Mobility promotes and jeopardizes biodiversity in rock-paper-scissors games

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Ludwig-Maximilians-Universität München LMU

Arnold Sommerfeld Center for Theoretical Physics

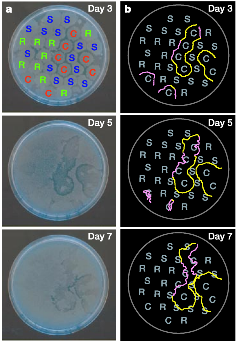
CeNS Center for NanoScience

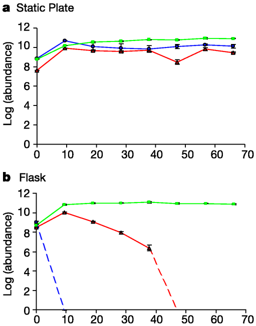
Colicinogenic Bacteria[1]

Three strands of E.coli:

- C: colicin-producing
- S: sensitive
- R: resistant

Different environments:
Plate versus Flask





Spatial structure matters!

Patterns?
Transition from Plate to Flask?

[1]B. Kerr et al., Nature **418** 171 (2002)

May-Leonard model

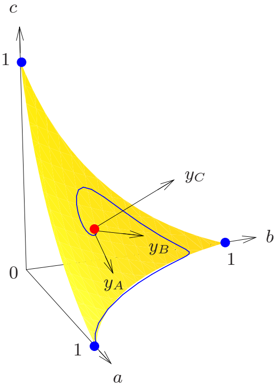
Species $A,\ B,\ C$ and empty sites $\oslash$

Cyclic dominance:

$$\begin{aligned} AB &\xrightarrow{\sigma} A\oslash \\ BC &\xrightarrow{\sigma} B\oslash \\ CA &\xrightarrow{\sigma} C\oslash \end{aligned}$$

Birth:

$$\begin{aligned} A\oslash &\xrightarrow{\mu} AA \\ B\oslash &\xrightarrow{\mu} BB \\ A\oslash &\xrightarrow{\mu} CC \end{aligned}$$



Rate equations (without spatial structure):

- Coexistence fixed point is unstable
- Trajectories spiral outwards on an invariant manifold
- Heteroclinic orbits

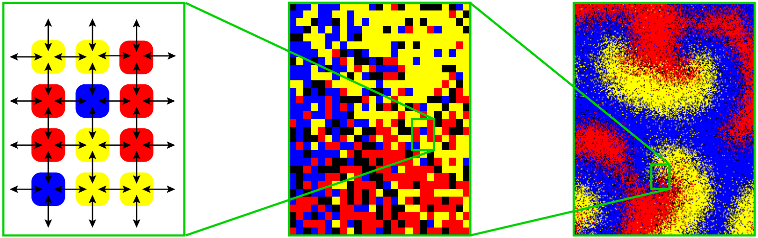
$\longrightarrow$ Loss of biodiversity

May-Leonard model on a lattice ($N = L^2$)[2]

Add migration:

$$\begin{aligned} A\oslash &\xrightarrow{\epsilon} \oslash A \\ AB &\xrightarrow{\epsilon} BA \\ \dots \end{aligned} \quad \Longrightarrow \text{macroscopic diffusion } D = \frac{1}{2L^2}\epsilon$$

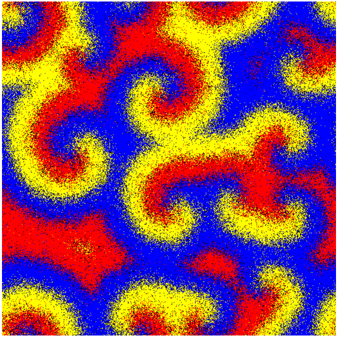
While stochastic effects dominate the system's appearance at small scales, regular spiral waves form at larger scales



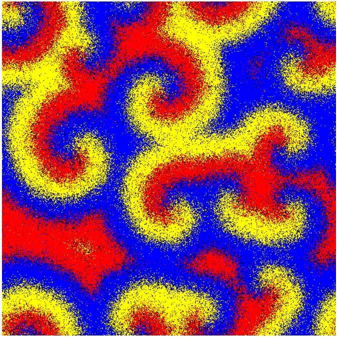
[2] T. Reichenbach, M. Mobilia, and E. Frey, Nature **448**, 1046 (2007)

Rotating spiral waves

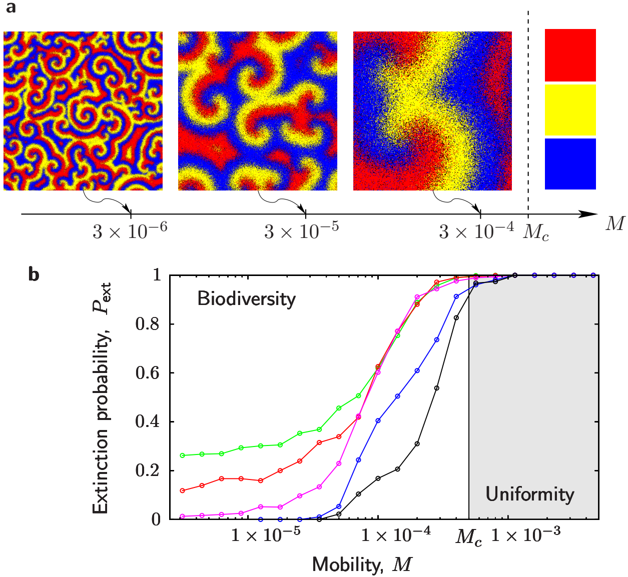
$D = 3 \times 10^{-5}$:



$D = 3 \times 10^{-4}$:



Biodiversity is lost above critical diffusivity



$P_{\text{ext}} = \{\text{Prob. of only one species after time } t \sim N\}$

Mathematical Description[3]

Look for a description in terms of local densities

$$\mathbf{s}(\mathbf{r}, t) = (a(\mathbf{r}, t), b(\mathbf{r}, t), c(\mathbf{r}, t))$$

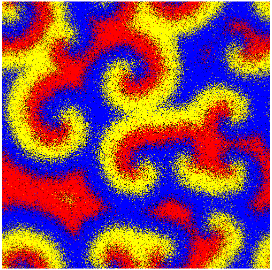
Continuum limit:
Stochastic partial differential equations:

$$\partial_t s_i(\mathbf{r}, t) = D\Delta s_i + \mathcal{A}_i(\mathbf{s}) + C_i(\mathbf{s})\xi_i$$

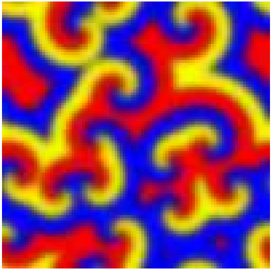
→ Numerically solvable (www.xmds.org)

Results from SPDE agree with lattice simulations (look at correlation functions)

Lattice simulations:



SPDE:



[3]T. Reichenbach, M. Mobilia, and E. Frey, accepted at Phys. Rev. Lett.

Complex Ginzburg-Landau Equation

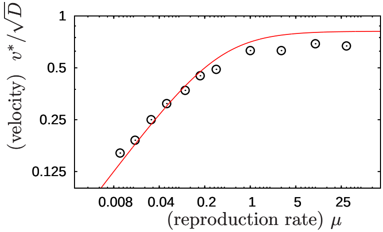
In the vicinity of the reactive fixed point, the PDE can be written as a complex Ginzburg-Landau equation:

$$\partial_t z = D\Delta z + c_1 z - (1 - ic_3)|z|^2 z$$

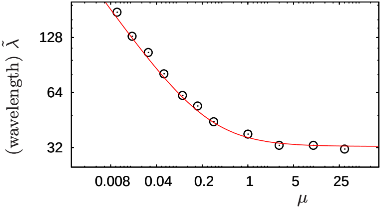
⇒ Analytic expressions for velocity, frequency, and wavelength of the spirals

Compare analytic results to results from SPDE:

Velocity:

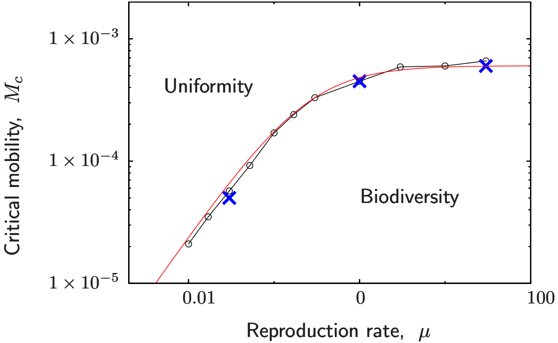


Wavelength:



State diagram

Numerical observation: there is a universal critical value λ_C above which the system looses biodiversity



blue: Lattice Simulations
red: CGLE
black: SPDE

Conclusions

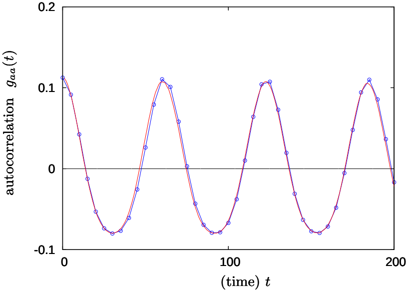
- Mixed populations: Fluctuations due to finite system size lead to uniformity
- Local interactions and diffusion: pattern formation and biodiversity
- There is a threshold value for the diffusion constant above which biodiversity is lost
- Analytic description possible via Complex Ginzburg Landau Equation

T. Reichenbach, M. Mobilia, and E. Frey,

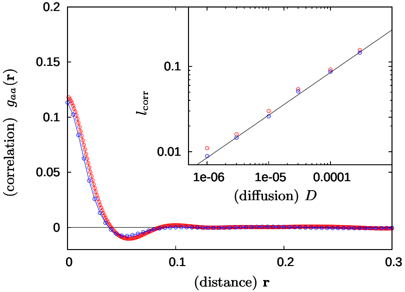
- Phys. Rev. E **74**, 051907 (2006)
- Nature **448**, 1046 (2007) F1000 Recommended F1000 Factor **3.2**
- arXiv:0710.0383 [q-bio.PE], accepted at Phys. Rev. Lett.

Correlation functions

In time: oscillations:



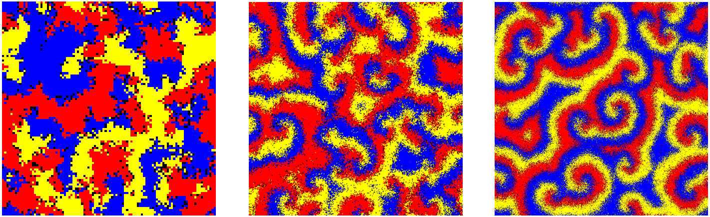
In space: correlation length:



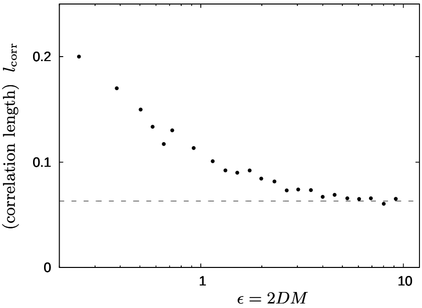
red: lattice simulations
blue: numerical solution of SPDE

Scaling of the correlation length: $l_{\mathrm{corr}} \sim \sqrt{D}$

Convergence to the continuum limit (where SPDE hold)



$L = 100(\epsilon = 0.2)$ $L = 300(\epsilon = 1.8)$ $L = 500(\epsilon = 5)$



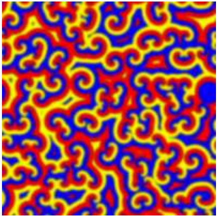
$\epsilon = 2DL^2$

Continuum description works already for $\epsilon \geq 5$ $(\beta = \gamma = 1)$

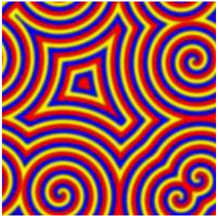
Stochastic versus deterministic PDE

In the SPDE, noise is $\sim 1/L$, thus tends to zero when $L \to \infty$. Neglect noise $\to$ deterministic PDE

Stoch. PDE



Det. PDE



- Deterministic PDE yield correct wavelength and frequency of the spirals
- Noise breaks large spirals up into a structure of short entangled spirals