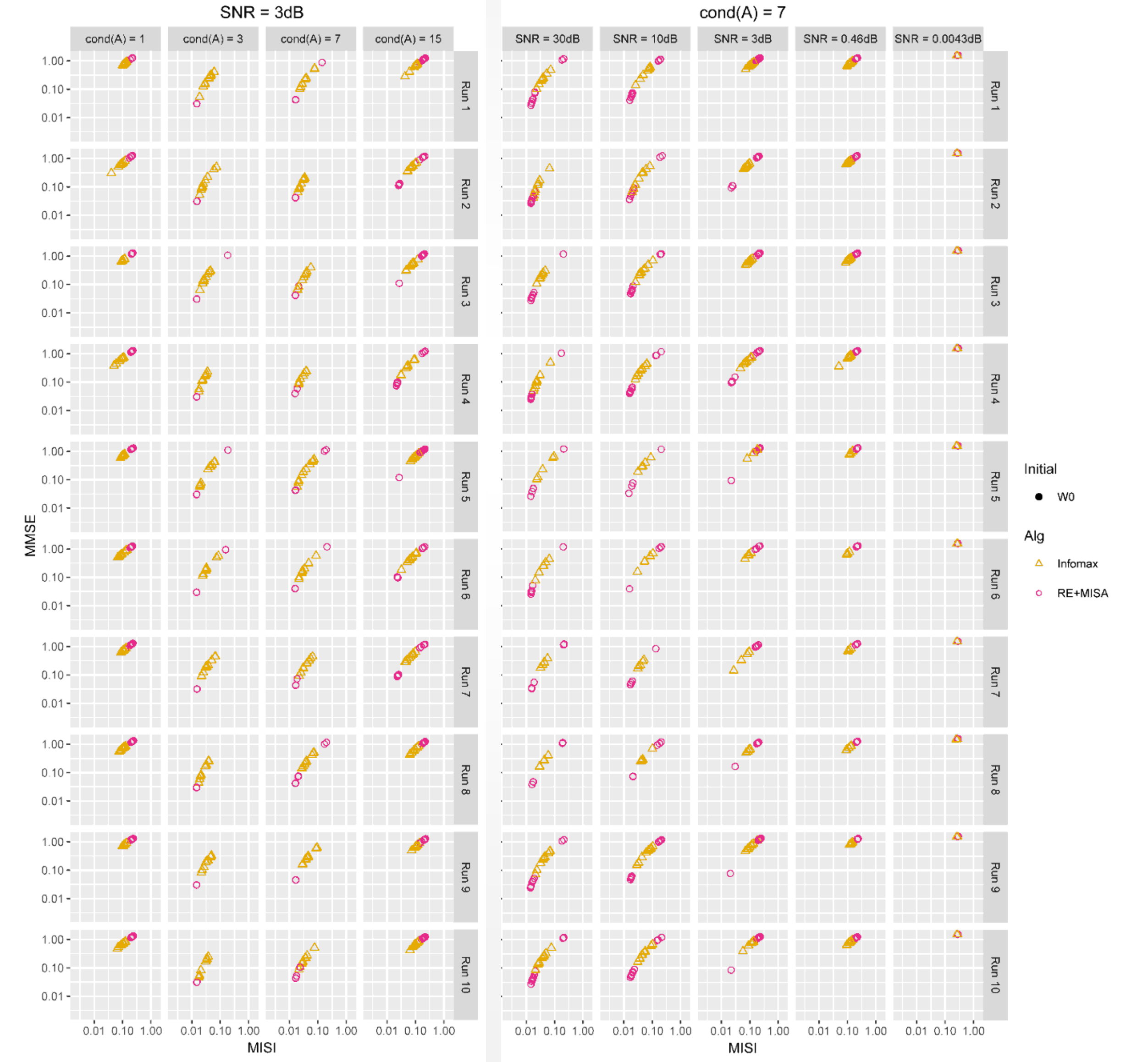
$\mathbf{x}$

ICA

Synthetic Data

ICA

- M = 1 dataset
- C = 75 hidden units into K = 75 one-dimensional subspaces
- N = 3500 examples sampled from a Laplace distribution
- Each run a new, unique (V × C) rectangular mixing matrix A (V = 8000)
- Initialized with ten different random row-orthogonal $\mathbf{W}_0$



Synthetic Data

ISA

– M = 1 Dataset

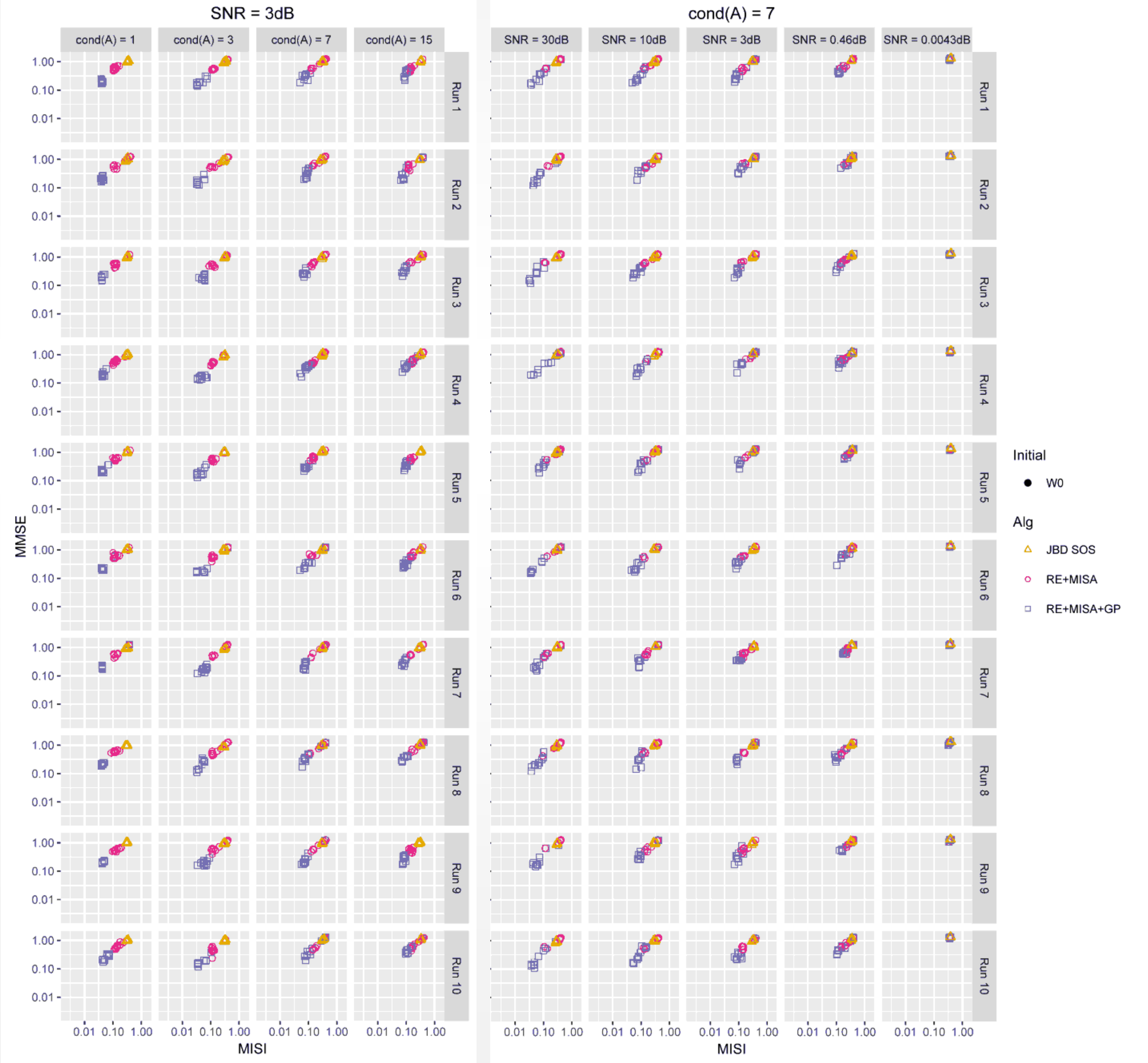
– C = 51 hidden units

– K = 18 d_k -dimensional subspaces, d_k = [1:5; 5:1; 1:5; 2; 2; 2]

– N = 5250 examples sampled from a Multivariate Laplace Distribution

– Each run a new, unique (V × C) rectangular mixing matrix A (V = 8000)

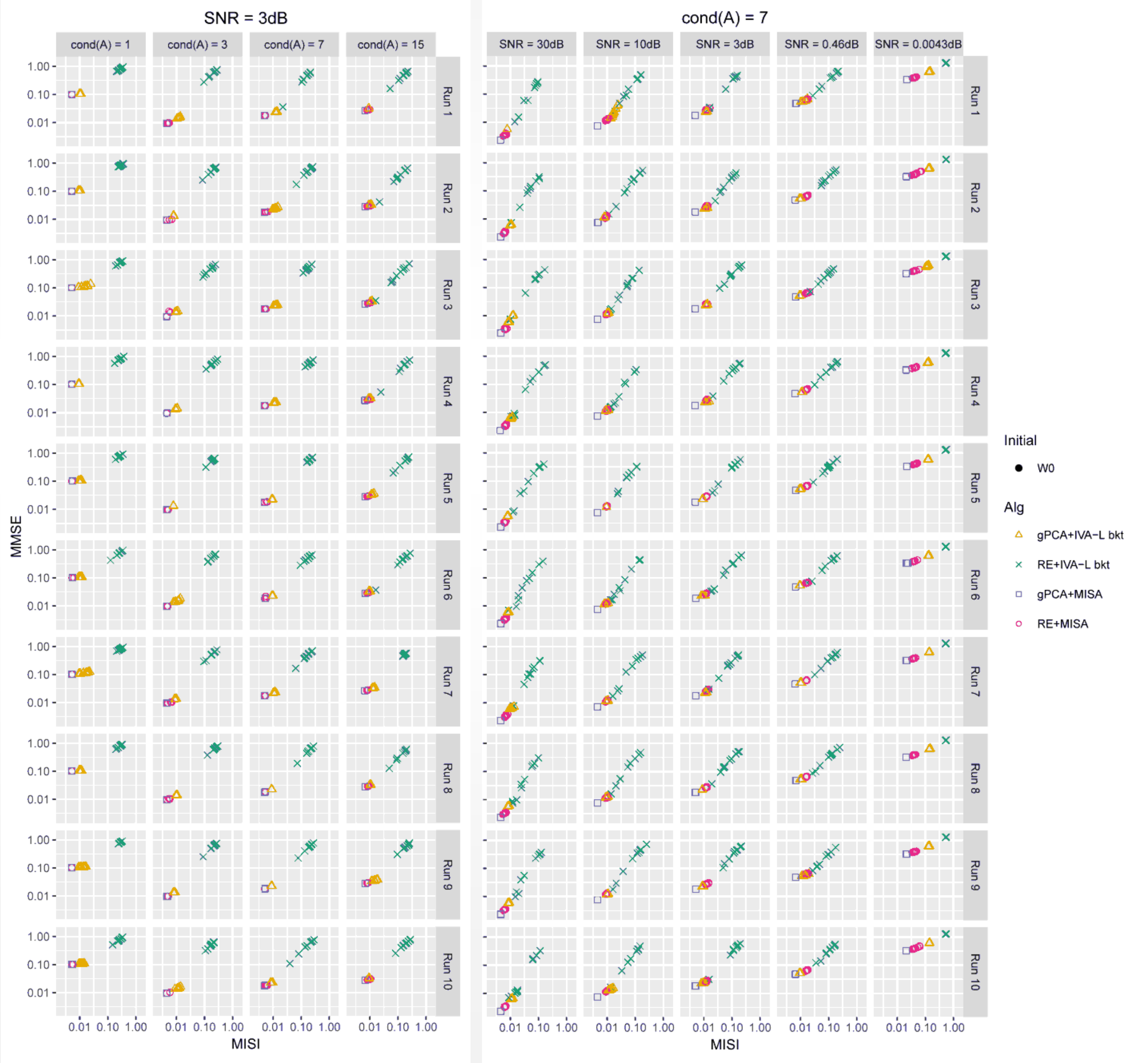
– Initialized with ten different random row-orthogonal $\mathbf{W}_0$



Synthetic Data

IVA

- M = 16 datasets
- C = 75 hidden units into K = 75 sixteen-dimensional subspaces
- N = 66000 examples sampled from a Multivariate Laplace Distribution
- Each run a new, unique (V × C) rectangular mixing matrix A (V = 250)
- Initialized with ten different random row-orthogonal $\mathbf{W}_0$



Hybrid Data

IVA

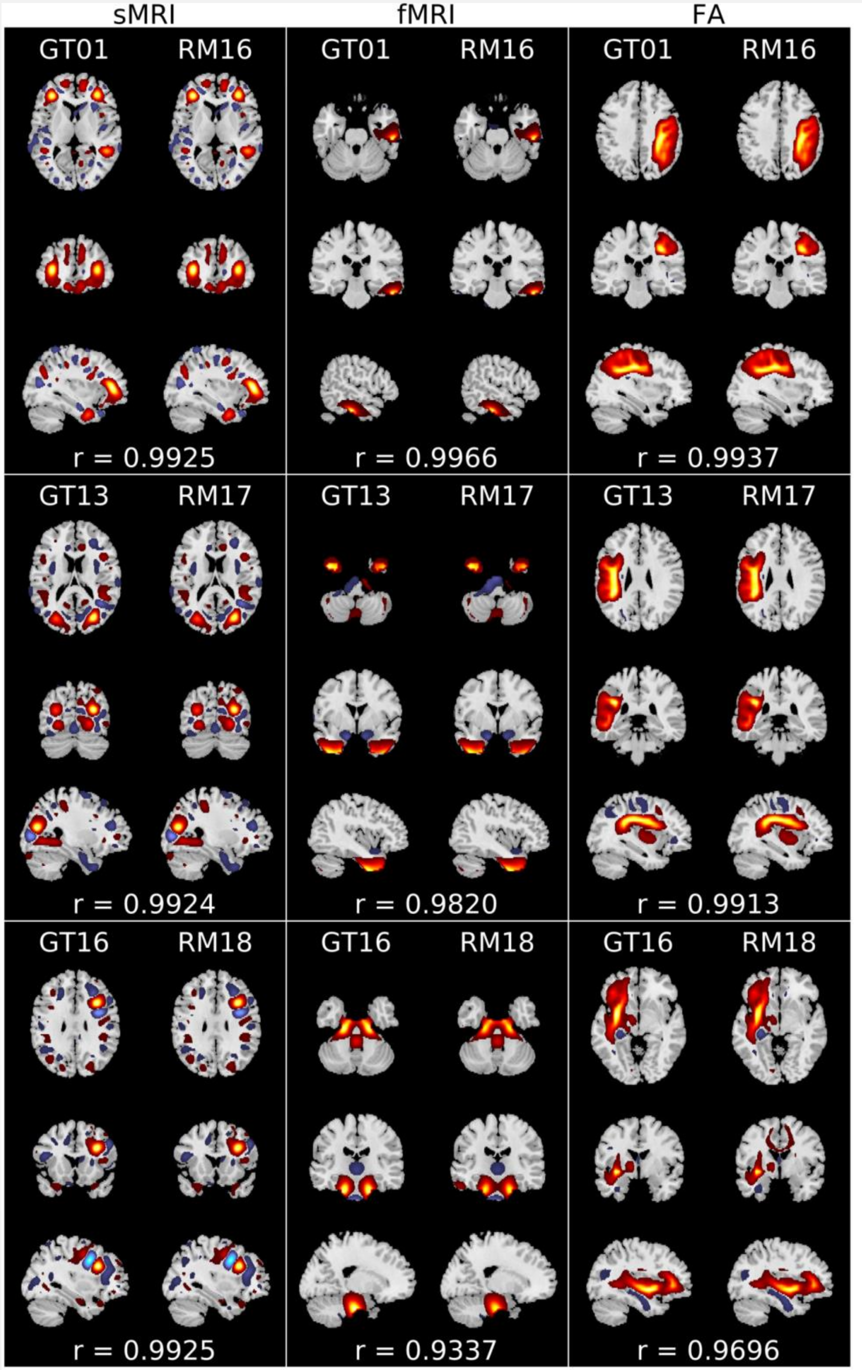
– M = 3 datasets
  – sMRI: V ≈ 300K voxels
  – fMRI: V ≈ 67K voxels
  – FA: V ≈ 15K voxels

– Mixing matrices $\mathbf{A}_m$ are the “real” part of the datasets
  – condition numbers: 1.52, 4.59, 1.63

– C = 20 hidden units into K = 20 three-dimensional subspaces

– N = 600 examples sampled from a Gaussian copula

– Additive Gaussian sensor noise: SNR = 3dB



What's Next?

- Explore deep non-linear models before subspace formation to identify optimal modality-specific depth.

- Learn the strictly sparse Subspace Assignment Matrix $\mathbf{P}$ automatically.

- Modality-specific architectures such as RNNs with hidden independent subspaces for modalities with sequential data