Repair Strategies for Minimizing the Risk of Cascading Failures in Electricity Networks

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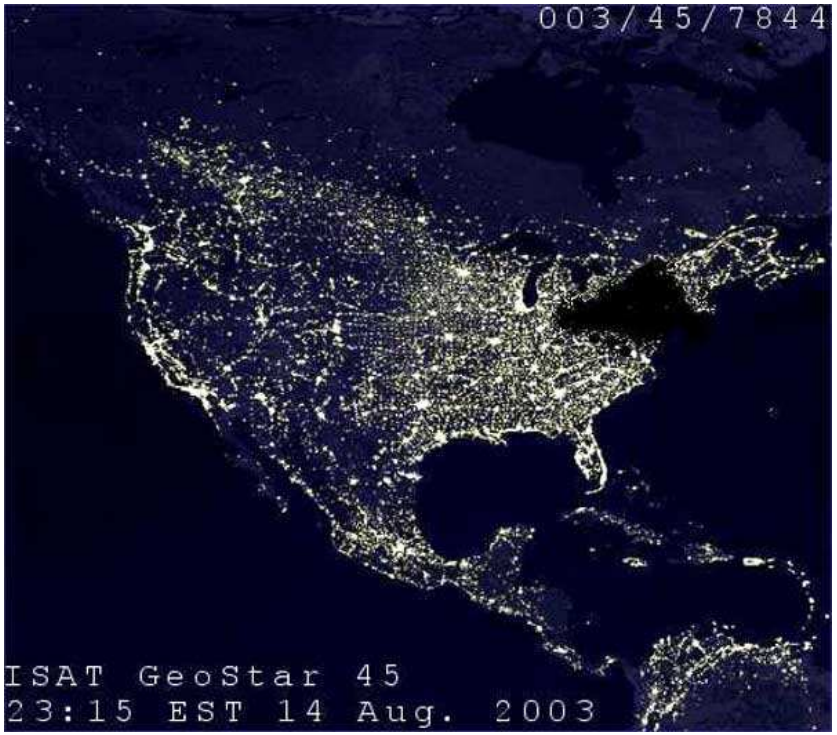
Institute for Operations Research, ETH Zurich

Critical Infrastructures as Complex Systems
Dresden, 5.10.2007

Reliability in Electricity Networks



Question

How should we react to emerging failures?

Reliability in Electricity Networks

Goal

We try to find repair strategies that allow to:

i. recover from blackouts
ii. prevent blackouts

$\Rightarrow$ We focus on combinatorial properties of the problems.

Overview

1. Blackout Recovery
2. Blackout Prevention

Overview

1 Blackout Recovery

2 Blackout Prevention

Blackout Recovery

Definition (Repair Strategy)

- *Input*: A power grid $G = (V, E)$ with cost $c(e)$ on every edge $e \in E$, a set $E_f \subseteq E$ of failed edges and a budget C.
- *Output*: A set of edges $E_r \subseteq E_f$ subject to the budget restriction $\sum_{e \in E_r} c(e) \leq C$.

Objective

Minimize the amount of unmet power demand.

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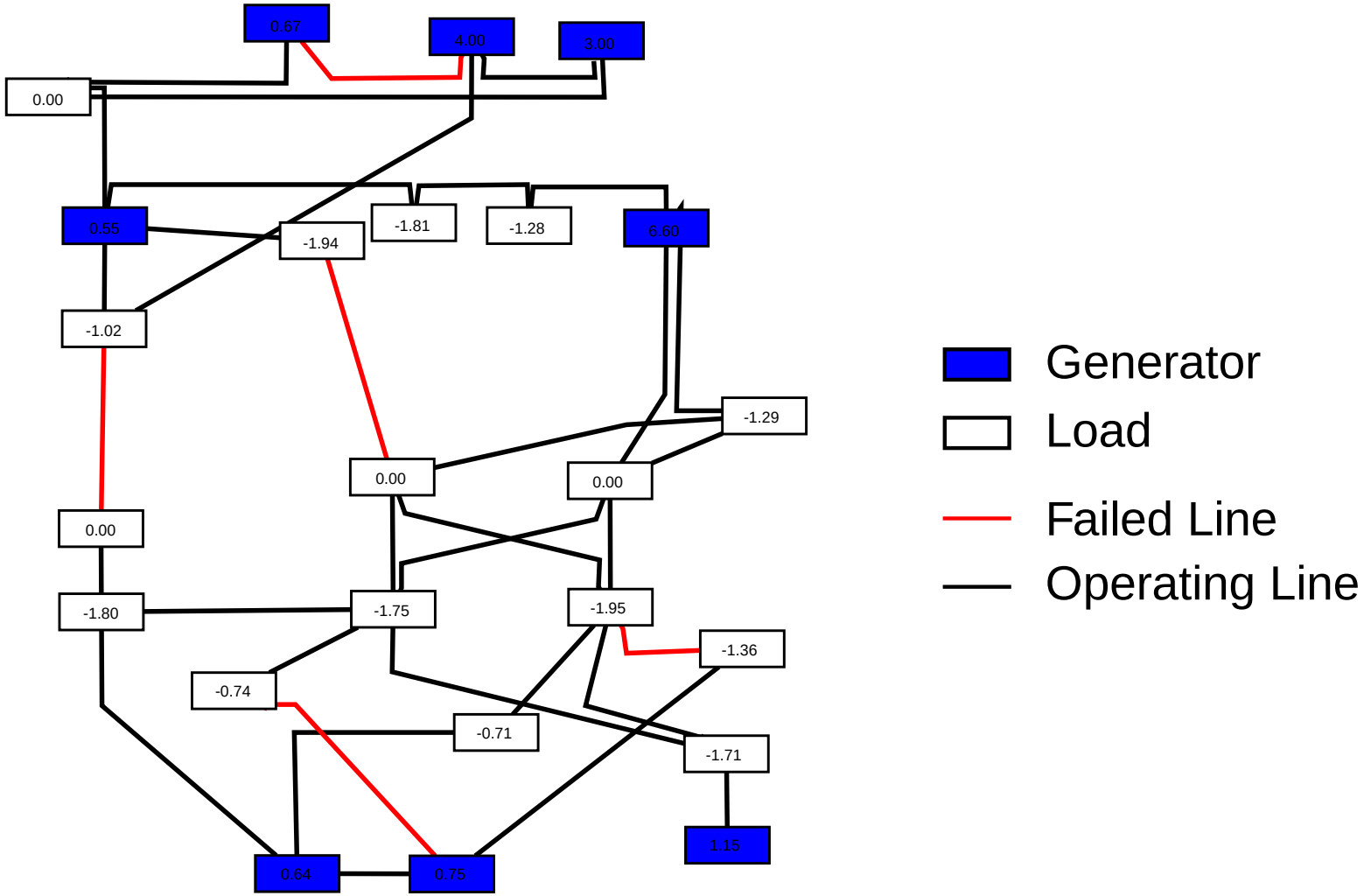
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Generator
Load
Failed Line
Operating Line

Power Flow Models

Model

How can we model the power flow on a transmission grid?

- Accurate: Complete set of physical equations.
- Approximate: *DC Power Flow* model.
- Abstract: *Network Flow* model.

Power Flow Models

Flow

A *flow* f is an assignment $f : E \to \mathbb{R}^+$.

Network Flow

Flow is constraint to:

- Capacity Limits
- Flow Conservation

DC Power Flow

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Find a repair strategy that maximizes the maximal *DC Power flow*.

Theorem

The decision problem corresponding to MaxDCFlowFixedCost *is $\mathcal{NP}$-hard.*

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Unless NP $\subseteq$ DTIME$(n^{O(\log\log n)})$, there is no approximation algorithm with ratio better than $\left(1-\frac{1}{e}\right)$ for the MaxFlowFixedCost *problem.*

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MaxFlowFixedCost

Theorem

The constructive problem corresponding to MaxFlowFixedCost *on unit capacity networks is in* $\mathcal{P}$.

Approximation Algorithm

Algorithm 1 MaxFlowFixedCostApprox(G,C)

Require: Graph $G = (V, E)$ with cost function $c : E \to \mathbb{Z}_0^+$, budget C
$u(e) \leftarrow U_{min}$ for every edge $e \in E$;
$A_{uc} \leftarrow$ optimal solution for this unit capacity network;
return A_{uc};

Theorem

The algorithm above is an $\frac{U_{min}}{U_{max}}$-approximation algorithm for MaxFlowFixedCost.

Approximation Algorithm

Algorithm 2 MaxFlowFixedCostApprox(G,C)

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Find a repair strategy that minimizes the risk of large blackouts.

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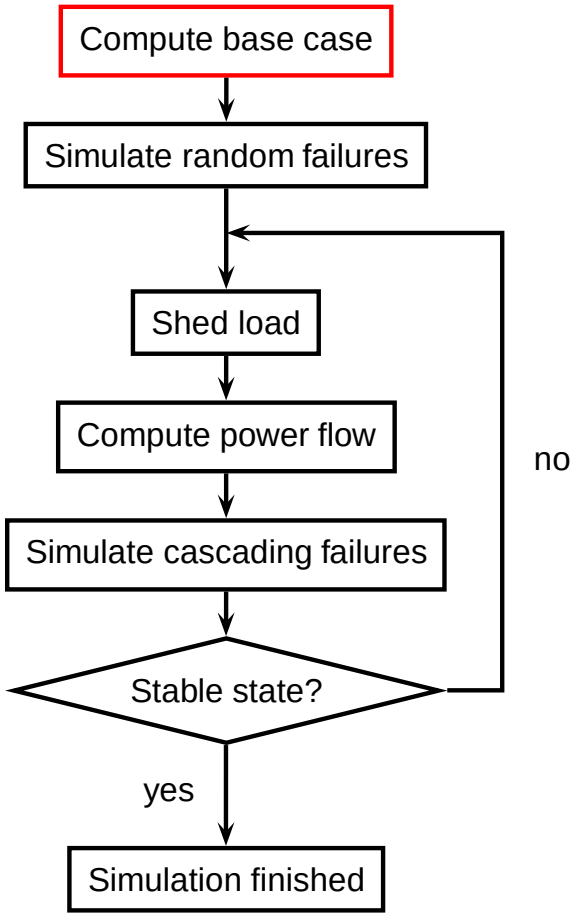
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Simulate random failures
Shed load
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Stable state?
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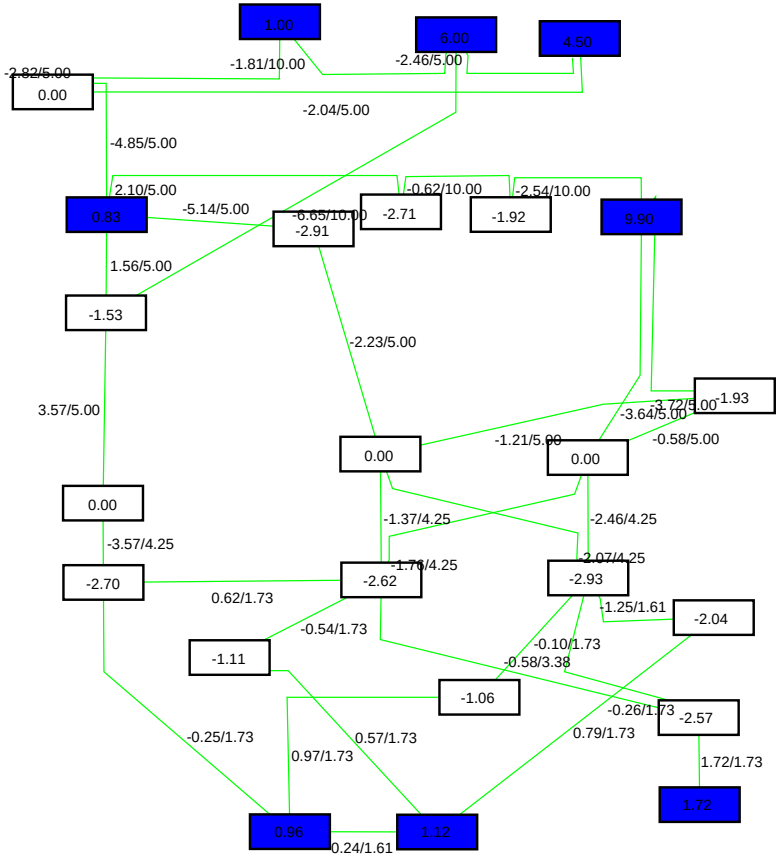
yes
Simulation finished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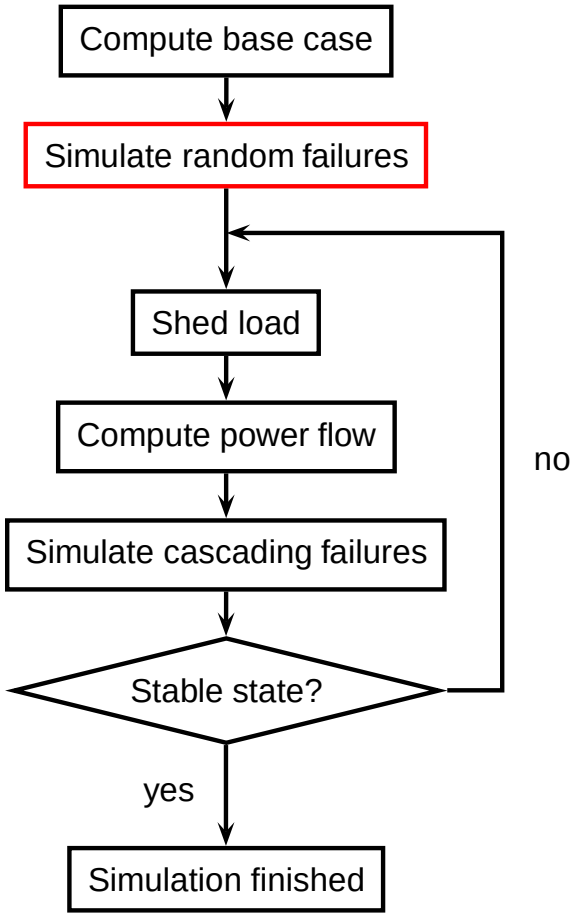
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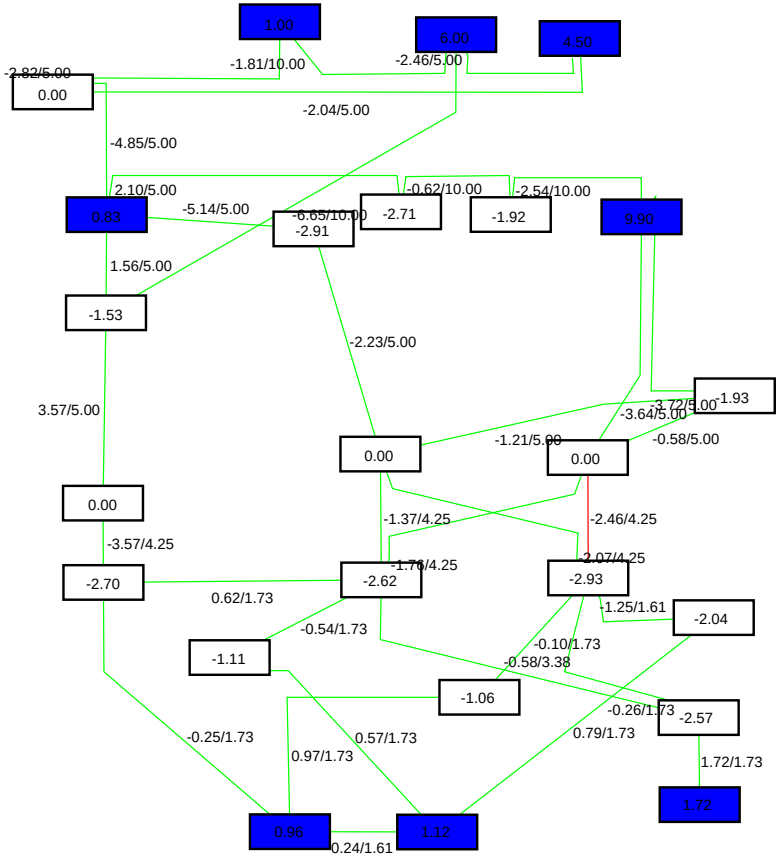
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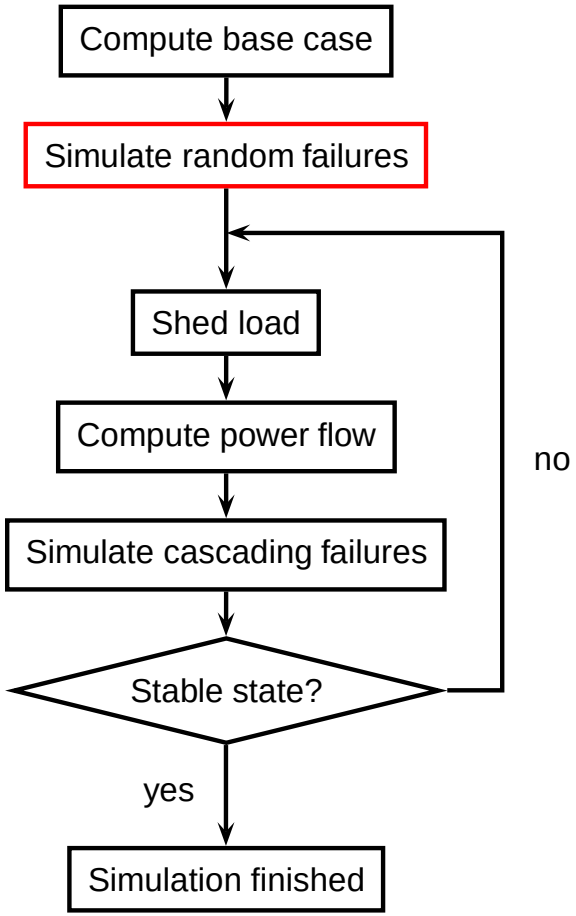
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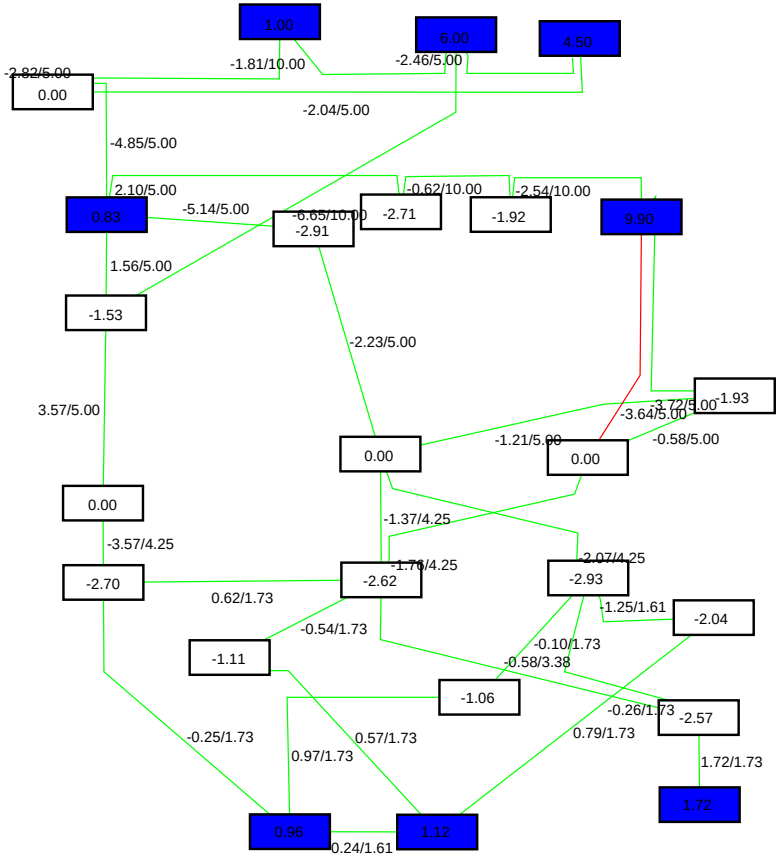
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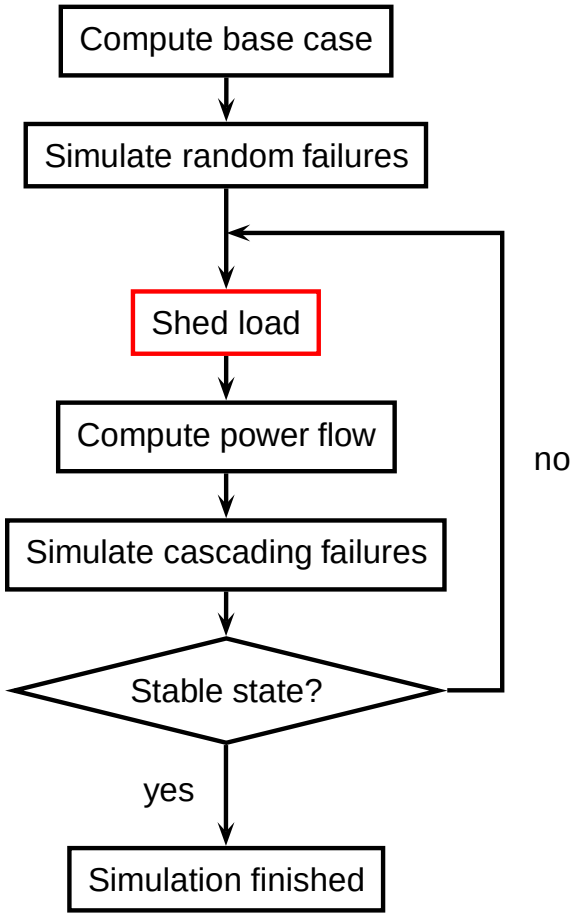
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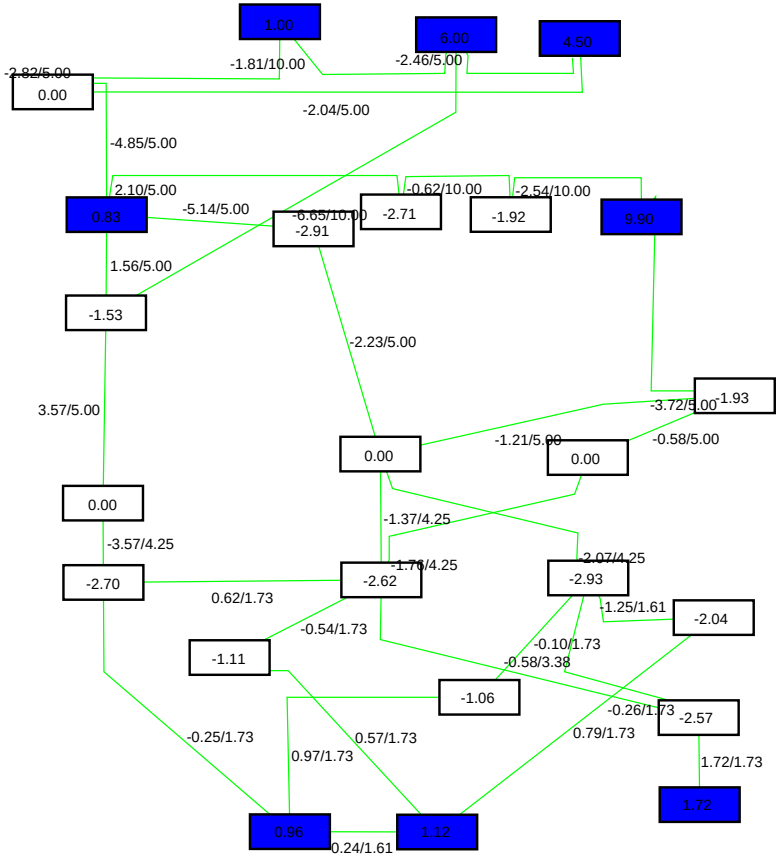
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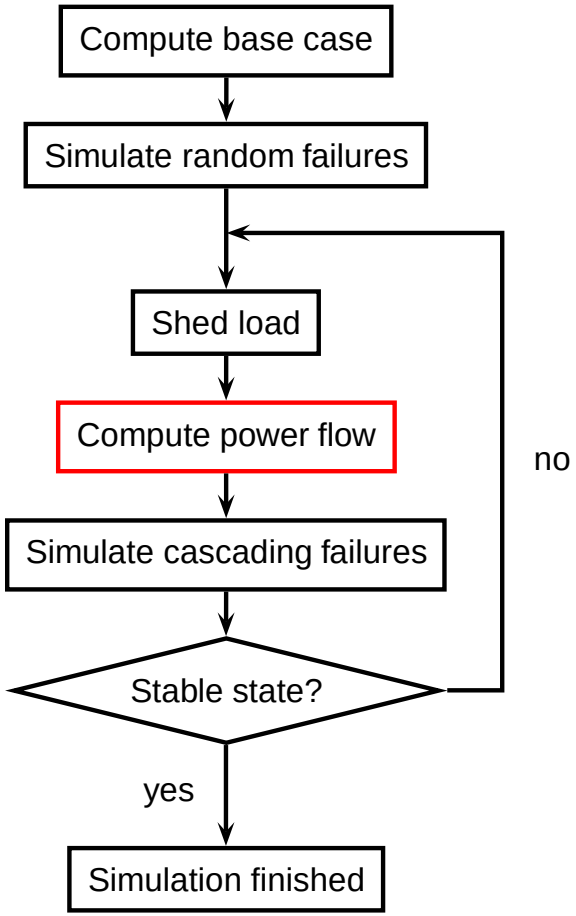
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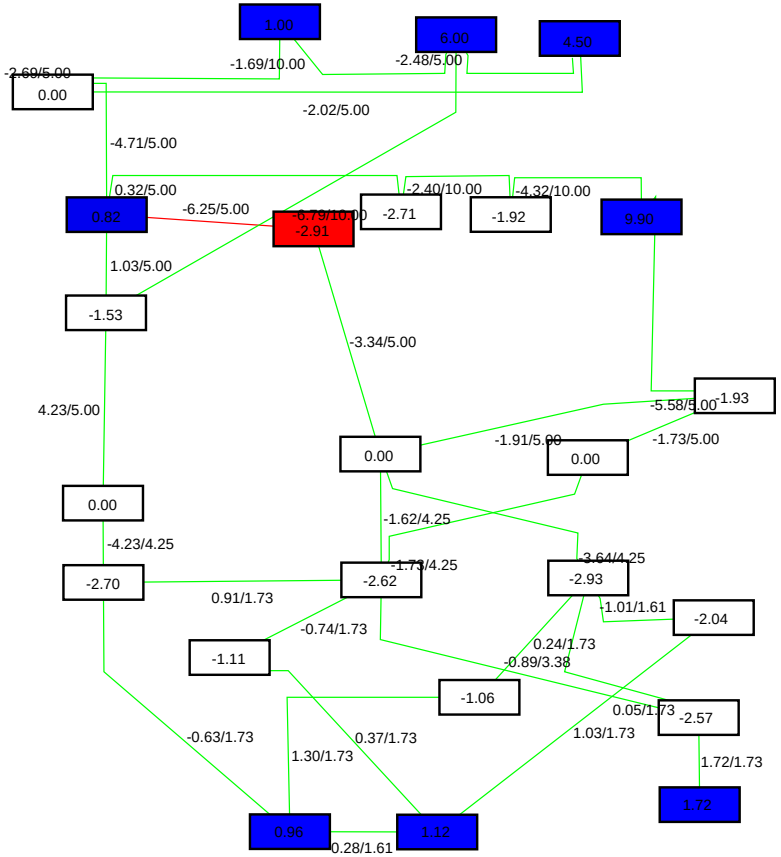
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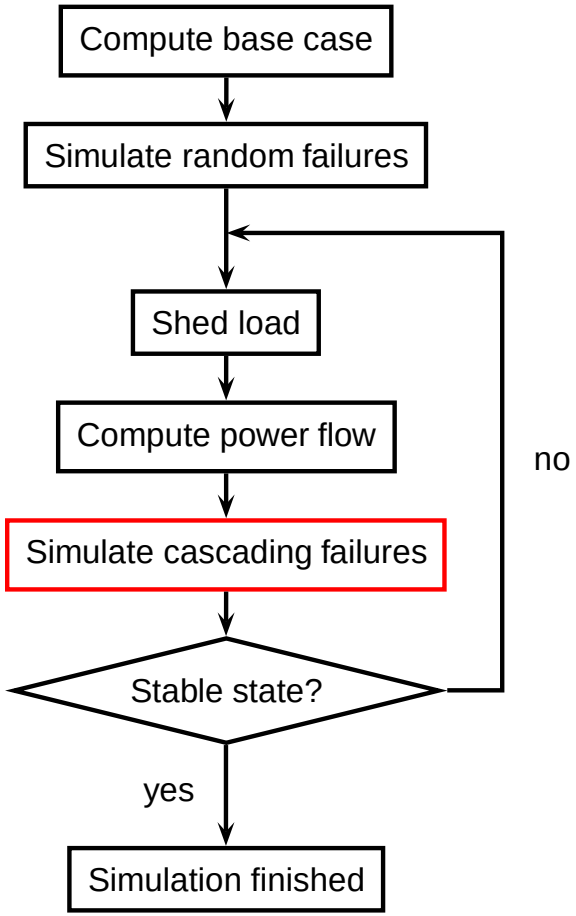
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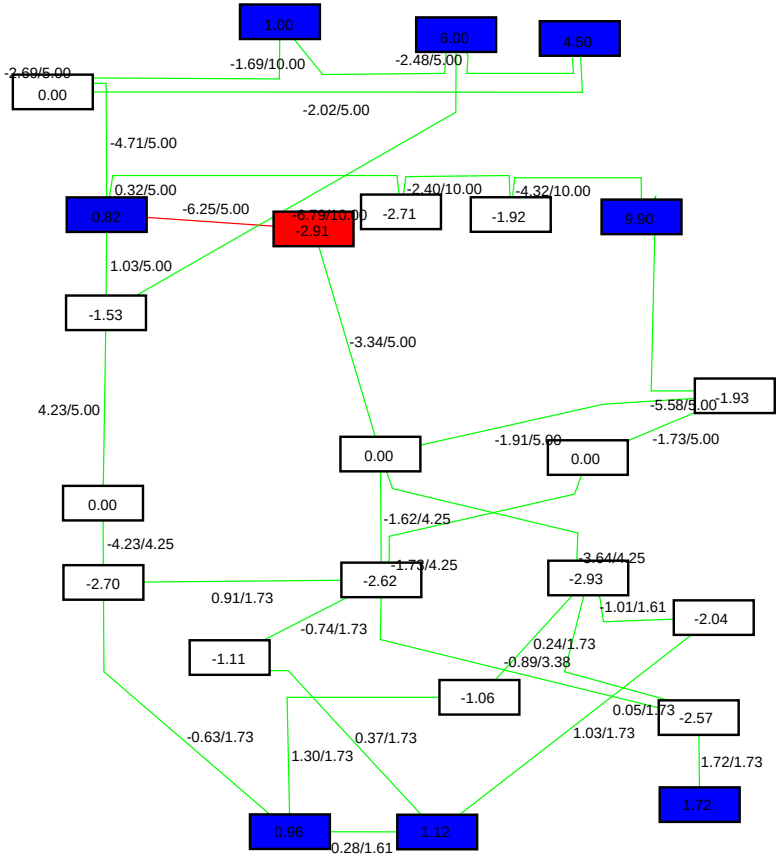
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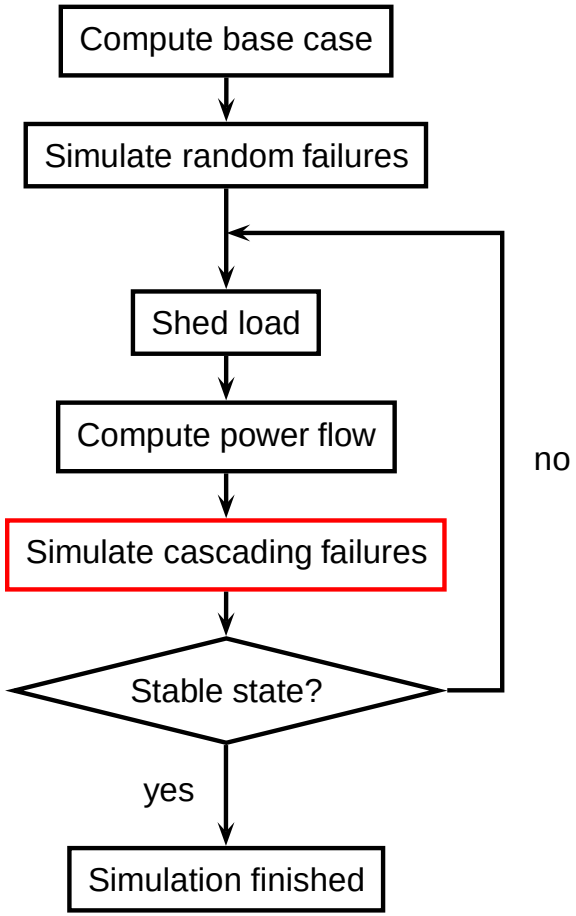
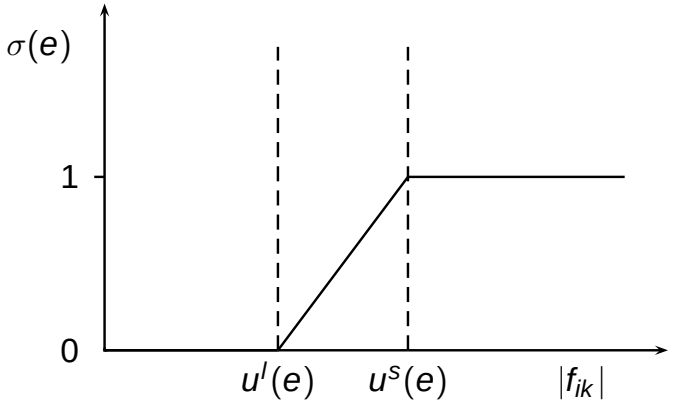
Blackout Model



We model the cascading failures by an independent probability of failure $\sigma(e)$ for each line.



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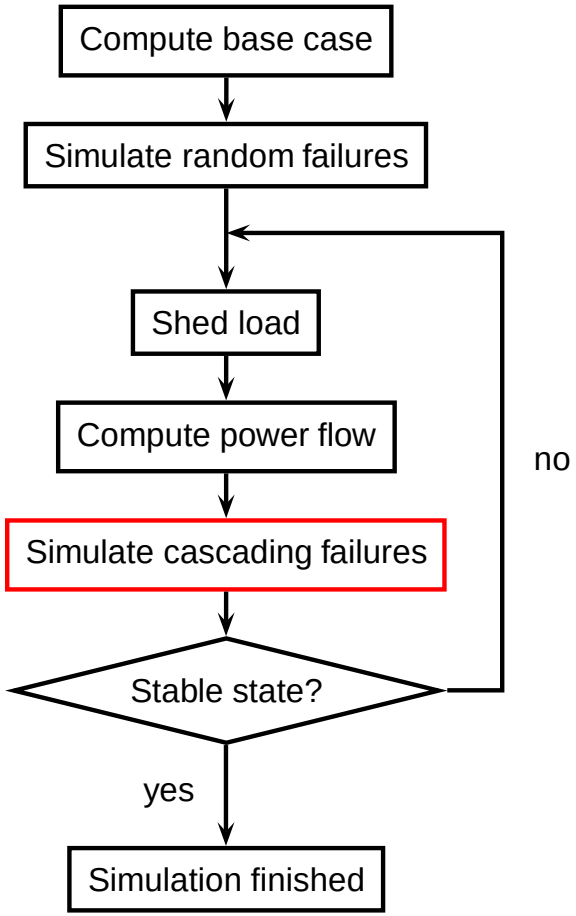
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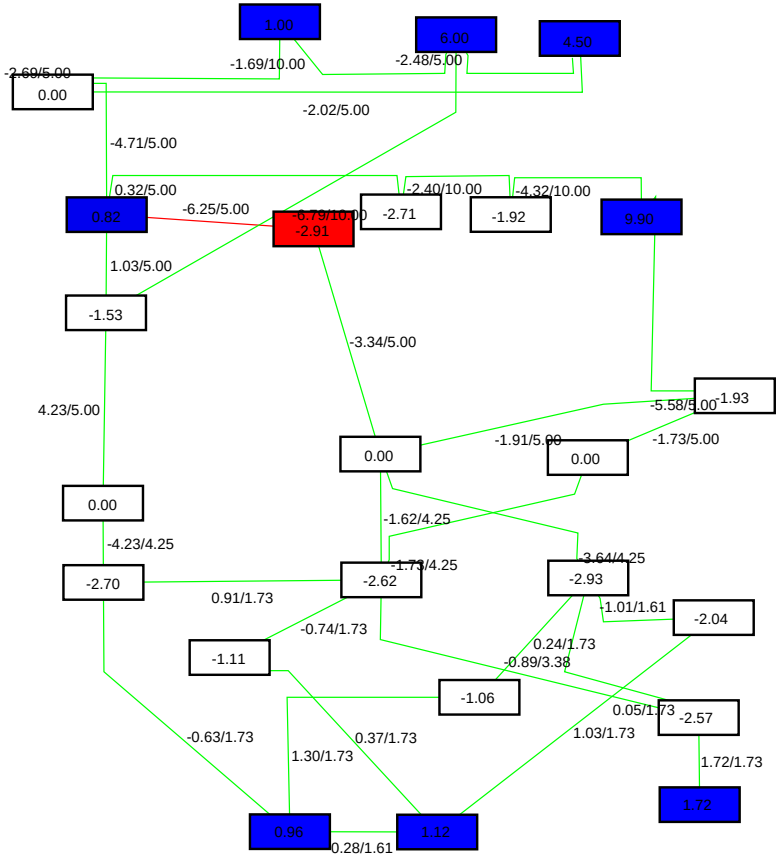
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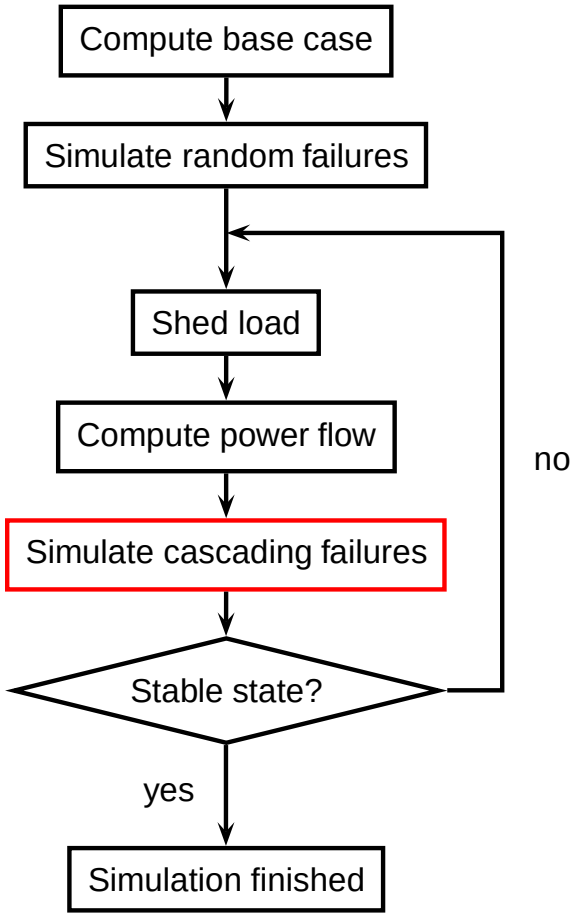
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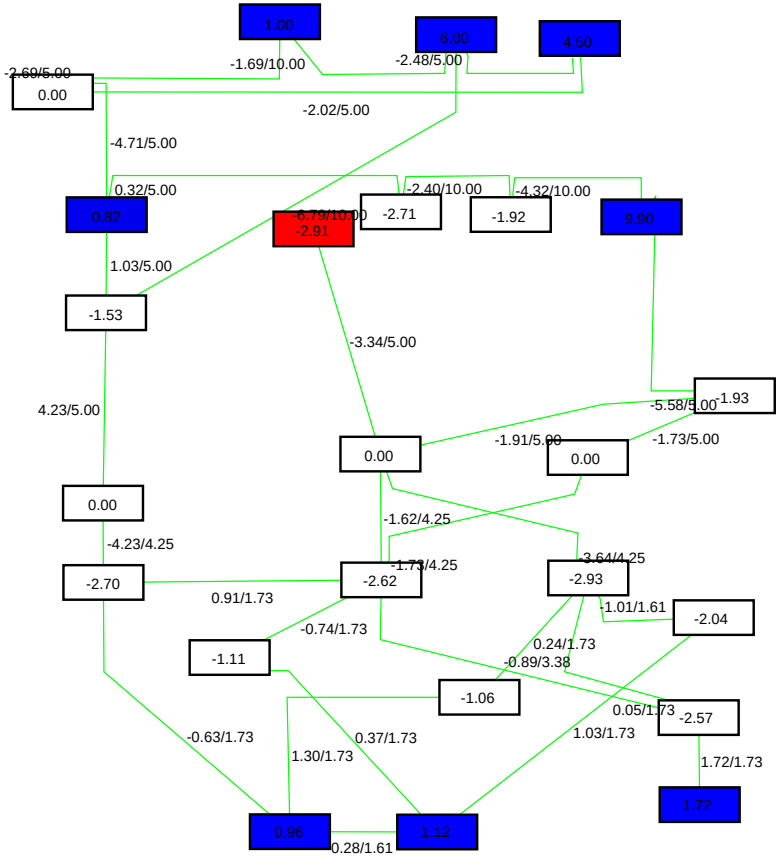
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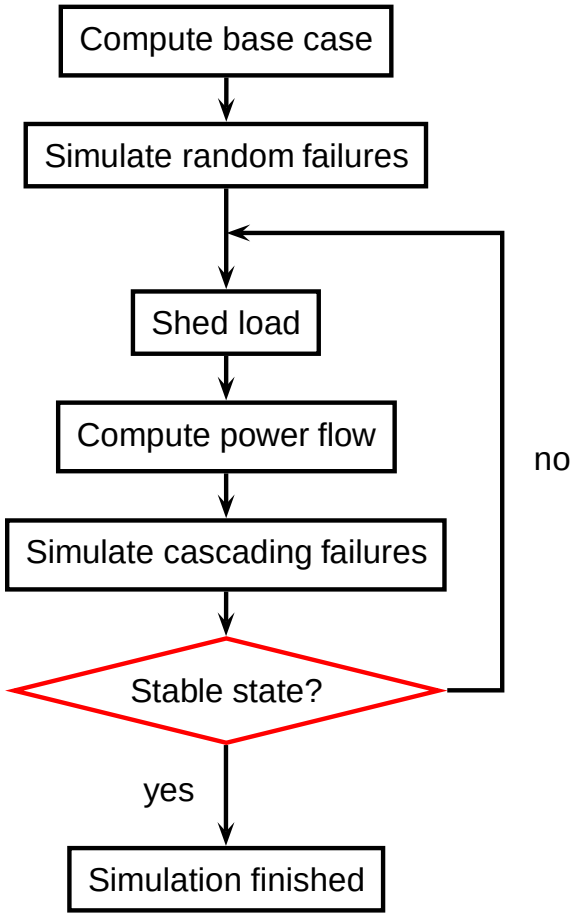
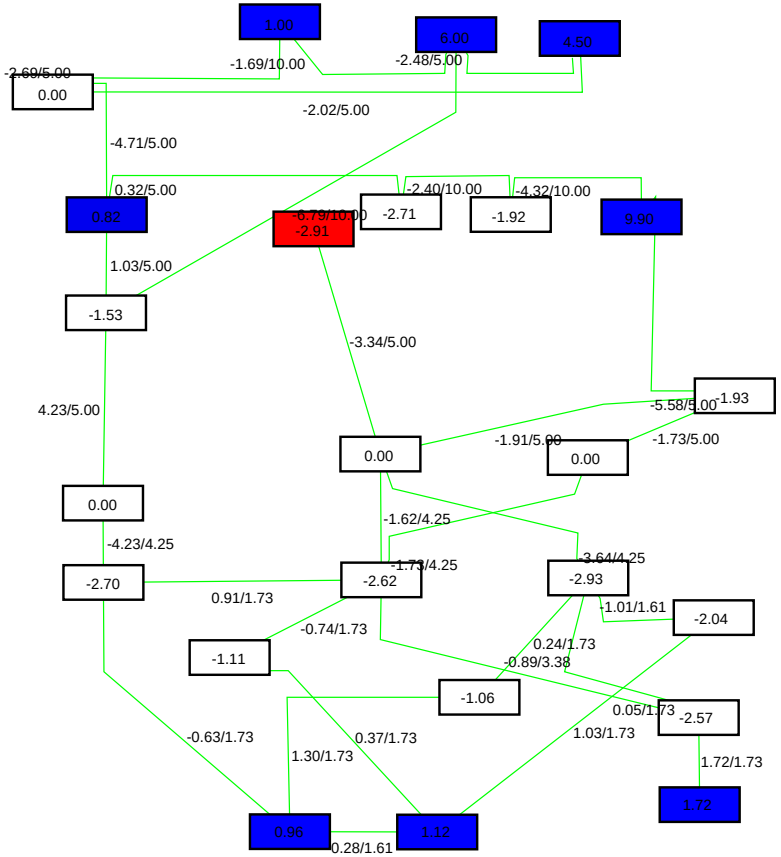
-2.02/5.00
-4.71/5.00
0.32/5.00
-2.40/10.00
-4.32/10.00
0.82
-6.79/10.00
-2.91
-2.71
-1.92
9.90
1.03/5.00
-1.53
-3.34/5.00
-1.93
-5.58/5.00
4.23/5.00
-1.91/5.00
-1.73/5.00
0.00
0.00
0.00
-1.62/4.25
-4.23/4.25
-1.73/4.25
-3.64/4.25
-2.70
-2.62
-2.93
0.91/1.73
-1.01/1.61
-2.04
-0.74/1.73
0.24/1.73
-1.11
-0.89/3.38
-1.06
0.05/1.73
-2.57
-0.63/1.73
0.37/1.73
1.03/1.73
1.30/1.73
1.72/1.73
1.72
0.96
1.12
0.28/1.61

Example


Compute base case
Simulate random failures
Shed load
Compute power flow
Simulate cascading failures
Stable state?
no
yes
Simulation finished


1.00
6.00
4.50
-1.69/10.00
-2.48/5.00
-2.69/5.00
0.00
-2.02/5.00
-4.71/5.00
0.32/5.00
-2.40/10.00
-4.32/10.00
0.82
-6.79/10.00
-2.91
-2.71
-1.92
9.90
1.03/5.00
-1.53
-3.34/5.00
-1.93
-5.58/5.00
4.23/5.00
0.00
-1.91/5.00
0.00
-1.73/5.00
0.00
-1.62/4.25
-4.23/4.25
-3.64/4.25
-1.73/4.25
-2.70
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-2.93
0.91/1.73
-1.01/1.61
-2.04
-0.74/1.73
0.24/1.73
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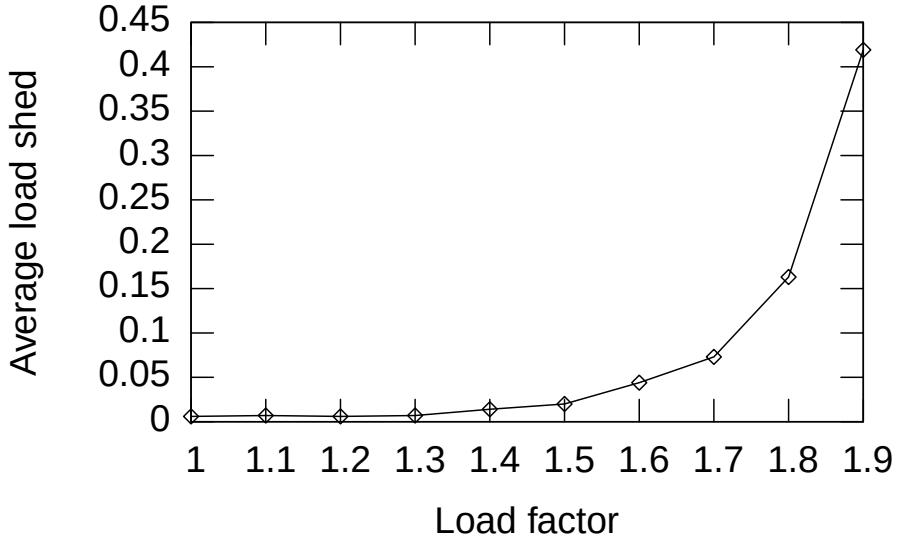
0.37/1.73
1.03/1.73
1.30/1.73
1.72/1.73
1.72
0.96
1.12
0.28/1.61

Loading Dependence



Observation

The blackout size sharply increases when uniformly scaling up the loads and generators.

Maximum Flow Strategy

Observation

The initial reserve capacities on the lines seem to be a good measure for the risk of cascading failures.

Maximum Flow Strategy

Maximizing the reserve capacity is a proxy for minimizing the blackout risk.

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Maximum Flow Strategy

Definition (Reserve Capacity)

Let the *reserve capacity* be the difference between the value of the maximum flow and the value of the current flow.

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Maximizing this measure for the reserve capacity given some budget corresponds to the MaxFlowFixedCost/MaxDCFlowFixedCost problem.

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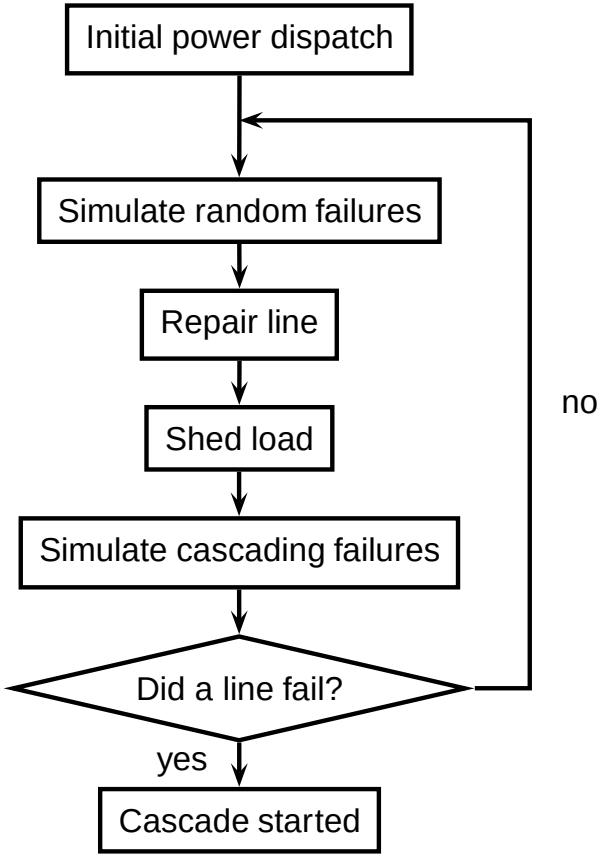
Blackout Prevention Model



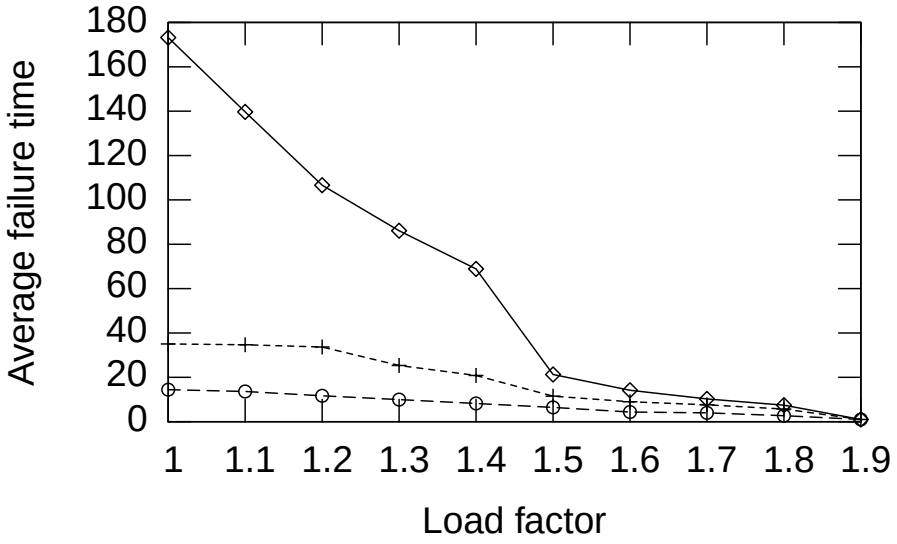
Idea

We test the ability of the maximum flow strategy to prevent cascading failures.

Definition (Failure time)

The *failure time* is the number of time steps elapsed until the first line gets tripped due to overloading.

Results



◇ Maximum Flow Strategy

+ Random Repair

○ Without Repair

Observation

The average failure time is up to five times bigger for the maximum flow strategy than for the random repair strategy.

Conclusion

Blackout Recovery

- *Blackout Recovery* is a computationally hard problem.

Blackout Prevention

- Maximizing the reserve capacity is a reasonable objective for *Blackout Prevention*.