Complexity of Coupled Map Lattices

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Complexity Measure

N. Ay et al. proposed a vector valued complexity measure

$$I := (I_1, \dots I_N),$$

which is computed from a discrete time series.

I_k quantifies the dependencies between k units, that cannot be explained by dependencies of any $k-1$ nodes.

- The exponential family $\mathcal{E}_k$ contains only distributions with interactions between at most k units.
- Components I_k are defined as Kullback-Leibler distances between projections to $\mathcal{E}_k$ and $\mathcal{E}_{k-1}$.
- Theoretical result: I_2 equals the multi-information and is maximal in synchronization.

We call a dynamics *complex*, if it has high values of I_k for $k \geq 3$.

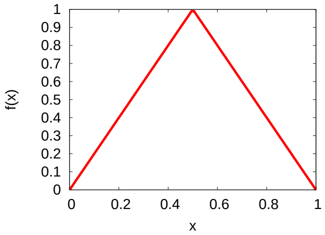
Our aim : To identify *complex* dynamics in a coupled map lattice.

Model System: Coupled Tent Maps

- coupled tent maps on a graph with adjacency matrix (g_{ij})
- discrete time $t = 0, 1, 2, \dots$ and real values $x_i(t) \in [0, 1]$.
- simultaneous updates according to

$$x_i(t+1) = \epsilon \sum_j \frac{g_{ij}}{k_i} f(x_j(t)) + (1-\epsilon) f(x_i(t))$$

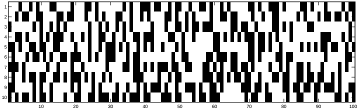
where f is the tent map.



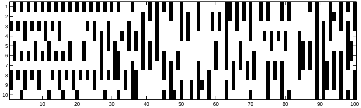
Main Example: Circle of 10 Nodes

Symbolic Dynamics of 10-circle

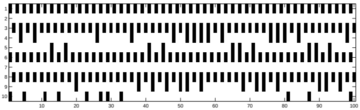
Assign to each node a symbol $\begin{cases} 0 & \text{if } x_i(t) \leq \frac{1}{2} \\ 1 & \text{otherwise} \end{cases}$.



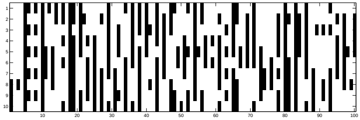
$\epsilon = 0.05$



$\epsilon = 0.464$

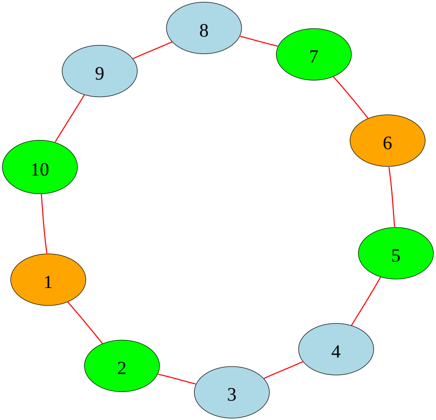


$\epsilon = 0.47$: *partial synchronization*

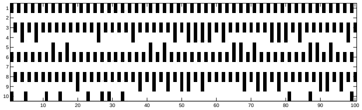


$\epsilon = 0.84$

Special Regime of 10 Node Circle at $\epsilon = 0.47$



Partial Synchronization of 10 nodes



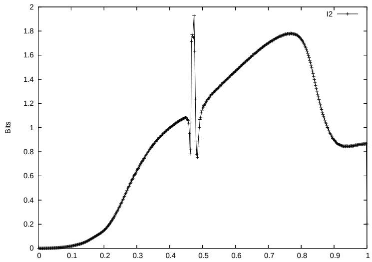
$t \longrightarrow$

$\epsilon = 0.47$

Partial synchronization.

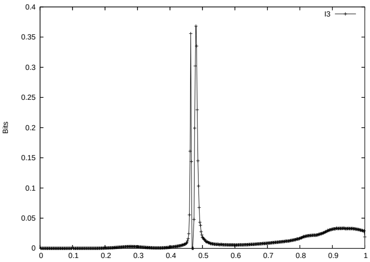
- 2 nodes constant
- 4 node almost quasiperiodic with large amplitude
- 4 nodes almost quasiperiodic with smaller amplitude

Results



$\epsilon \longrightarrow$

I_2 *for the 10-cycle*



$\epsilon \longrightarrow$

I_3 *for the 10-cycle*

- Vector I detects the synchronization
- “Complex Dynamics” on the edge of synchronization

Poster (#301): I_4, I_5, I_6, full graph, $\ldots$