Computation of the MLE for bivariate interval censored data

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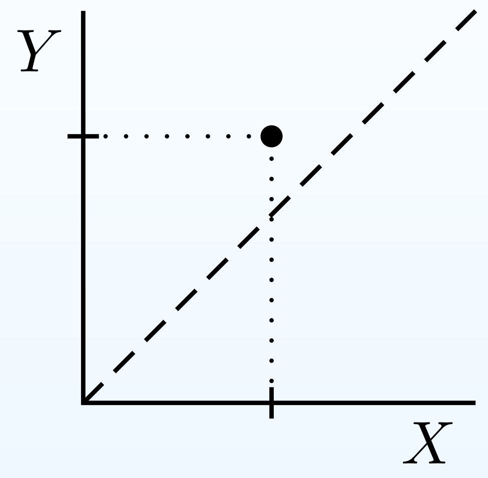
Bivariate interval censored data: an example

- We want to estimate the joint distribution function of (X, Y), where:
  - X: time of HIV infection
  - Y: time of onset of AIDS



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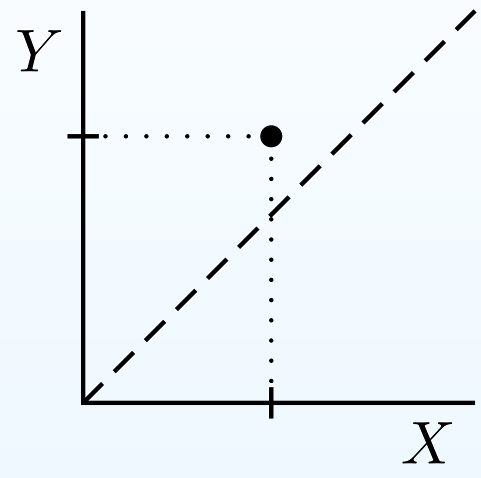

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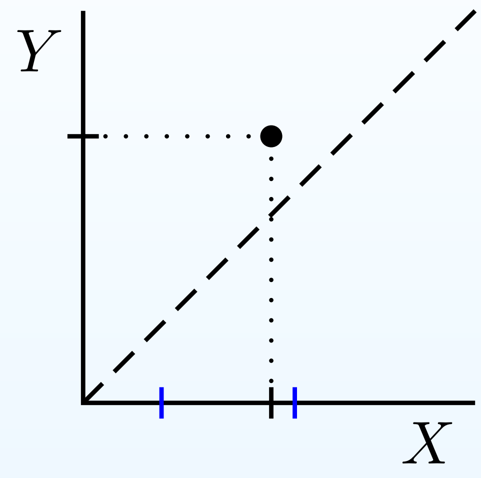

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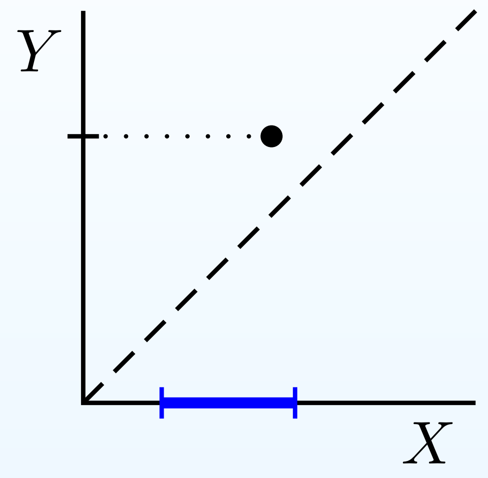

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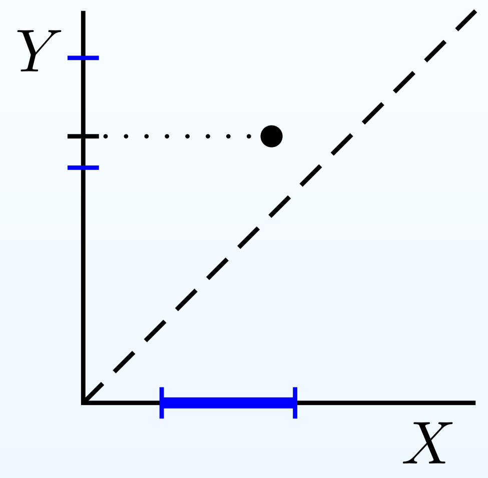

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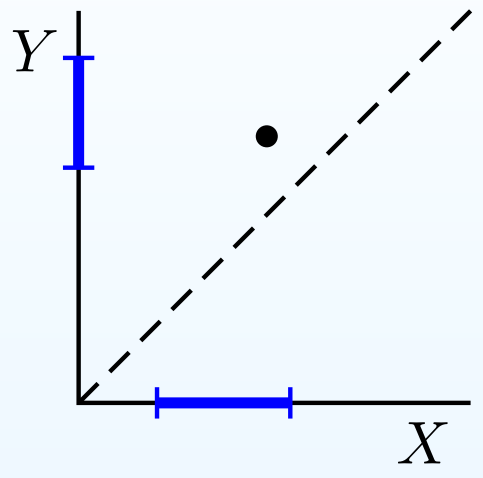

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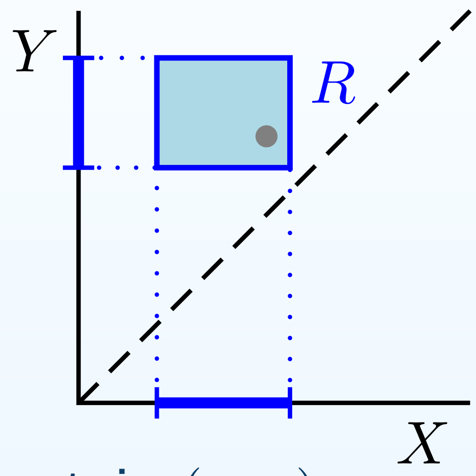

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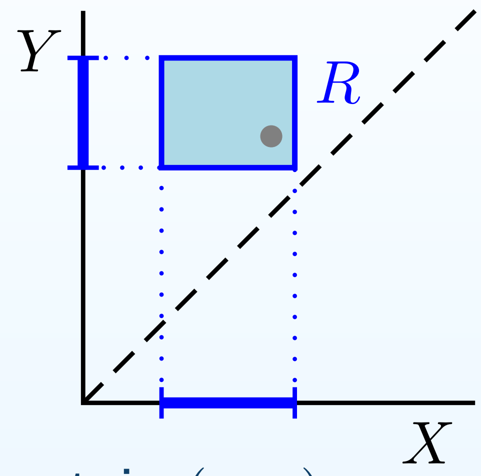
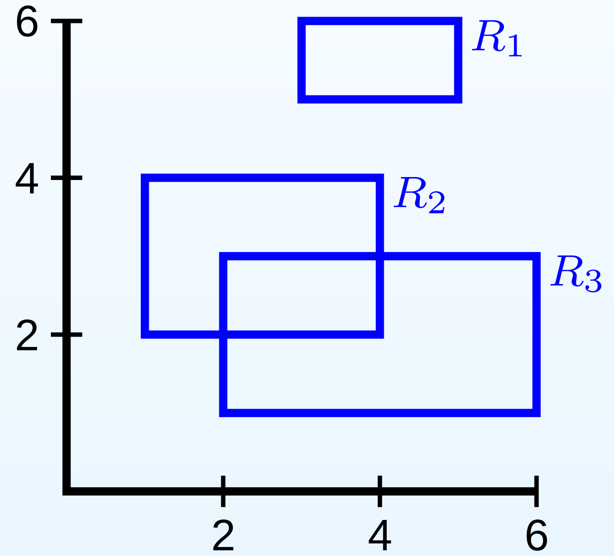


- X and Y can be interval censored. Instead of a realization (x, y), we observe an *observation rectangle* R that is known to contain (x, y).
- Goal: based on n i.i.d observation rectangles $R_1, \ldots, R_n$ we want to compute the MLE for the joint distribution function of (X, Y).

The nonparametric maximum likelihood estimator

$\max_{F \in \mathcal{F}} \sum_{i=1}^{n} \log(P_F(R_i))$



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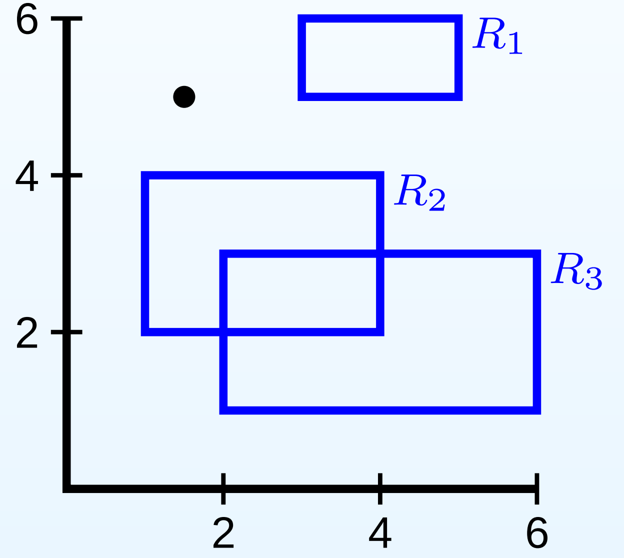

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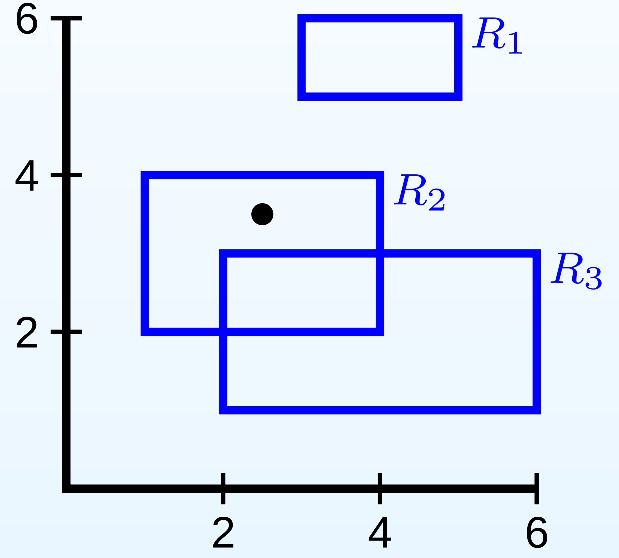

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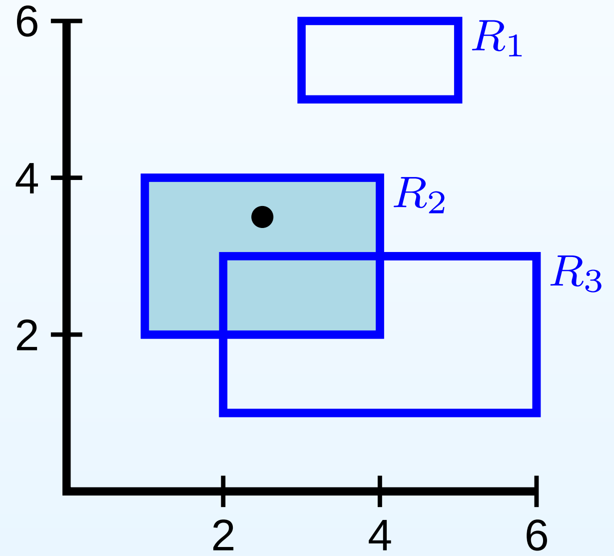

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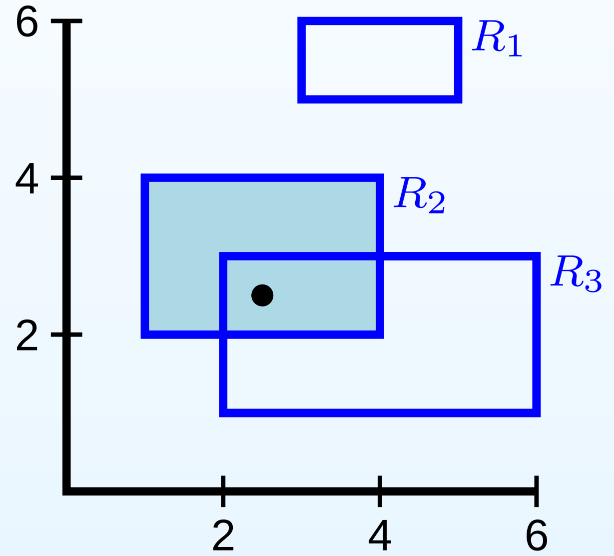

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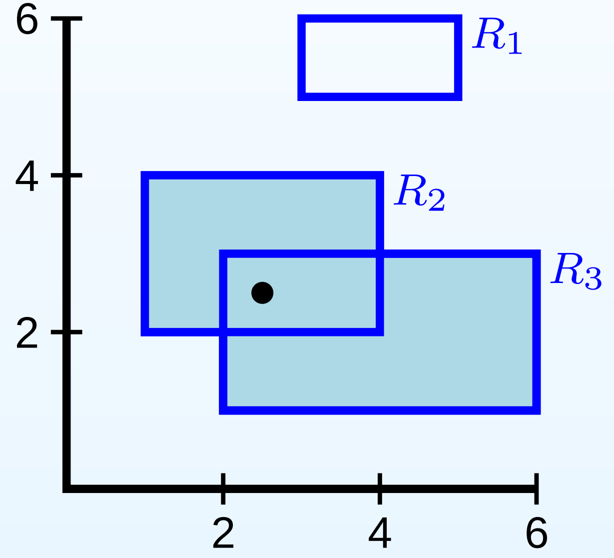

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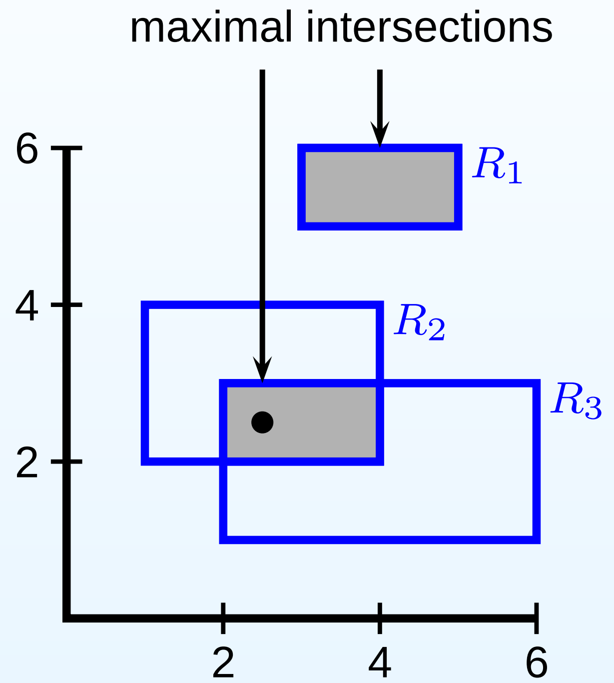

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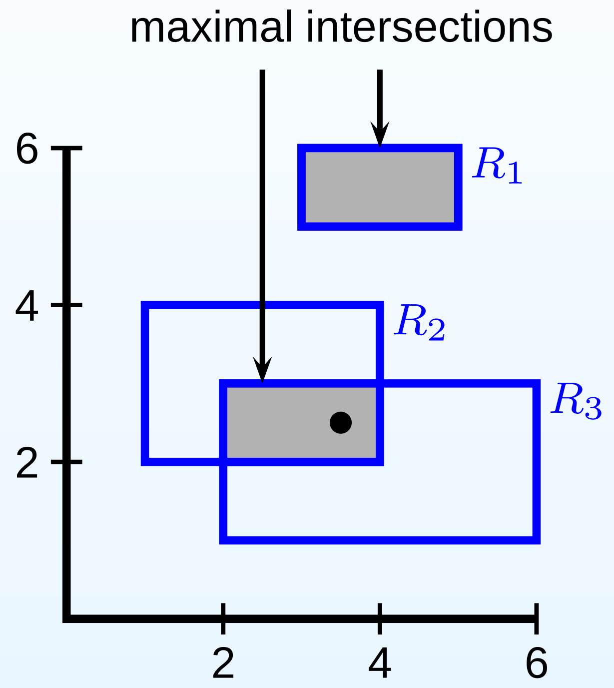

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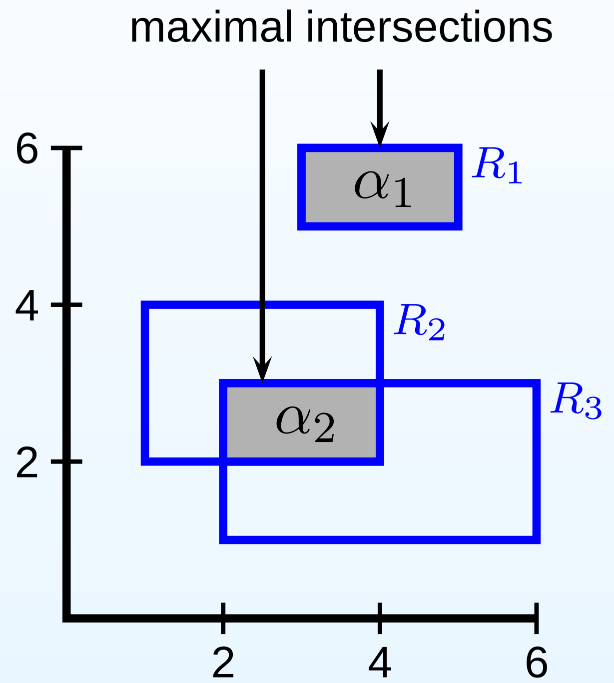

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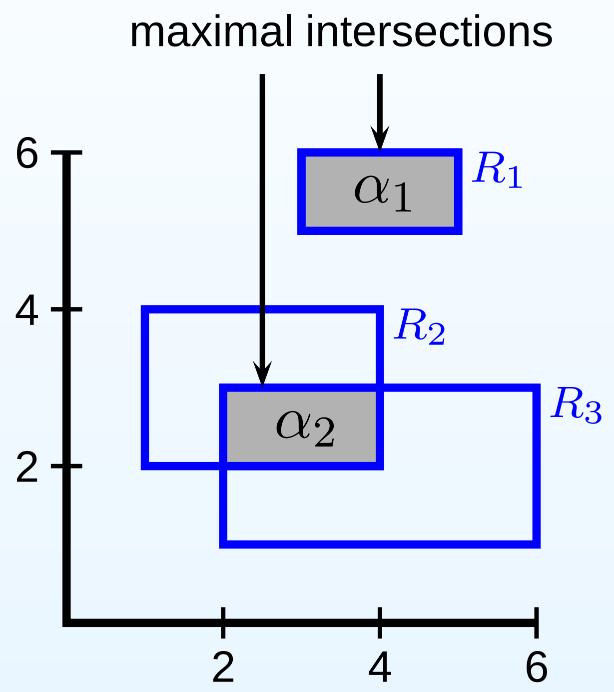

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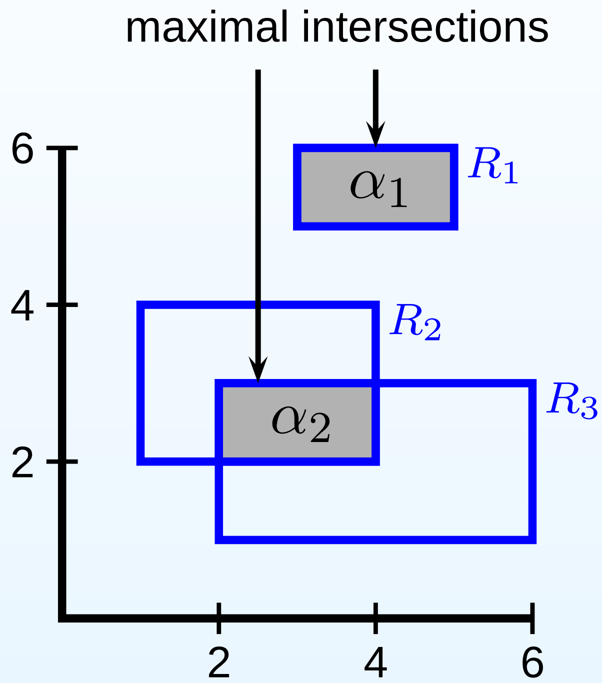

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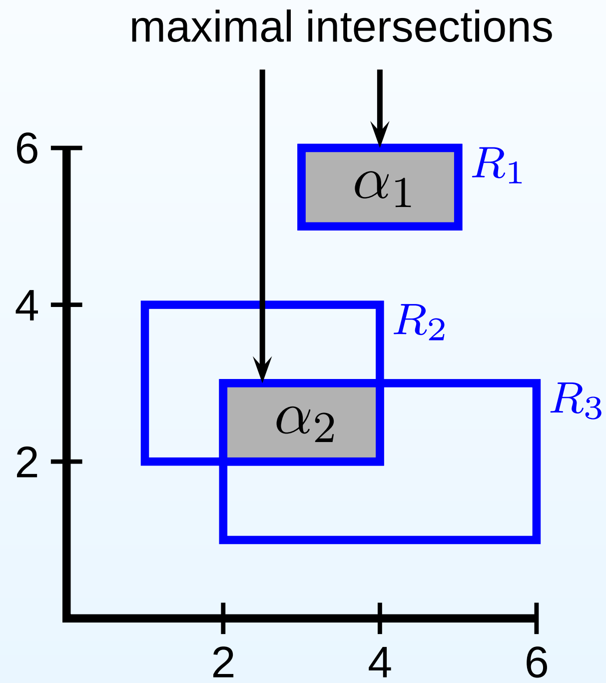
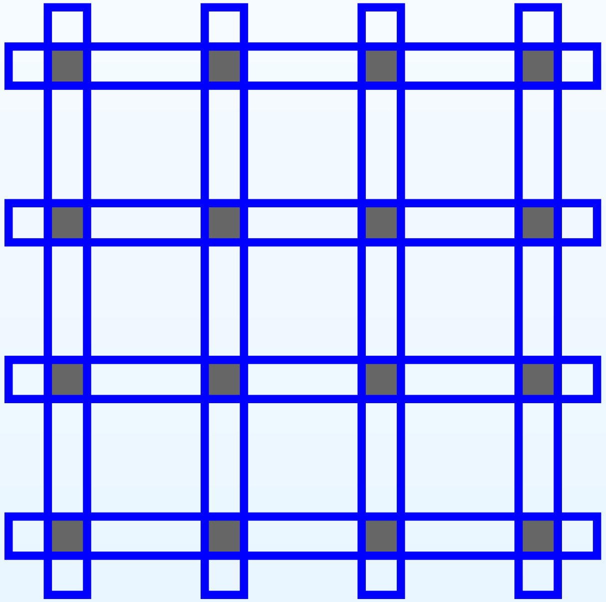


Computation of the MLE:

- Reduction step: find maximal intersections
- Optimization step: solve optimization problem in α

Difficulty in the computation of the MLE

- Number of maximal intersections can be very large:
  - For bivariate censored data: $O(n^2)$



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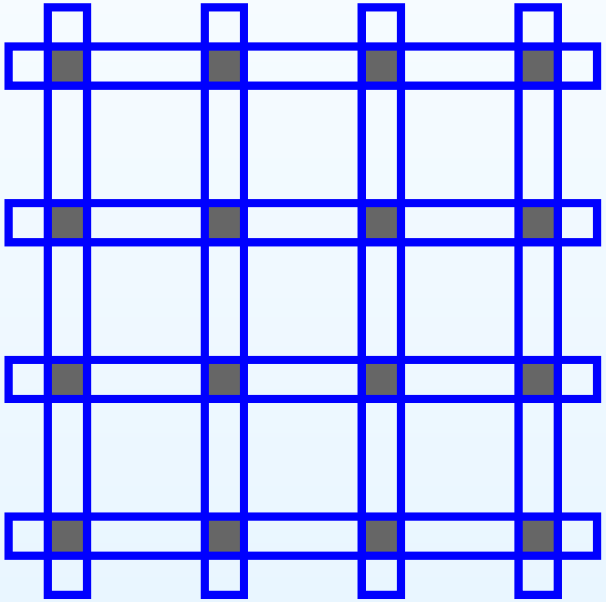

  - For d-variate censored data: $O(n^d)$

Outline

- Reduction step
- Optimization step
- R-package ‘MLEcens’

Reduction step

Previous work:

- Betensky and Finkelstein (1999)
- Song (2001)
- Gentleman and Vandal (2001), time complexity $O(n^5)$
- Bogaerts and Lesaffre (2004), time complexity $O(n^3)$

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Related algorithm: finding the maximum number of rectangles having a non-empty intersection:

- Lee (1983), time complexity $O(n \log n)$

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Related algorithm: finding the maximum number of rectangles having a non-empty intersection:

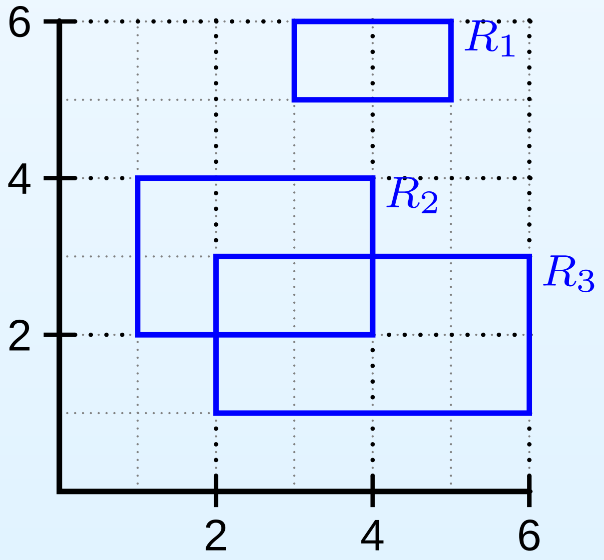
- Lee (1983), time complexity $O(n \log n)$

Our reduction algorithm, motivated by Lee (1983):

- Height Map Algorithm, time complexity $O(n^2)$

Height Map Algorithm

- Definition: $A_j \neq \emptyset$ is a *maximal intersection* if and only if $A_j = \cap_{i \in \beta_j} R_i$ for some set $\beta_j \subset \{1, \ldots, n\}$ and there is no strict superset $\beta_j^* \subset \{1, \ldots, n\}$ of β_j for which $\cap_{i \in \beta_j^*} R_i \neq \emptyset$.



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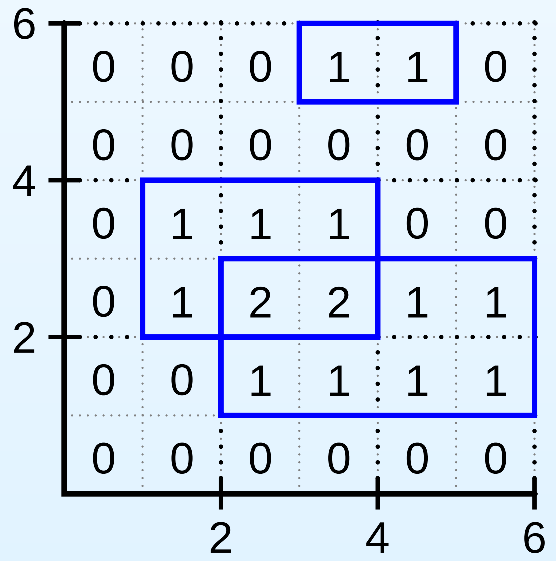
- Basic idea of the Height Map Algorithm:
  - Define a height map of the observed sets:



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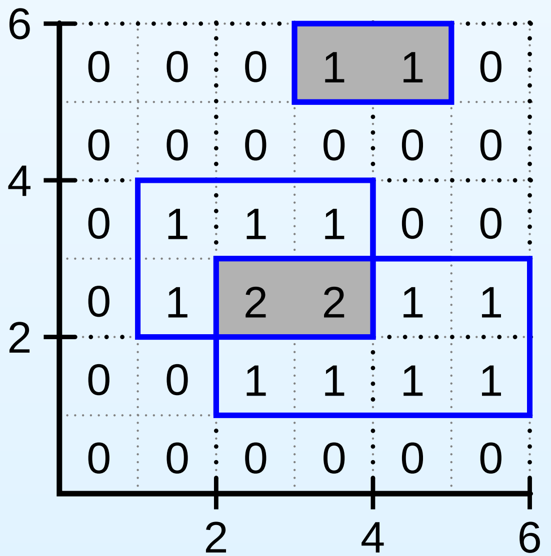
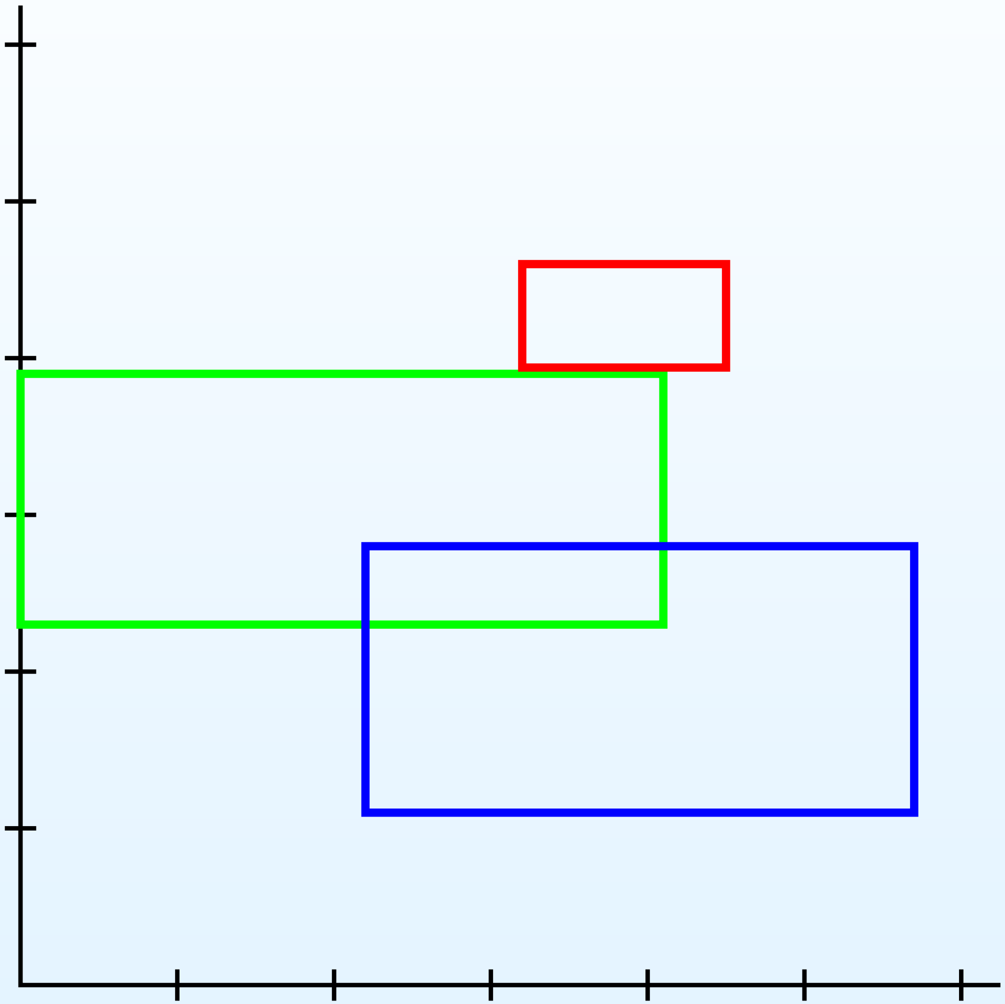
  - Define a height map of the observed sets:
  - The maximal intersections are exactly the local maximum regions of the height map



The algorithm

- Transform the observation rectangles into “canonical rectangles"
- Find local maximum regions of the height map of the canonical rectangles (by sweeping)
- Transform local maxima back to original coordinates

Transform rectangles into canonical rectangles



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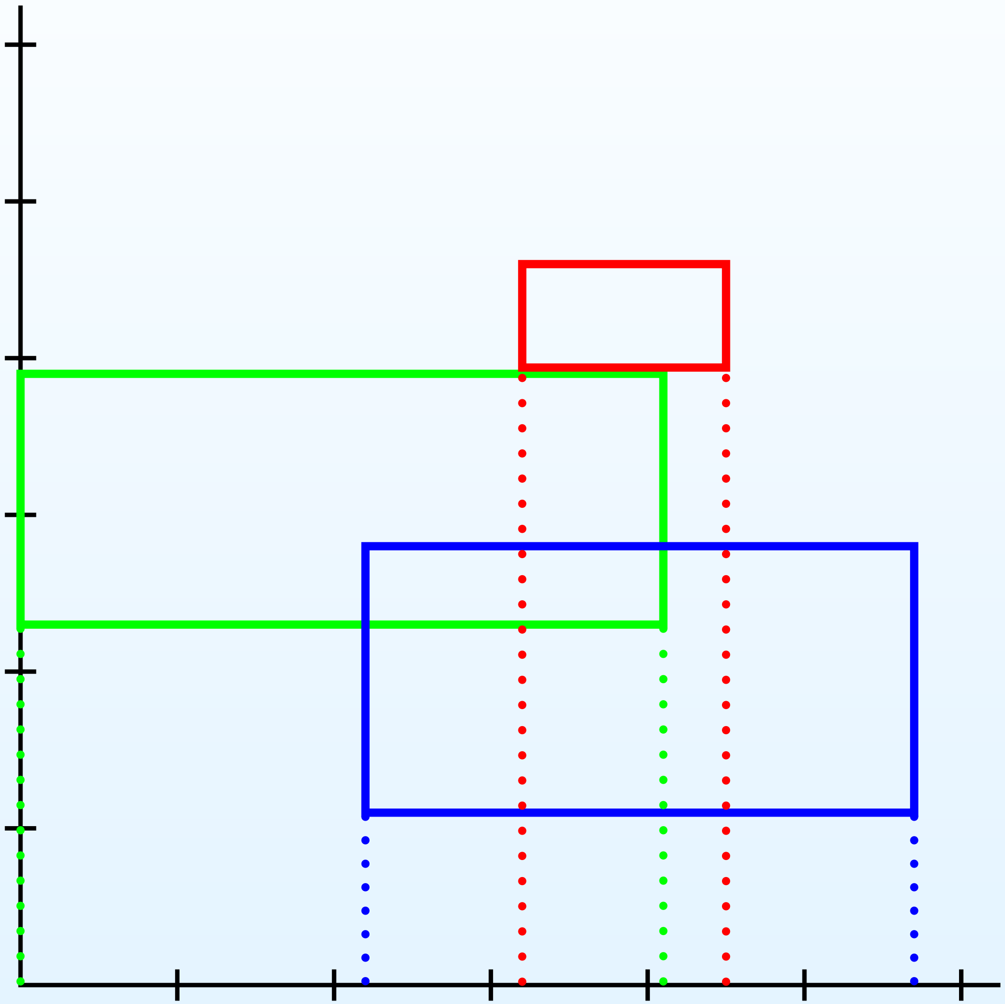

- Replace x-coordinates by their order statistics.

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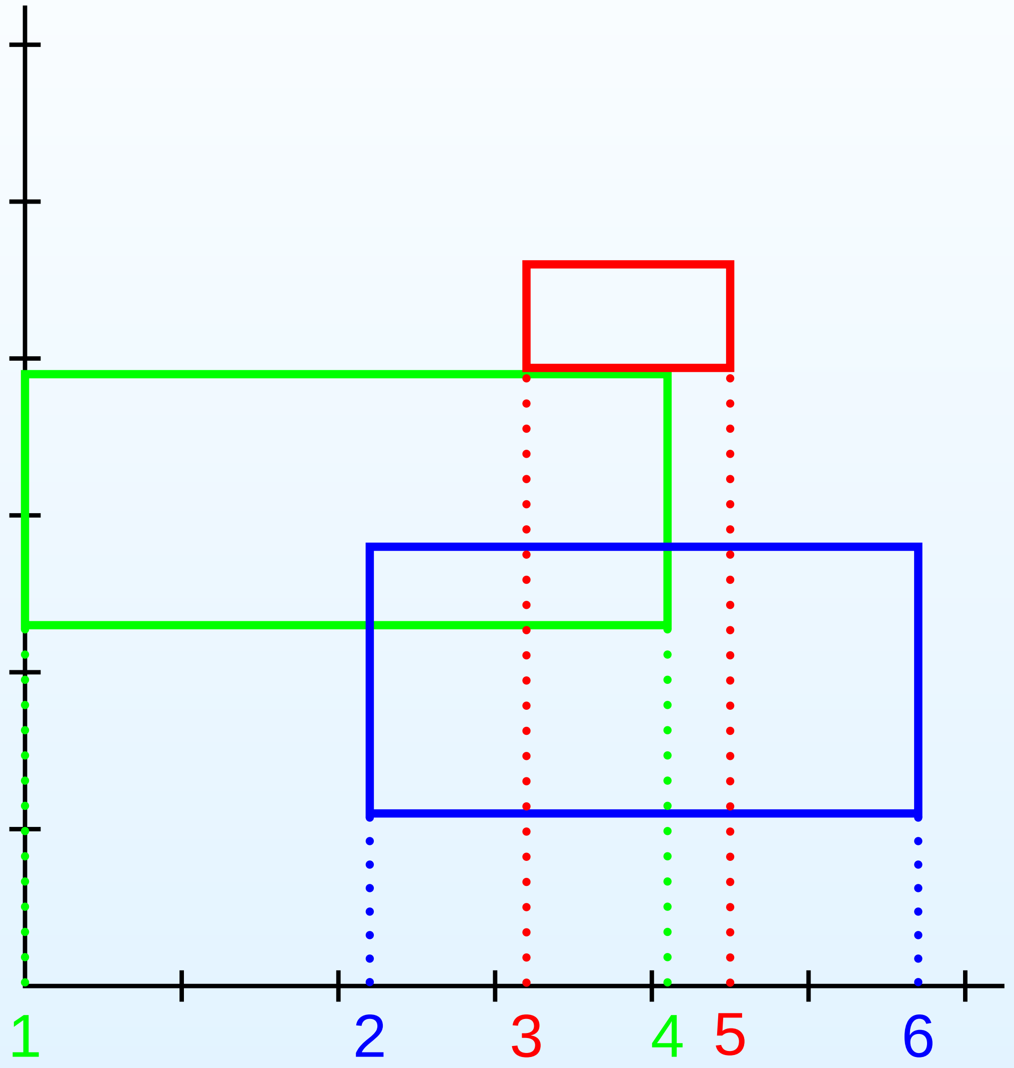

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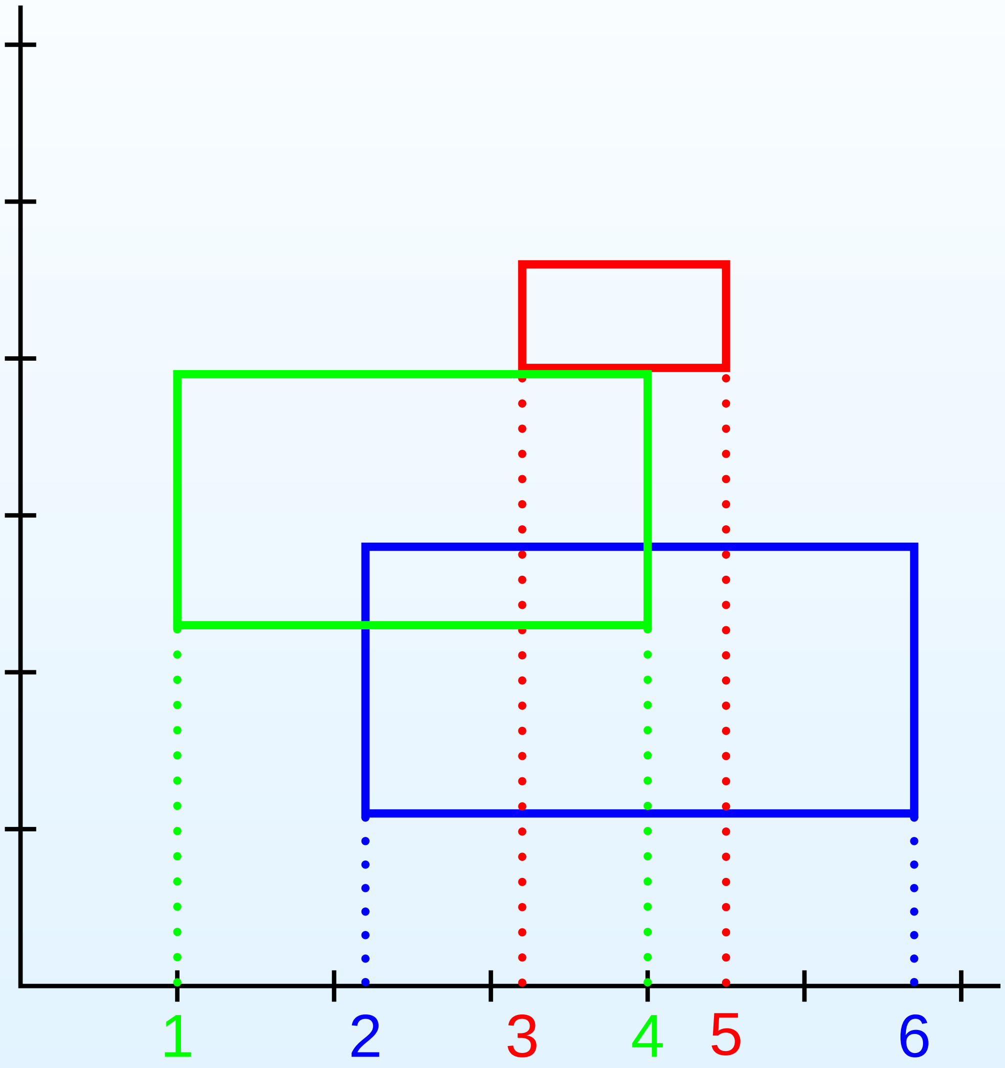

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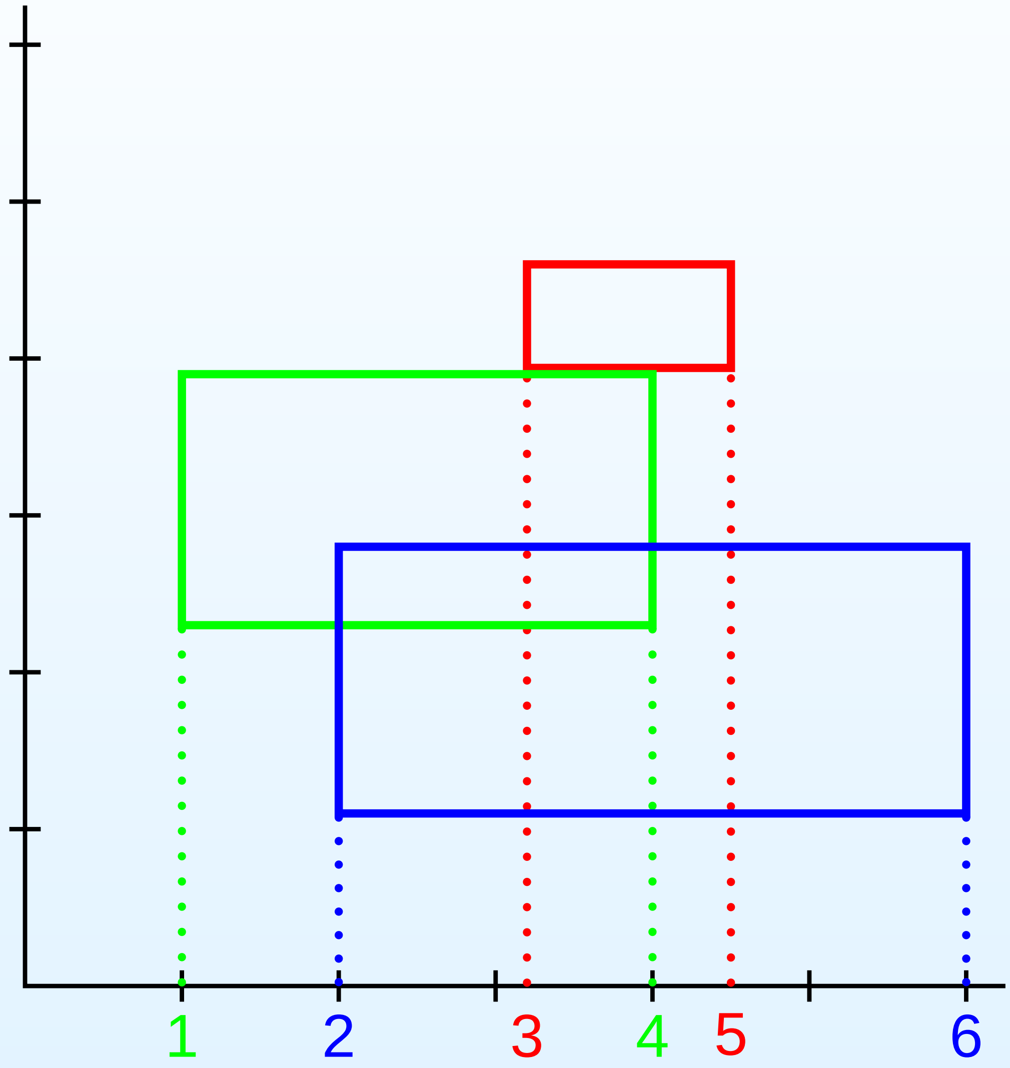

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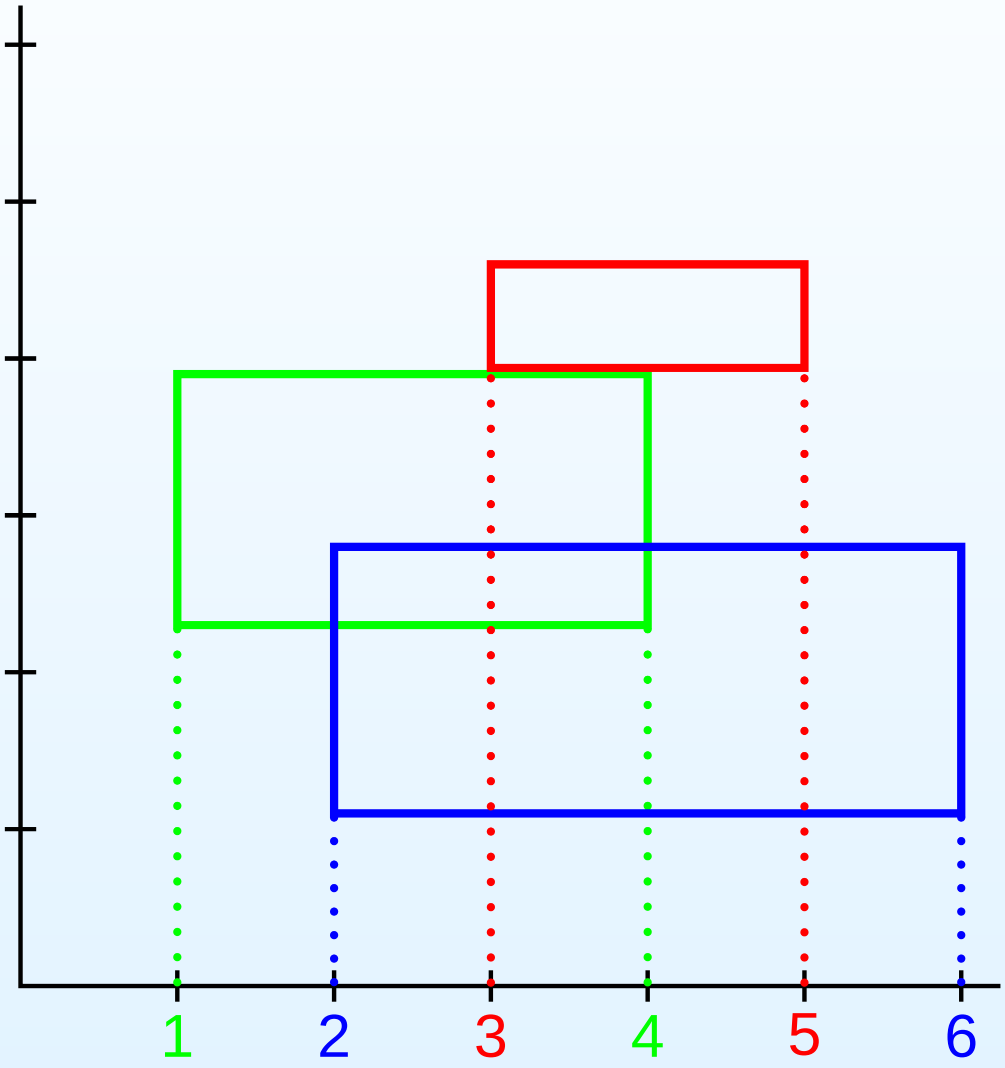

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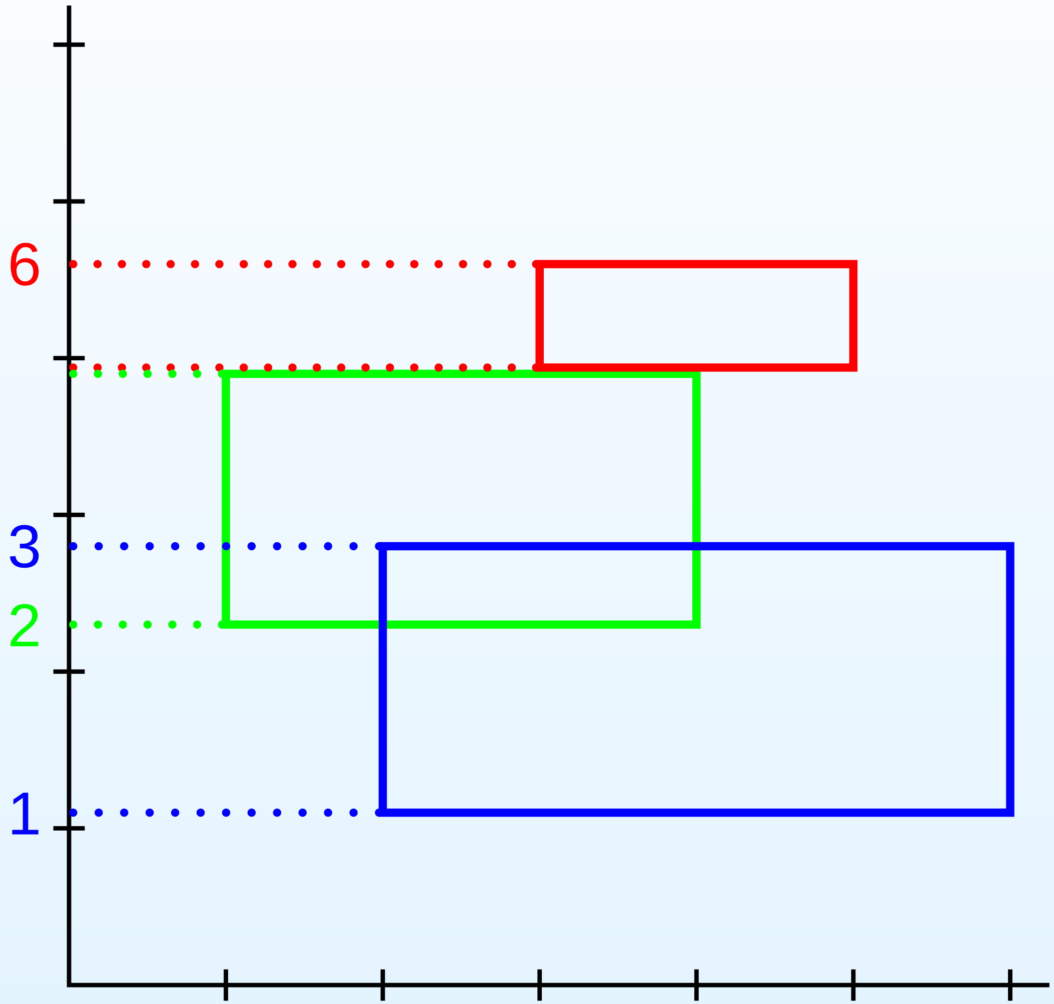

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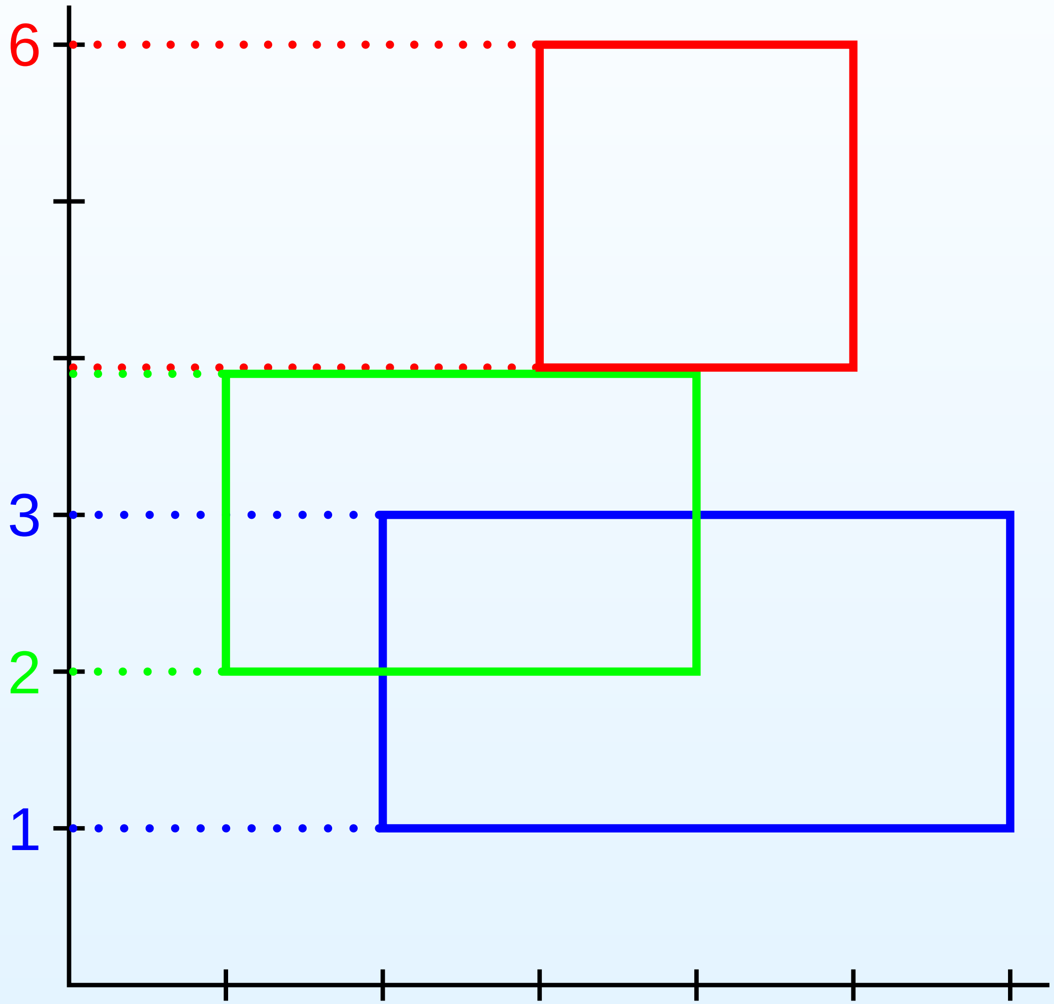

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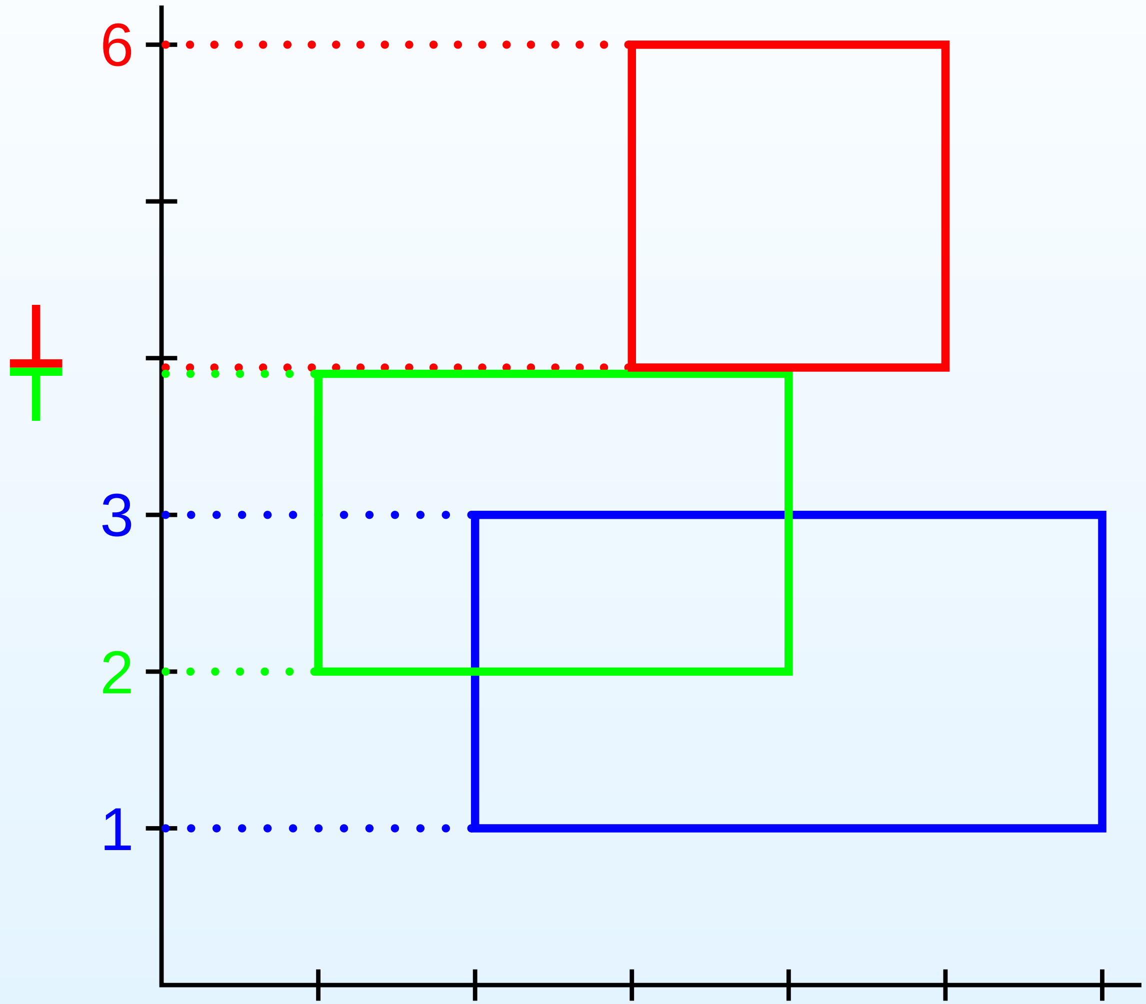

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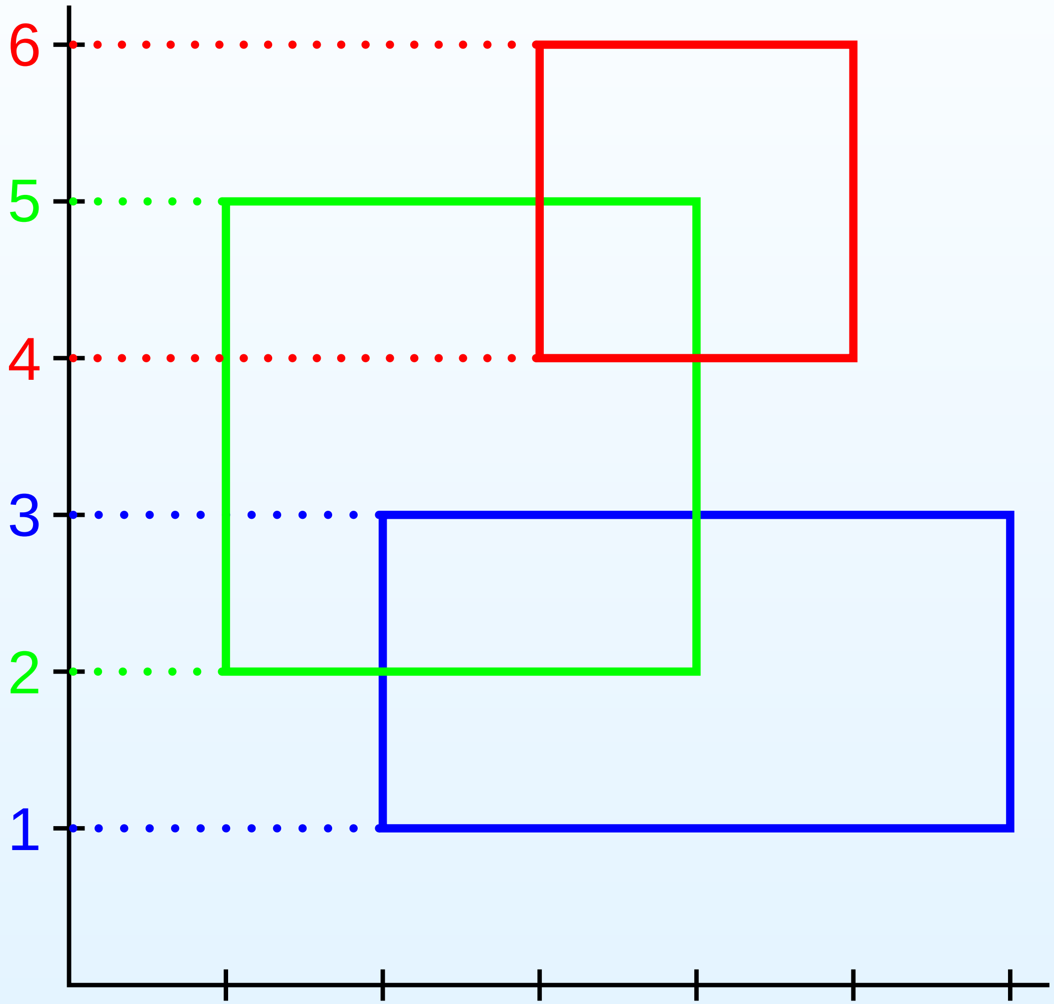

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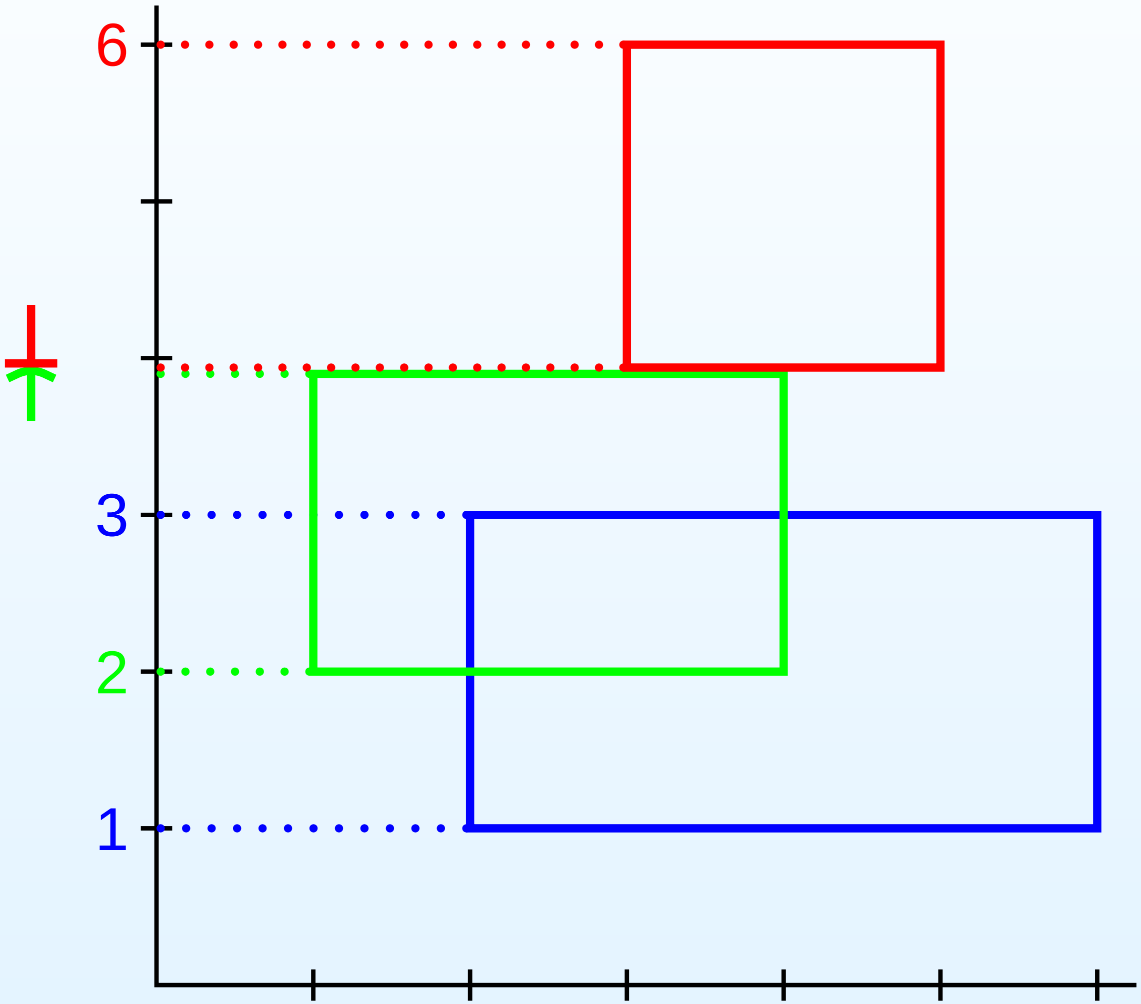

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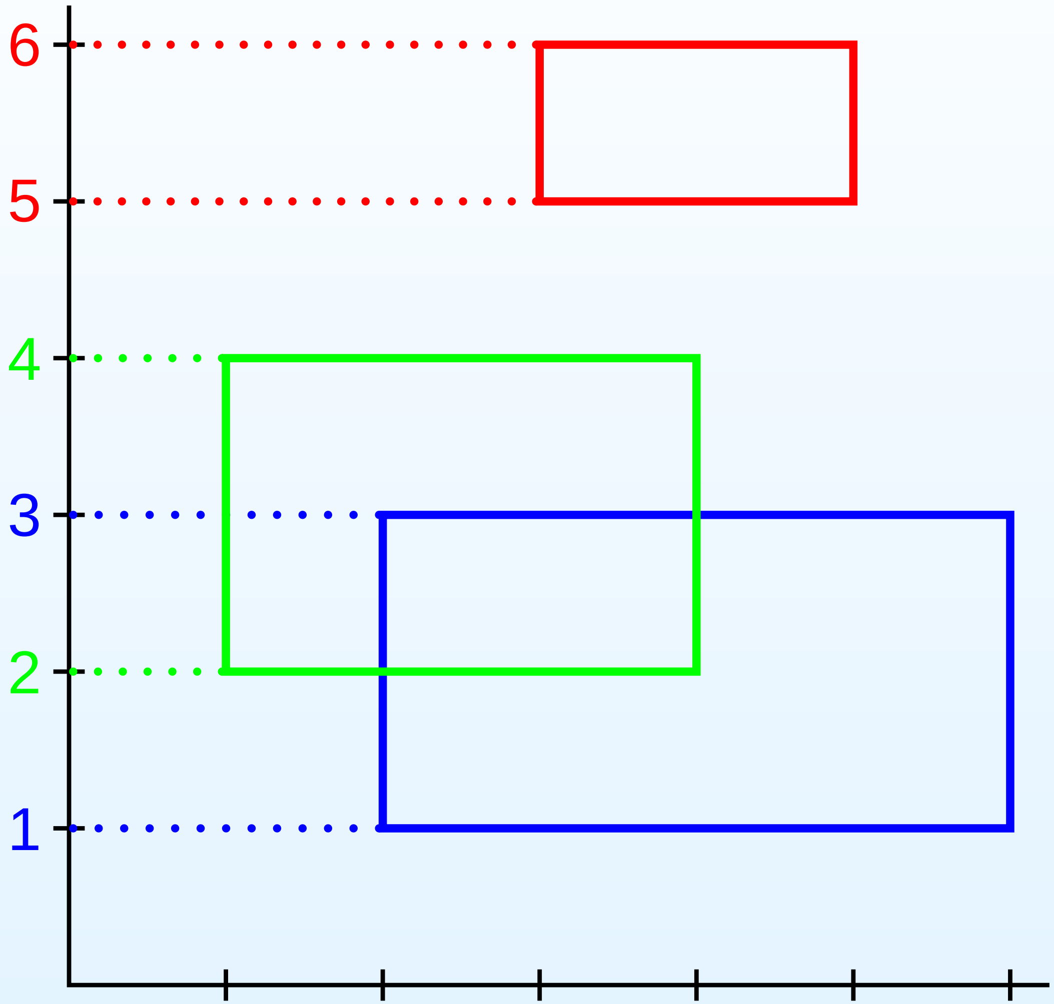

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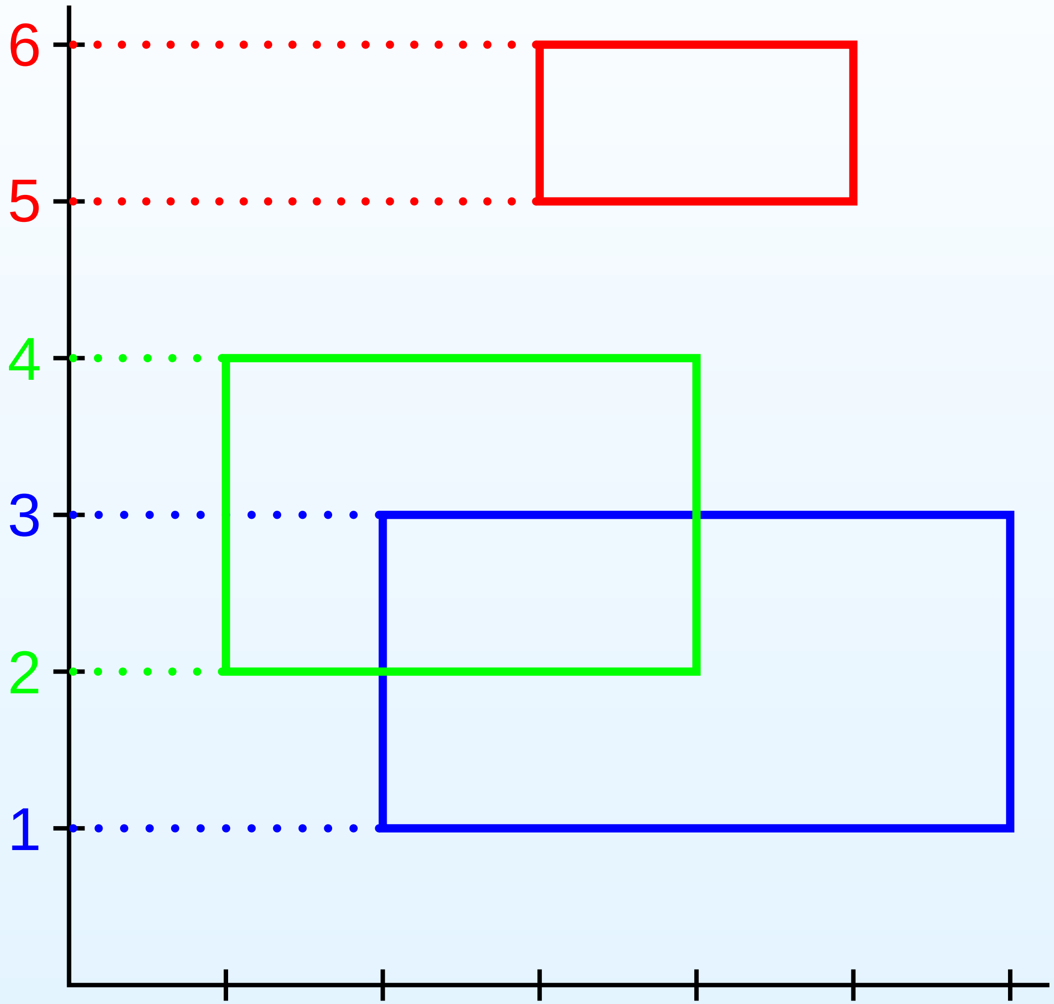

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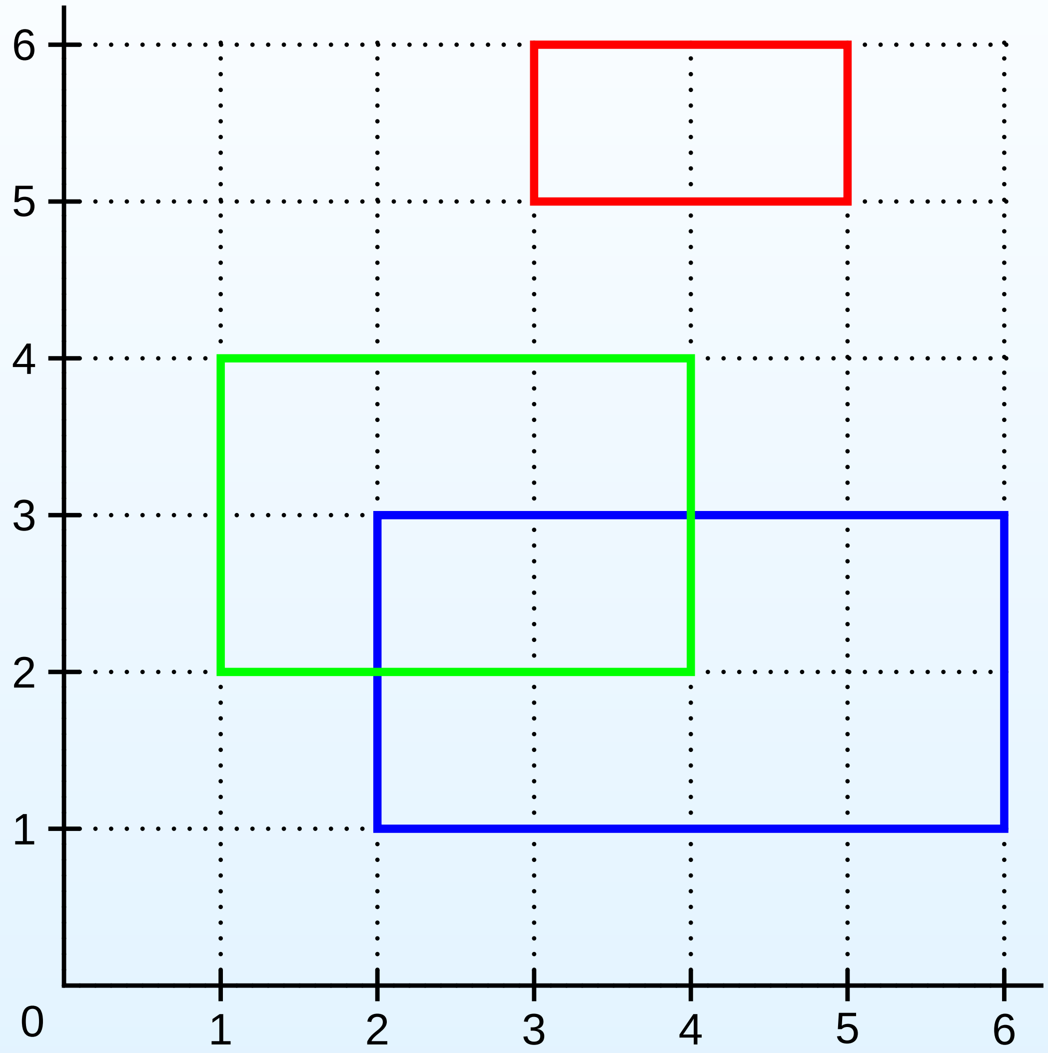
- Replace y-coordinates by their order statistics.
- All x-coodinates are different and take values in $\{1, 2, \ldots, 2n\}$ (same for y-coordinates).
- Intersection structure of the original and canonical rectangles is identical.

Why use canonical rectangles?

- We break possible ties early, so that we don't have to worry about this anymore
- It is easier and faster to work with integer coordinates

Find local maxima by sweeping through the height map



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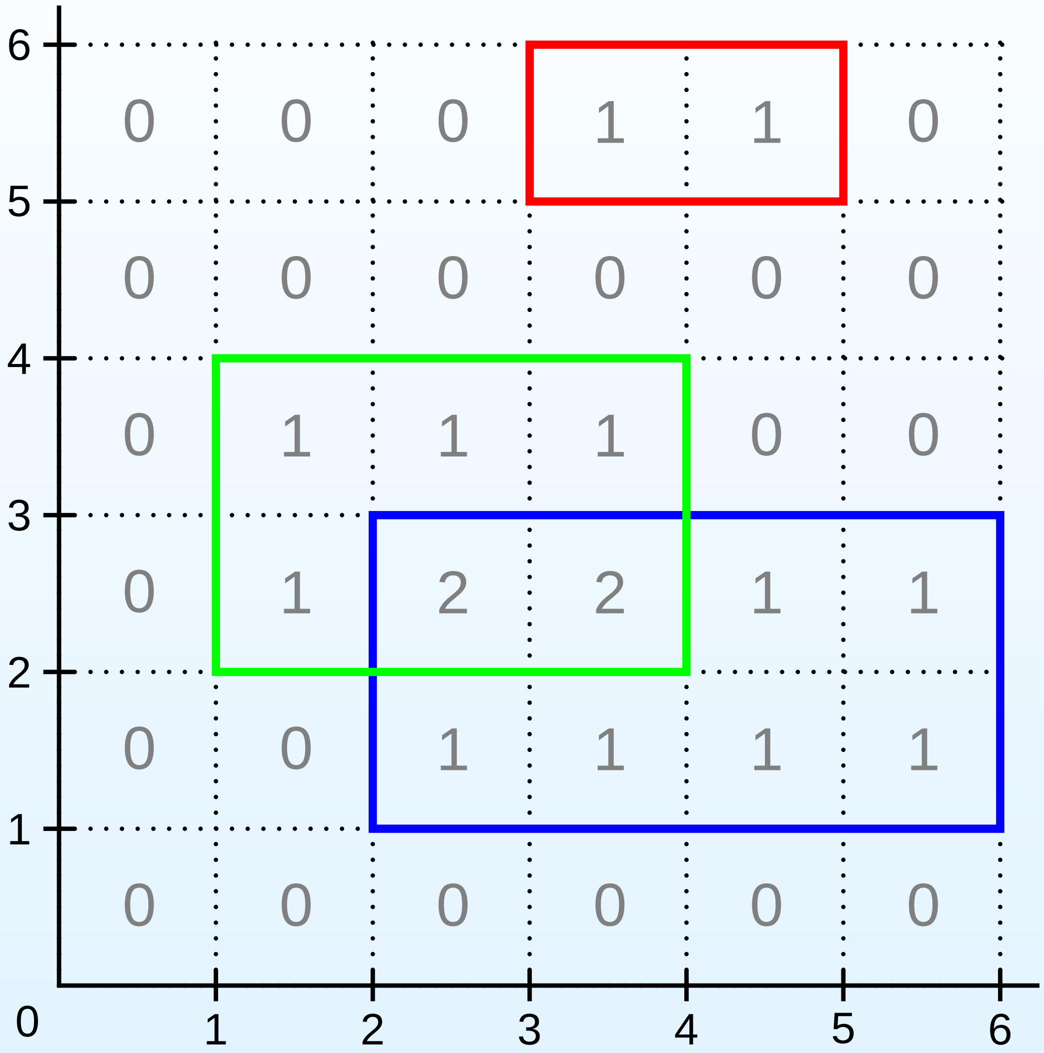

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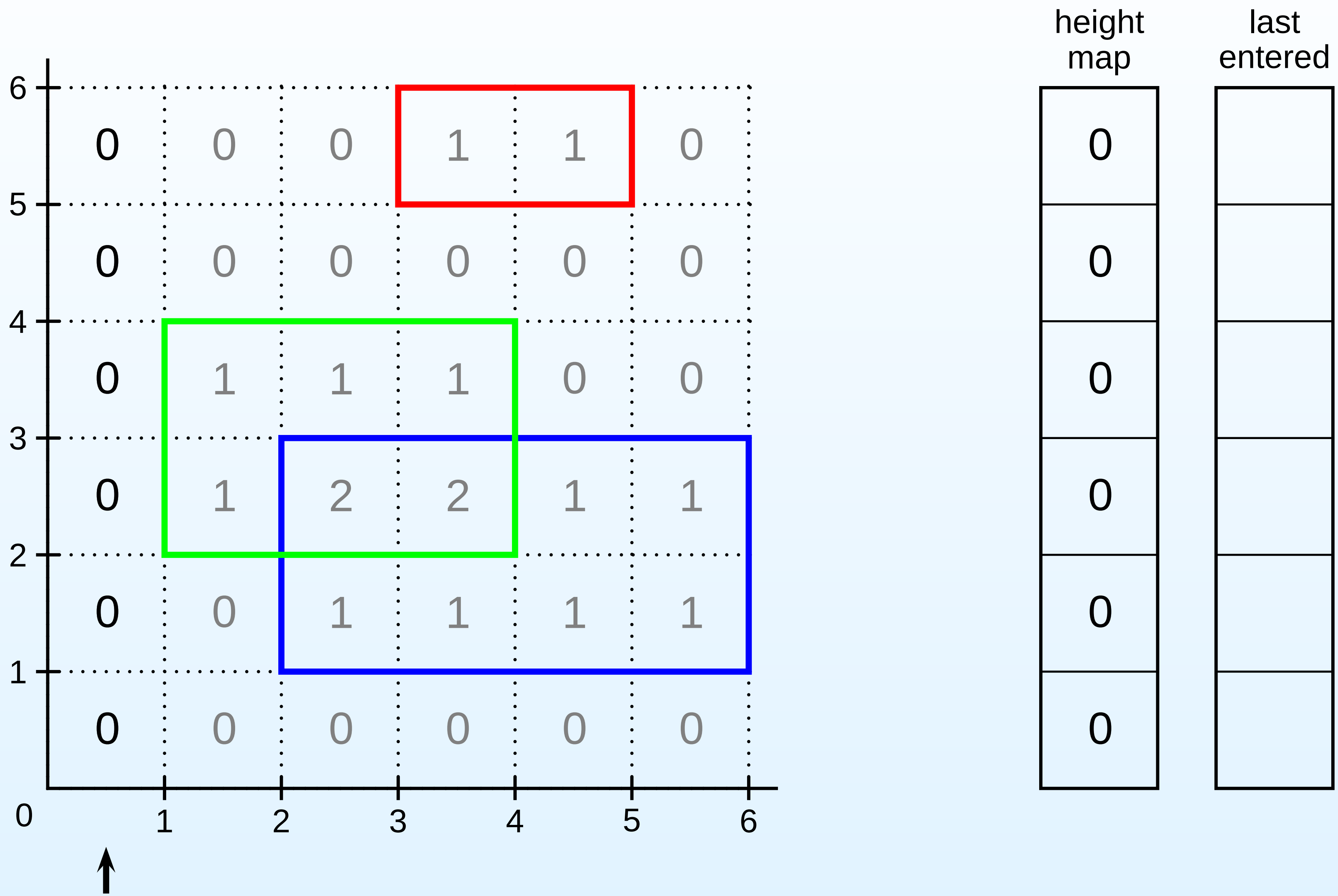

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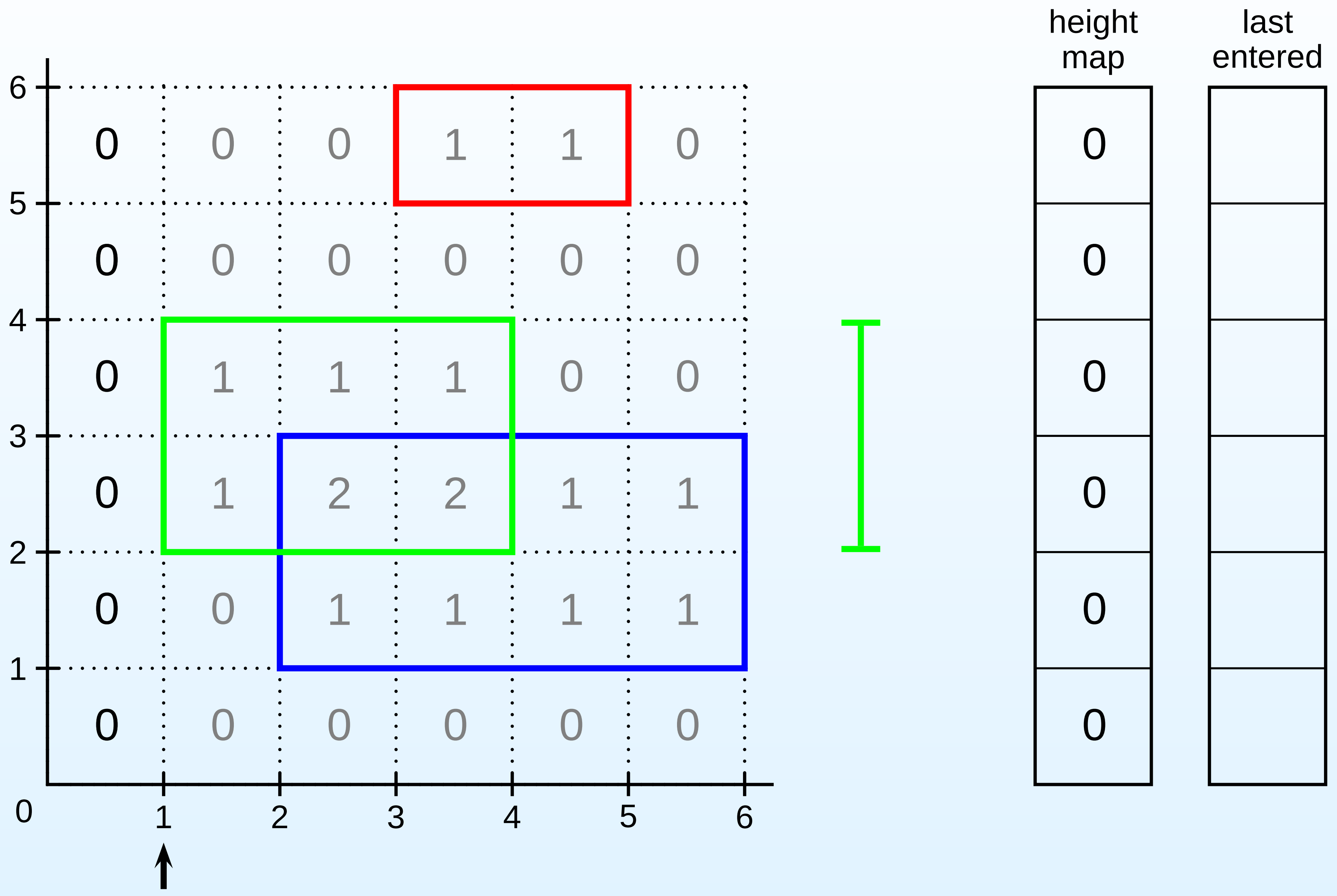

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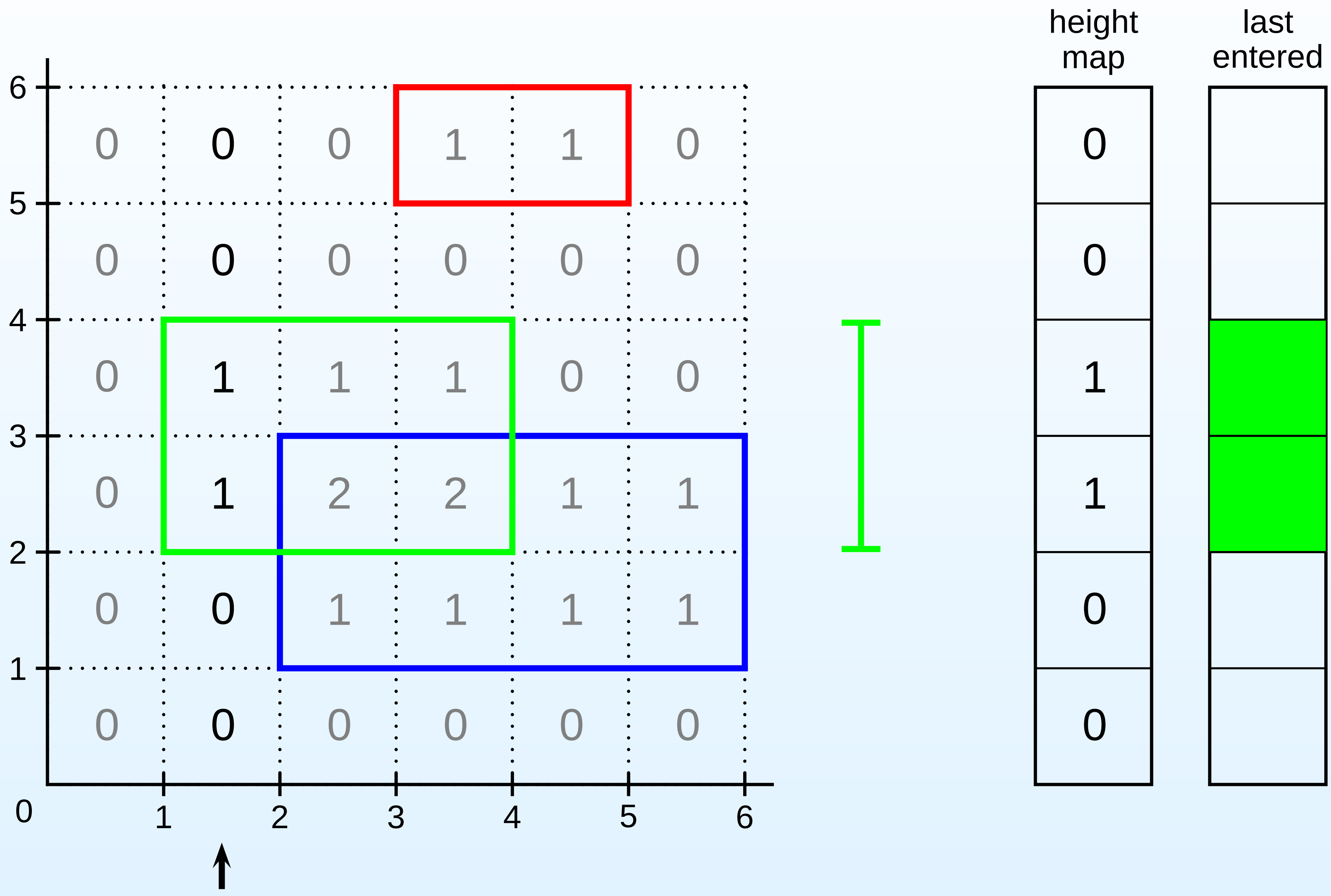

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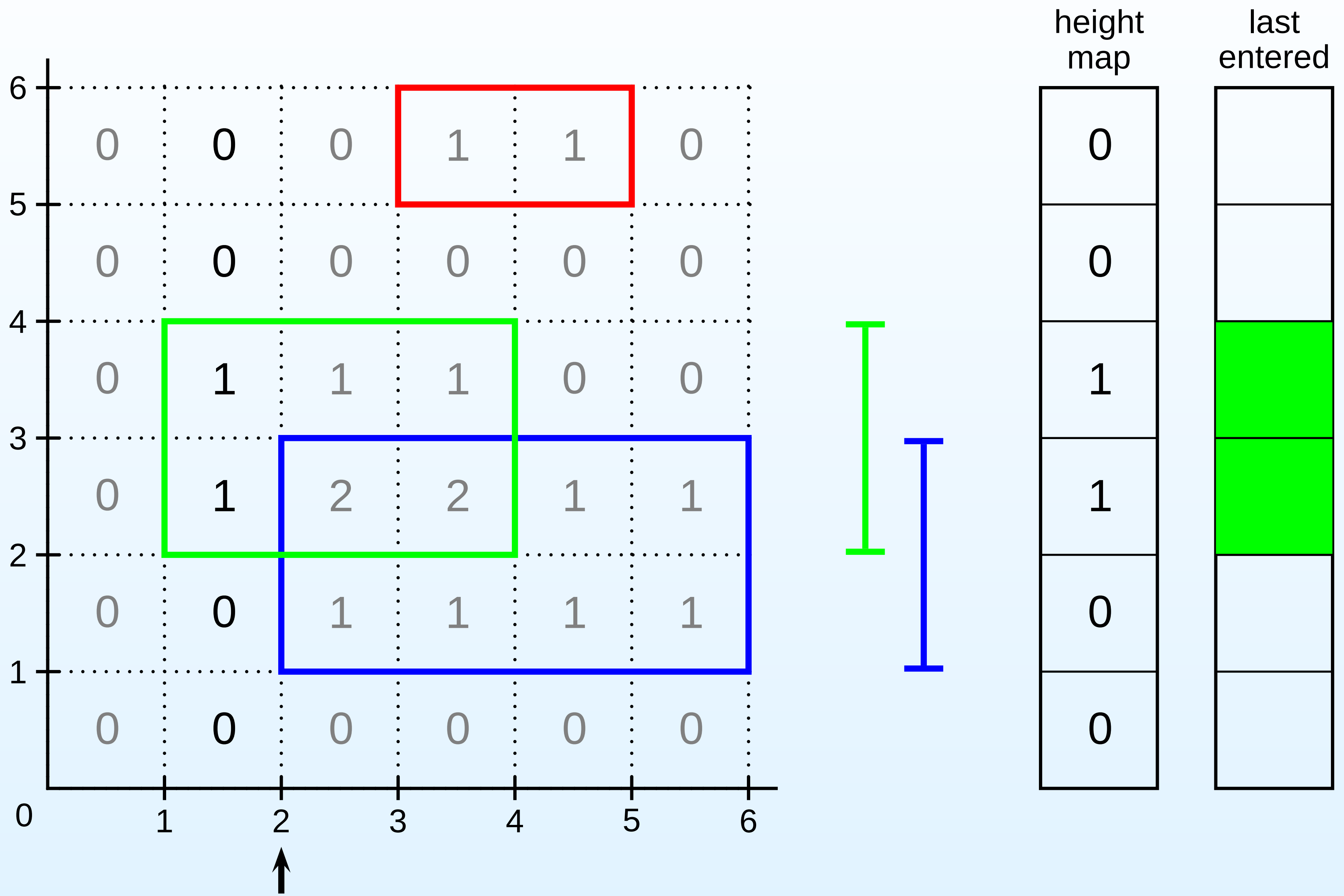

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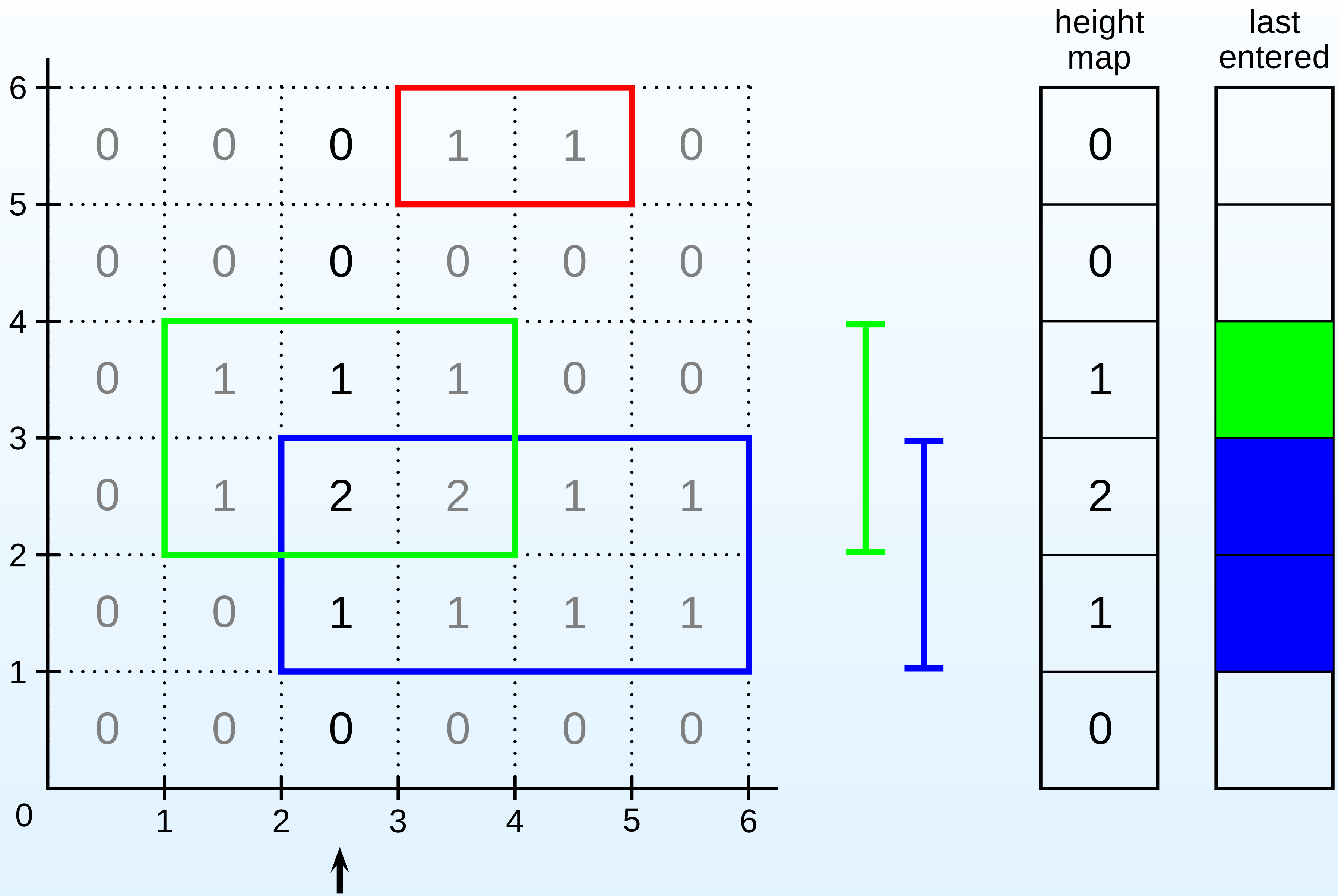

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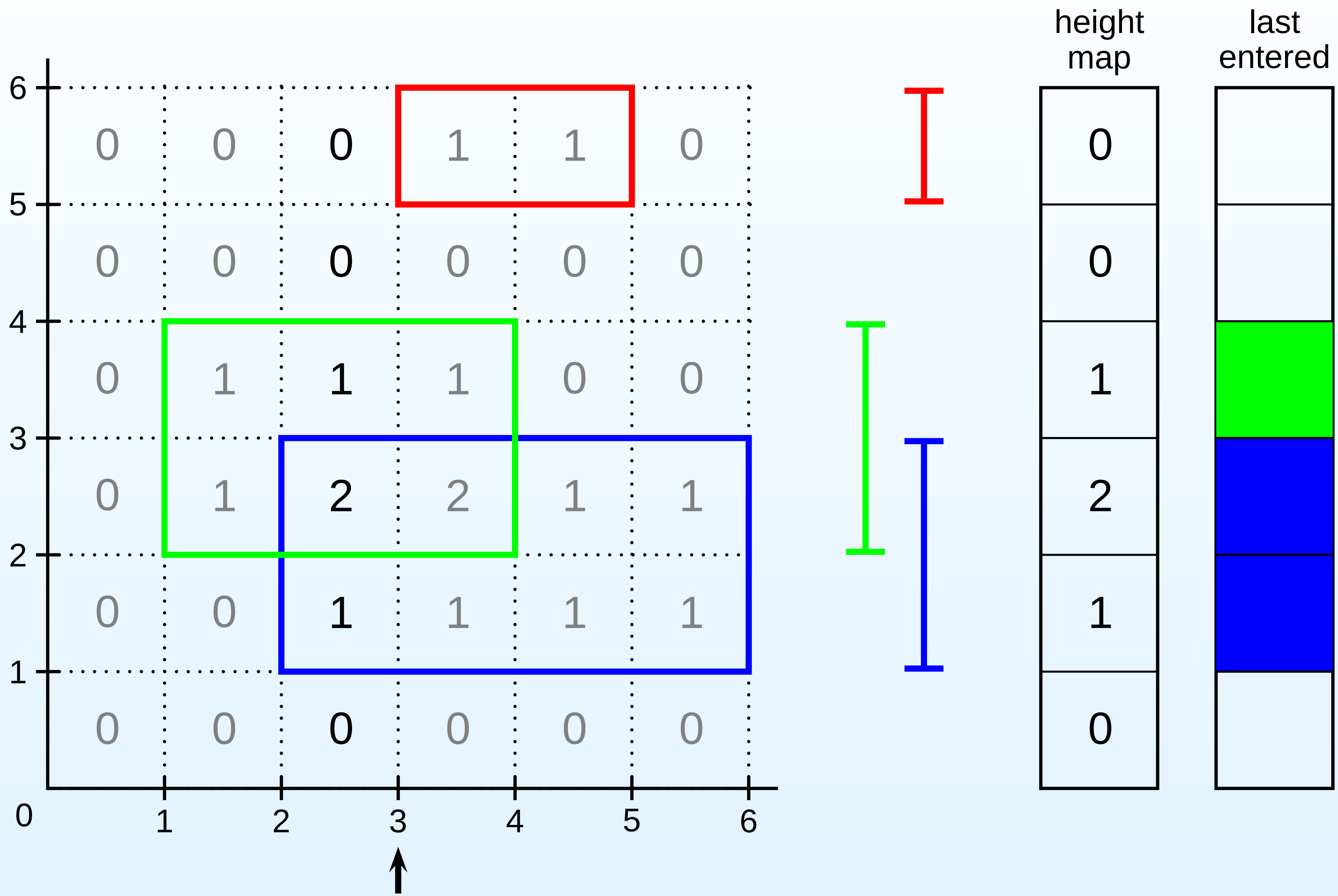

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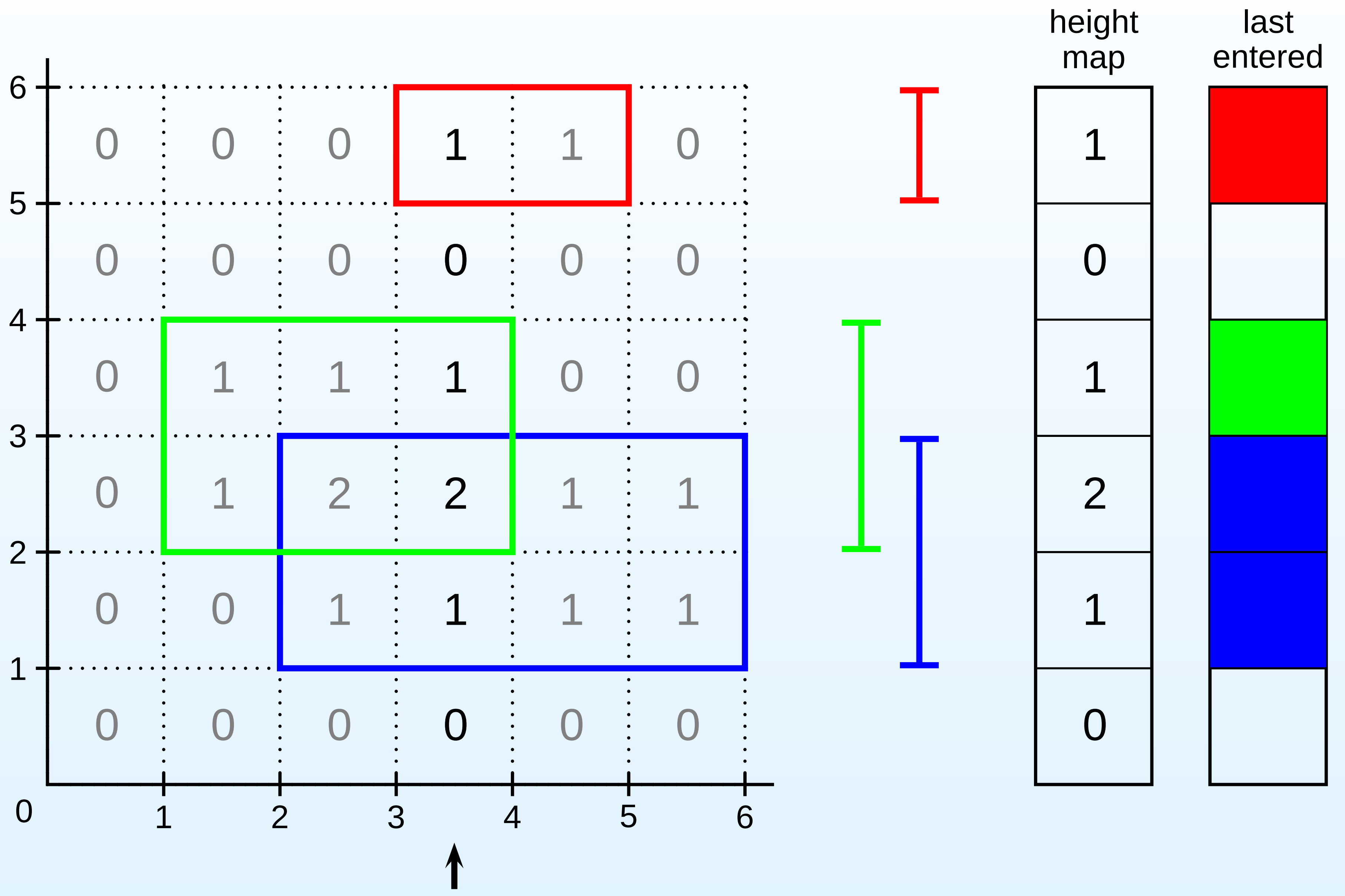

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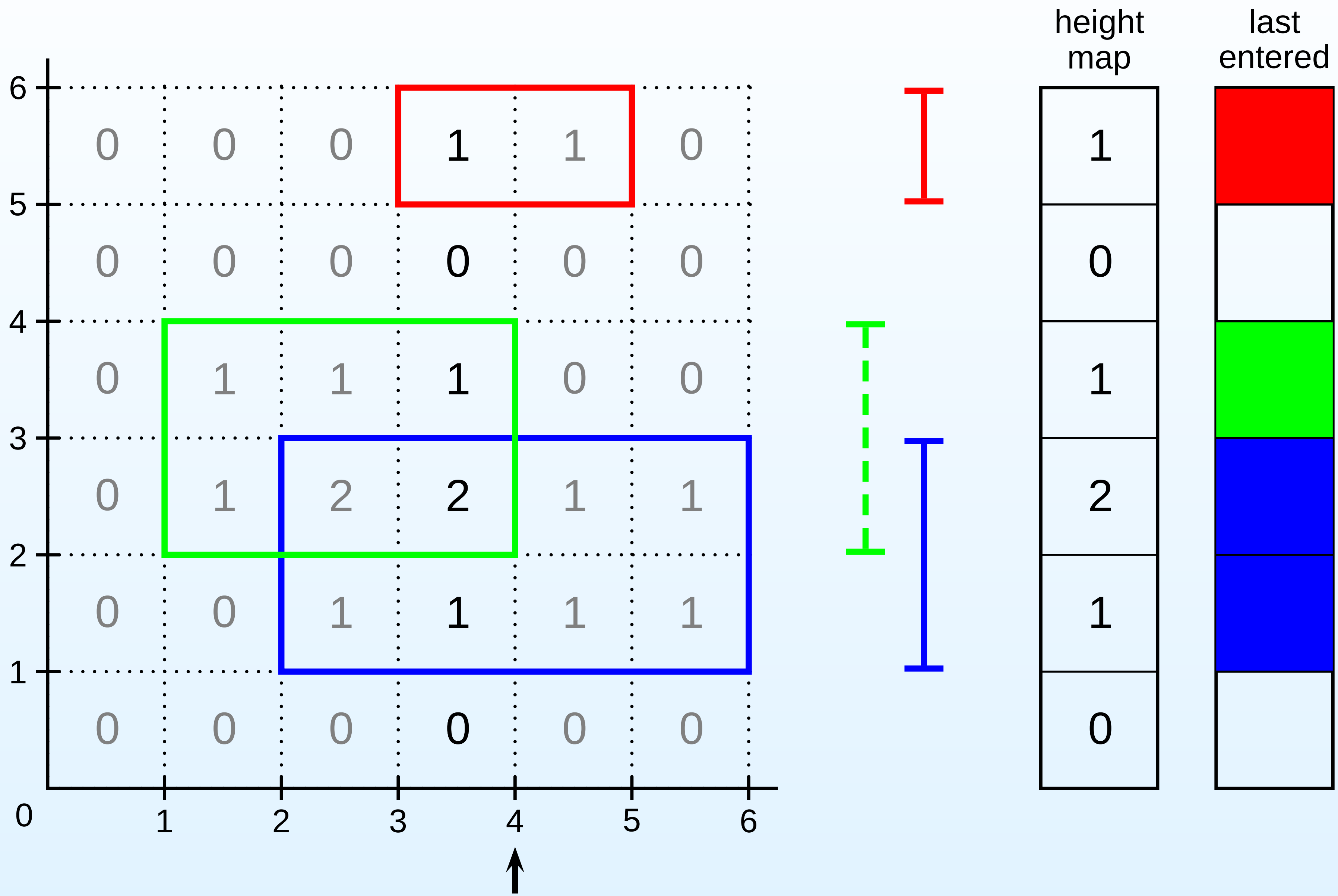

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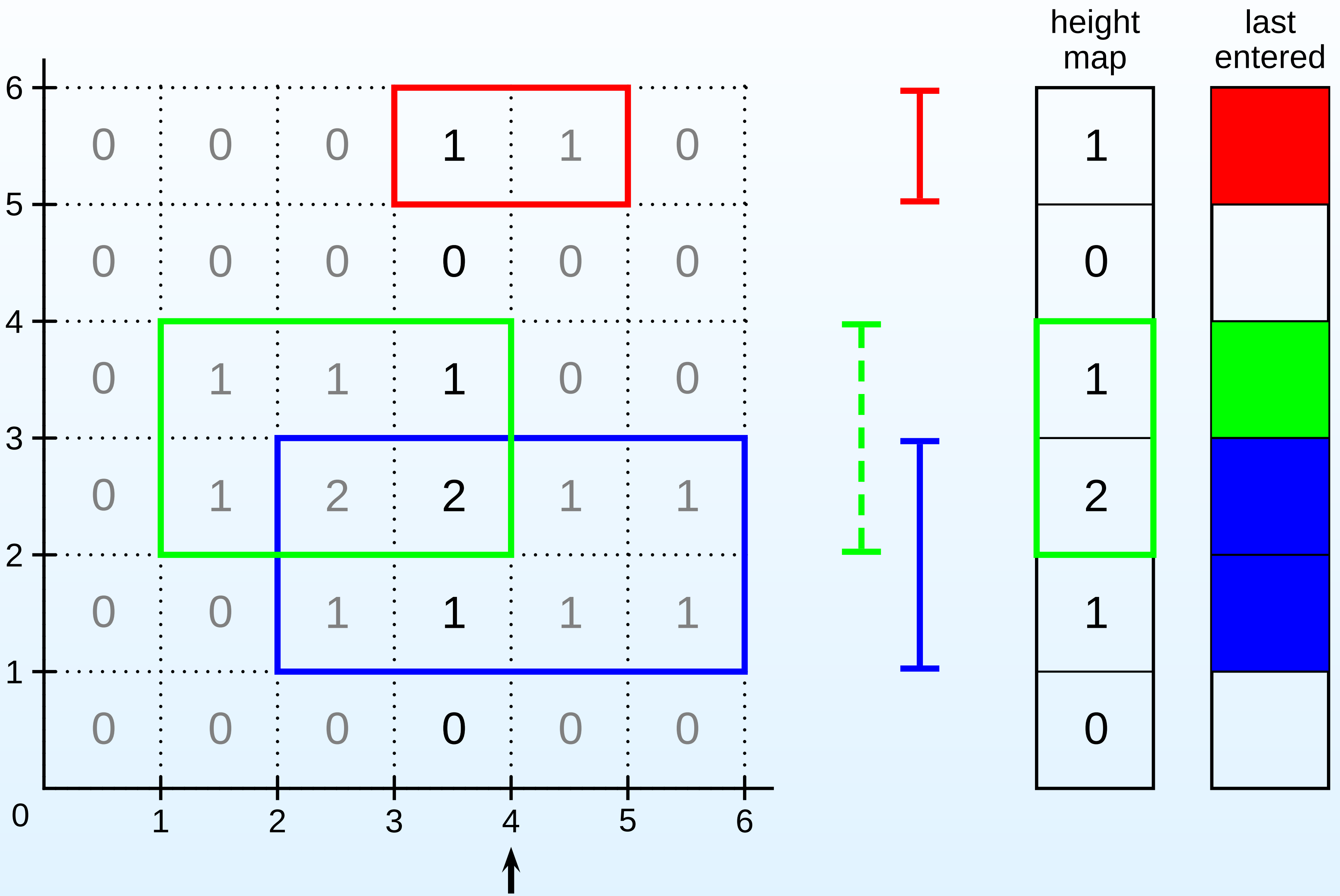

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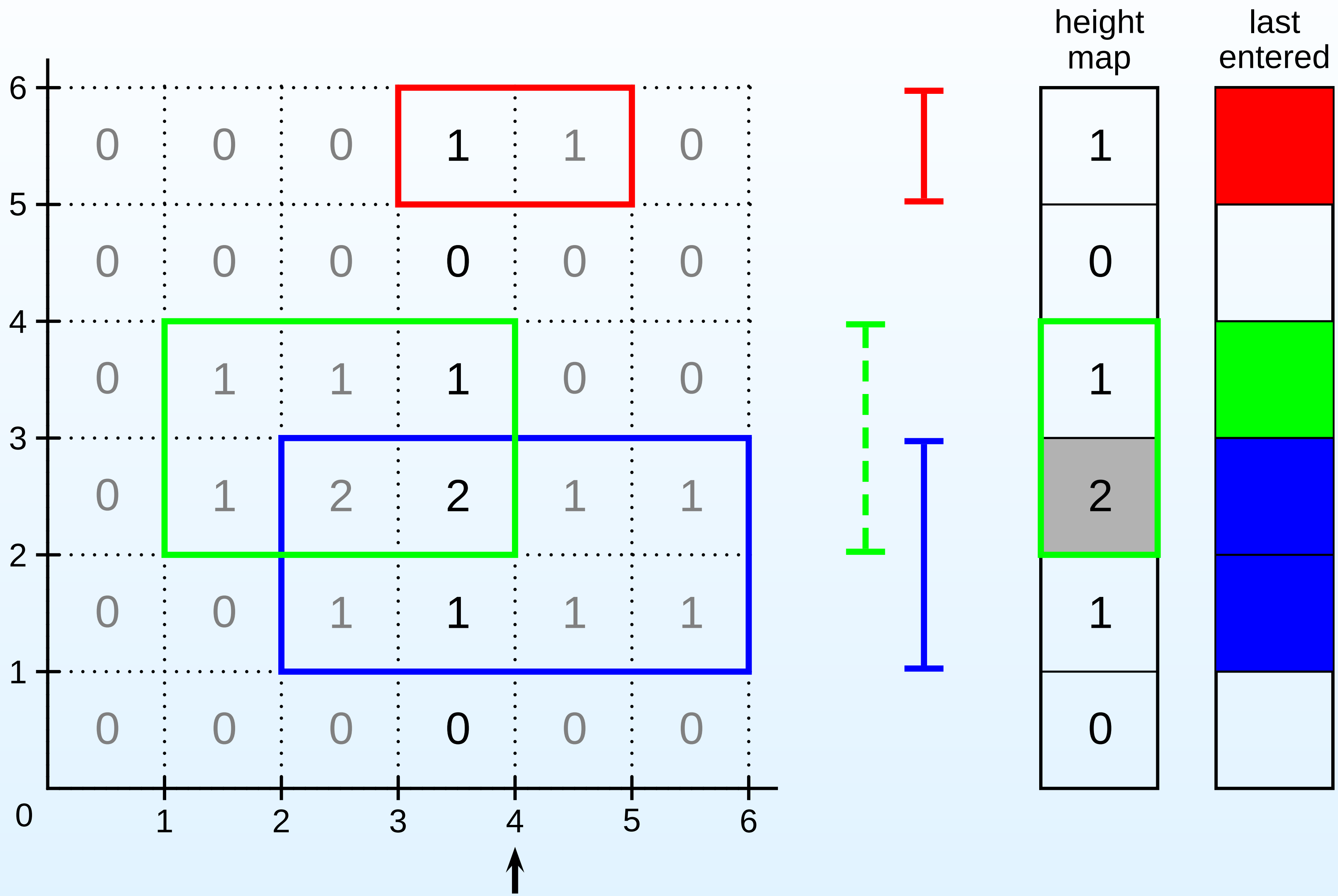

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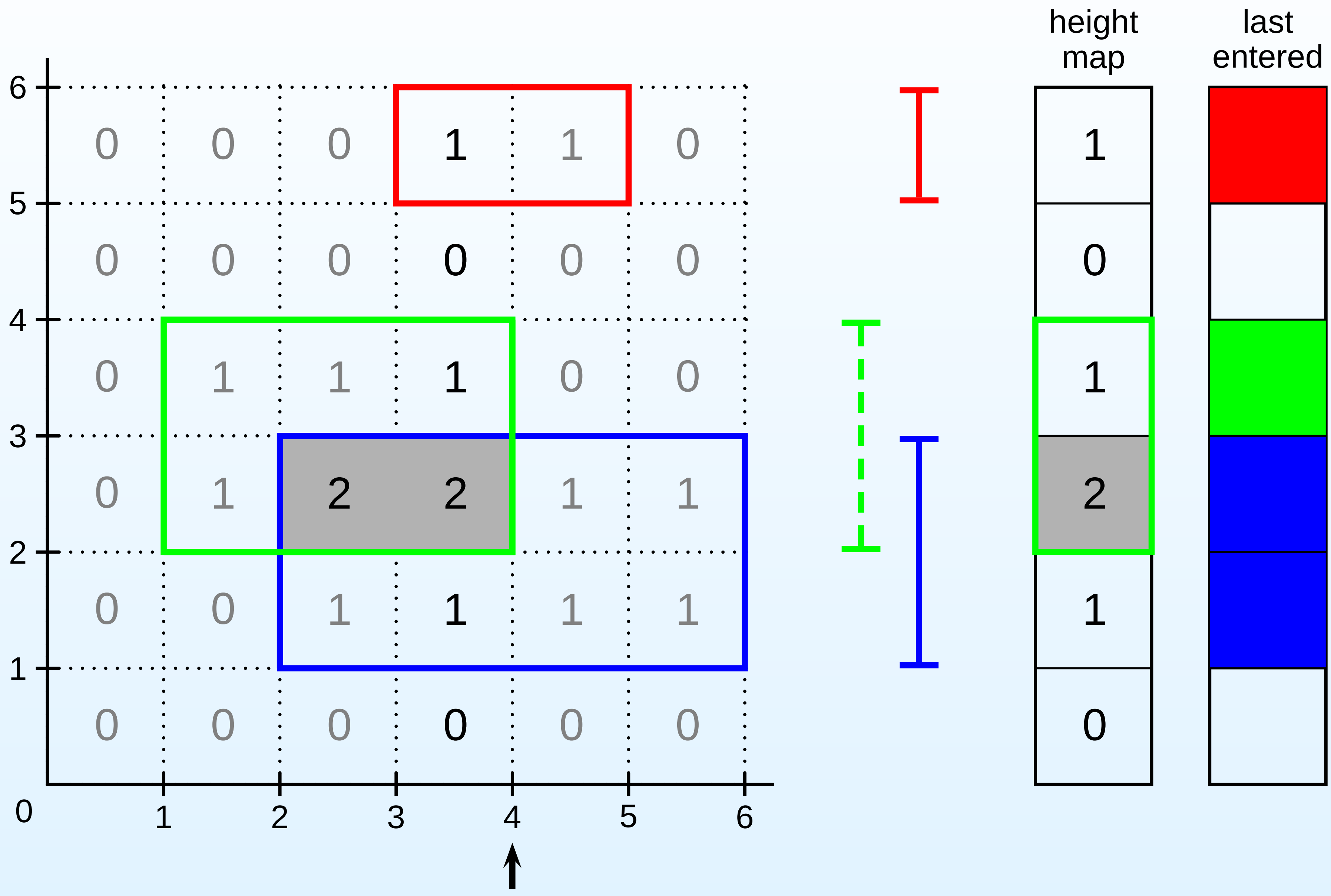

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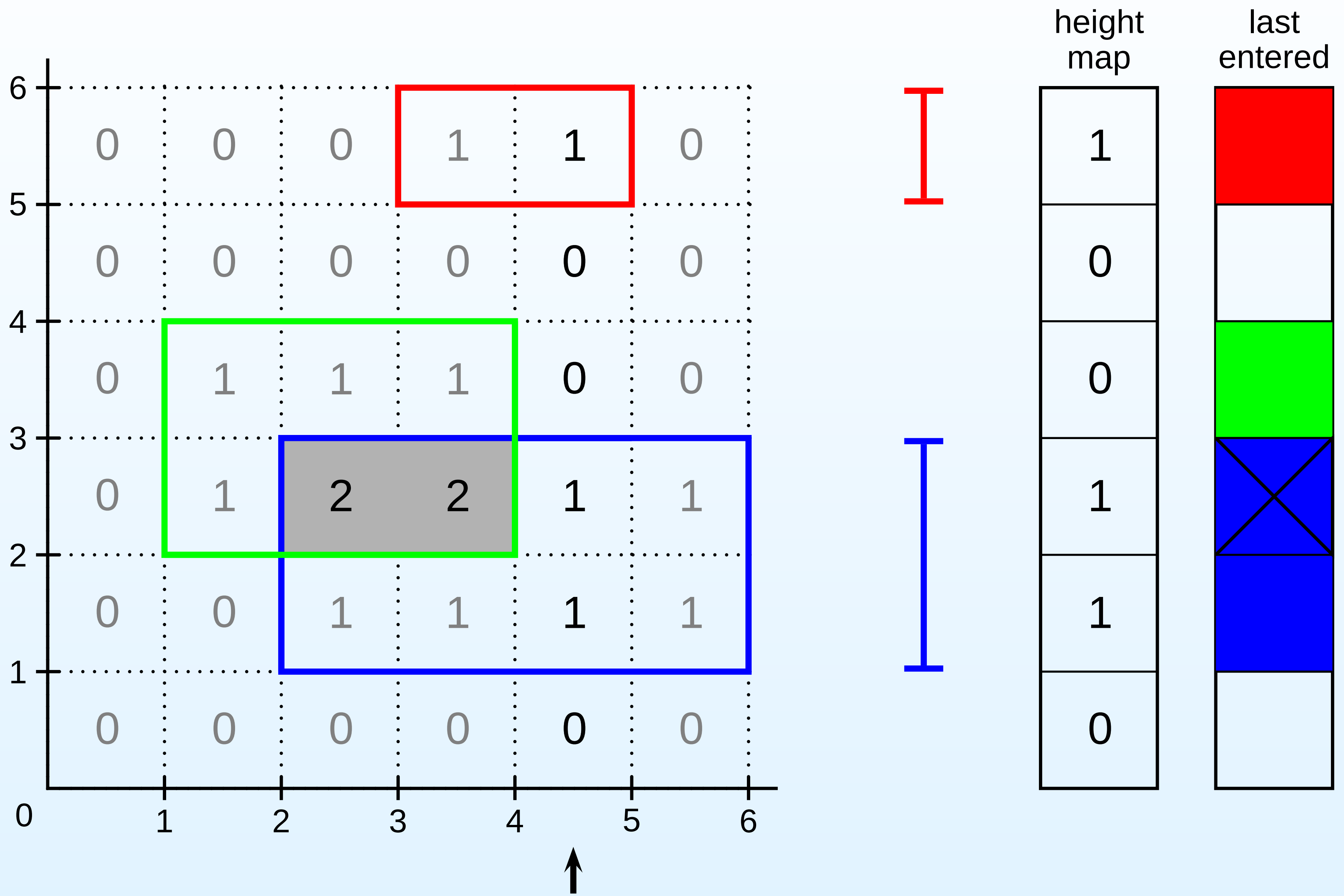

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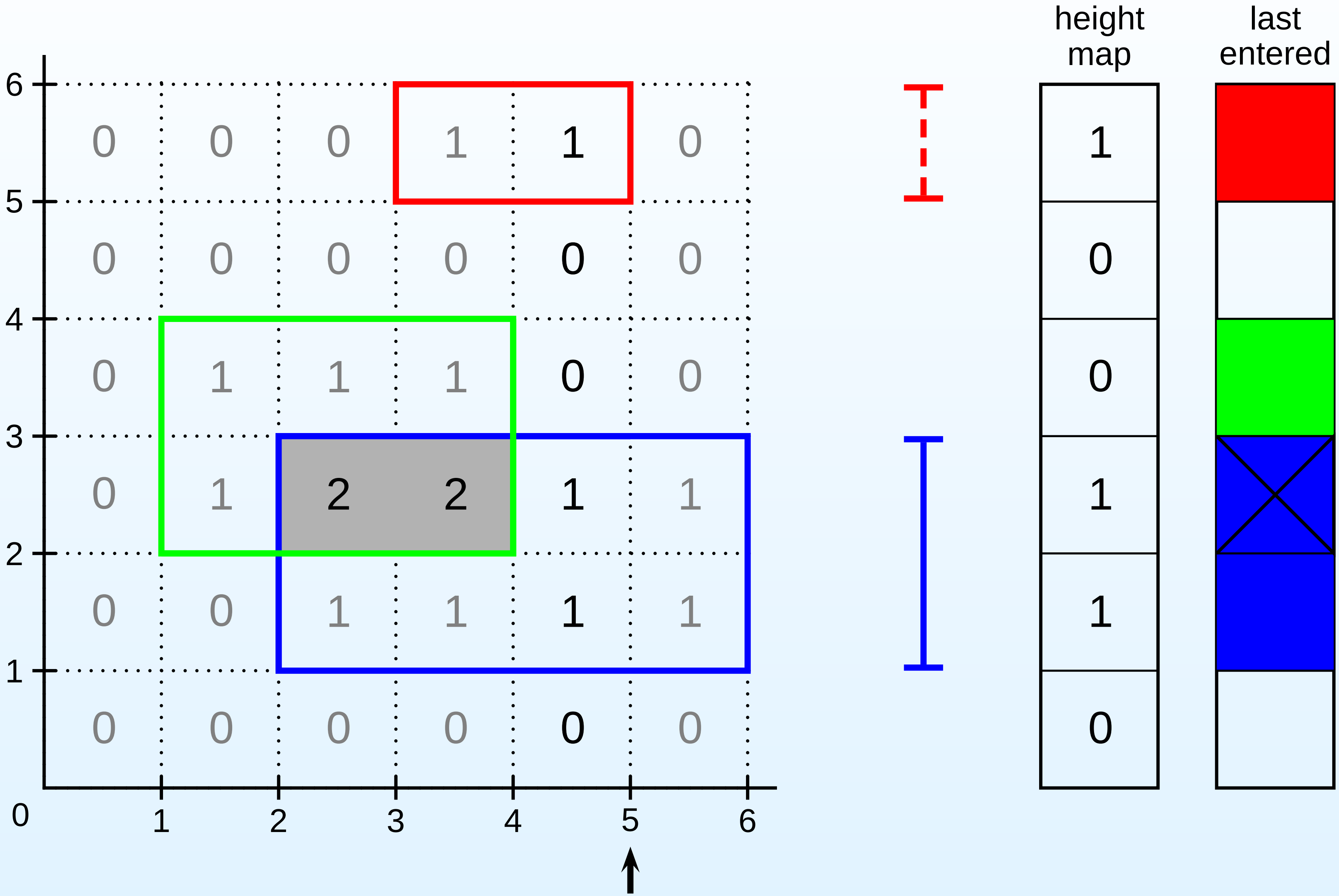

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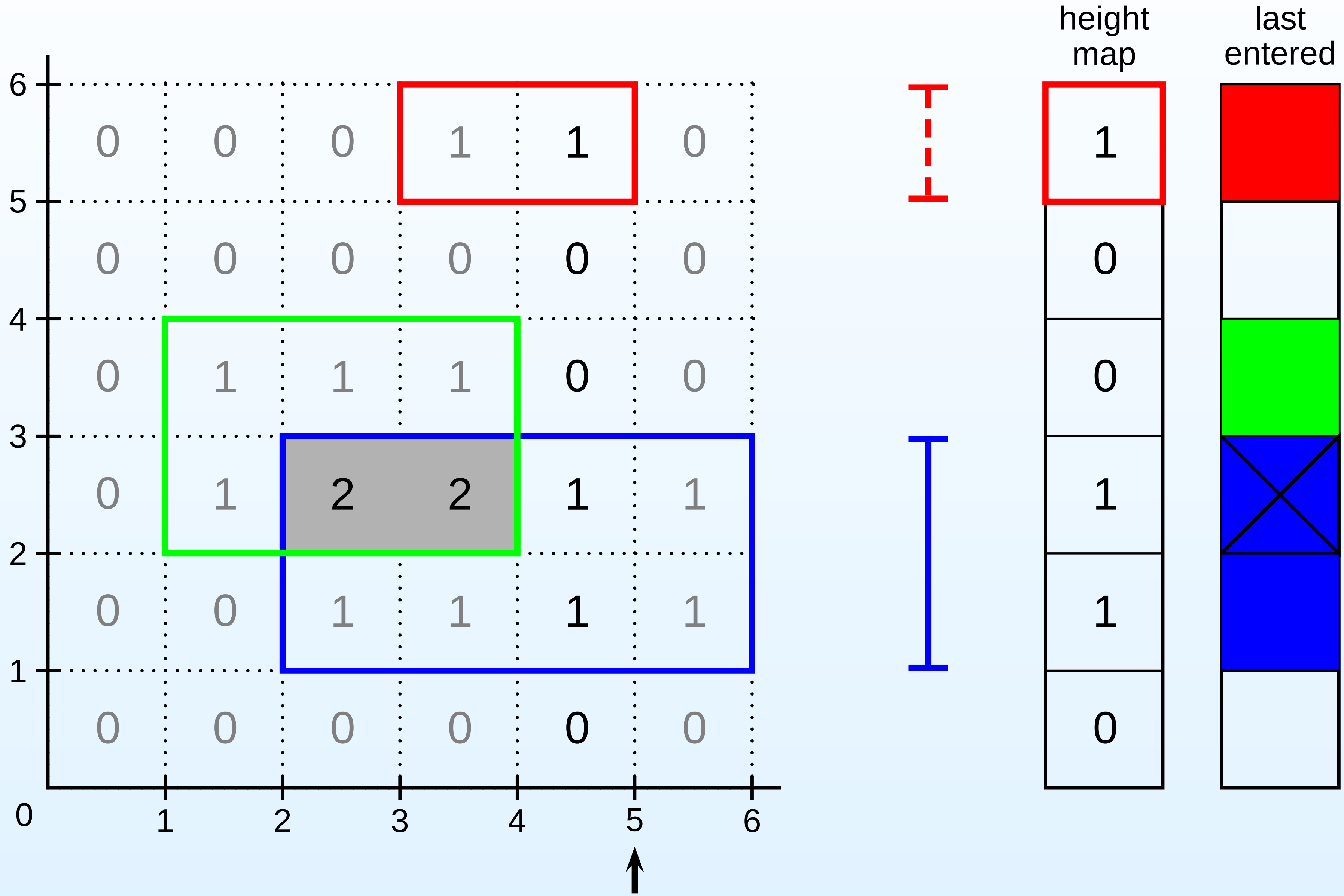

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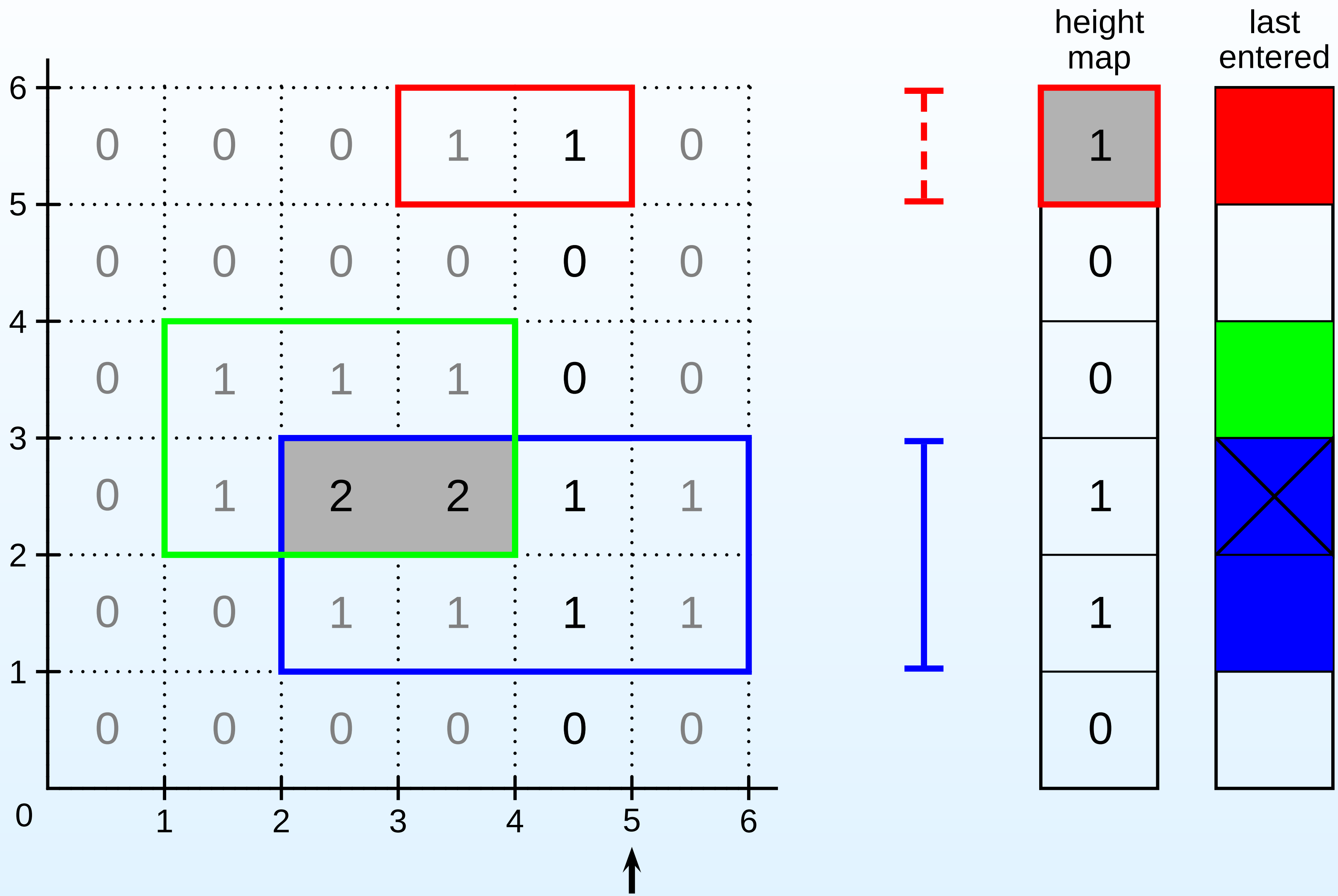

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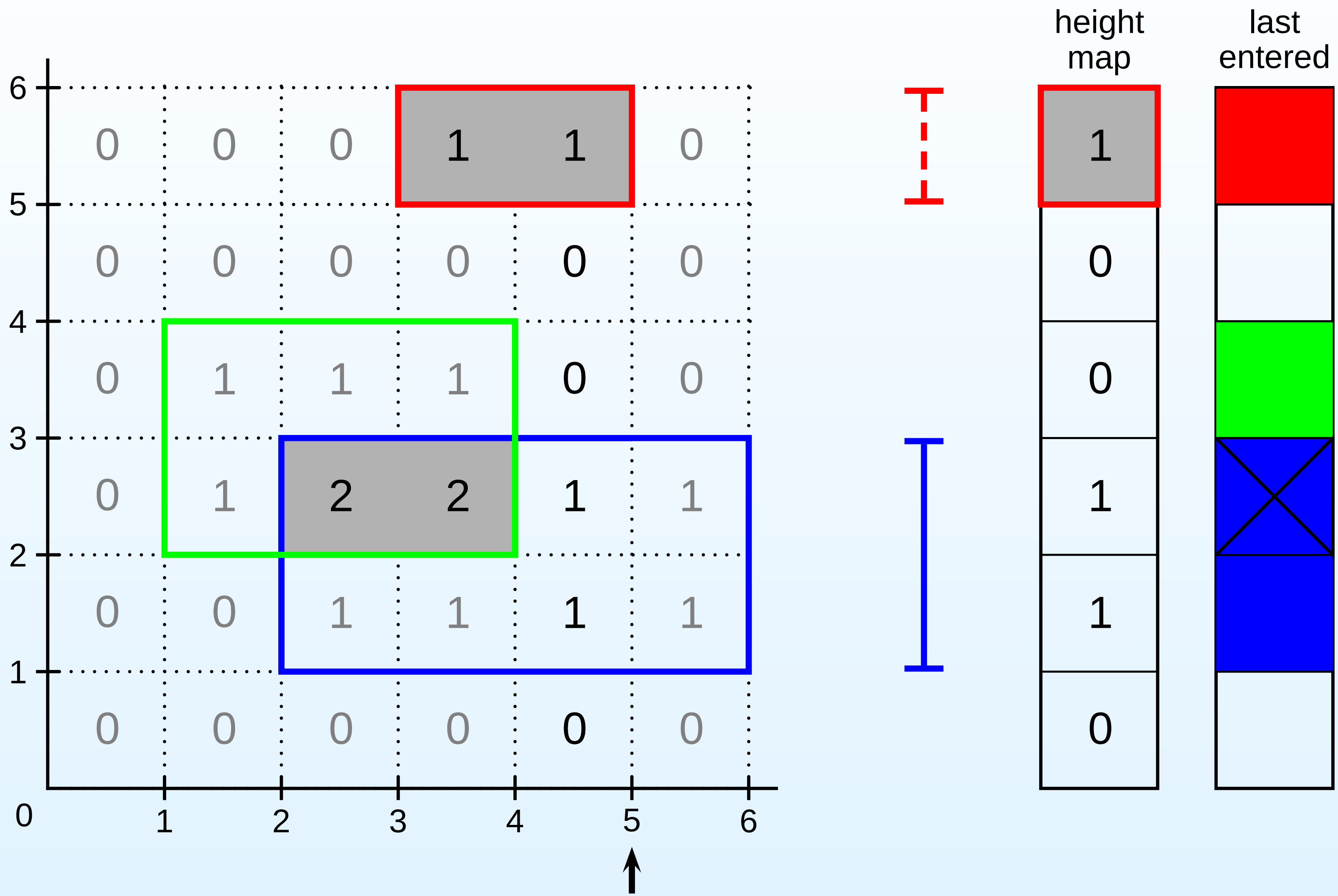

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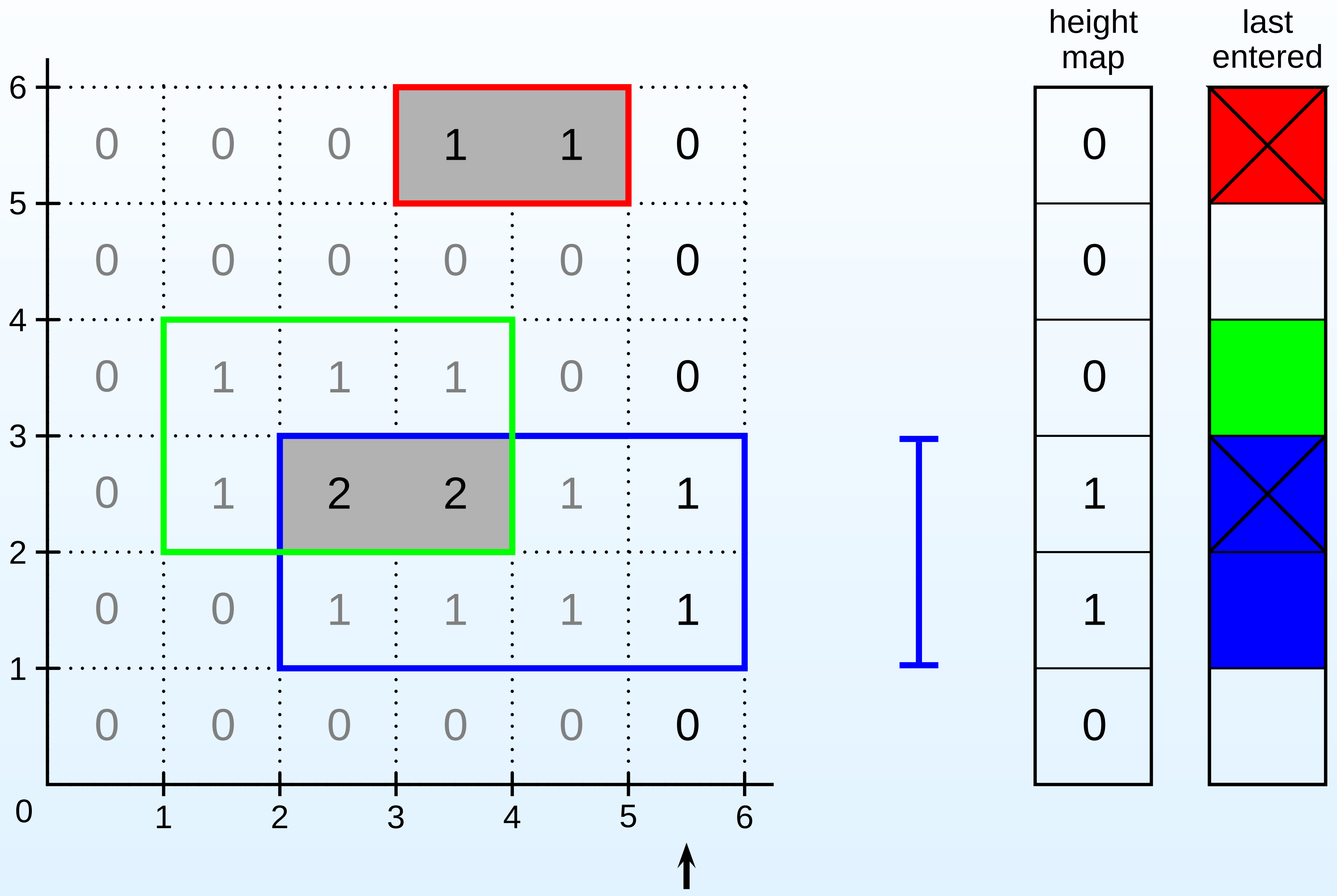

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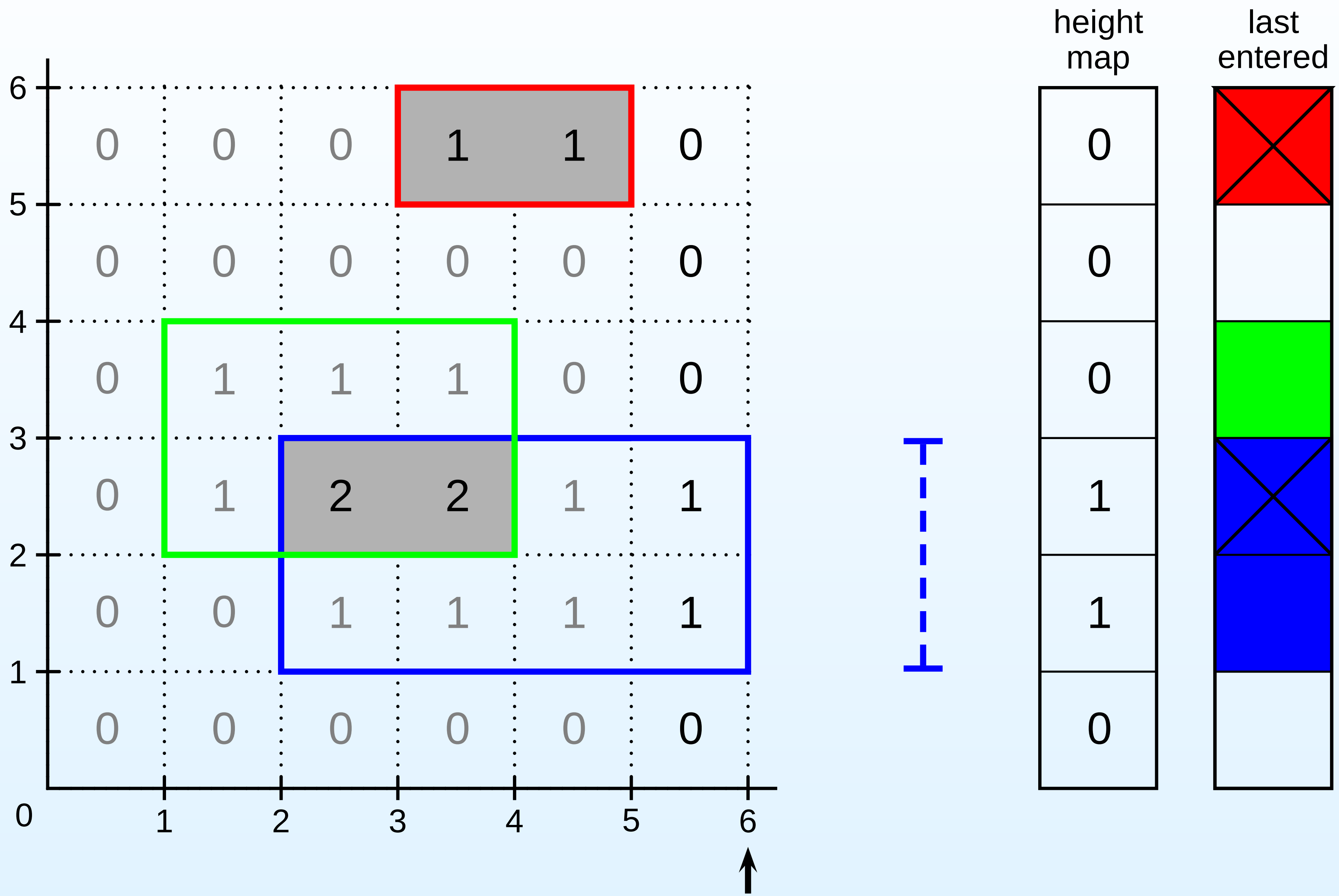

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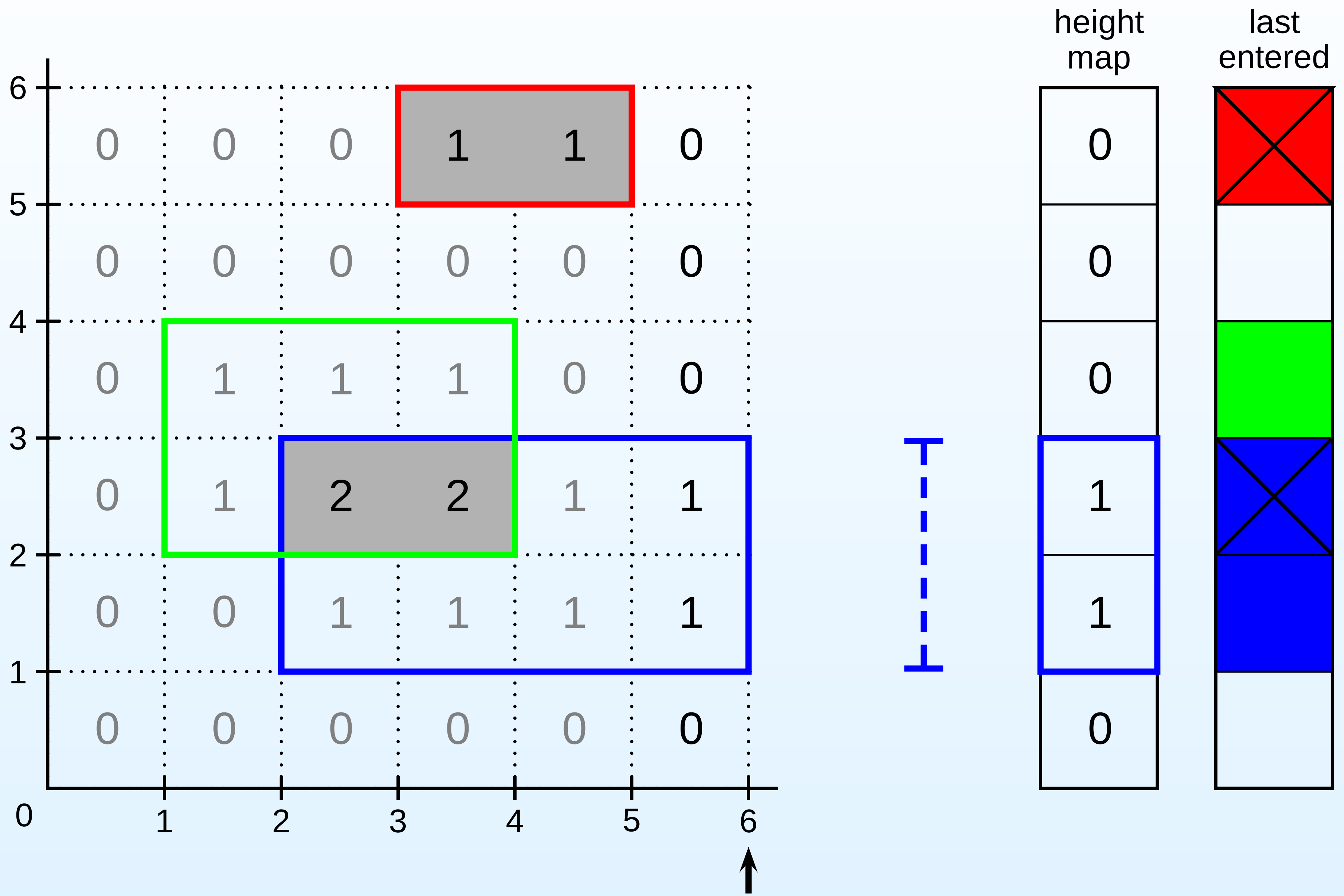

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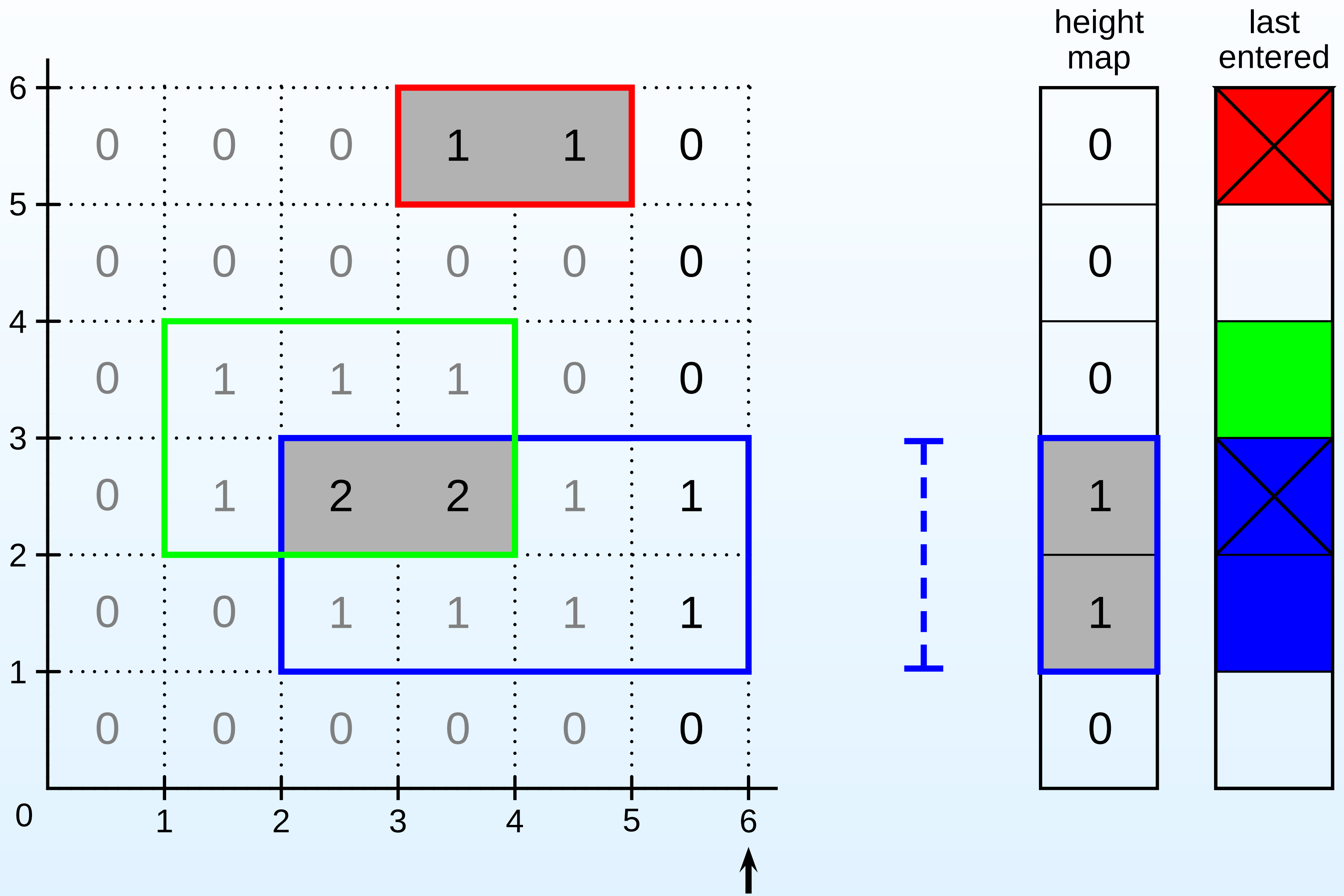

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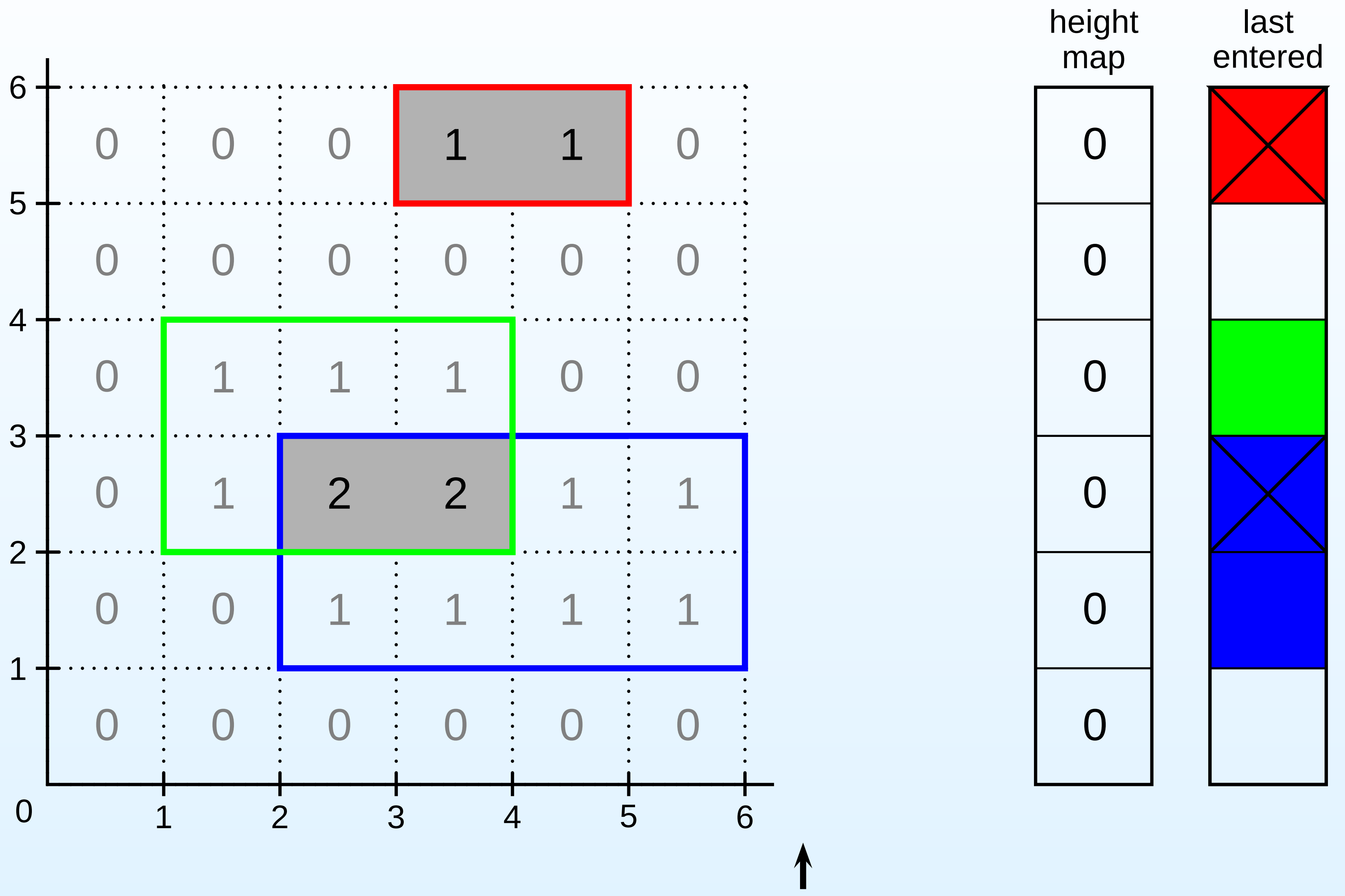

Time and space complexity of the algorithm

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- time complexity: $O(n^2)$
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Simulation study

Bivariate current status data from a simple exponential model:

- Variables of interest: $X, Y \sim \text{Exp}(1)$
- Observation times: $U, V \sim \text{Exp}(1)$
- X, Y, U, V mutually independent
- 50 simulations for sample sizes

$$50 \quad 100 \quad 250 \quad 500 \quad 1{,}000 \quad 2{,}500 \quad 5{,}000 \quad 10{,}000$$

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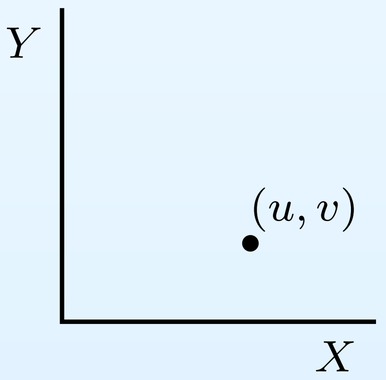

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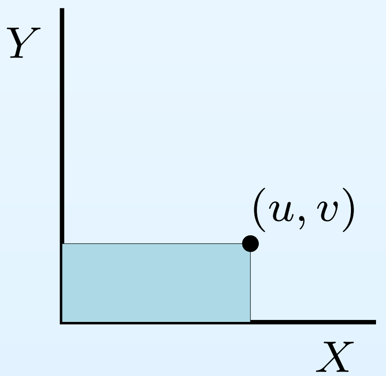

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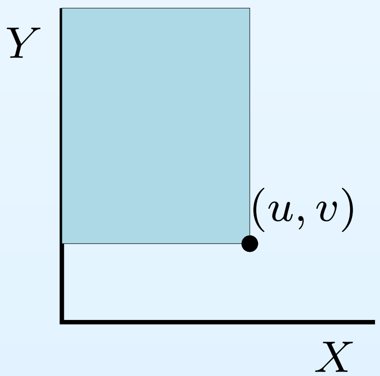

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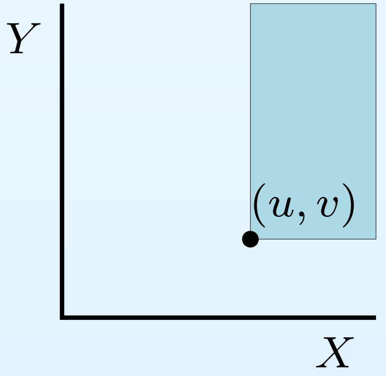

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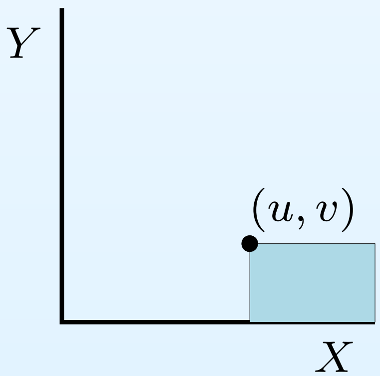

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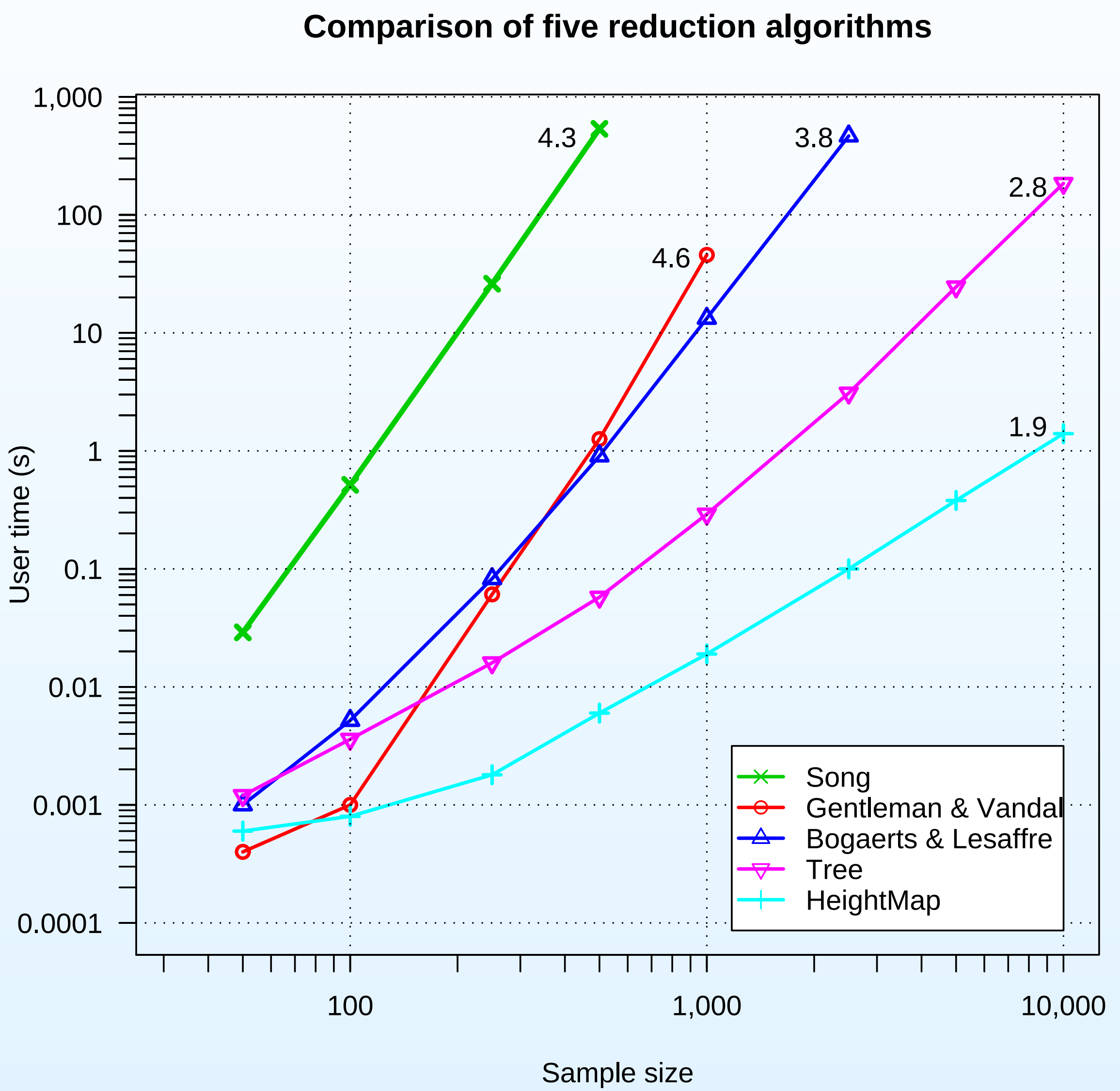
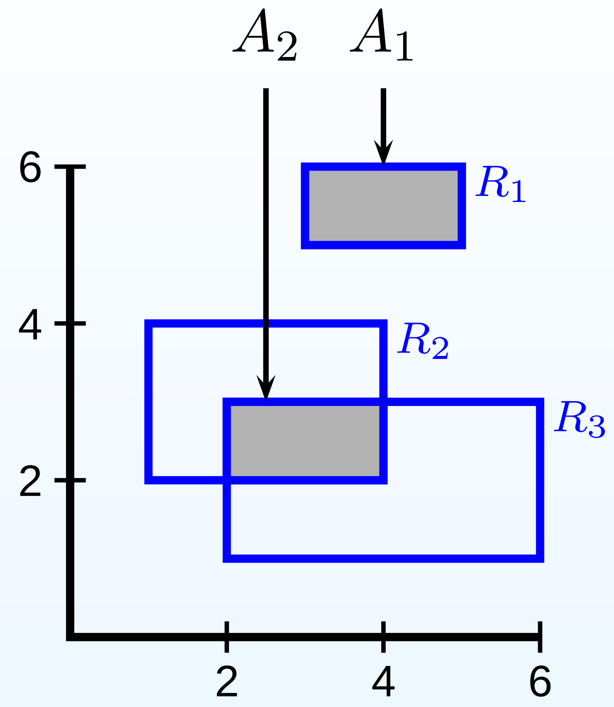


Computing the MLE: optimization step

$\max_{F \in \mathcal{F}} \sum_{i=1}^{n} \log(P_F(R_i))$



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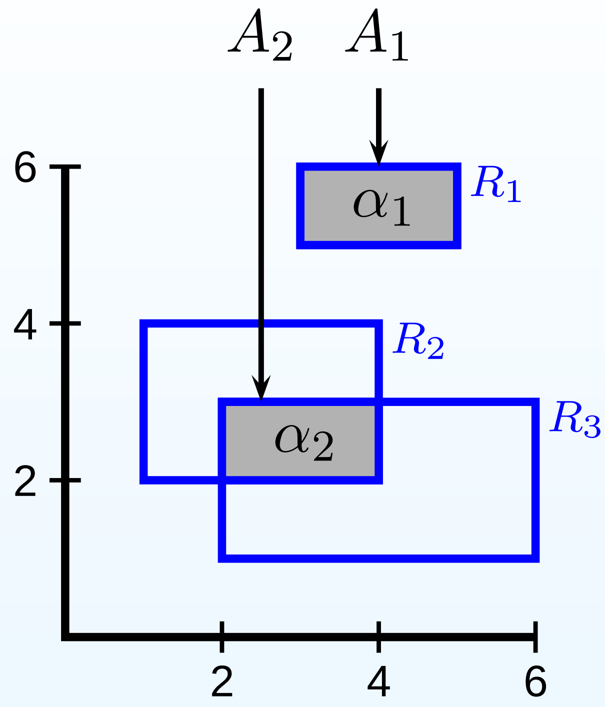

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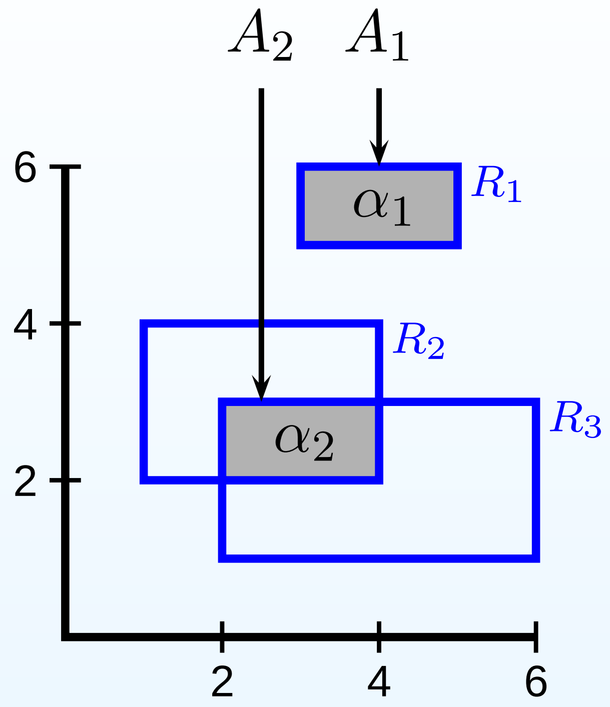

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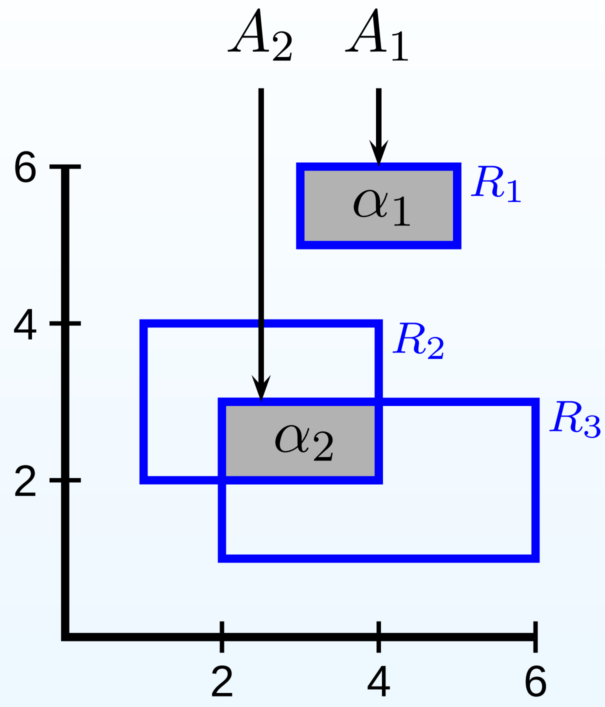

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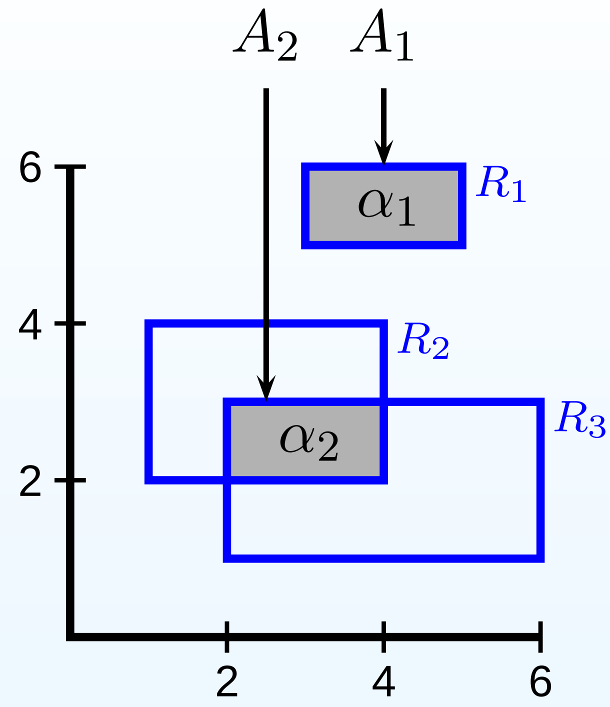

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$\min_{\alpha\in\mathcal{A}^*} \left[-\frac{1}{n}\sum_{i=1}^{n} \log(C^T\alpha)_i + \sum_{j=1}^{m} \alpha_j\right] = \min_{\alpha\in\mathcal{A}^*} \phi(\alpha)$

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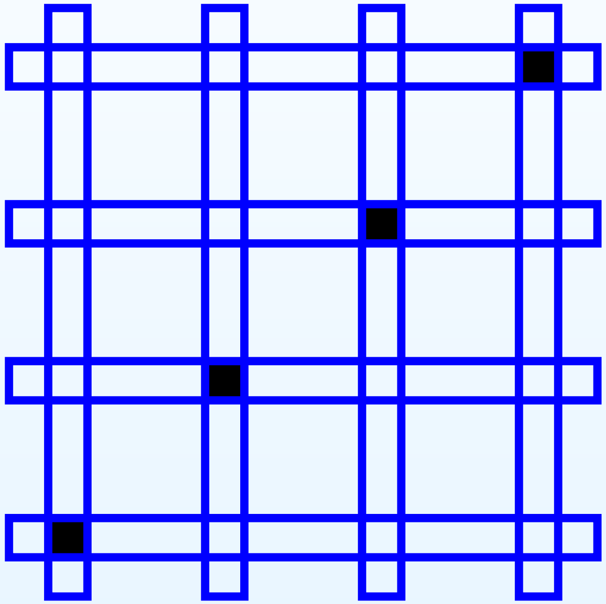

mass $1/4$

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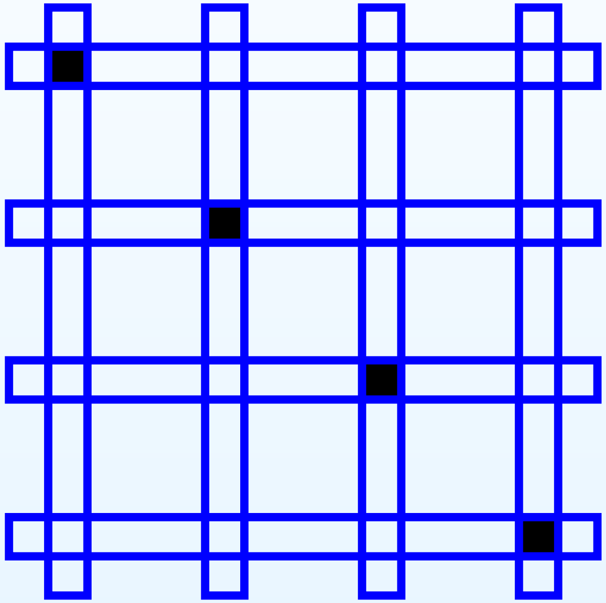

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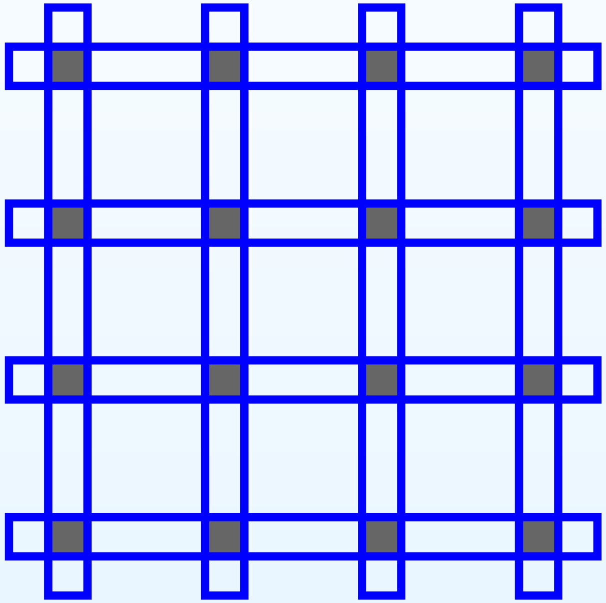

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- We compute $\hat{\alpha}$ with an iterative algorithm, and stop if $(*)$ is satisfied within some tolerance ϵ, e.g. $\epsilon = 10^{-10}$

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R-packages for interval censored data

- Existing packages:
  - `Icens` (Gentleman and Vandal):
    several functions for univariate/bivariate interval censored data
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- New package `MLEcens`
  - Includes package `bicreduc` (which is no longer maintained)
  - Some overlap with `Icens`, but `MLEcens` has:
    - Very fast reduction step
    - Fast and stable optimization step
    - Various new plotting functions

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  - We finally chose to use the open source BLAS (Basic Linear Algebra Subprograms) and LAPACK (Linear Algebra PACKage) libraries that come with R

Overview of `MLEcens`

- Available functions:
  - Basic plot function: `plotRects`
  - Canonical rectangles: `real2canon, canon2real`
  - Reduction step: `reduc, plotHM, plotCM`
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- Demonstration...

Possible extensions of R-package

- 3d-plotting functions
- Specialized algorithms for univariate interval censored data
- Extension to 3-dimensional interval censored data
- . . .

I am happy to modify the package. So please let me know if you are interested in any extensions, or if you have any other feedback/suggestions.

References

- Reduction step:
  - Maathuis (2005). “Reduction algorithm for the NPMLE for distribution function of bivariate interval censored data", *JCGS* **14** 352–362.
- Optimization step:
  - Groeneboom, Jongbloed and Wellner (2007). “The support reduction algorithm for computing nonparametric function estimates in mixture models", *Submitted*.
  - Maathuis (2003). “Nonparametric maximum likelihood estimation for bivariate censored data", Master’s Thesis, Delft University of Technology, The Netherlands.
- R-package:
  - Maathuis (2007), “R-package `MLEcens`", CRAN.

Thanks!

Presentation (including R-code), papers, and R-package
are posted on my website:

http://stat.ethz.ch/∼maathuis