Applications of Machine Learning to the Game of Go

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(working with **Thore Graepel, Ralf Herbrich, David MacKay**)

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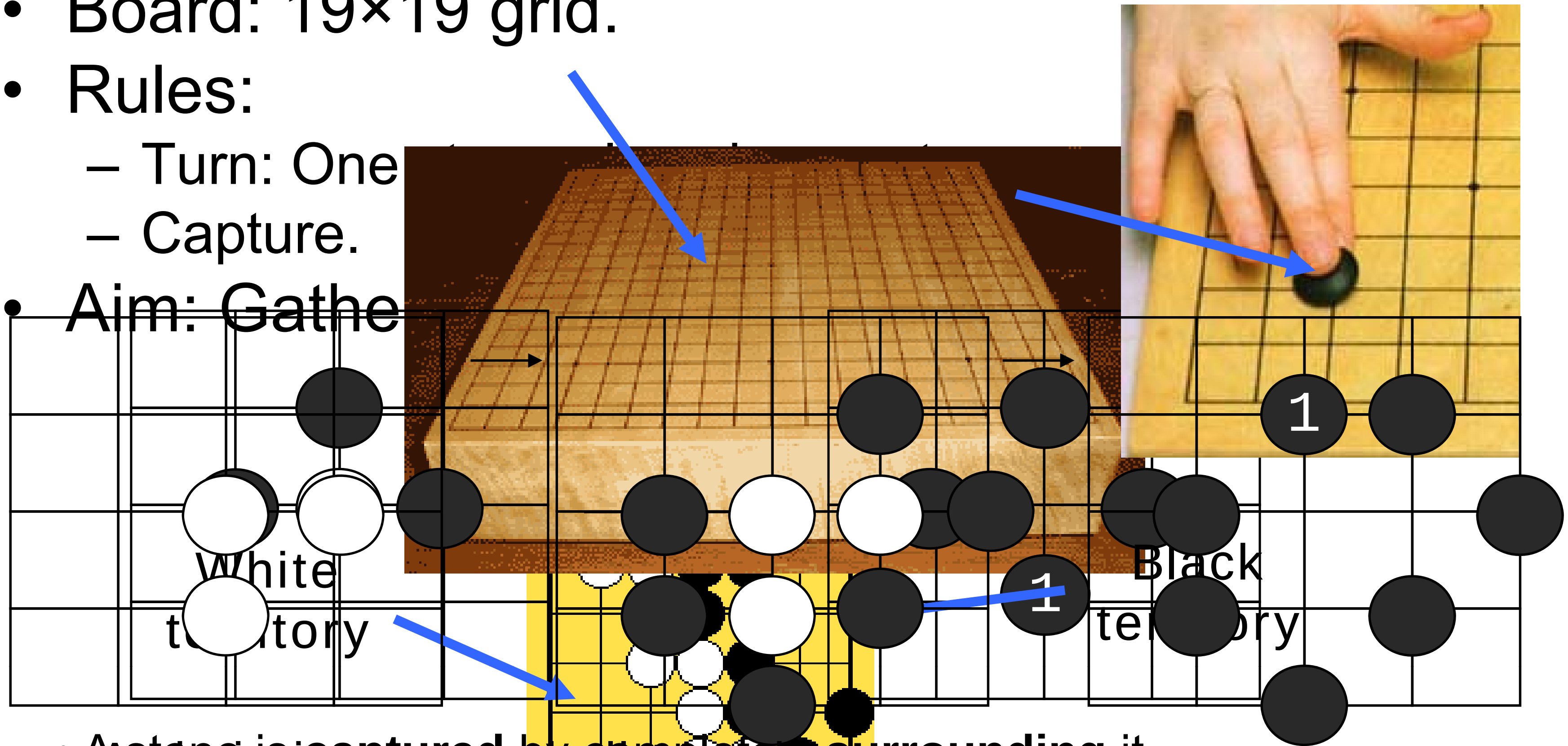
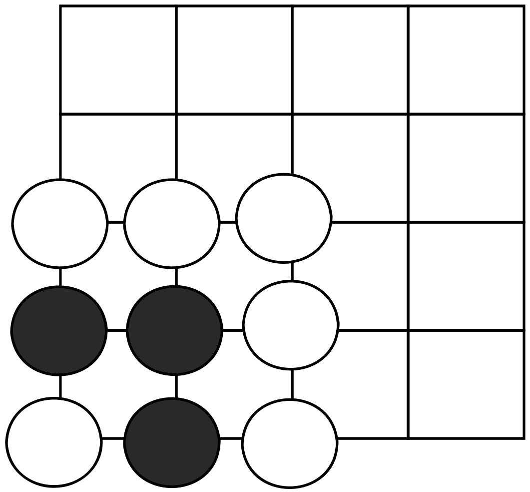
The Game of Go

- Started about 4000 years ago in ancient China.
- About 60 million players worldwide.
- 2 Players: Black and White.
- Board: 19×19 grid.
- Rules:
  - Turn: One
  - Capture.
- Aim: Gathe

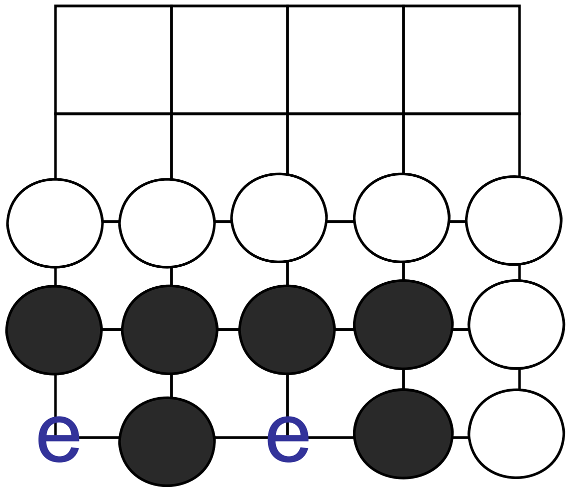


- A stone is **captured** by completely **surrounding** it.
- A chain is captured all together.

One eye = death



Two eyes = life



Computer Go

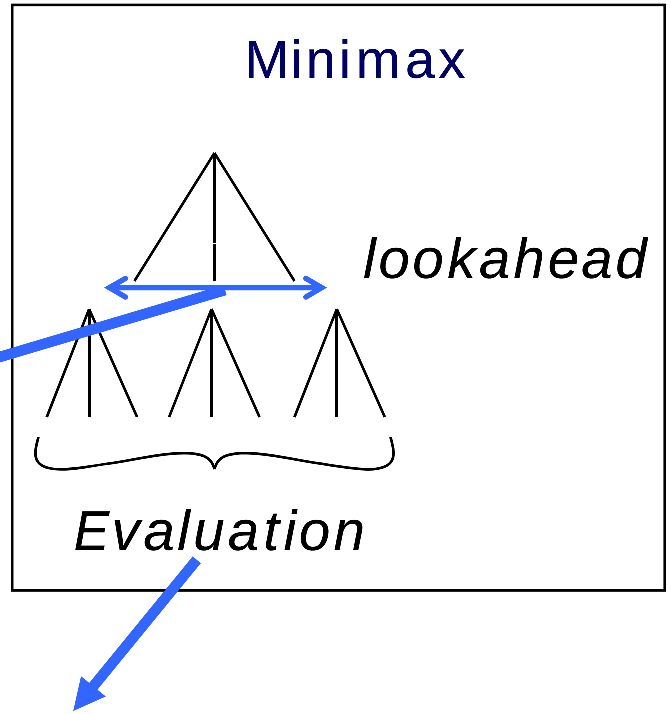
- 5th November 1997:
  Gary Kasparov beaten by Deep Blue.

Kasparov ponders his next move
(CNN)

- Best Go programs cannot beat amateurs.
- Go recognised as grand challenge for AI.
- Müller, M. (2002). Computer Go. *Artificial Intelligence, 134.*

Computer Go

- Minimax search defeated.
- High **Branching Factor**.
  - Go: ~200
  - Chess: ~35
- Complex **Position Evaluation.**
  - Stone’s value derived from configuration of surrounding stones.



Use of Knowledge in Computer Go

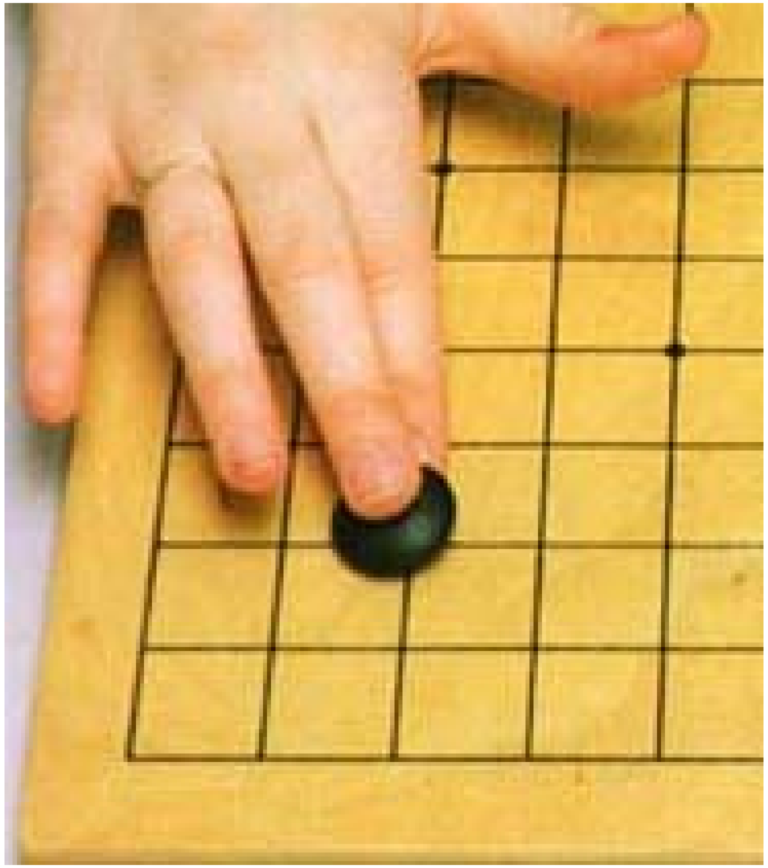
- Trade off between search and knowledge.
- Most current Go programs use hand-coded knowledge.
  1. Slow knowledge acquisition.
  2. Tuning hand-crafted heuristics difficult.
- Previous approaches to automated knowledge acquisition:
  - Neural Networks (Erik van der Werf et al., 2002).
  - Exact pattern matching (Frank de Groot, 2005), (Bouzy, 2002)

Uncertainty in Go

- Go is game of perfect information.
- Complexity of game tree +
  limited computer speed → uncertainty.
- 味 'aji' = 'taste'.
- Represent uncertainty using probabilities.

Move Prediction

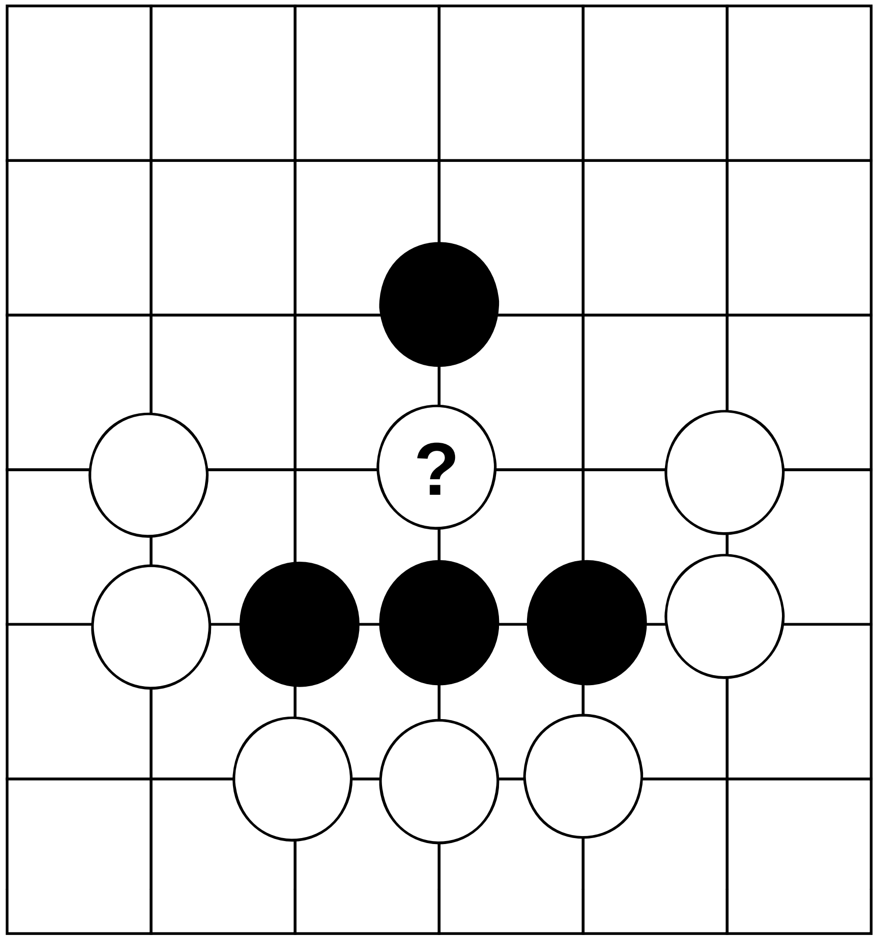
Learning from Expert Game Records



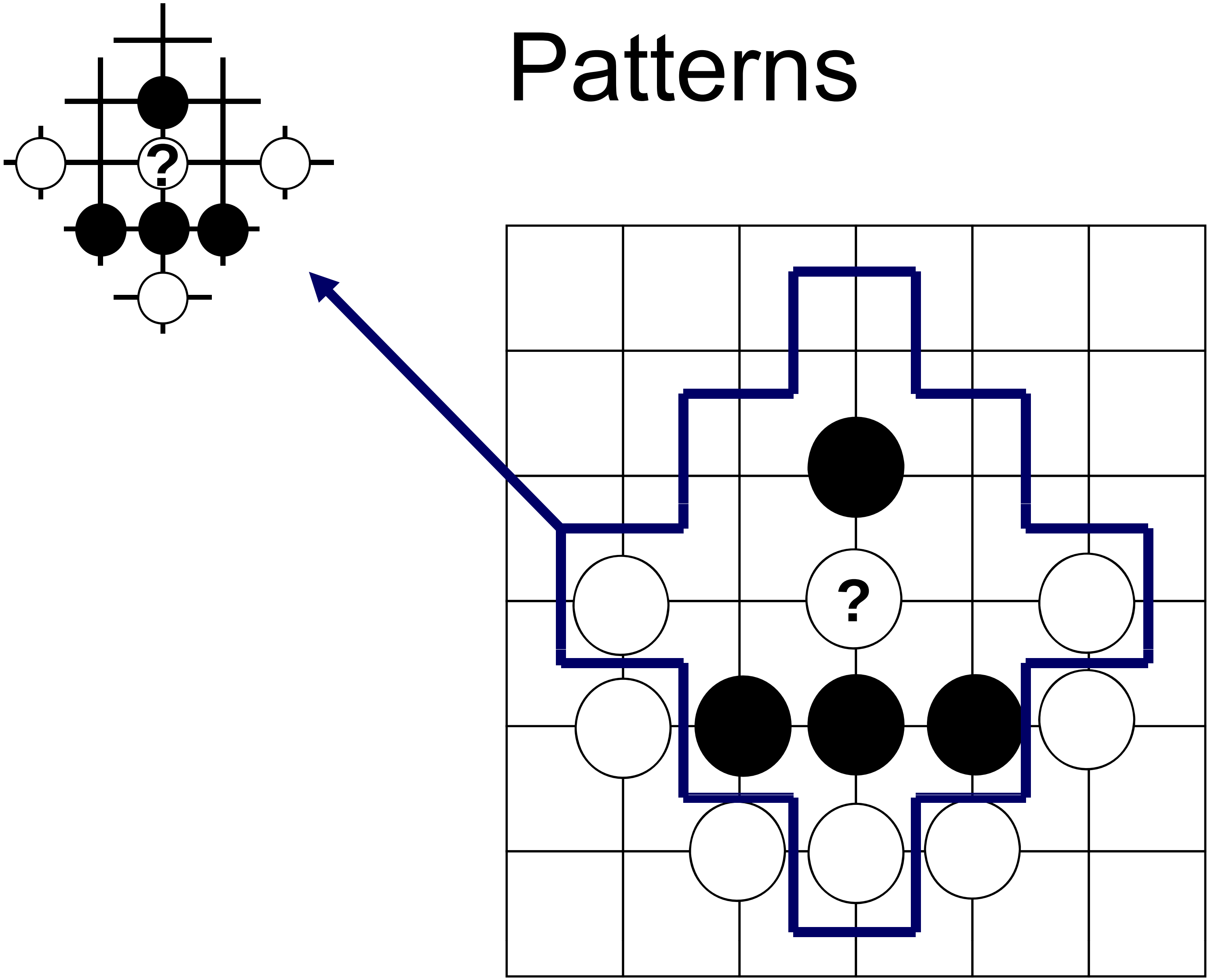
Pattern Matching for Move Prediction

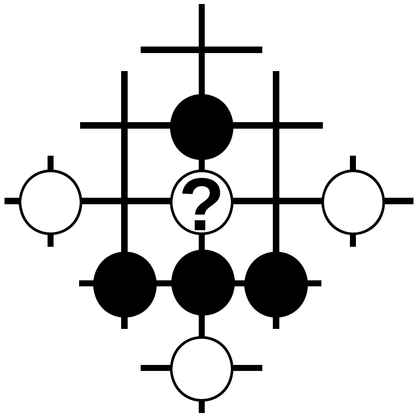
- Move associated with a set of *patterns*.
  - Exact arrangement of stones.
  - Centred on proposed move.
- Sequence of nested templates.

Patterns

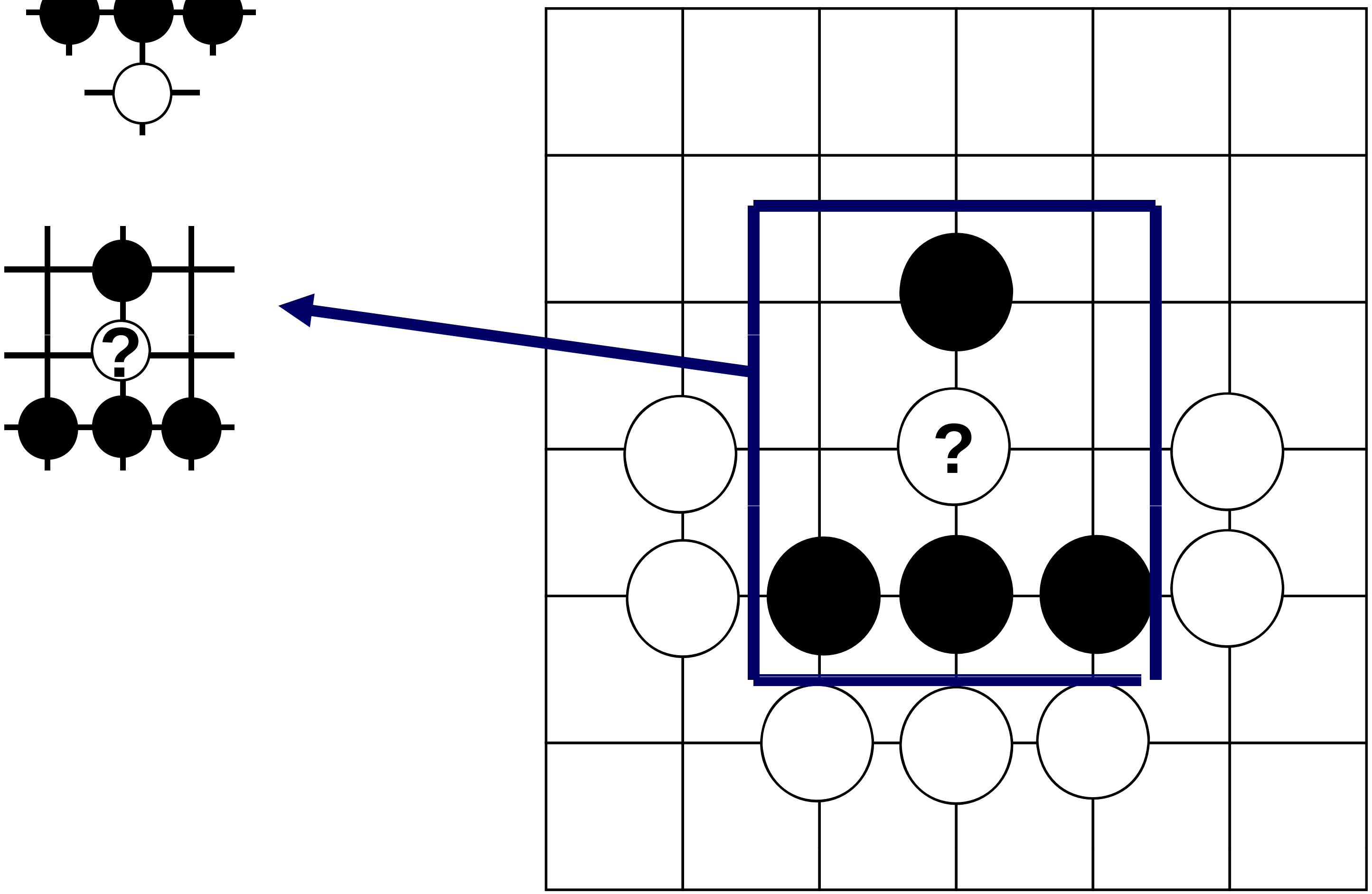


Patterns

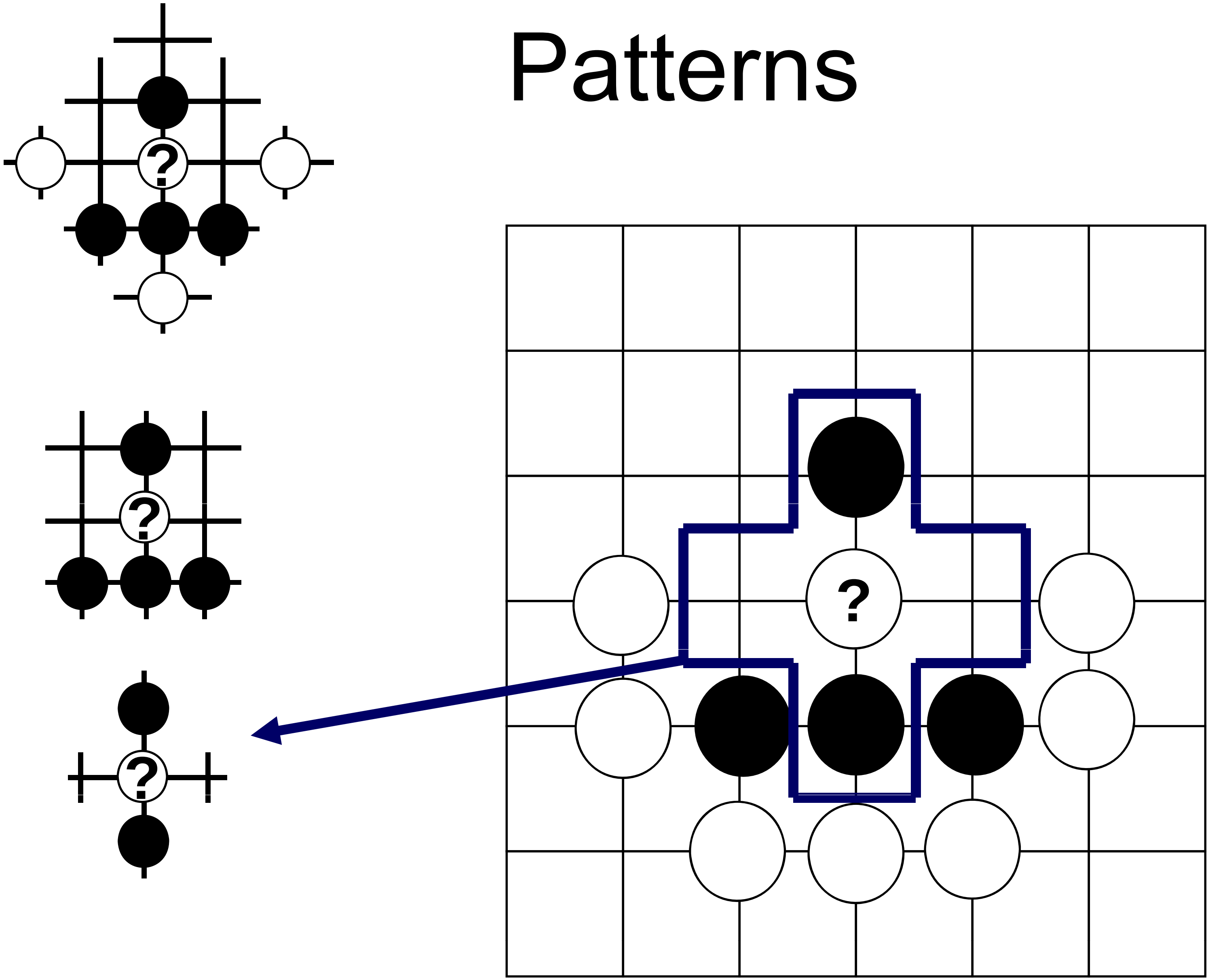




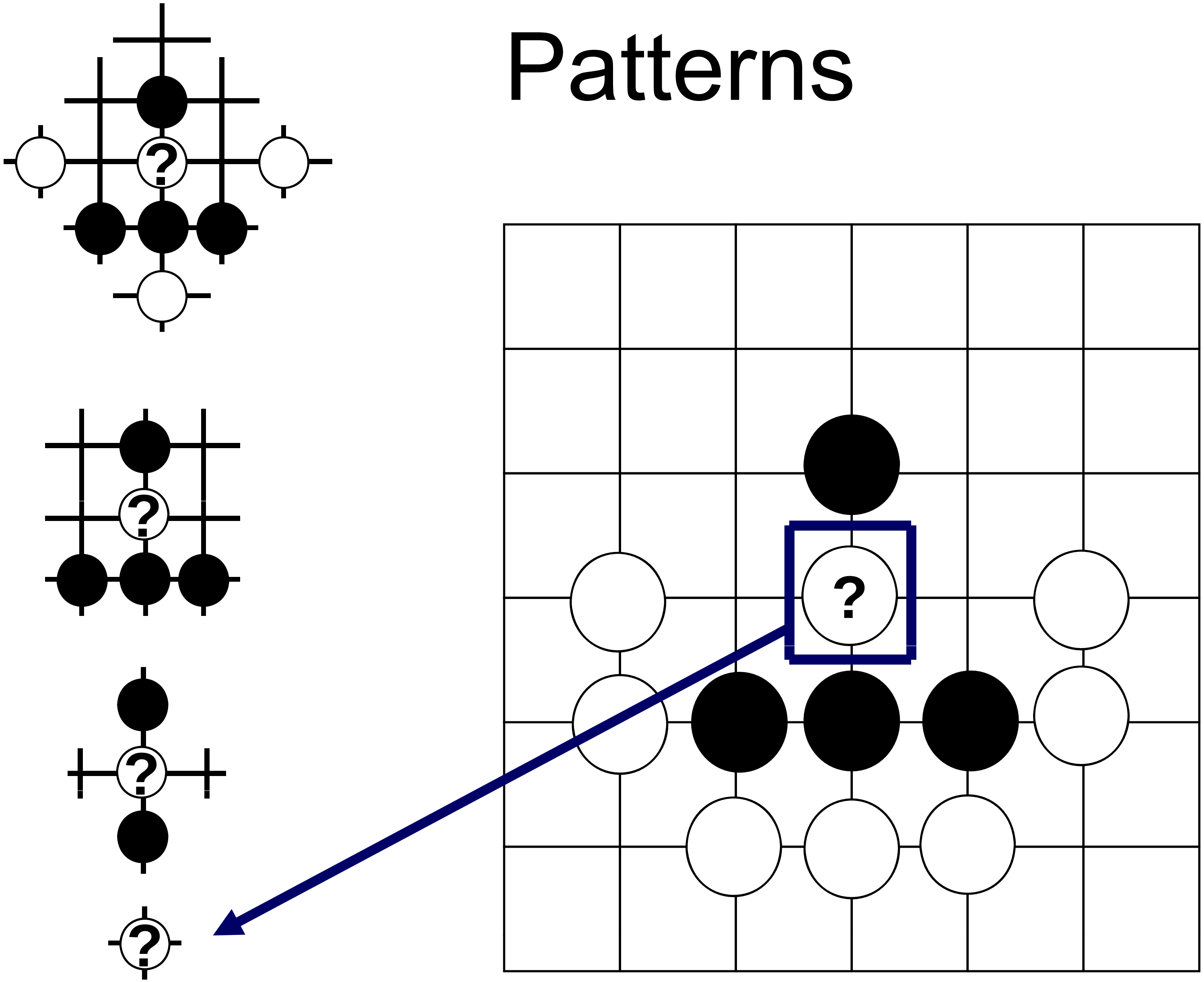
Patterns

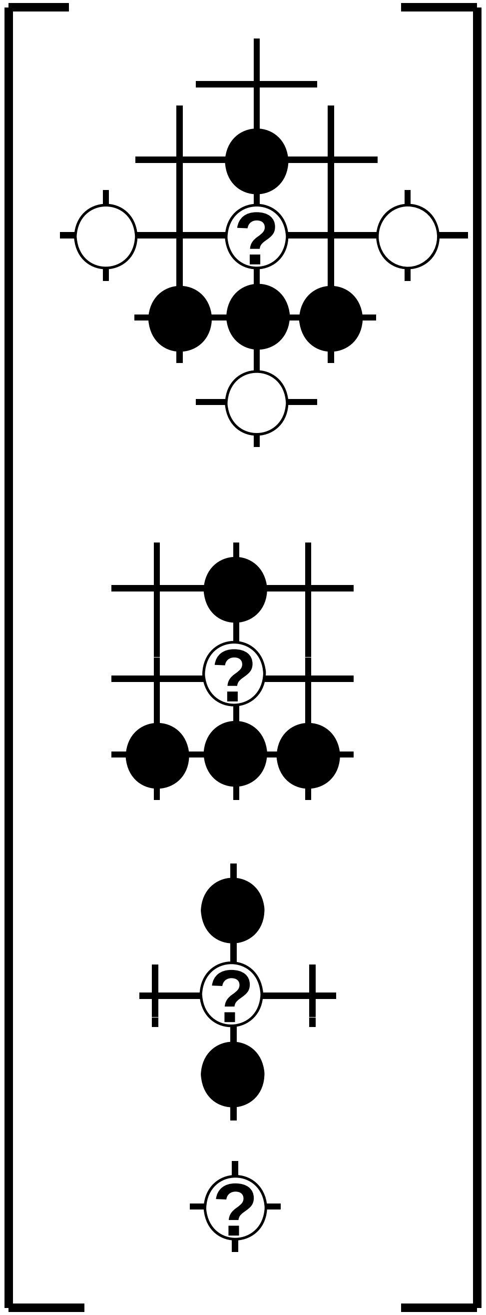


Patterns

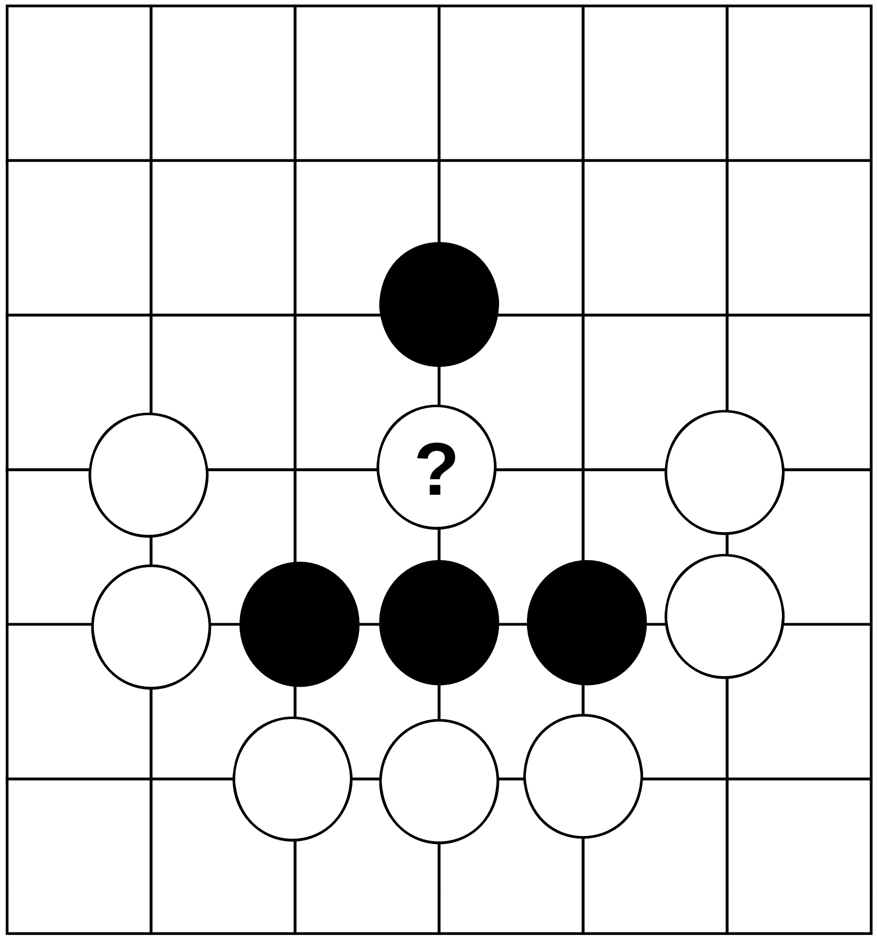


Patterns

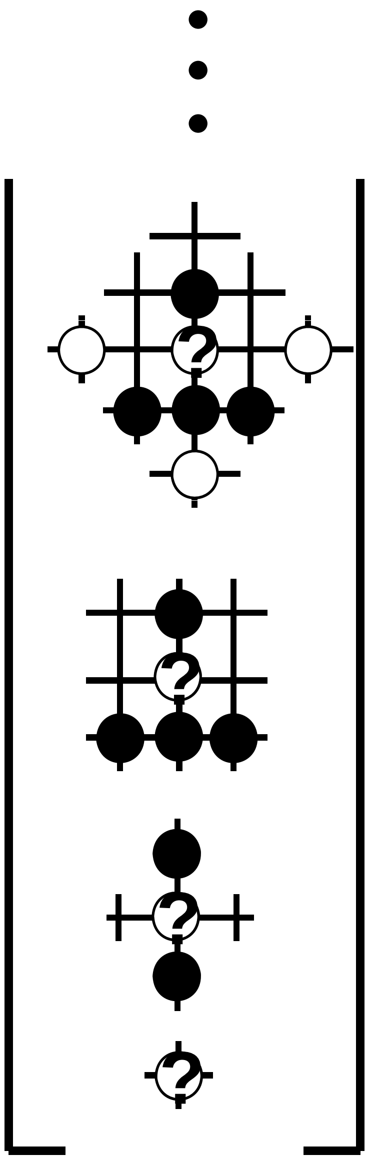




Patterns



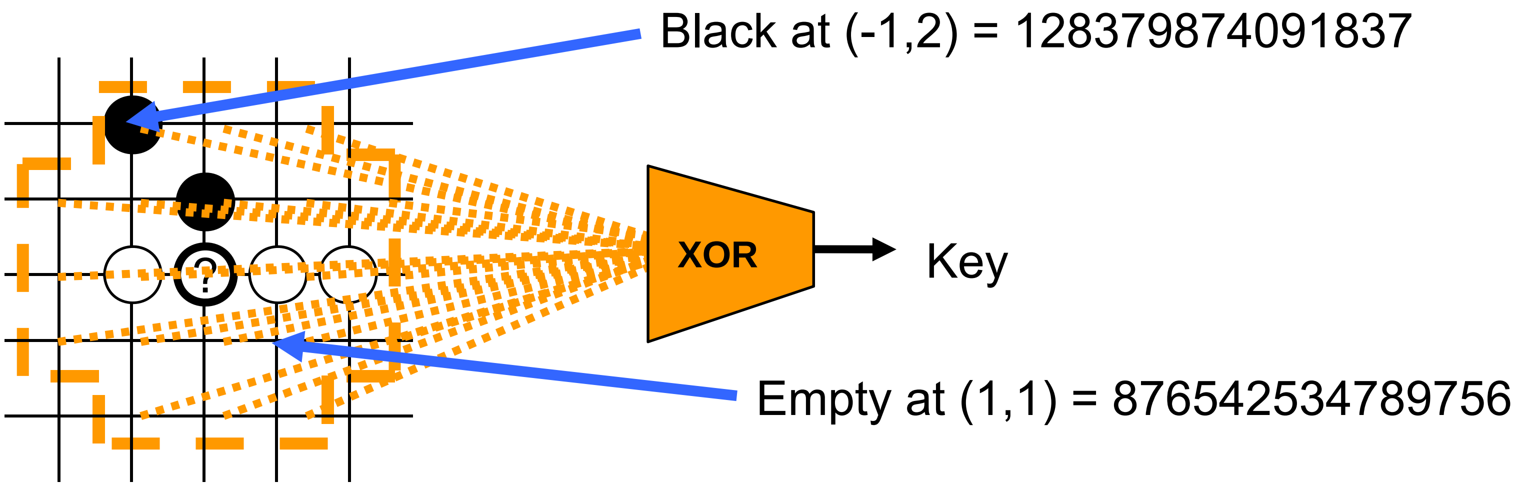
Patterns



- 13 Pattern Sizes
  - Smallest is vertex only.
  - Biggest is full board.

Pattern Matching

- Pattern information stored in hash table.
  - Constant time access.
  - No need to store patterns explicitly.
- Need rapid incremental hash function.
  - Commutative.
  - Reversible.
- 64 bit random numbers for each template vertex: One for each of {**black, white, empty, off**}.
- Combine with XOR (Zobrist, 1970).



Pattern Hash Key

- Pattern information stored in hash table.
  - Access in constant time.
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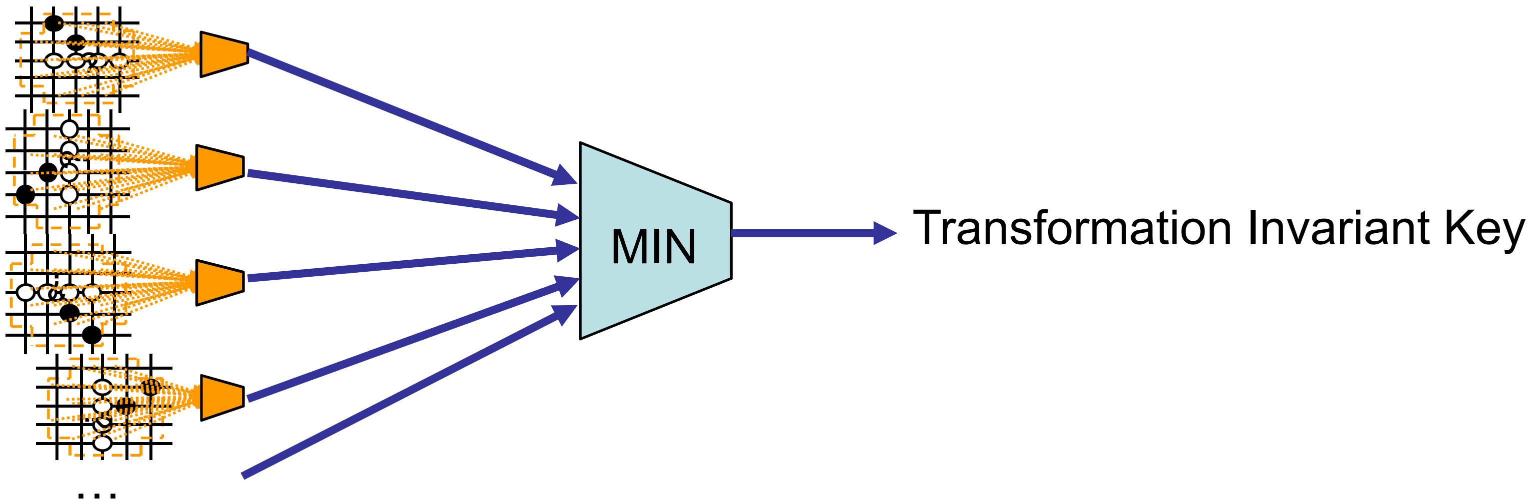
  - Commutative.
  - Reversible.
- 64 bit random numbers for each template vertex: One for each of {**black, white, empty, off**}.
- Combine with XOR (Zobrist, 1970).
- Min of transformed patterns gives invariance.

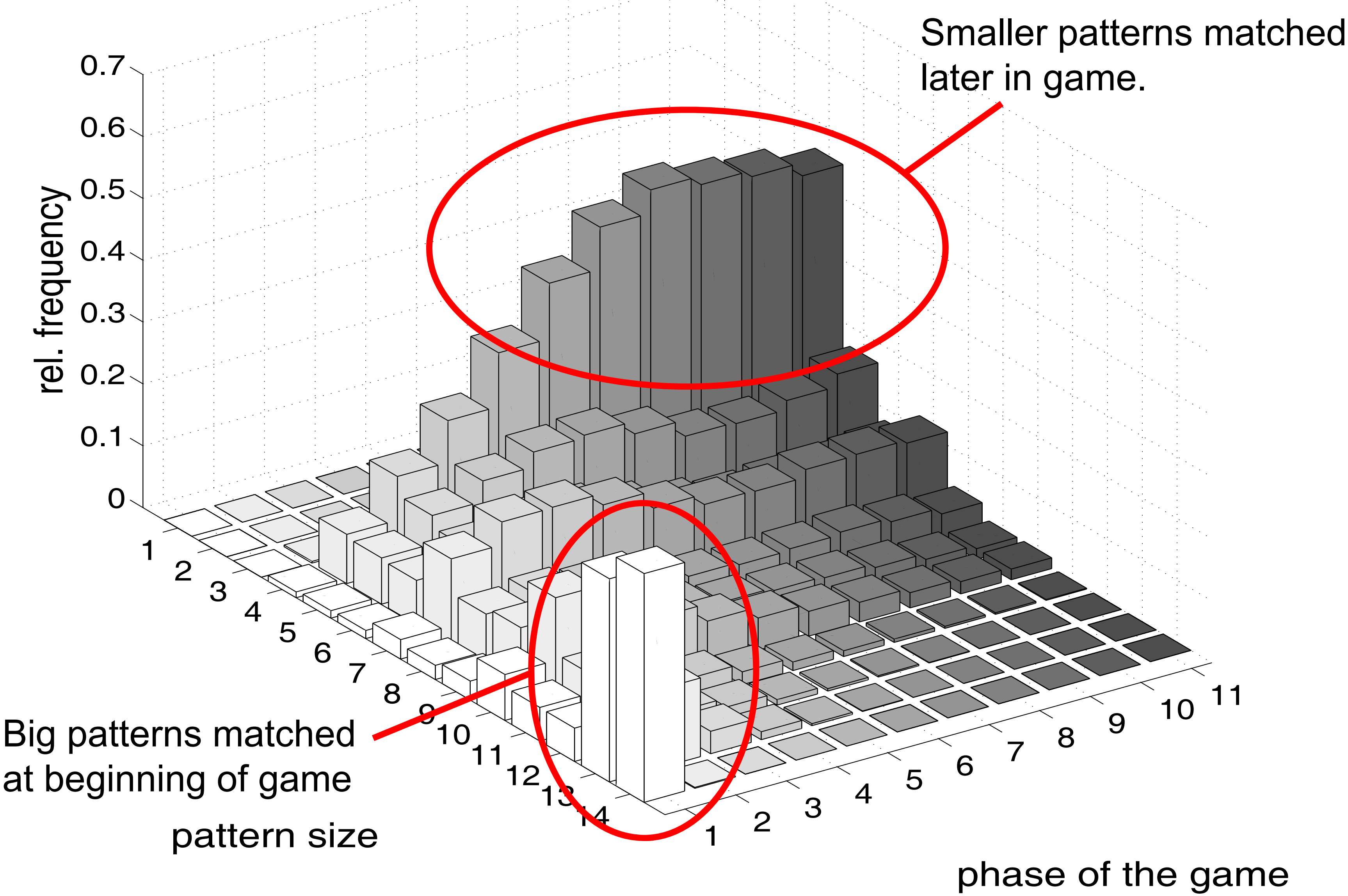


Harvesting

- Automatically Harvest from Game Records.
- 180,000 games × 250 moves × 13 pattern sizes…

  …gives 600 million potential patterns.
- Need to limit number stored.
  - Space.
  - Generalisation.
- Keep all patterns played more than *n* times.
- *Bloom filter*:
  Approximate test for set membership with minimal memory footprint. (Bloom, 1970).

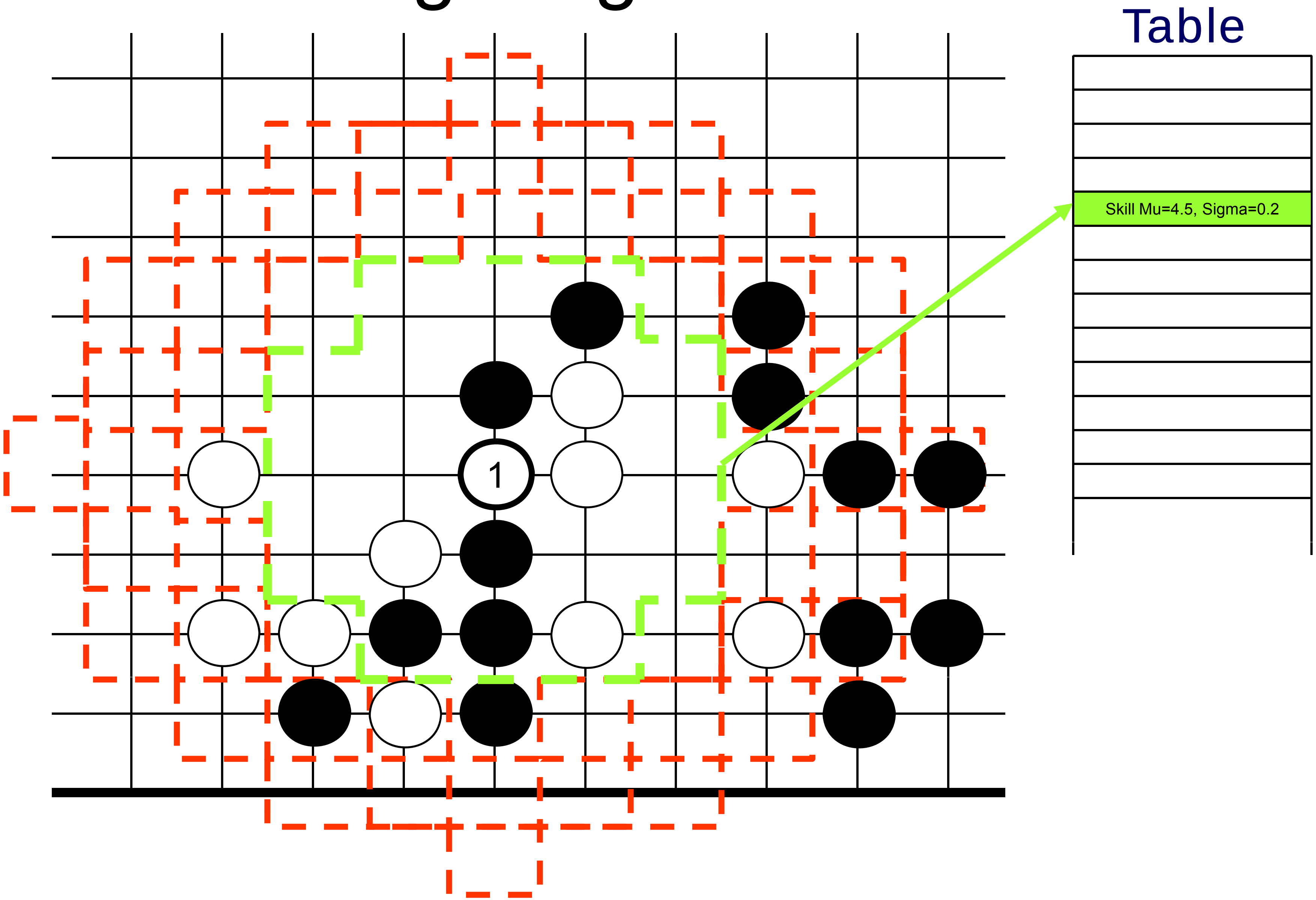
Relative Frequencies of Pattern Sizes



Training

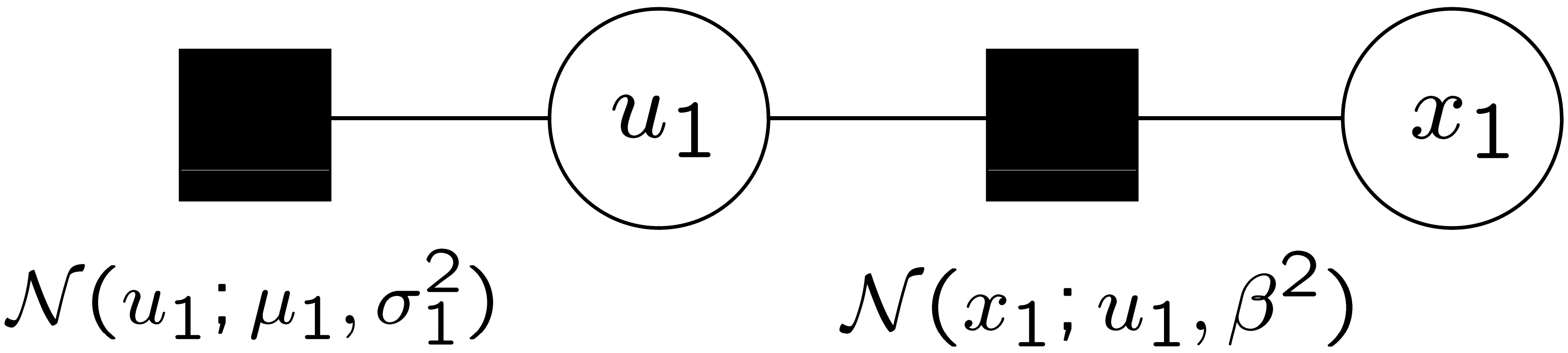
- First Phase: **Harvest**
- Second Phase: **Training**
- Use same games for both.
- Represent move by **largest pattern only**.

Matching Largest Pattern

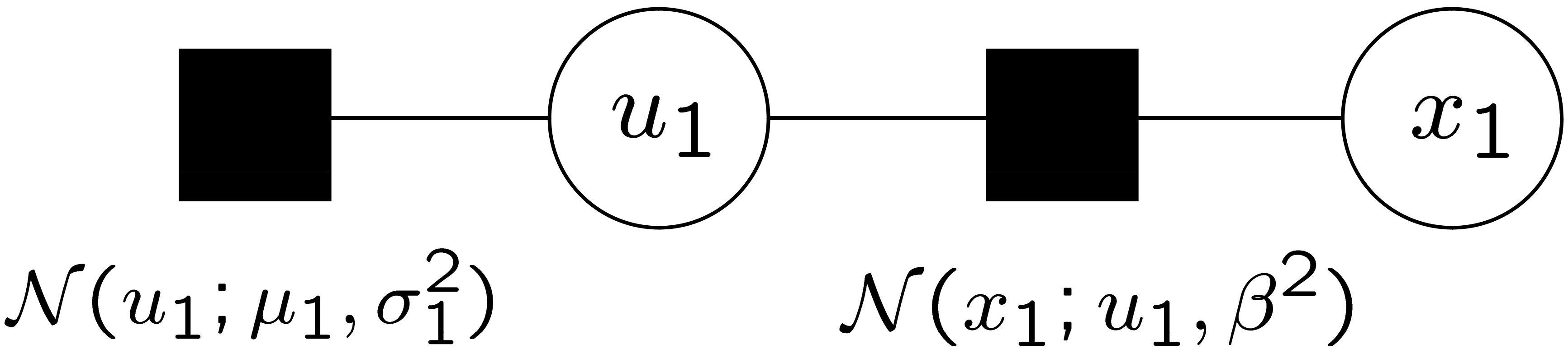


Bayesian Ranking Model

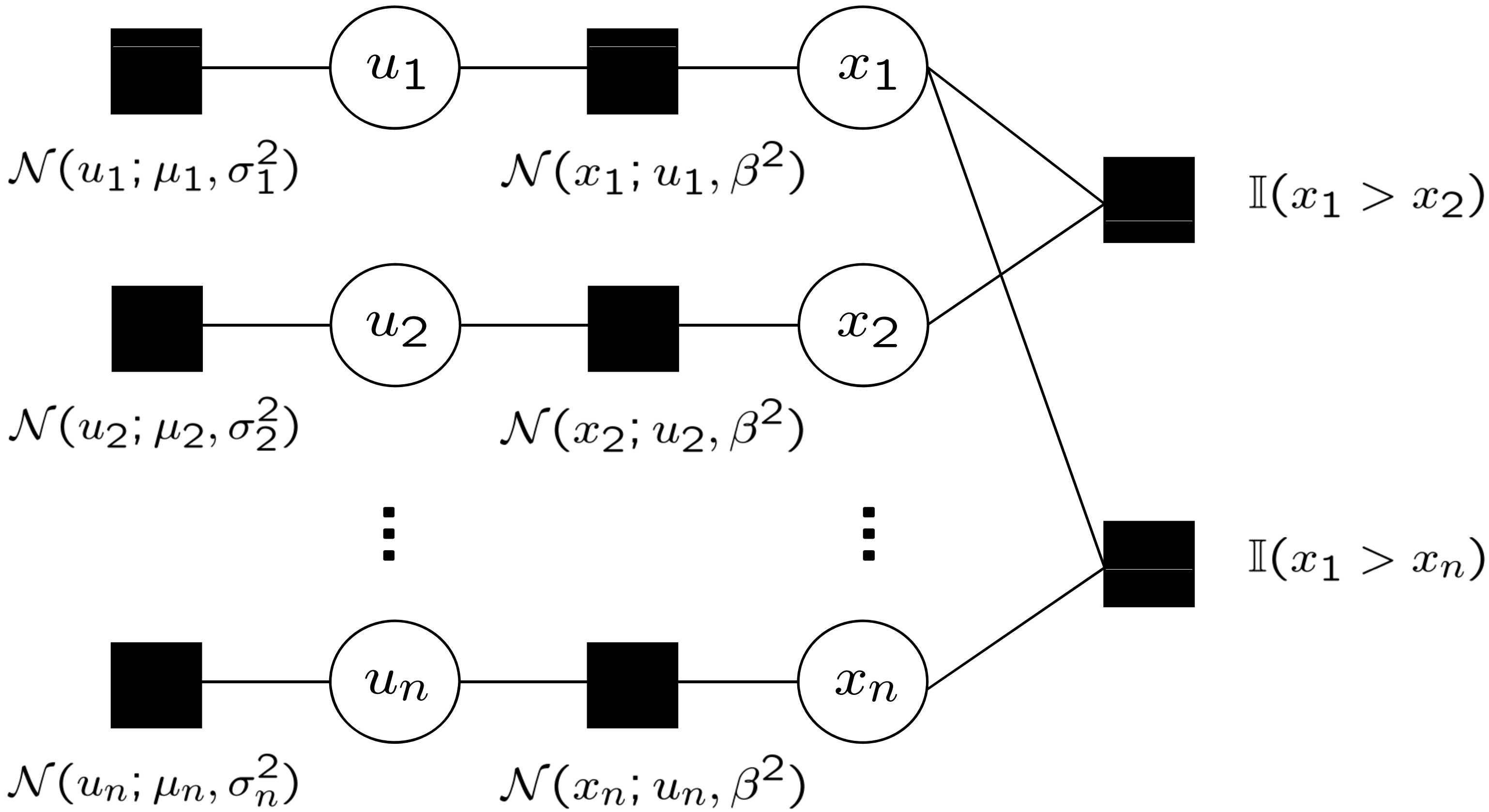
- Pattern value: $u_1 \sim \mathcal{N}(u_1; \mu_1, \sigma_1^2)$
- Latent urgency: $x_1 \sim \mathcal{N}(x_1; u_1, \beta^2)$



Bayesian Ranking Model



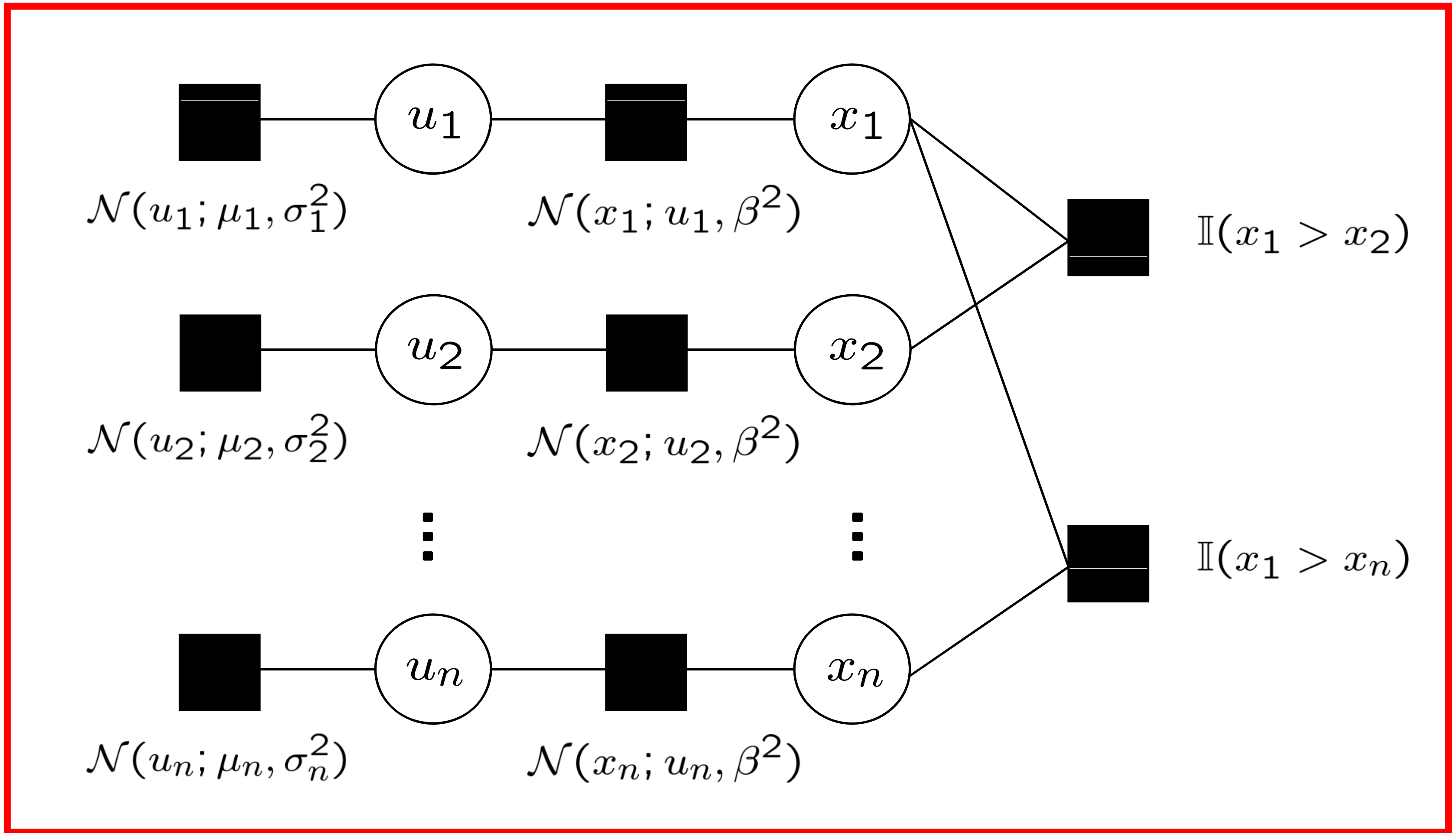
Bayesian Ranking Model



Training Example:

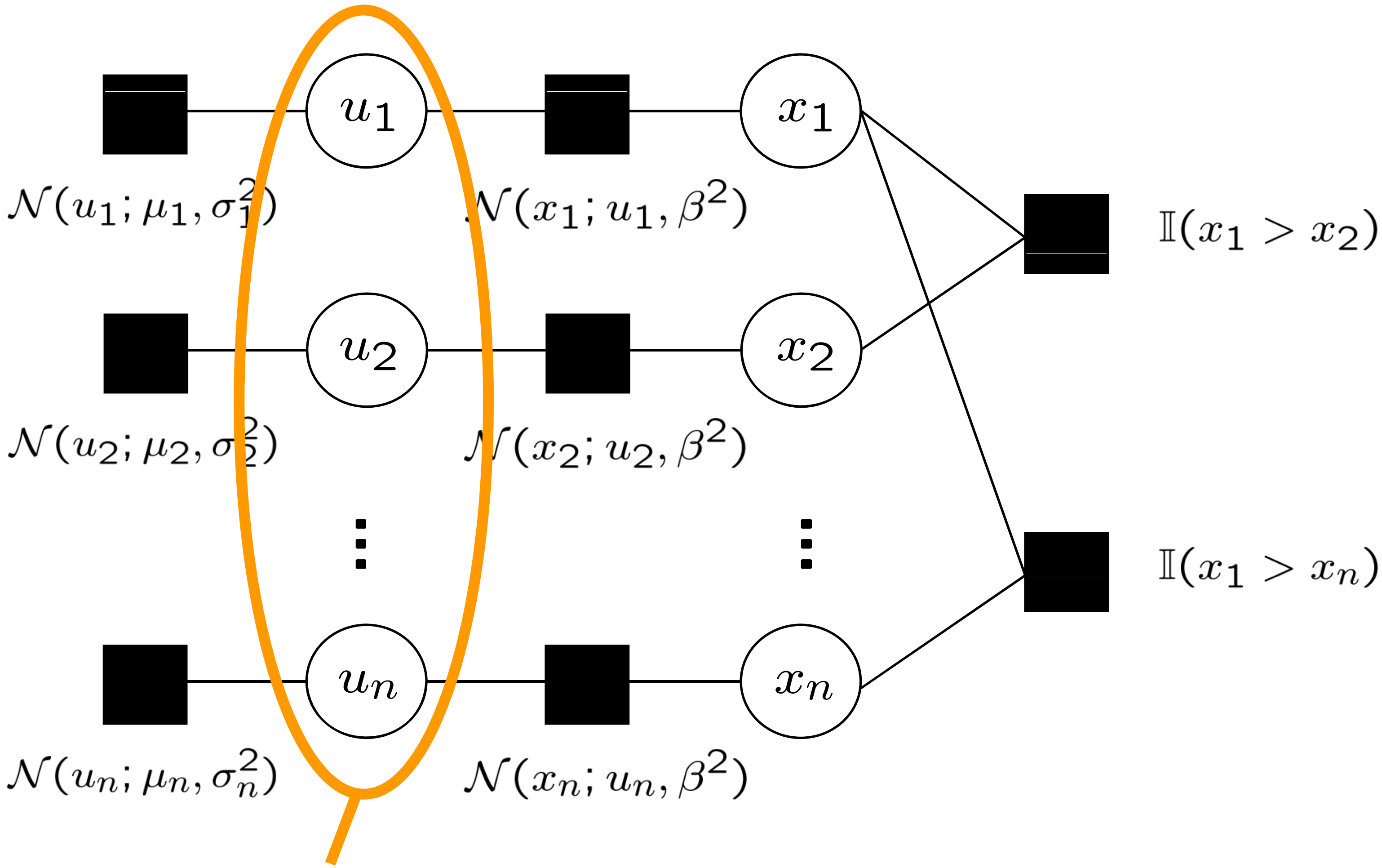
- Chosen move (pattern).
- Set of moves (patterns) not chosen.

Bayesian Ranking Model



$p(\mathbf{u}, \mathbf{x} | \text{move, position})$

Bayesian Ranking Model



$$p(\mathbf{u}|\text{move, position}) = \int p(\mathbf{u}, \mathbf{x}|\text{move, position}) d\mathbf{x}$$

Online Learning from Expert Games

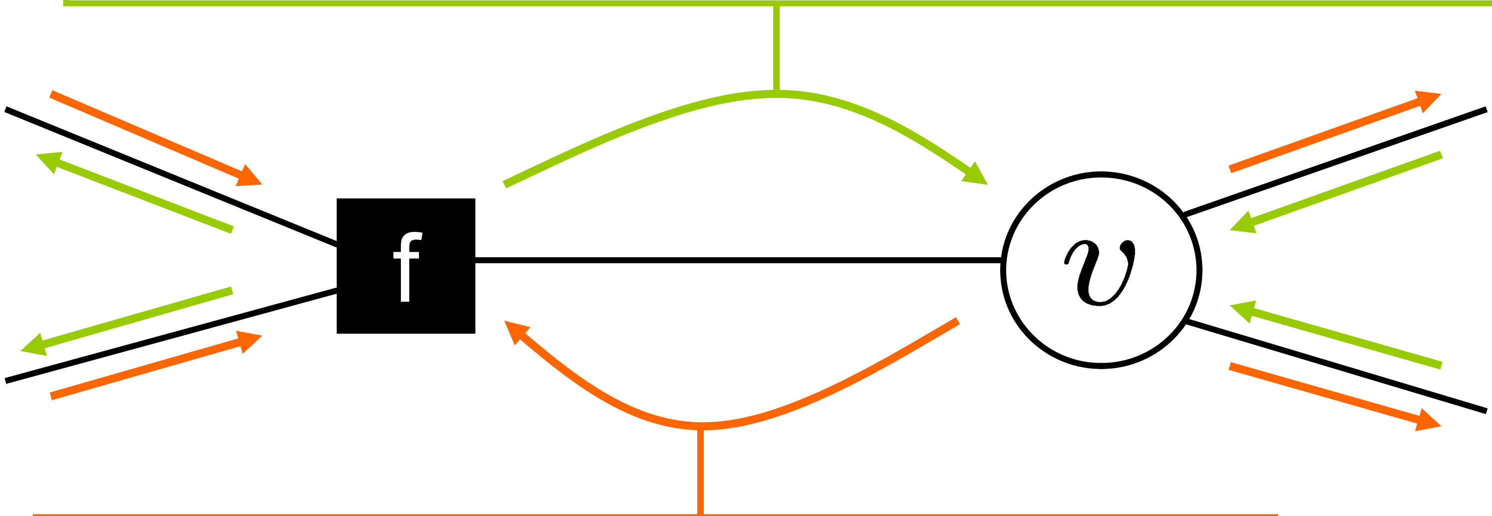
- Training Example:
  - Chosen move (pattern).
  - Set of moves (patterns) not chosen.
- Posterior:

$$p(\mathbf{u}|\text{move}, \text{position}) = \int p(\mathbf{u}, \mathbf{x}|\text{move}, \text{position}) d\mathbf{x}$$

- Approximate (Gaussian) posterior determined by Gaussian message passing.
- Online Learning (Assumed Density Filtering):
  - After each training example we have new μ_i and σ_i for each pattern.
  - Replace values and go to next position.

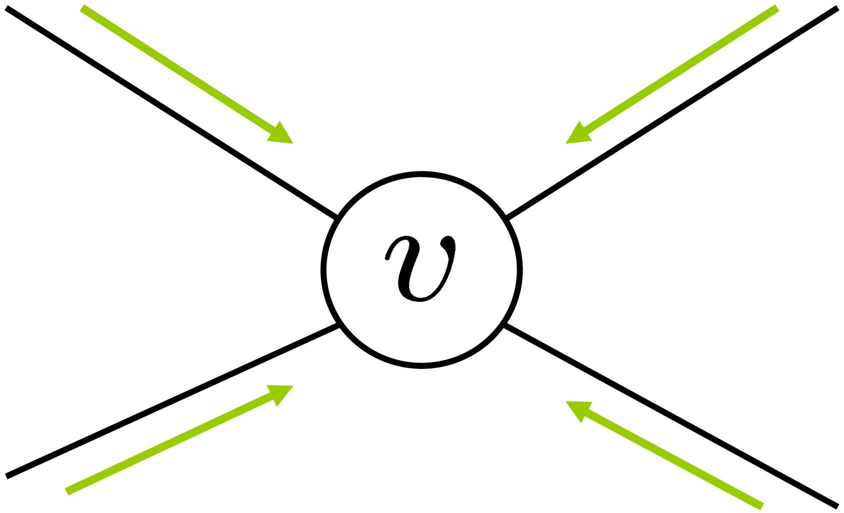
Message Passing

$$m_{f \to v}(v) = \int f(\mathbf{v}) \prod_{v_j \in \text{neigh}(f) \setminus v}^{n} m_{v_j \to f}(v_j) d\mathbf{v}$$



$$m_{v \to f}(v) = \prod_{f_j \in \text{neigh}(v) \setminus f} m_{f_j \to v}(v)$$

Marginal Calculation



$$p(v) = \prod_{f_k \in \mathsf{neigh}(v)} m_{f_k \longrightarrow v}(v)$$

Gaussian Message Passing

- All messages Gaussian!
- Most factors Gaussian.
- Messages from ‘ordering’ factors are approximated:
  - Expectation propagation.
- True Marginal Distribution:

$$p(v_i) = m_{f_k \to v_i}(v_i) \cdot m_{v_i \to f_k}(v_i)$$

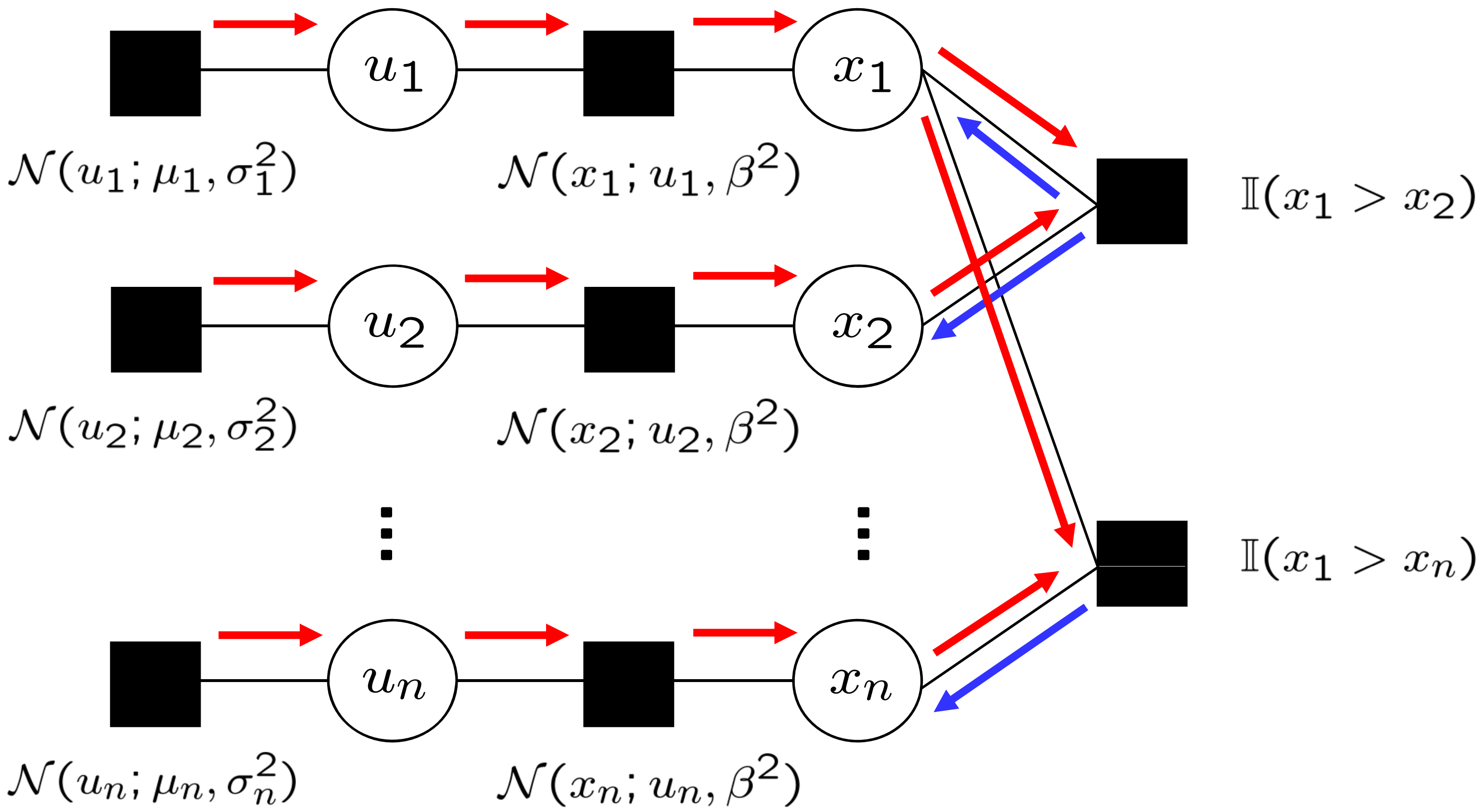
- Approximation:

$$q(v_i) = \hat{m}_{f_k \to v_i}(v_i) \cdot m_{v_i \to f_k}(v_i)$$

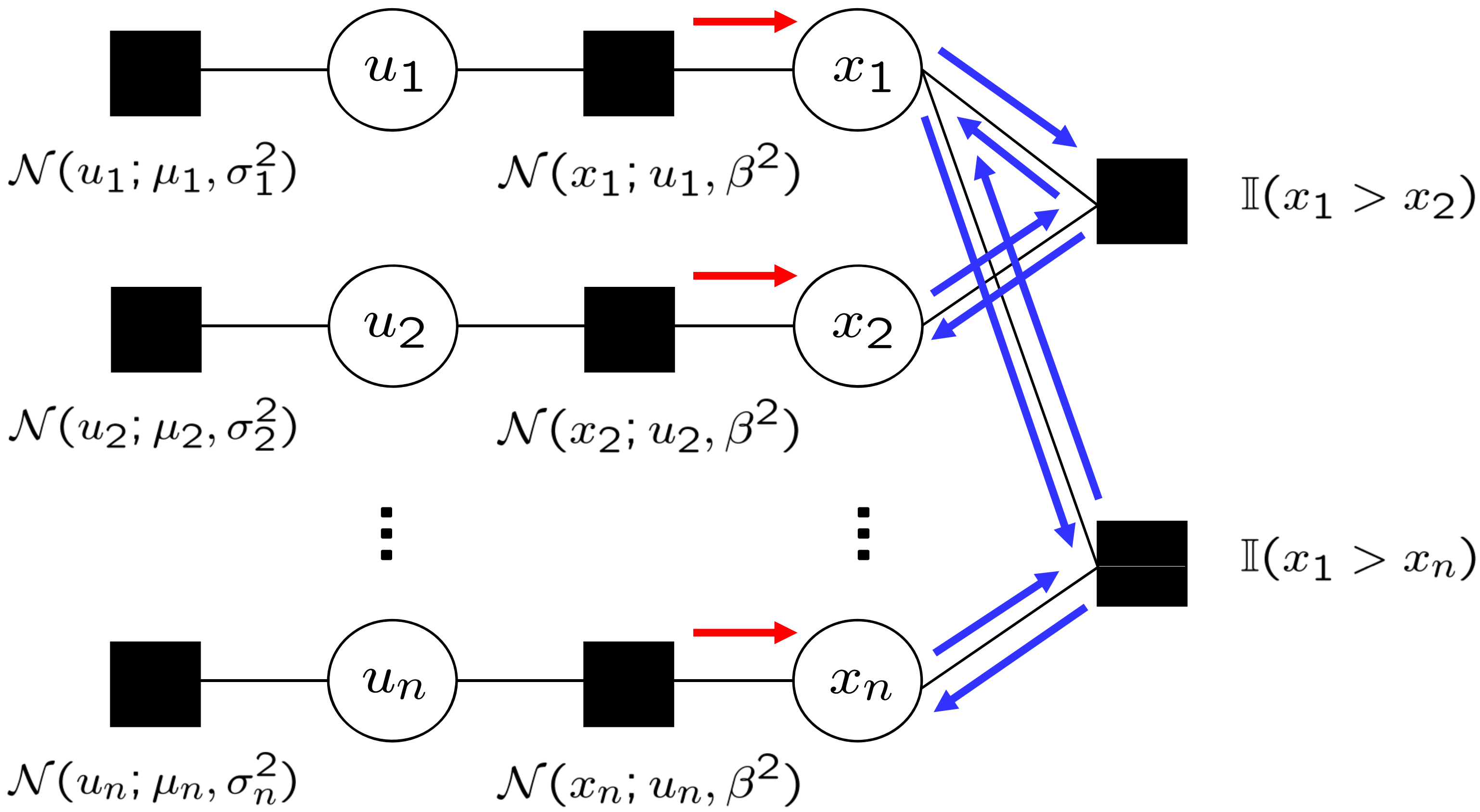
- Moment Match:

$$\hat{m}_{f_k \to v_i}(v_i) = \frac{\mathrm{MM}\left[m_{f_k \to v_i}(v_i) \cdot m_{v_i \to f_k}(v_i)\right]}{m_{v_i \to f_k}(v_i)}$$

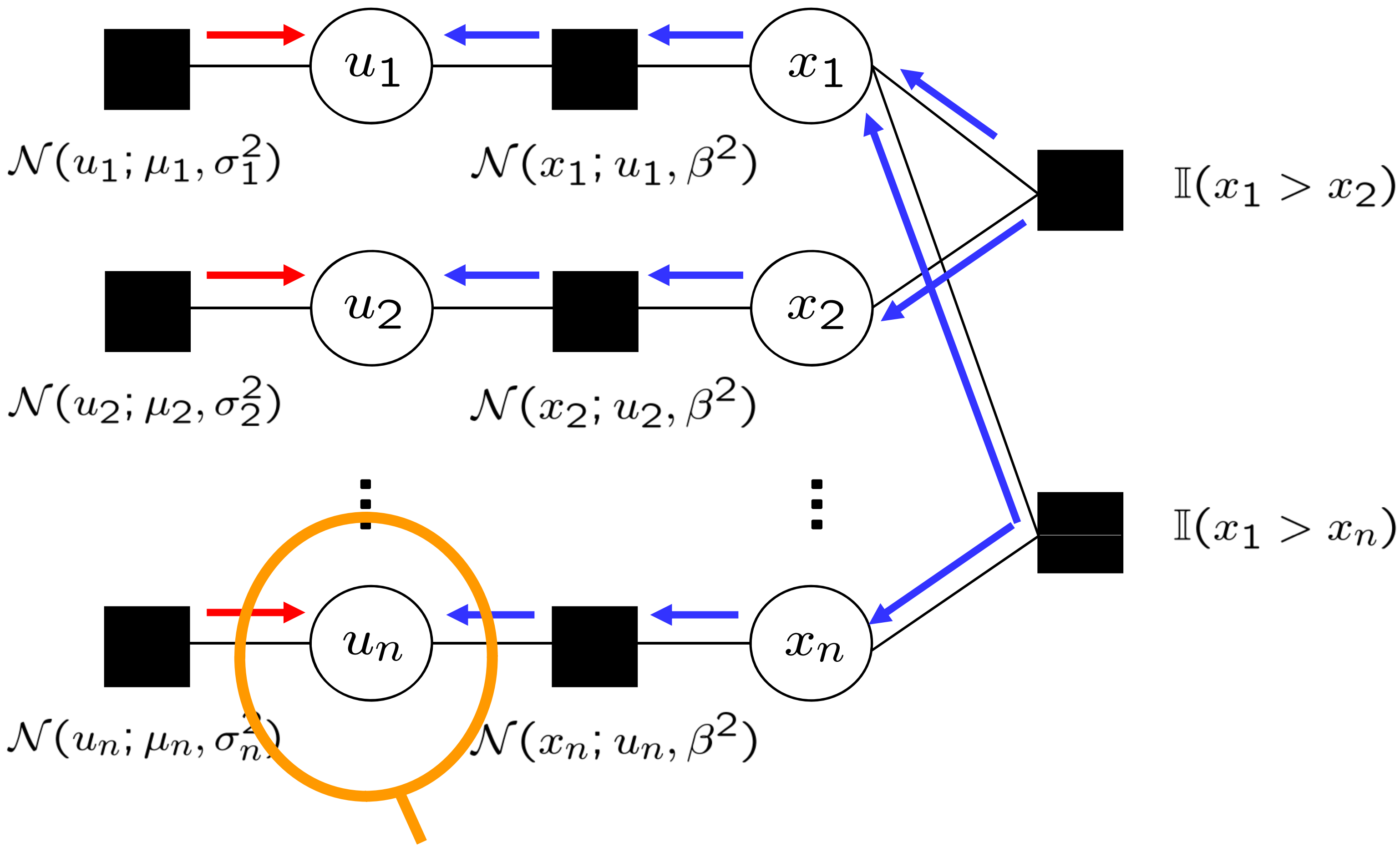
Gaussian Message Passing



Gaussian Message Passing
– EP Approximation

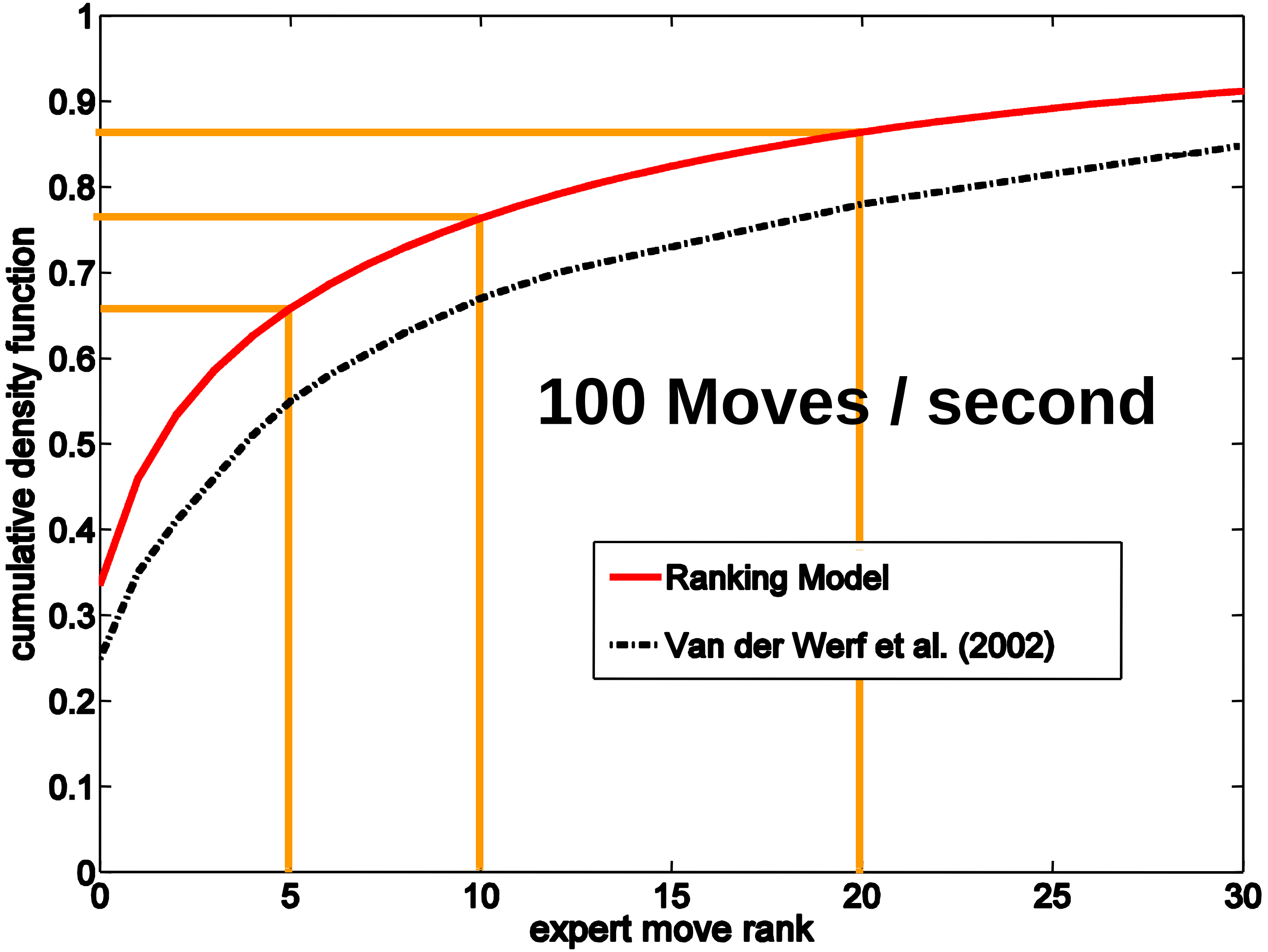


Gaussian Message Passing
- Posterior Calculation

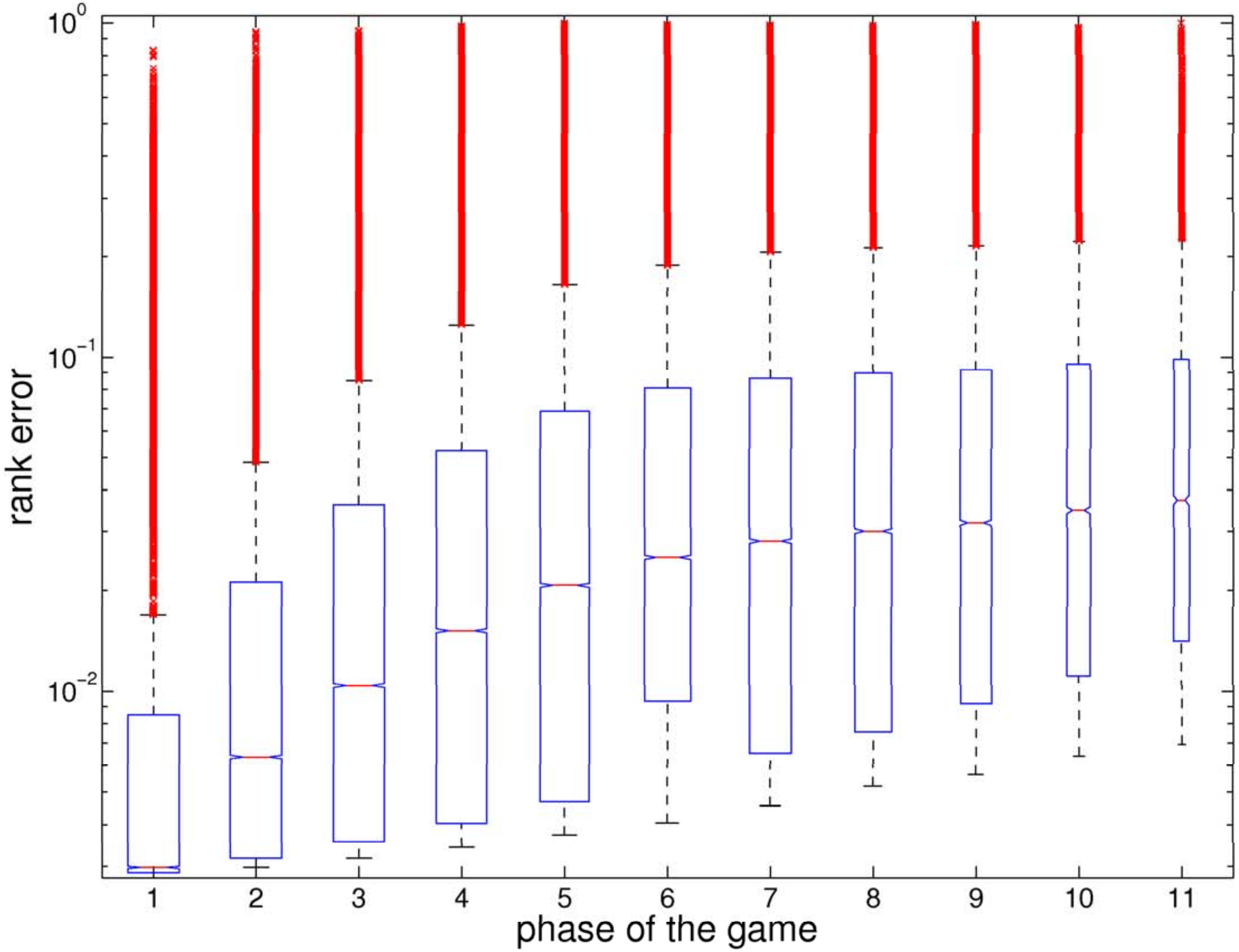


$$p(u_i|\text{move}, \text{position}) = \prod_{f \in \text{neigh}(u_i)} m_{f \to u_i}(u_i)$$

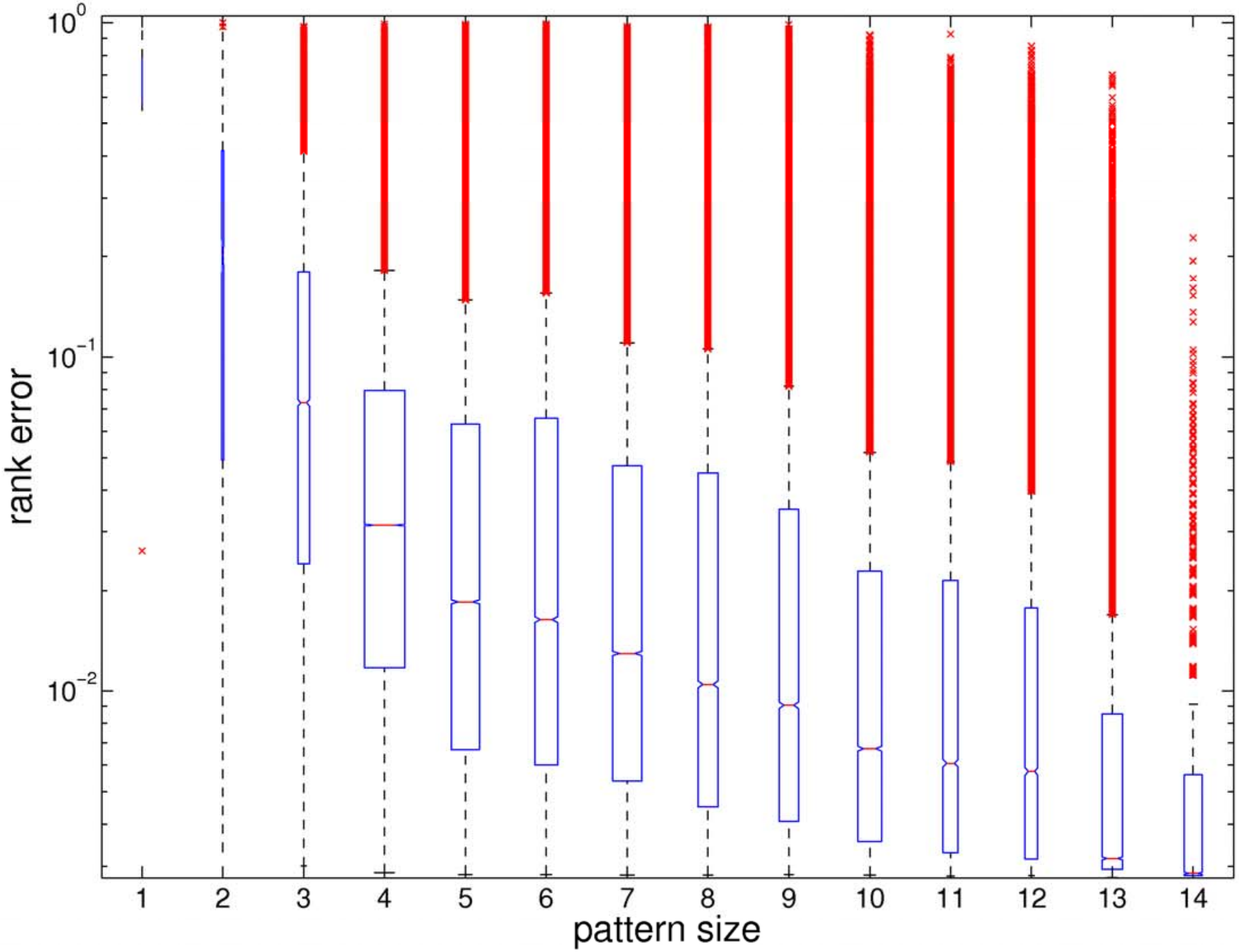
Move Prediction Performance



Rank Error vs Game Phase



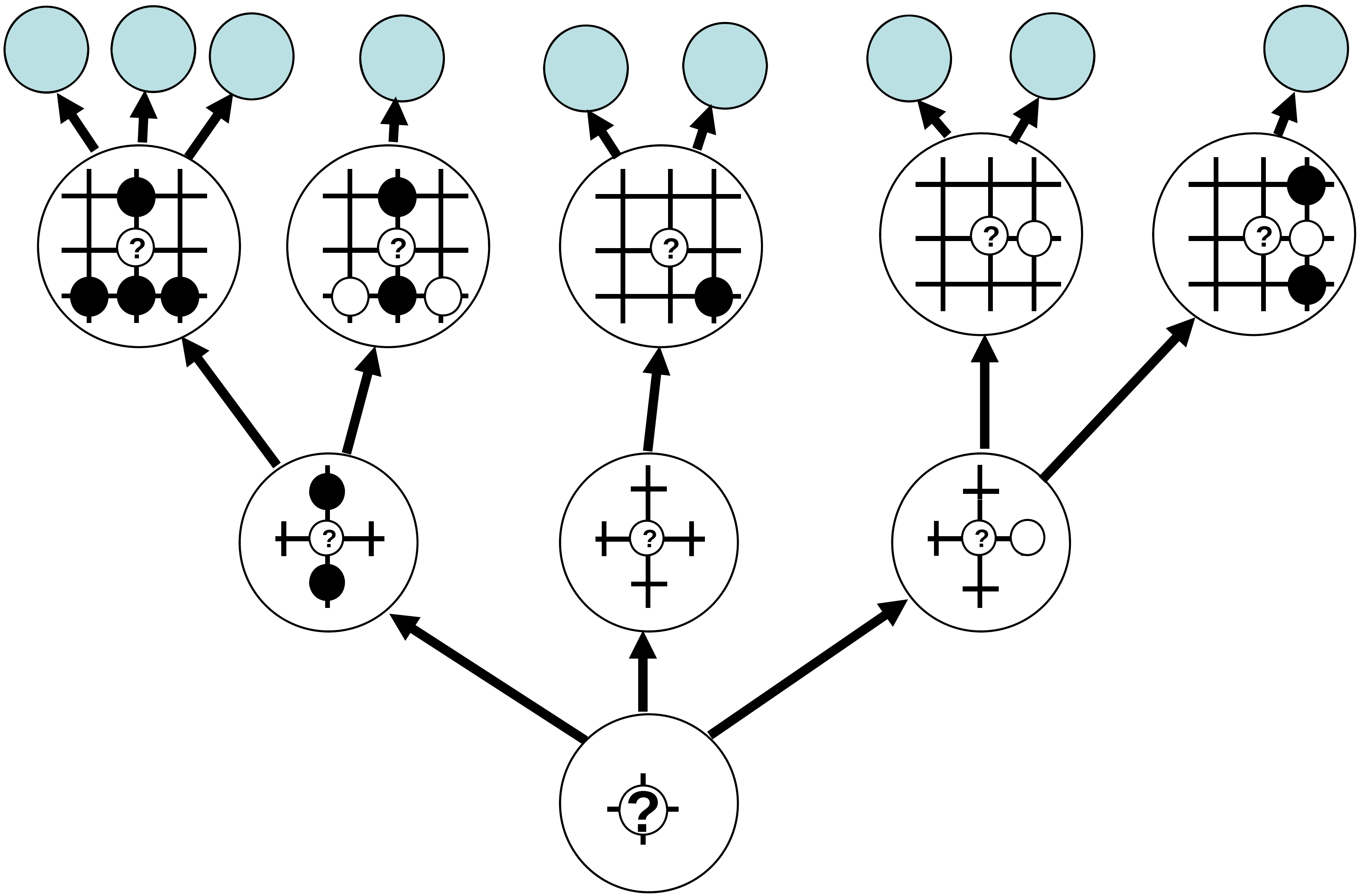
Rank Error vs Pattern Size



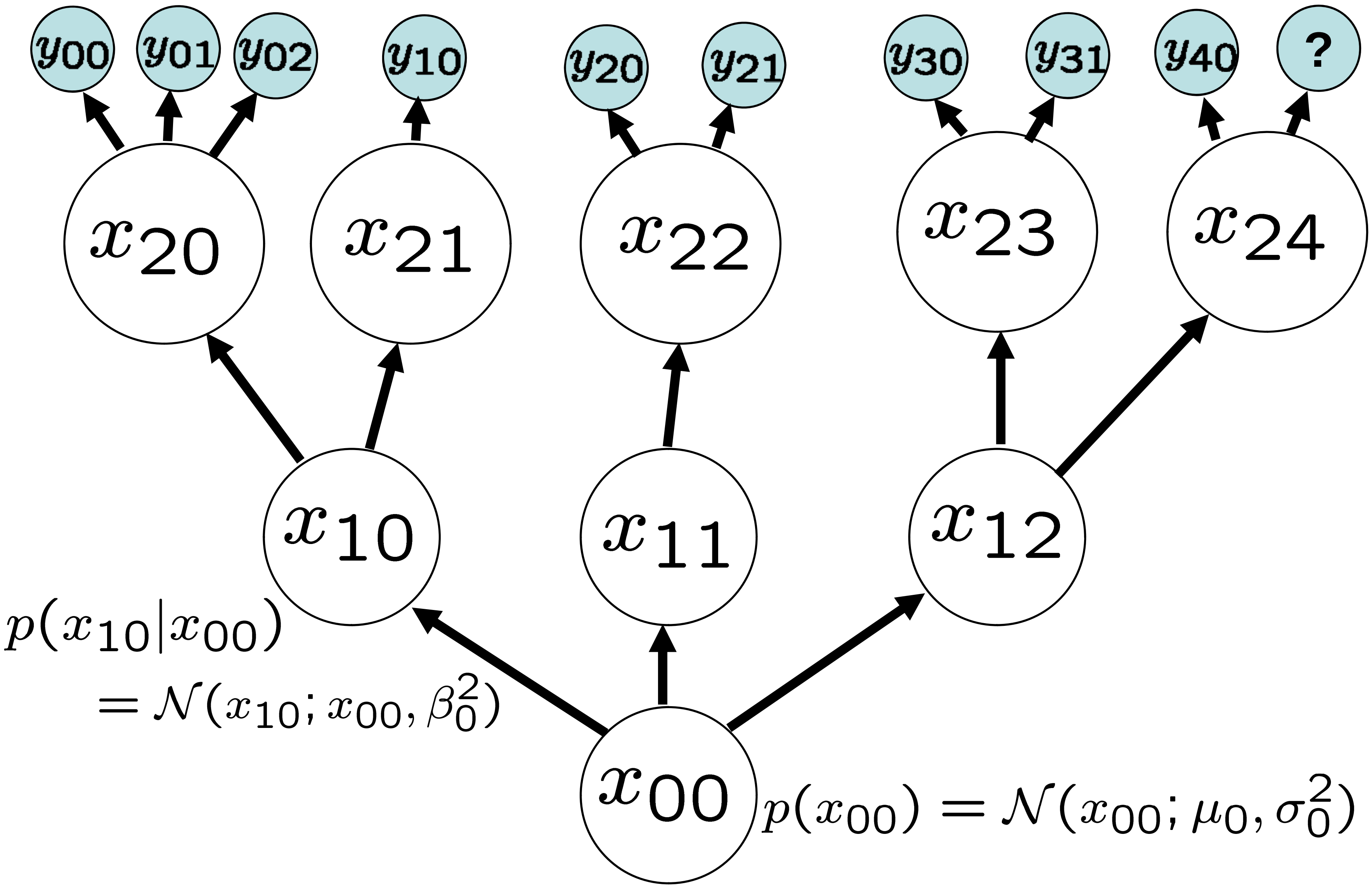
Hierarchical Gaussian Model of Move Values

- Use all patterns that match – not just biggest.
  - Evidence from larger pattern should dominate small pattern at same location.
  - However – big patterns seen less frequently.
- Hierarchical Model of move values.

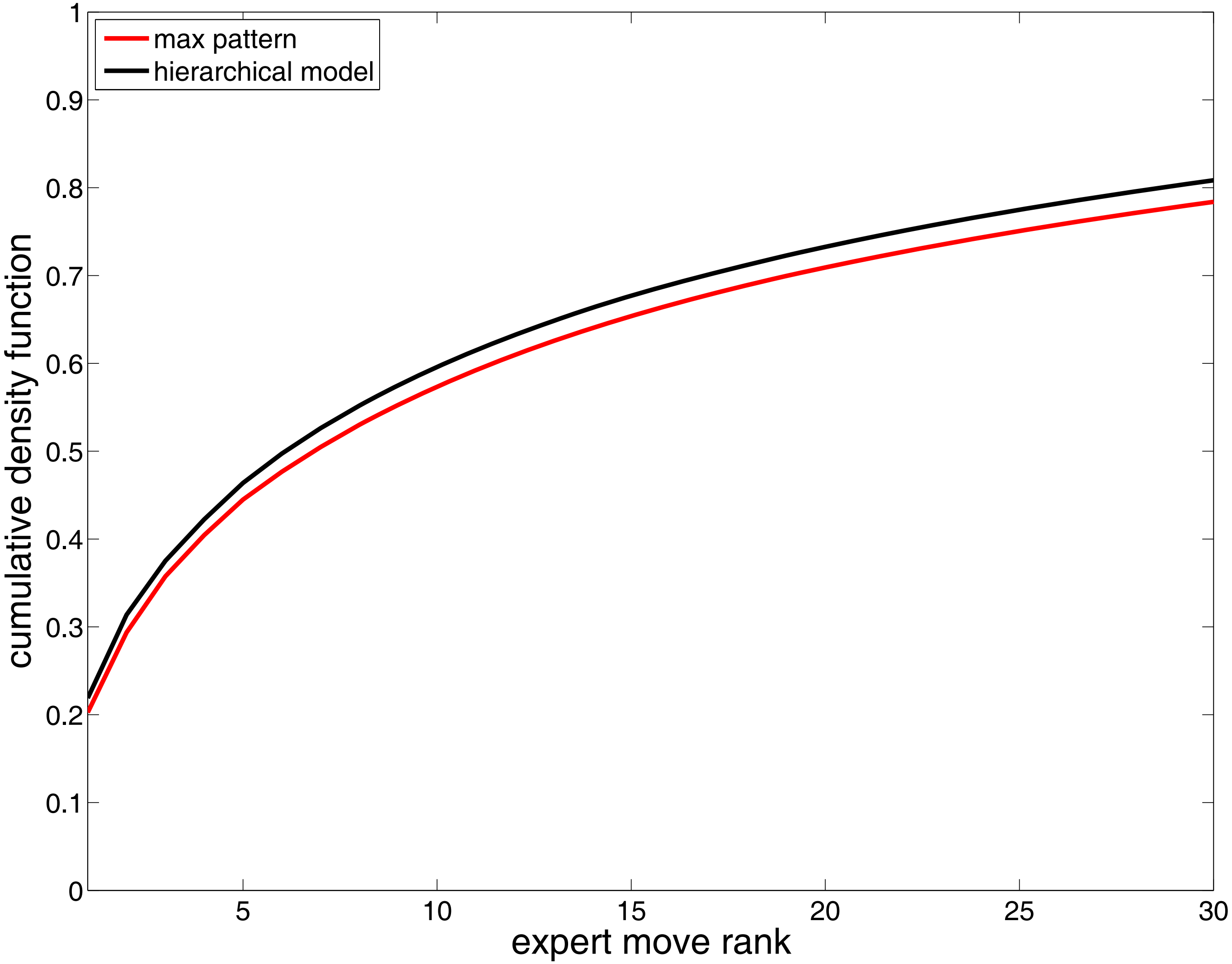
Pattern Hierarchy



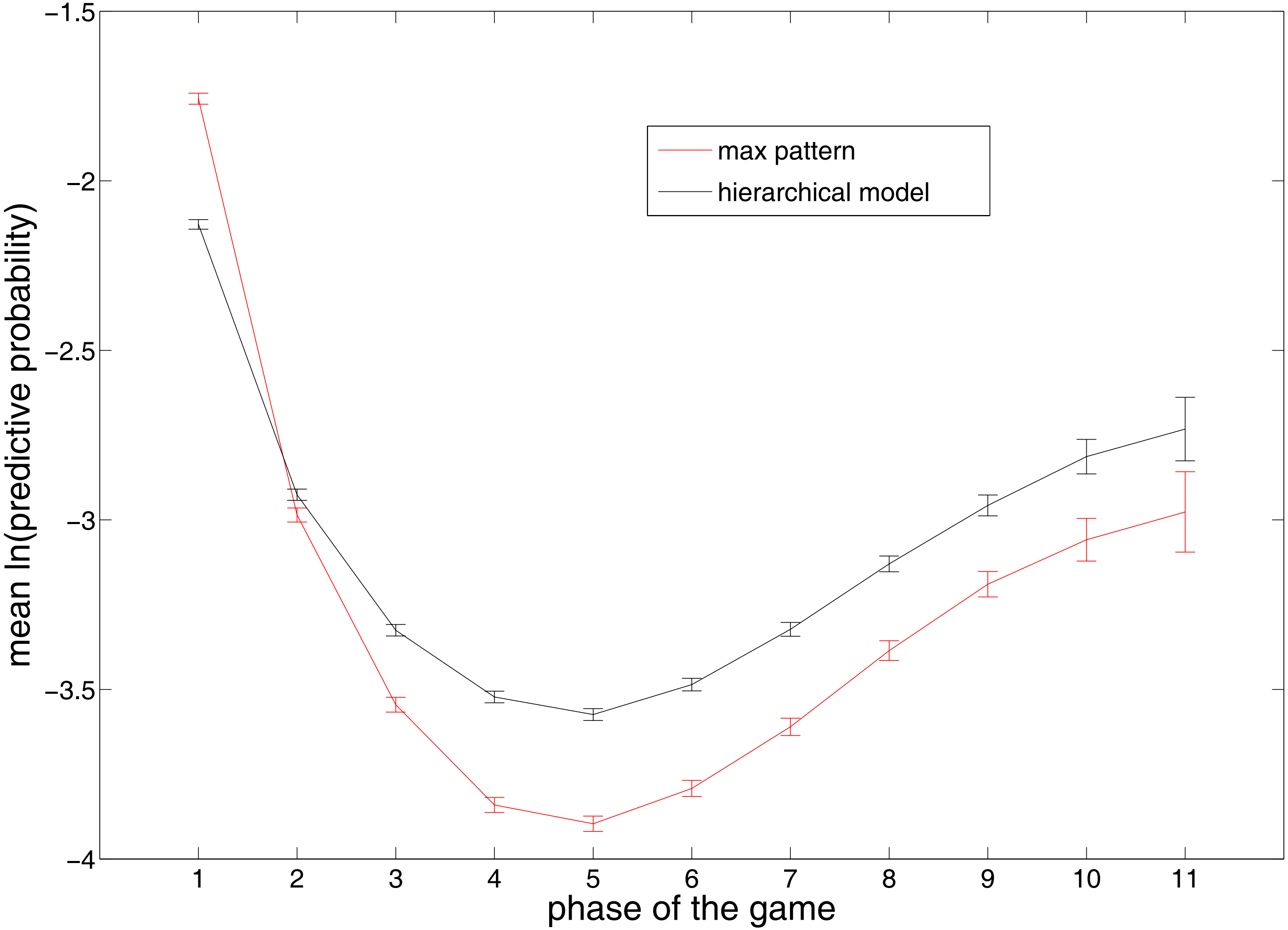
Hierarchical Gaussian Model



Move Prediction Performance



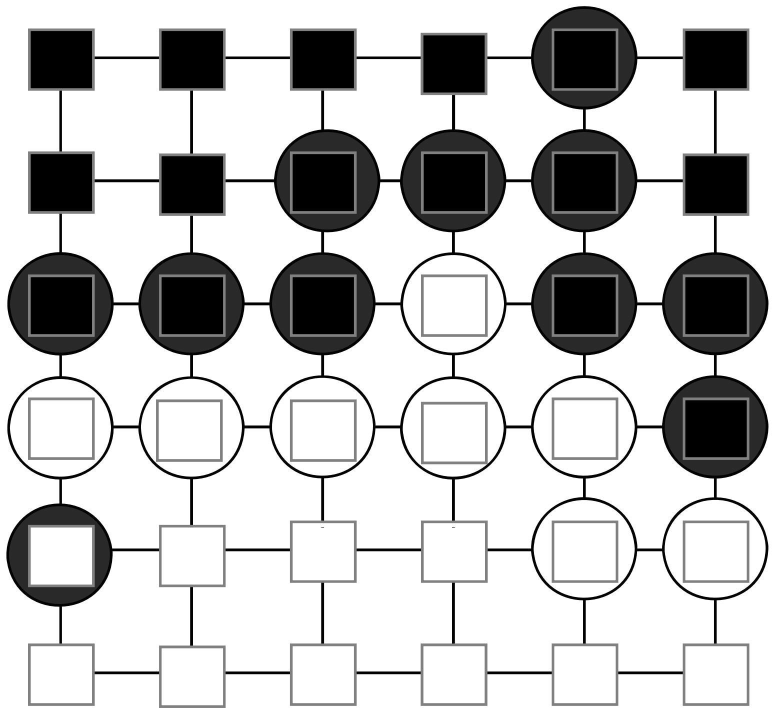
Predictive Probability



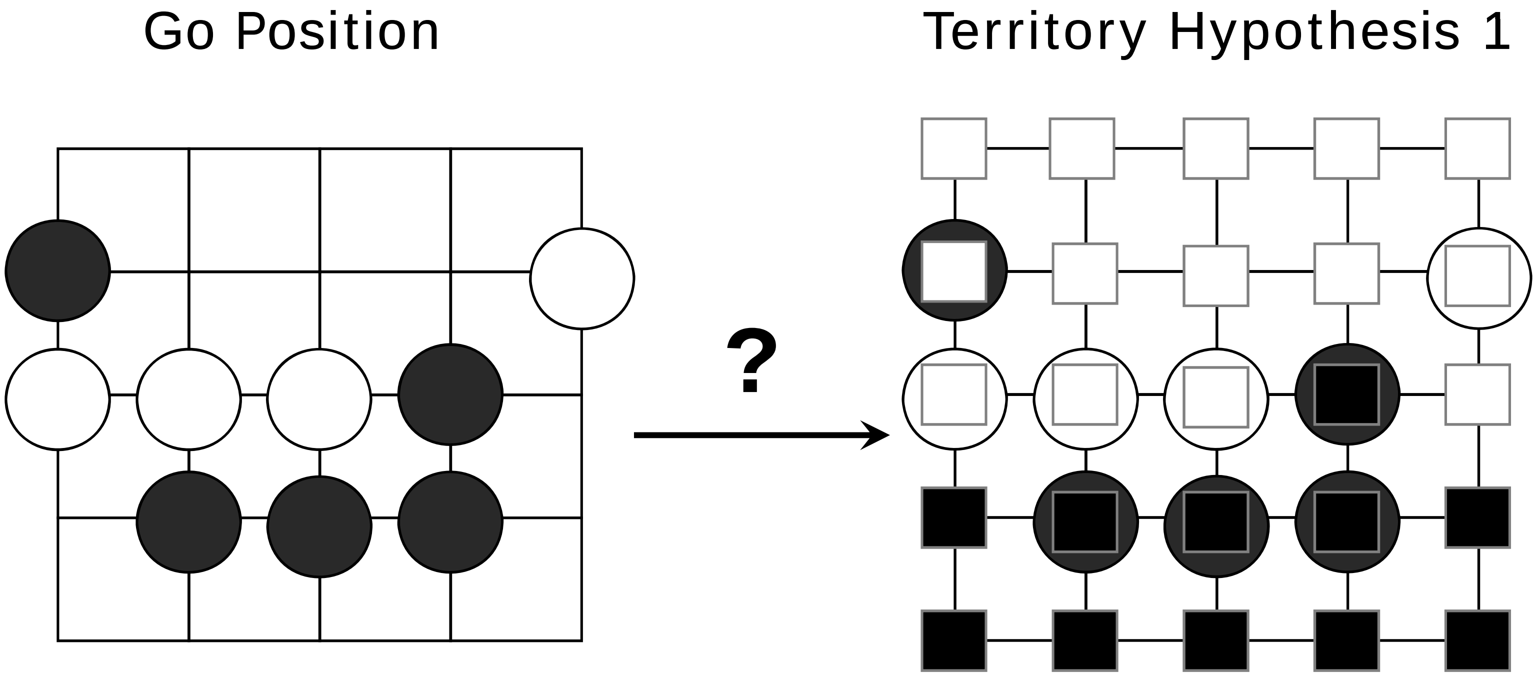
Territory Prediction

Territory

1. Empty intersections surrounded.
2. The stones themselves (Chinese method)

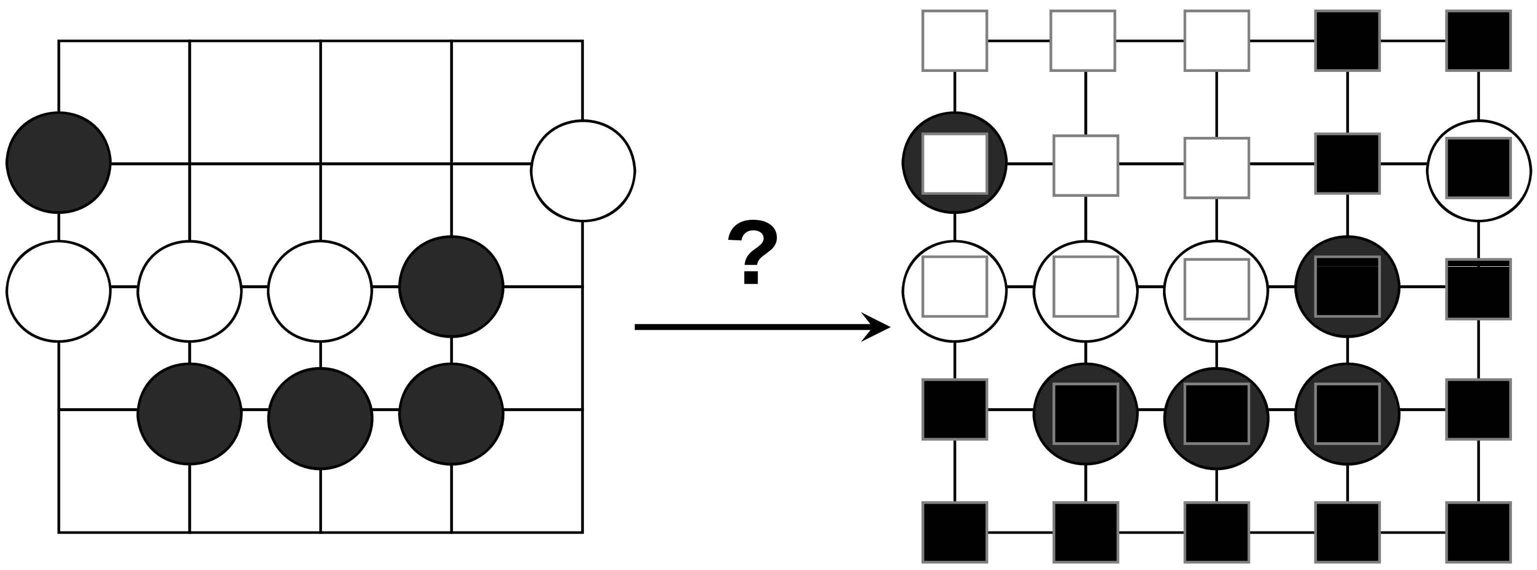


Predicting Territory



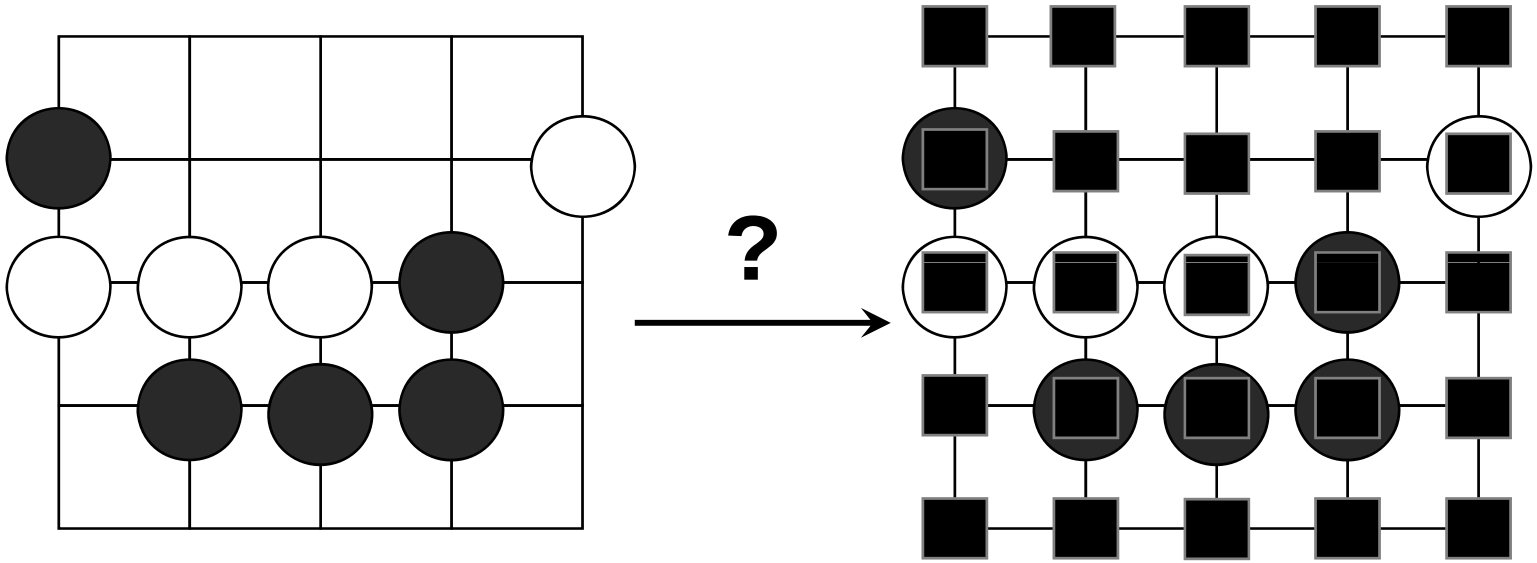
Predicting Territory

Territory Hypothesis 2



Predicting Territory

Territory Hypothesis 3



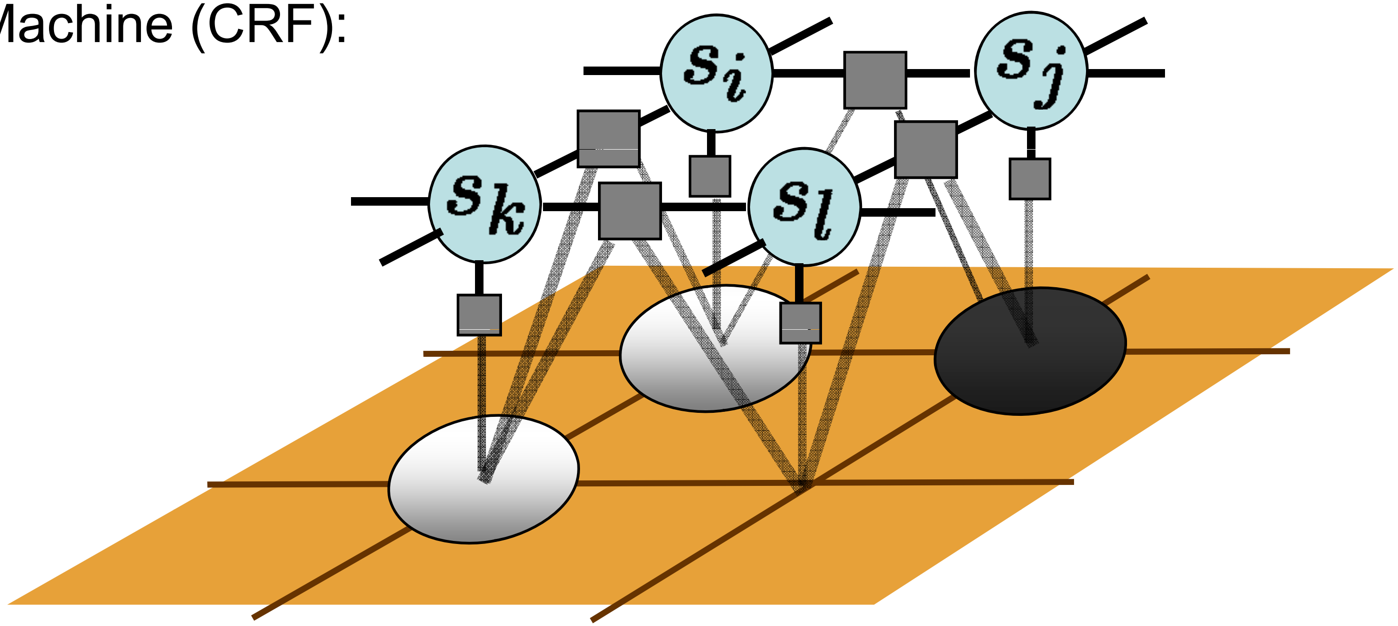
Predicting Territory

- Board Position: $\mathbf{c} \in \{\text{black}, \text{white}, \text{empty}\}^N$
- Territory Outcome: $\mathbf{s} \in \{+1, -1\}^N$

- Model Distribution: $P(\mathbf{s}|\mathbf{c})$

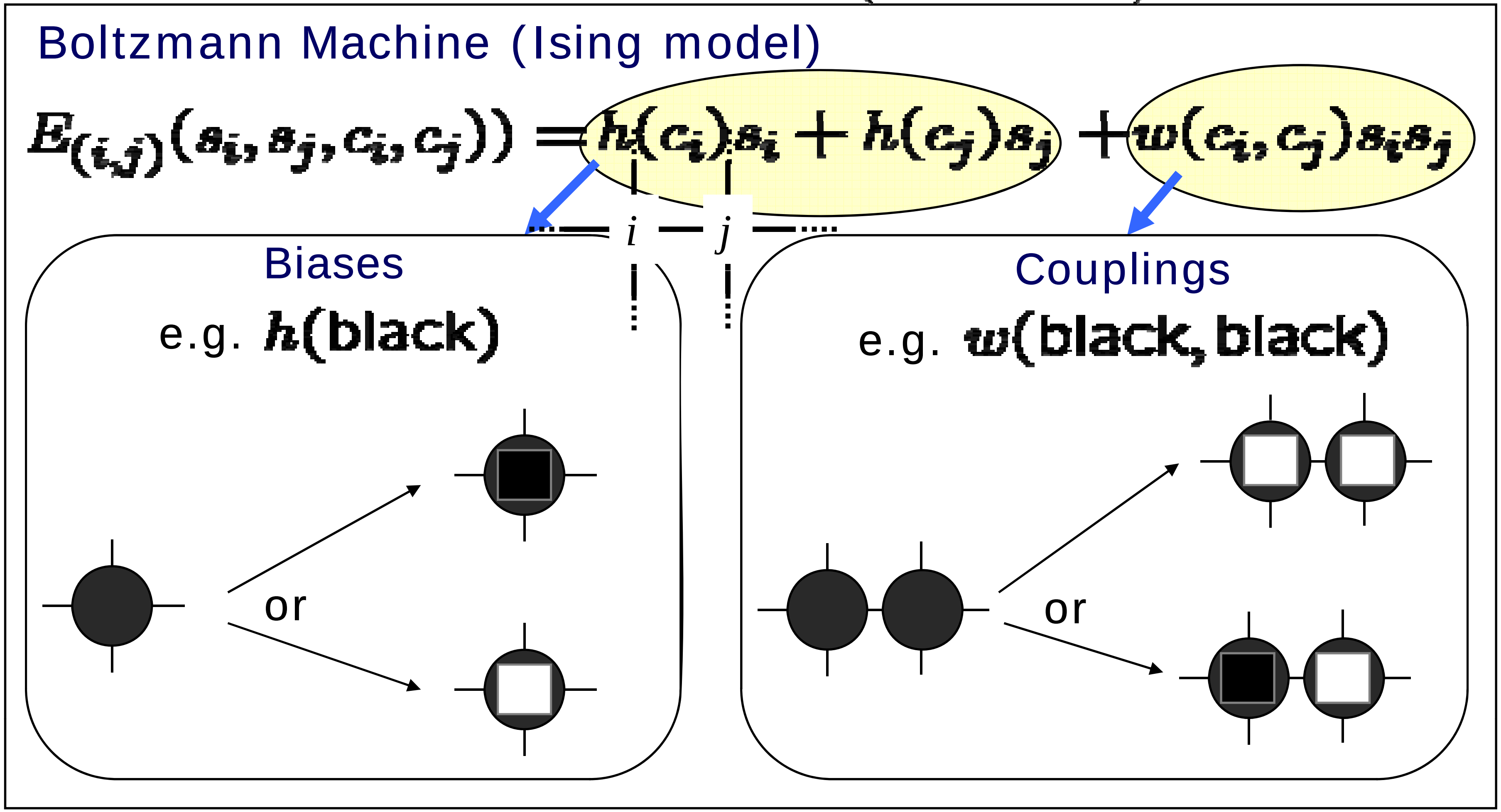
- E(Black Score) $= \sum_i \langle s_i \rangle_{P(\mathbf{s}|\mathbf{c})}$

- Boltzmann Machine (CRF):



Territory Prediction Model

$$P(\mathbf{s}|\mathbf{c}) = \frac{1}{Z(\mathbf{c},\theta)} \exp\left(\sum_{(i,j)} E_{(i,j)}\right)$$



Game Records

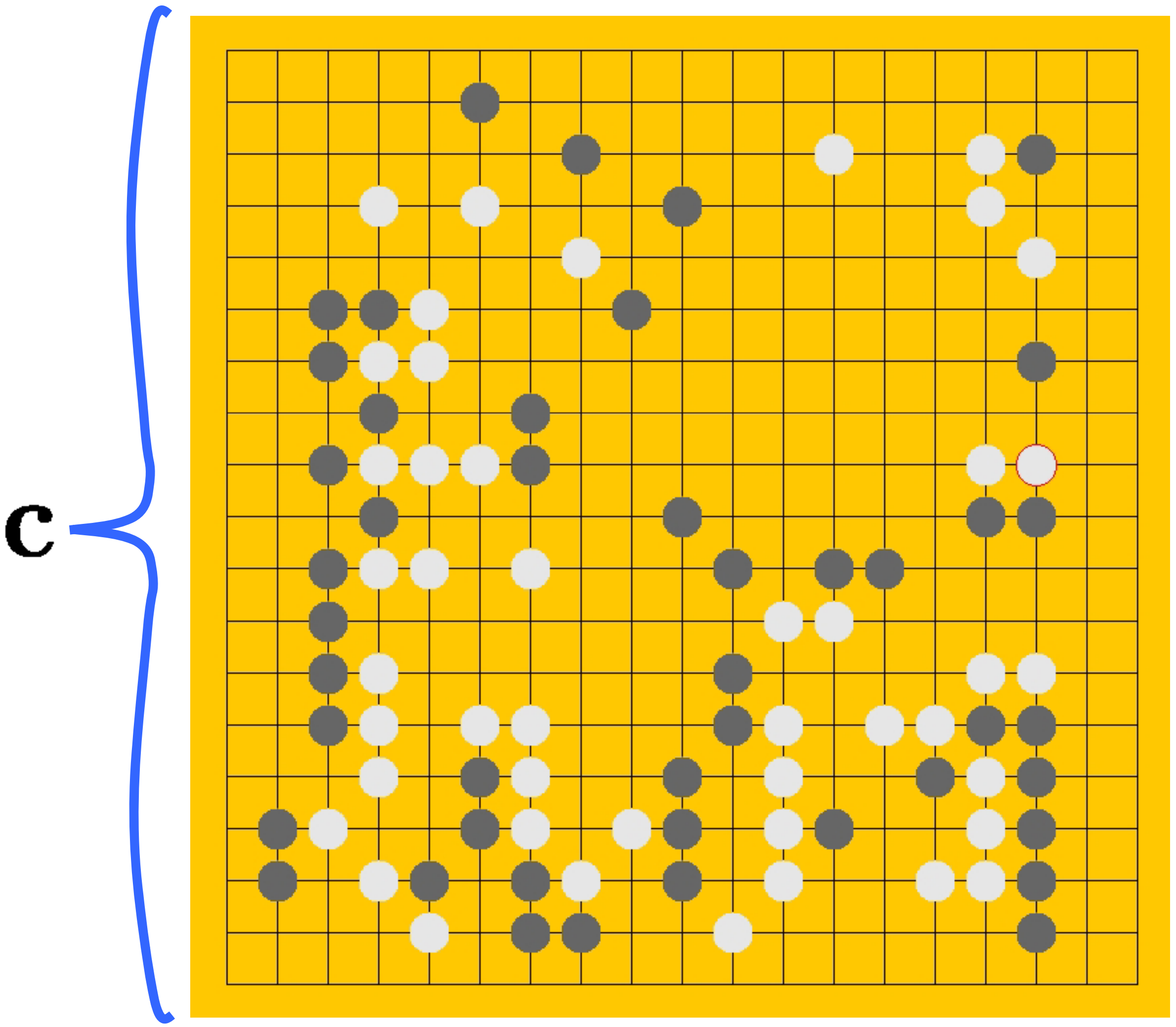
- 1,000,000 expert games.
  - Some labelled with final territory outcome.
- Training Data is pair of game position, c_i , + territory outcome, s_i.
- Maximise Log-Likelihood:

$$\sum_i \ln P(\mathbf{s}_i|\mathbf{c}_i, \mathbf{w})$$

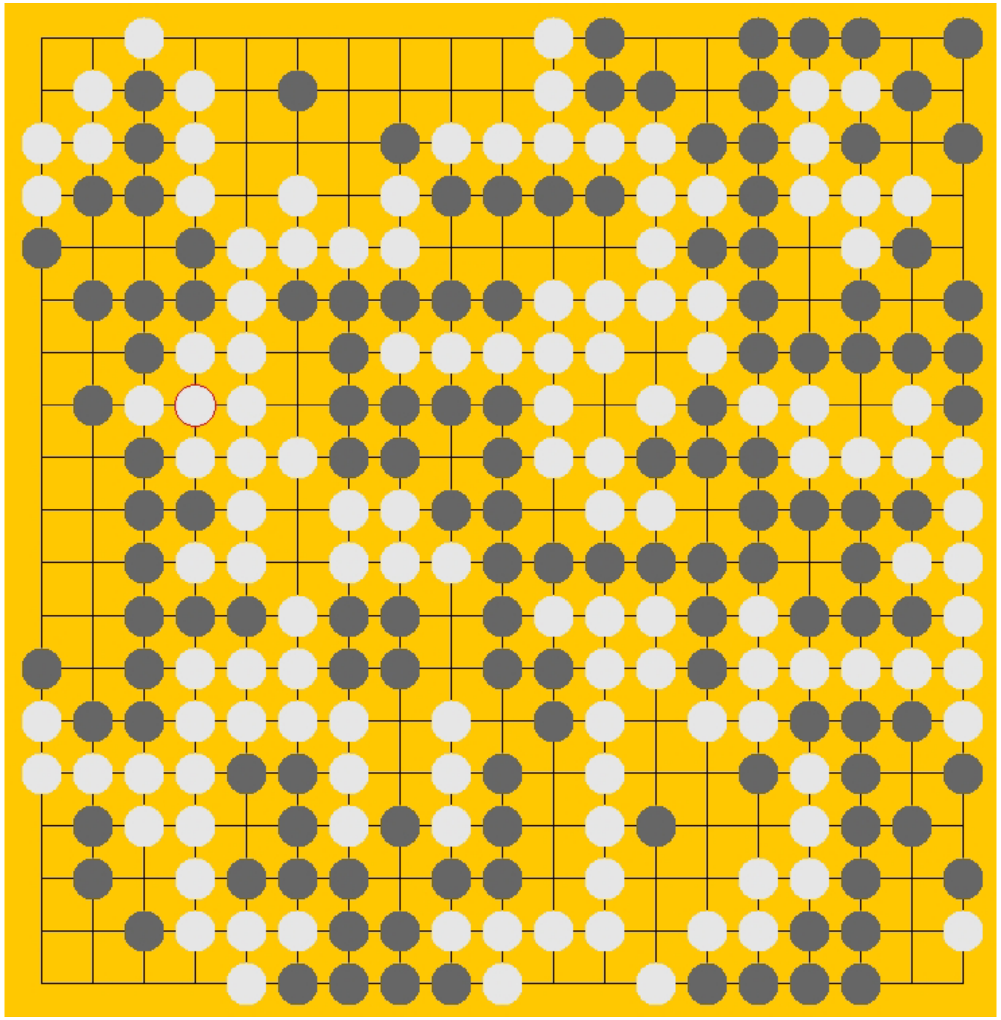
- Inference by Swendsen-Wang sampling

1000,000 Expert Games…

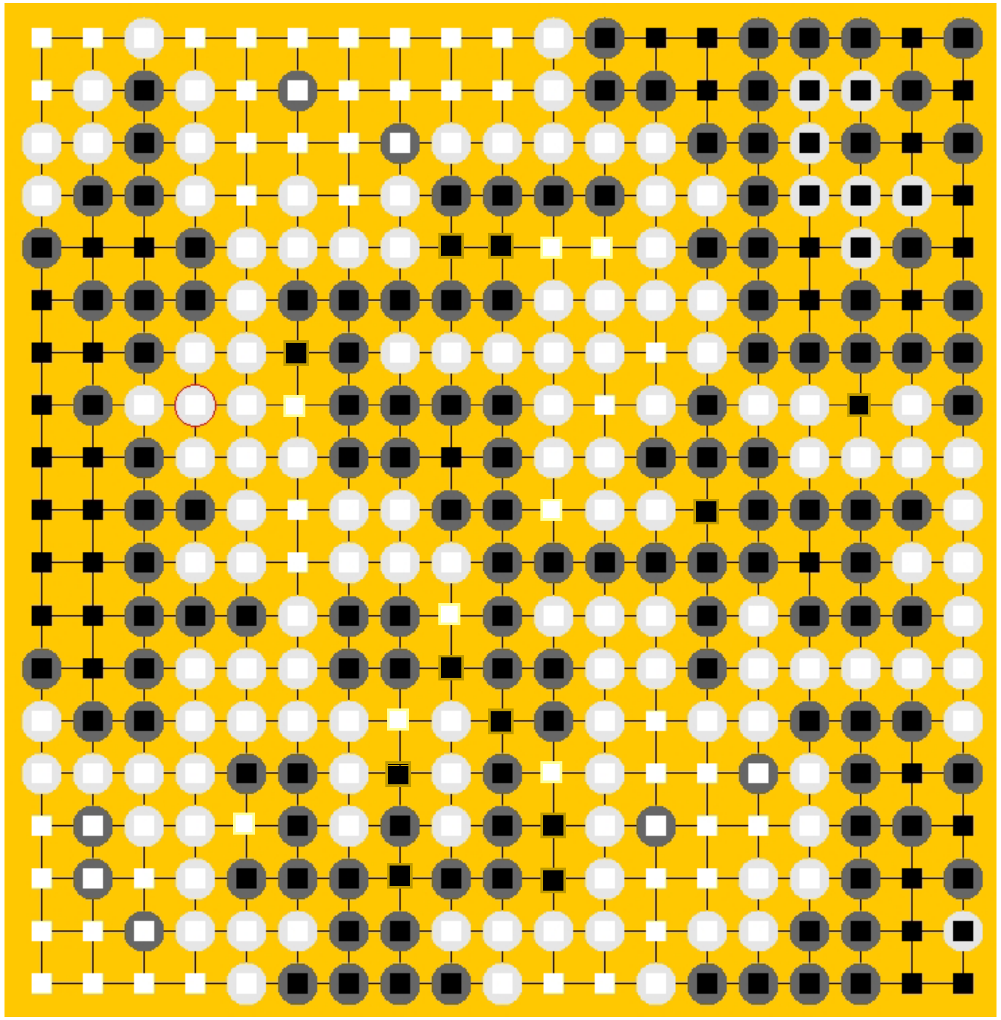
So Yokoku [6d] (black) -VS- (white) [9d] Kato Masao



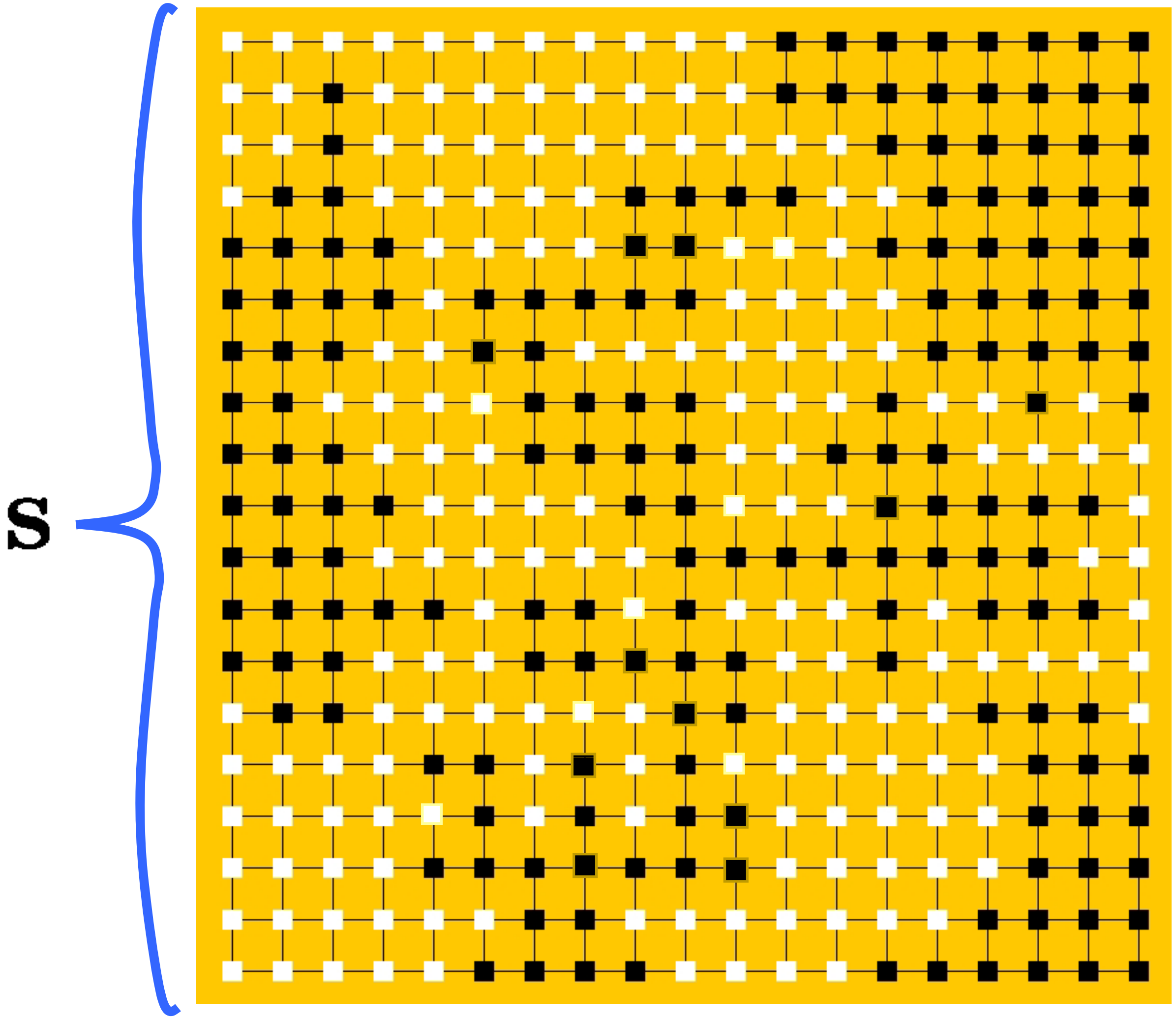
Final Position



Final Position + Territory



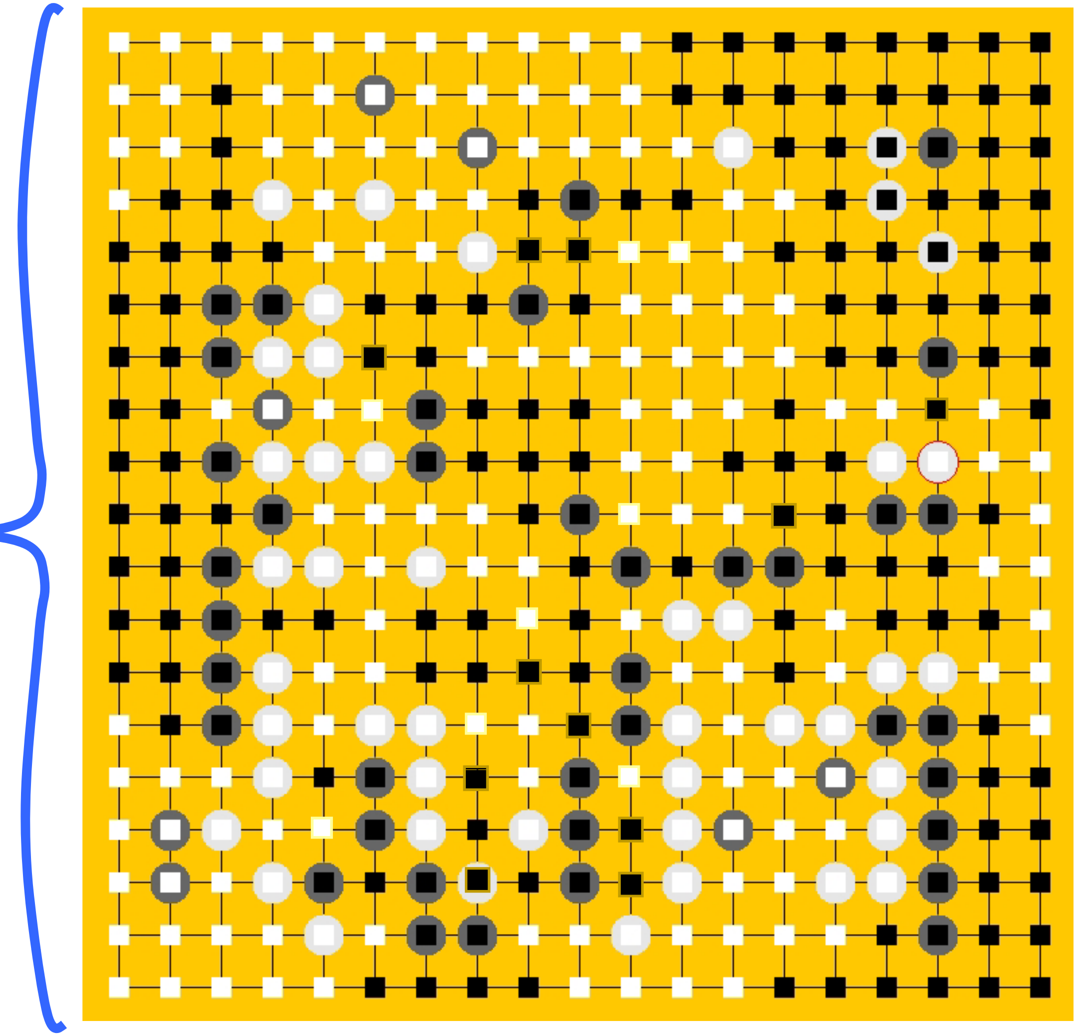
Final Territory Outcome



Position + Final Territory

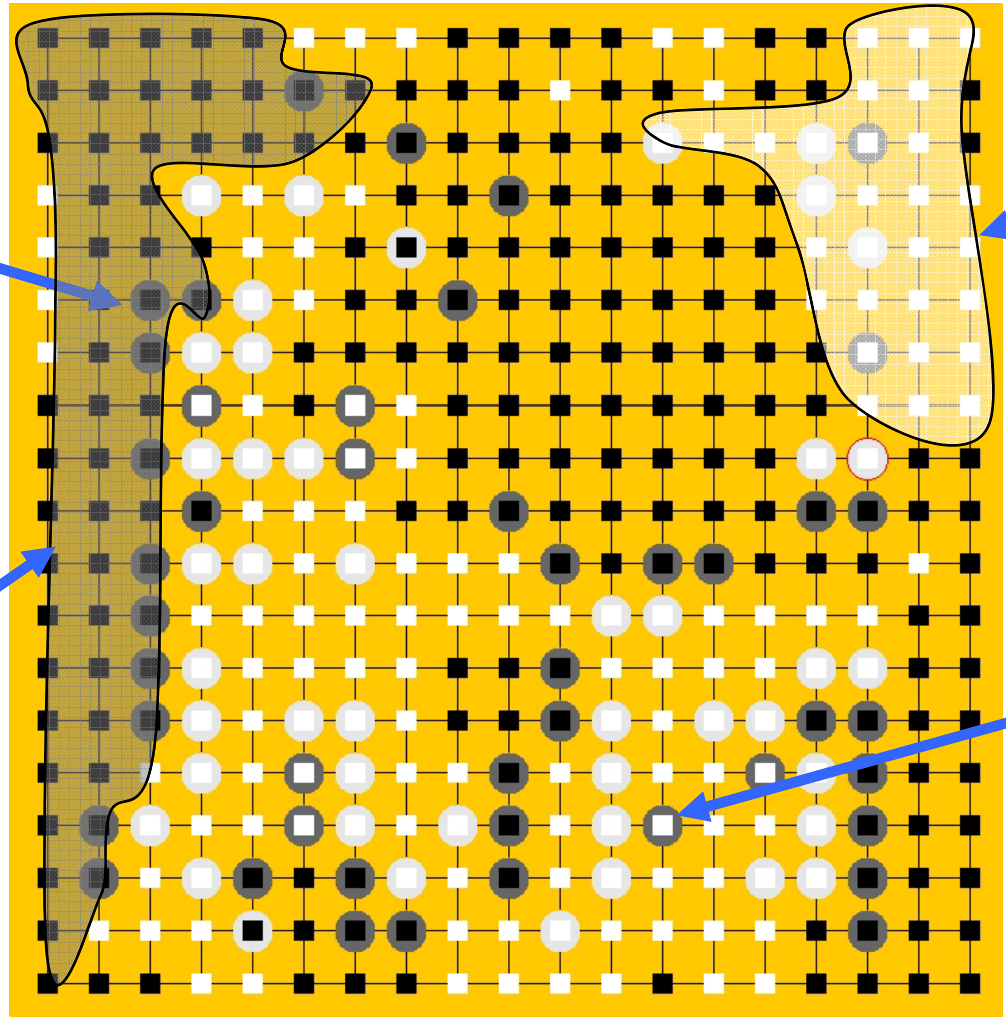
Train CRF by Maximum Likelihood

$\mathbf{s}|\mathbf{c}$



Position + Boltzmann Sample

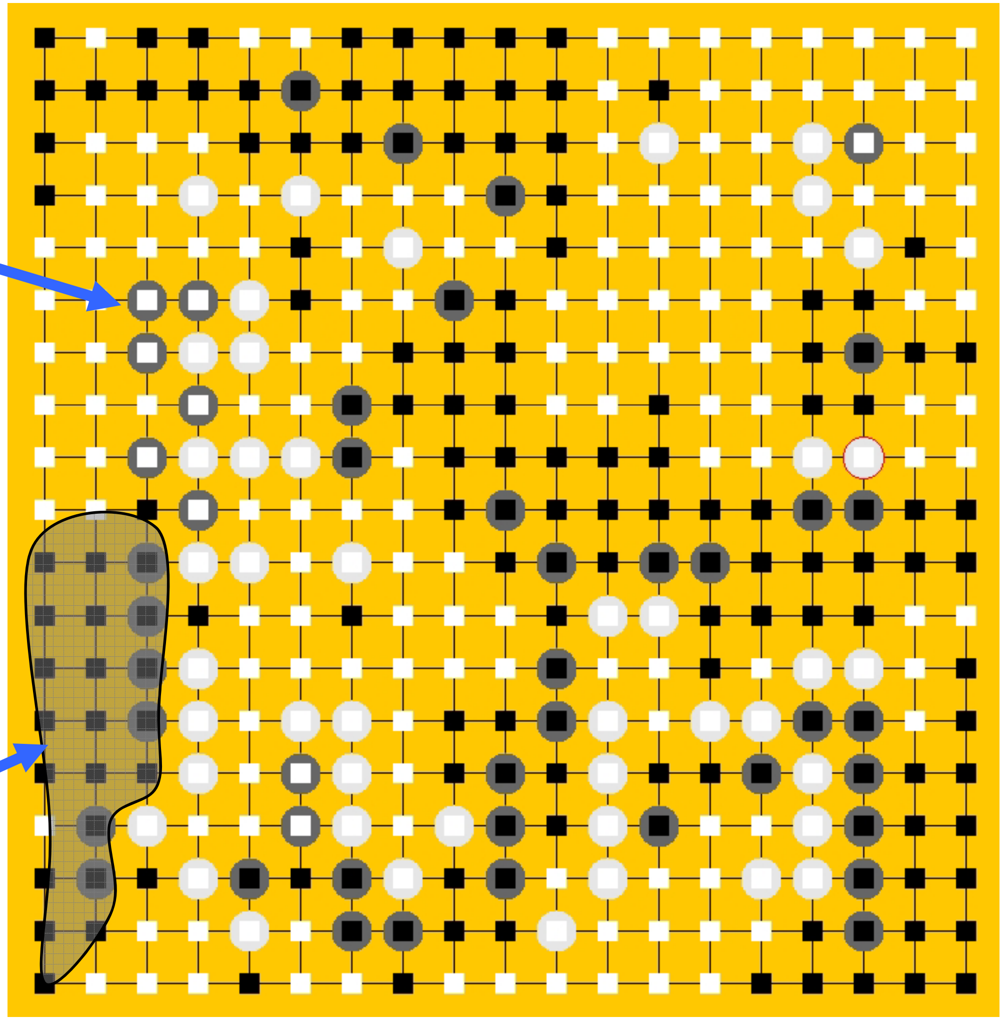
(Generated by Swendsen-Wang)



Position + Boltzmann Sample

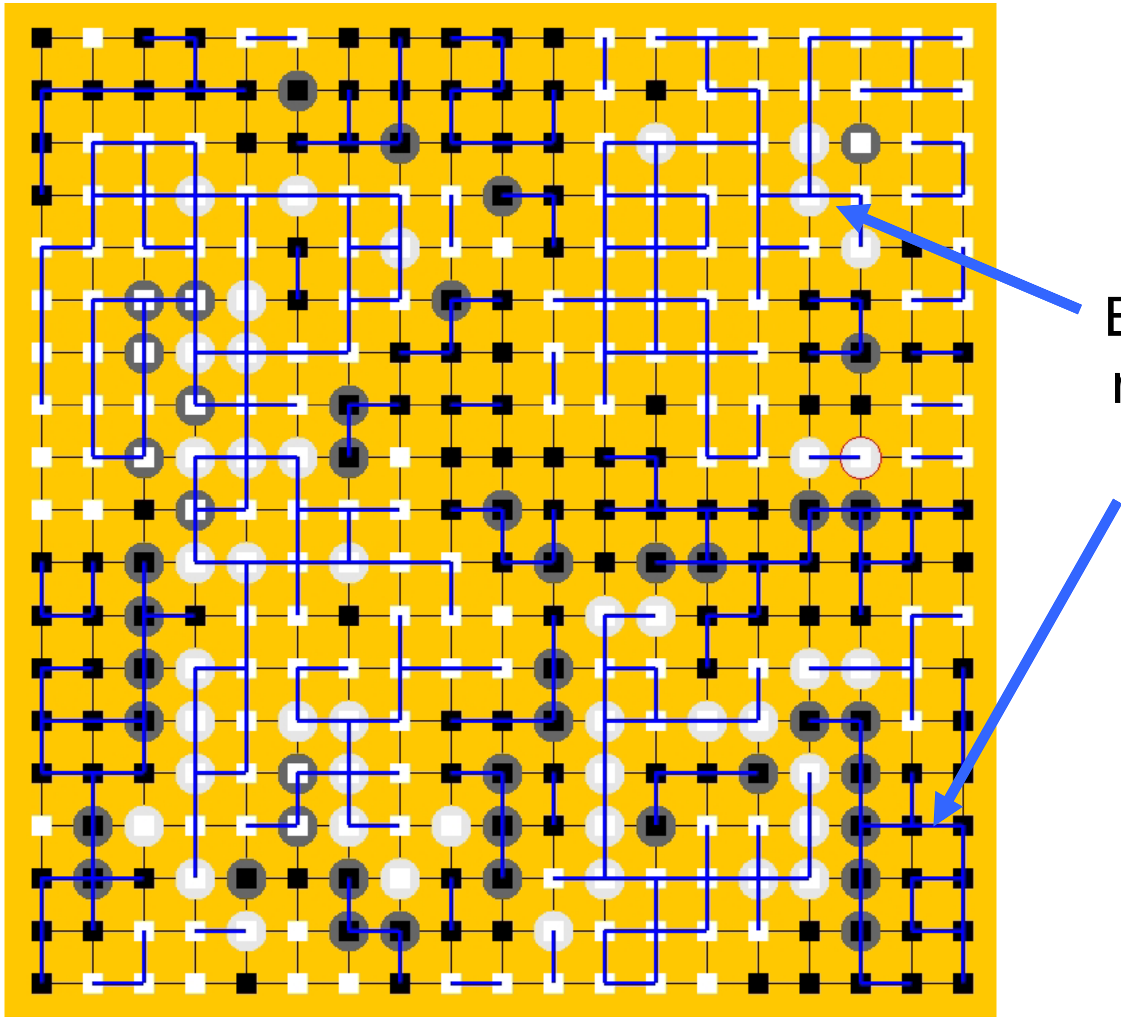
(Generated by Swendsen-Wang)

Hypothesis: These stones are captured



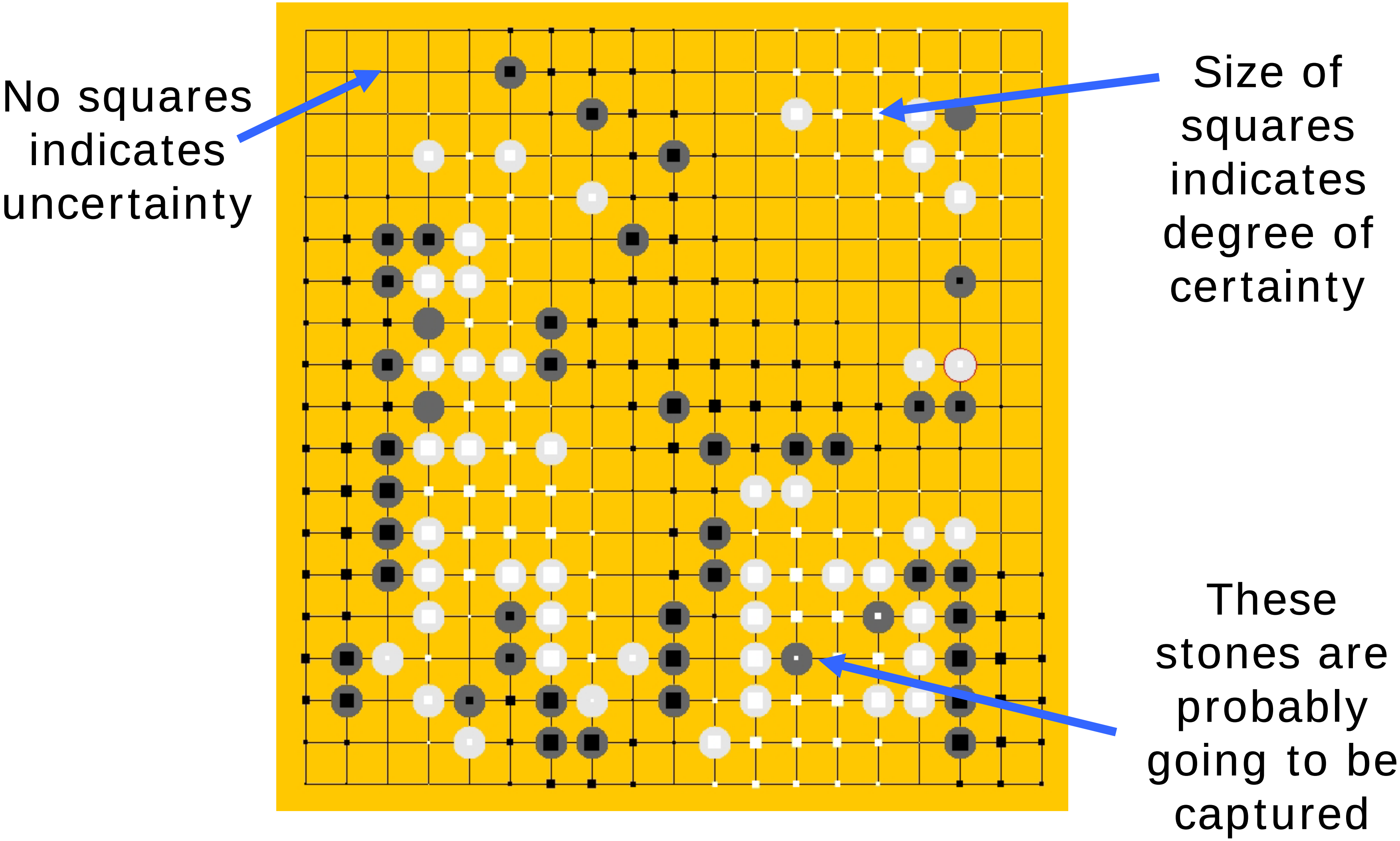
black controls this region

Sample with Swendsen-Wang Clusters Shown



Bonds link regions of common fate

Boltzmann Machine – Expectation over Swendsen-Wang Samples



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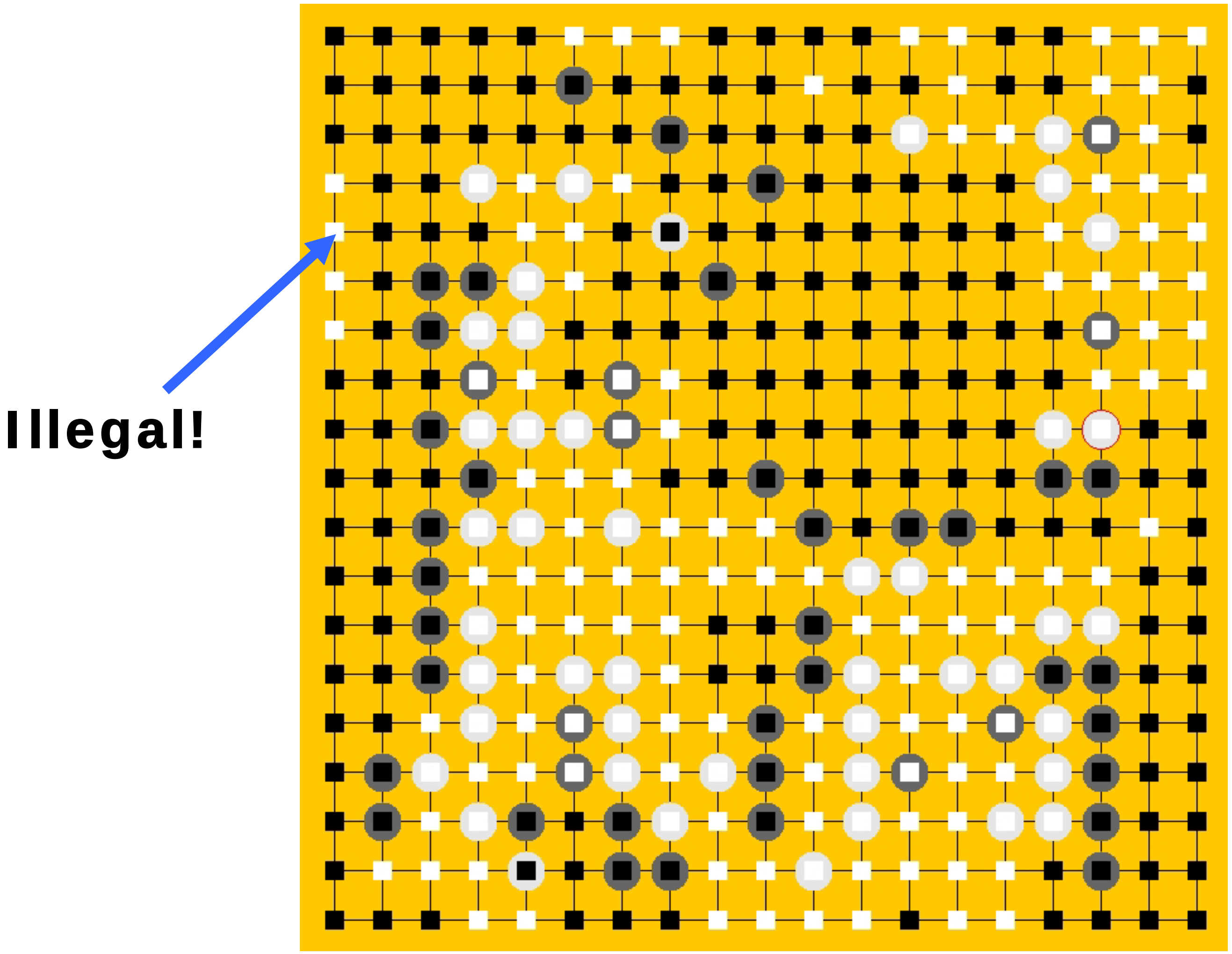
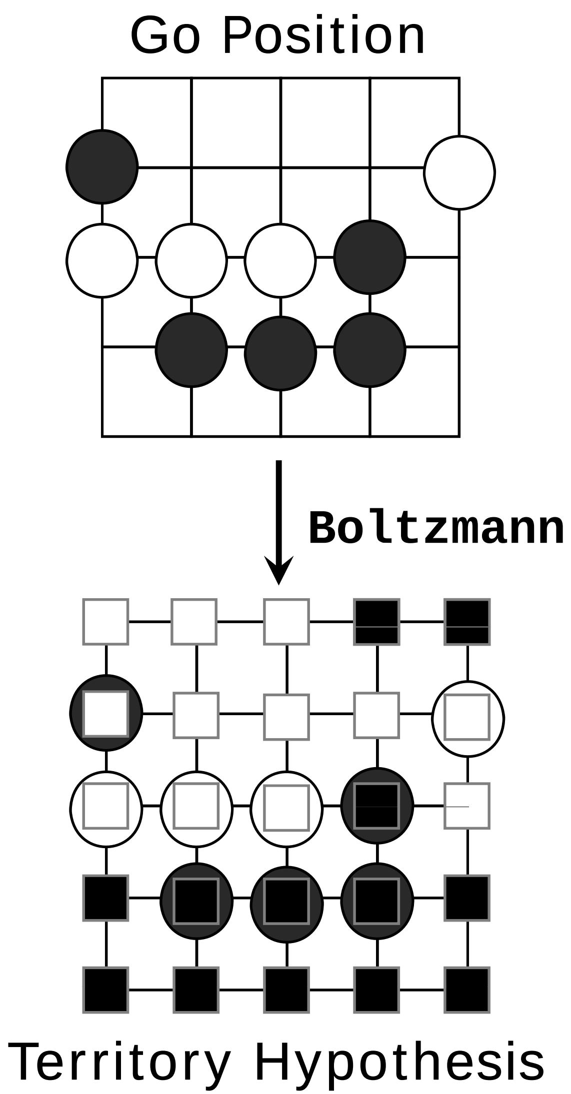
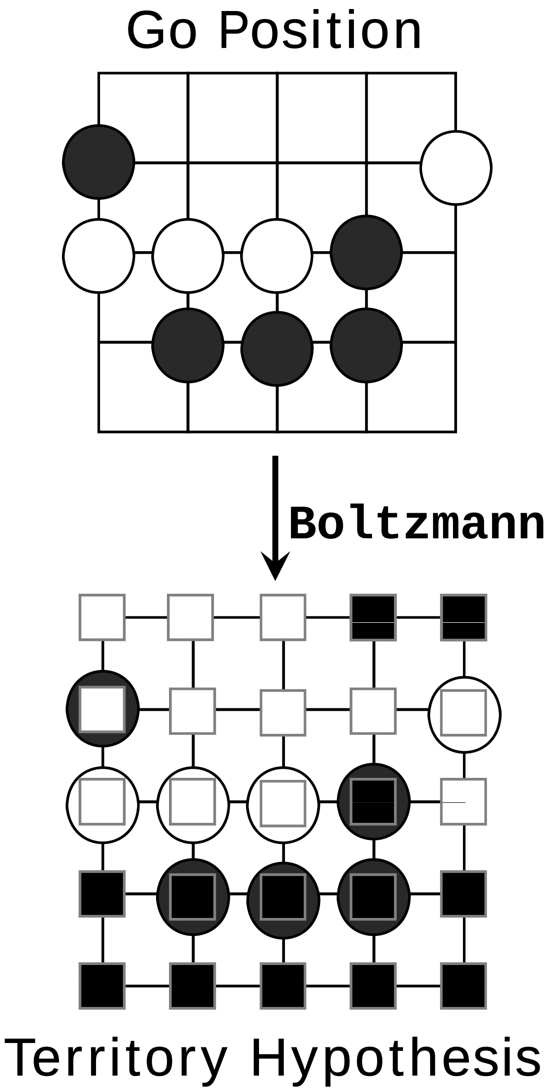


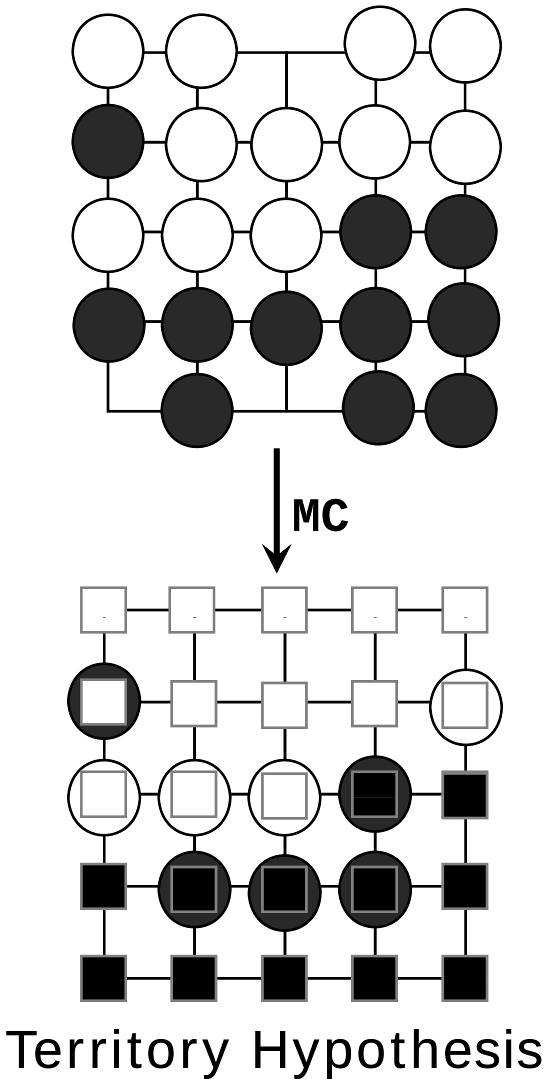
Monte Carlo Go
(work in progress)

Monte Carlo Go

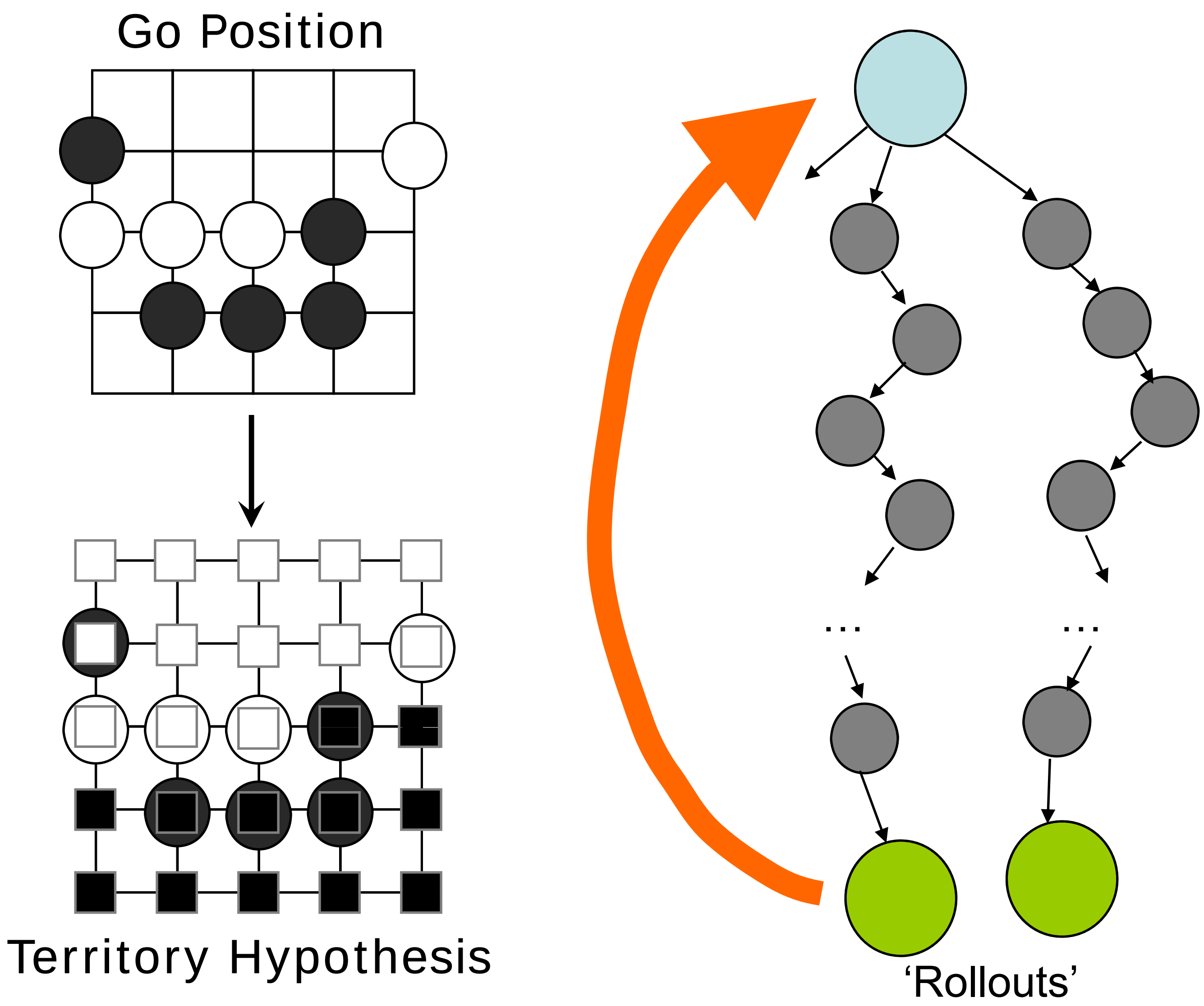


Monte Carlo Go





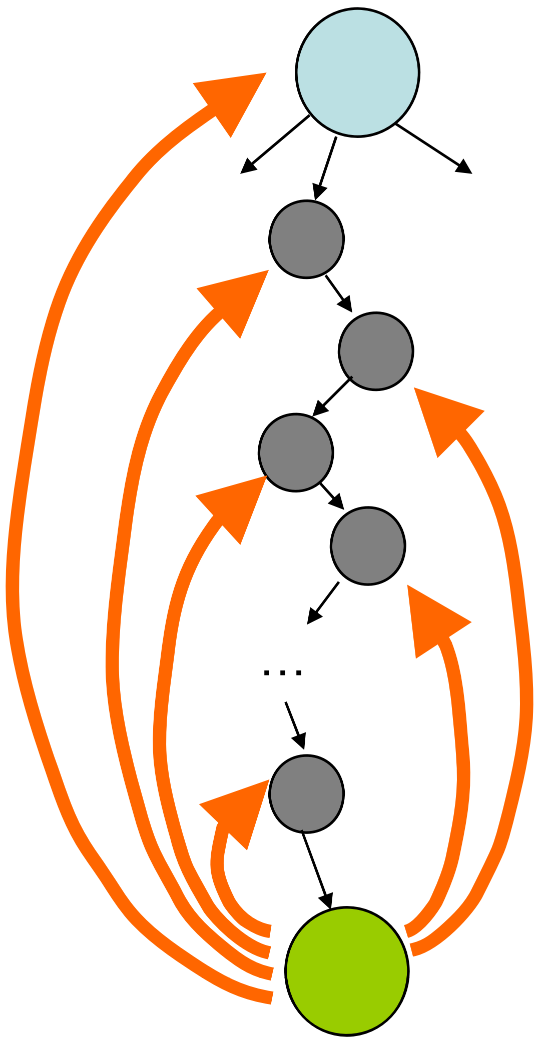
Monte Carlo Go



Monte Carlo Go

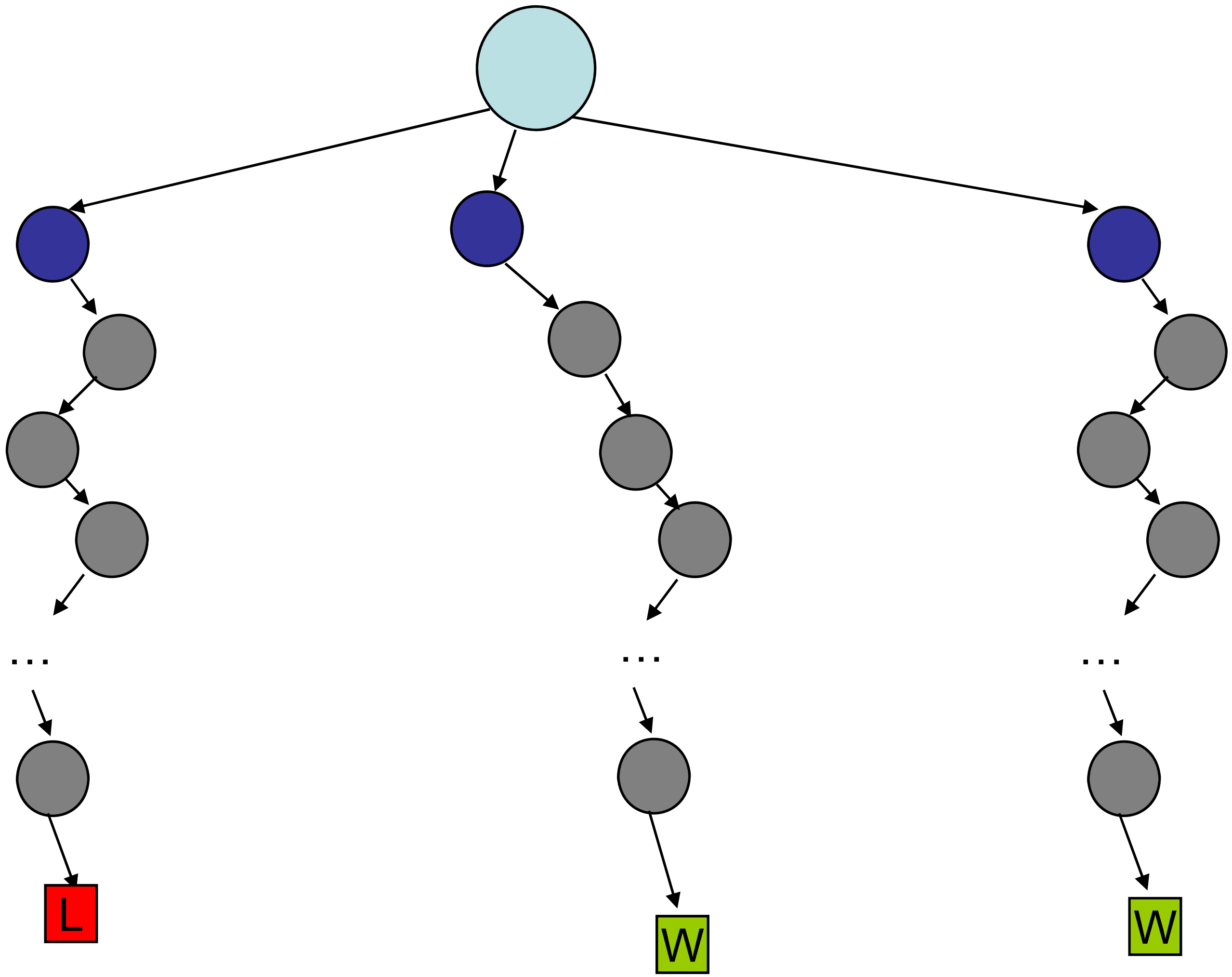
- ‘Rollout’ or ‘Playout’
  - Complete game from current position to end.
  - Not fill in own eyes.
  - Score at final position easily calculated.
- 1 sample = 1 rollout
- Brugmann’s Monte Carlo Go (MC)
  - For each available move m sample s rollouts.
  - At each position, moves selected uniformly at random.
  - Rollout Value: $X_{m,i} \in \{1, 0\}$ (win or loss)
  - Move Value: $\overline{X}_m = \frac{1}{s} \sum_i X_{m,i}$

Adaptive Monte Carlo Planning

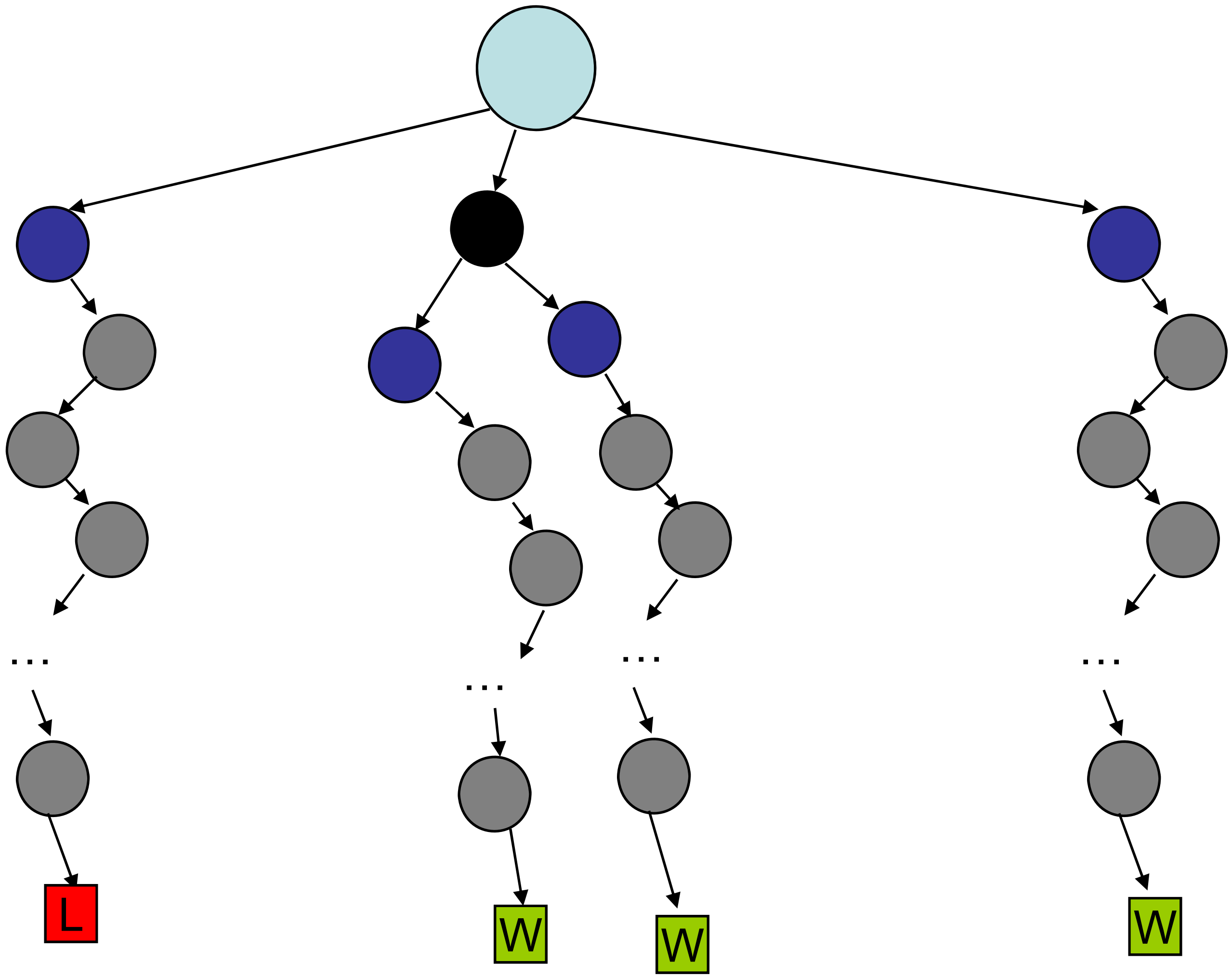


- Update values of all positions in rollout.
  - Store value (distribution) for each node.
  - Store tree in memory.
- Bootstrap policy.
  - UCT.
  - Strong Play on small boards E.g. 'MoGo' (Silvain Gelly)

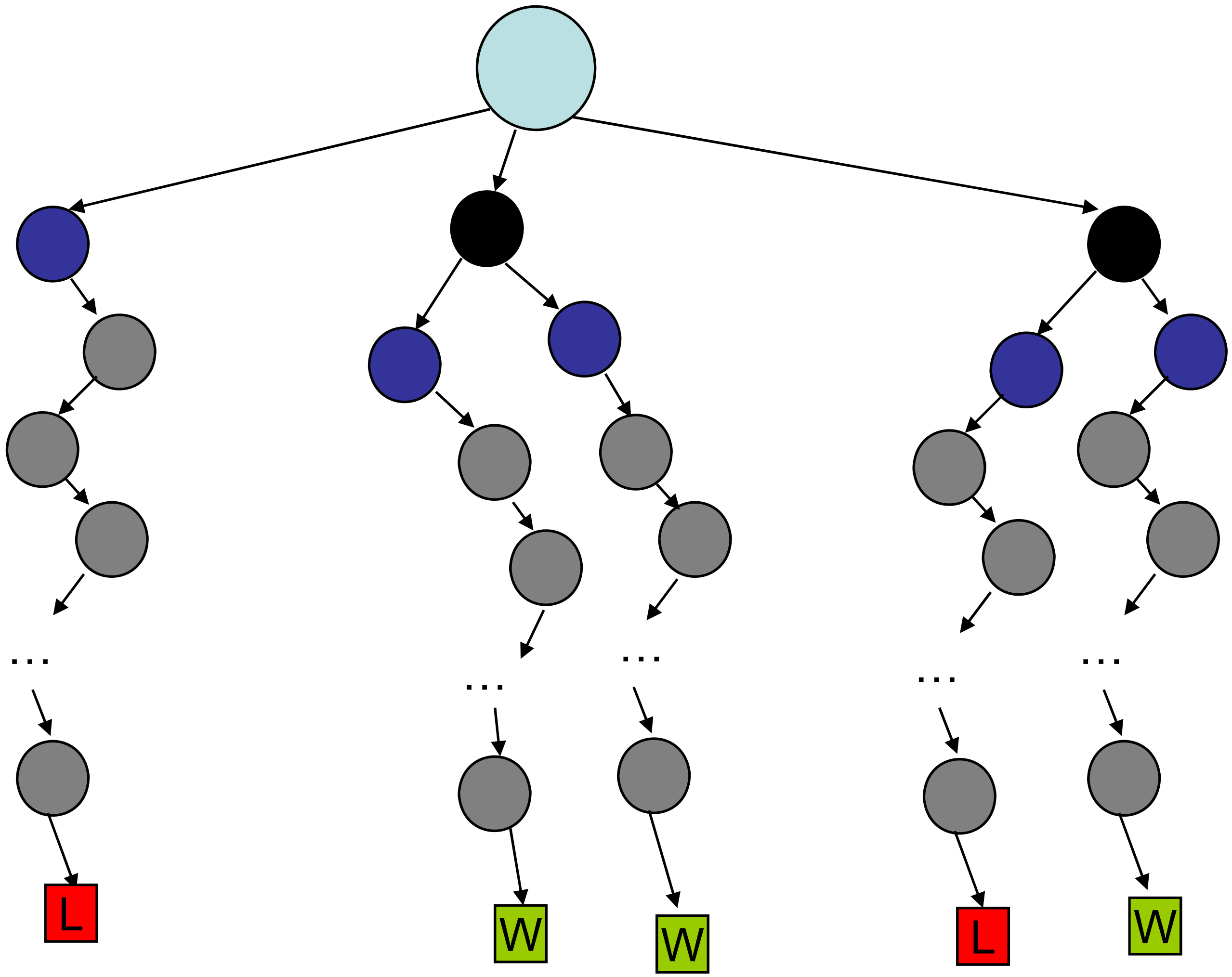
Adaptive Monte Carlo Go



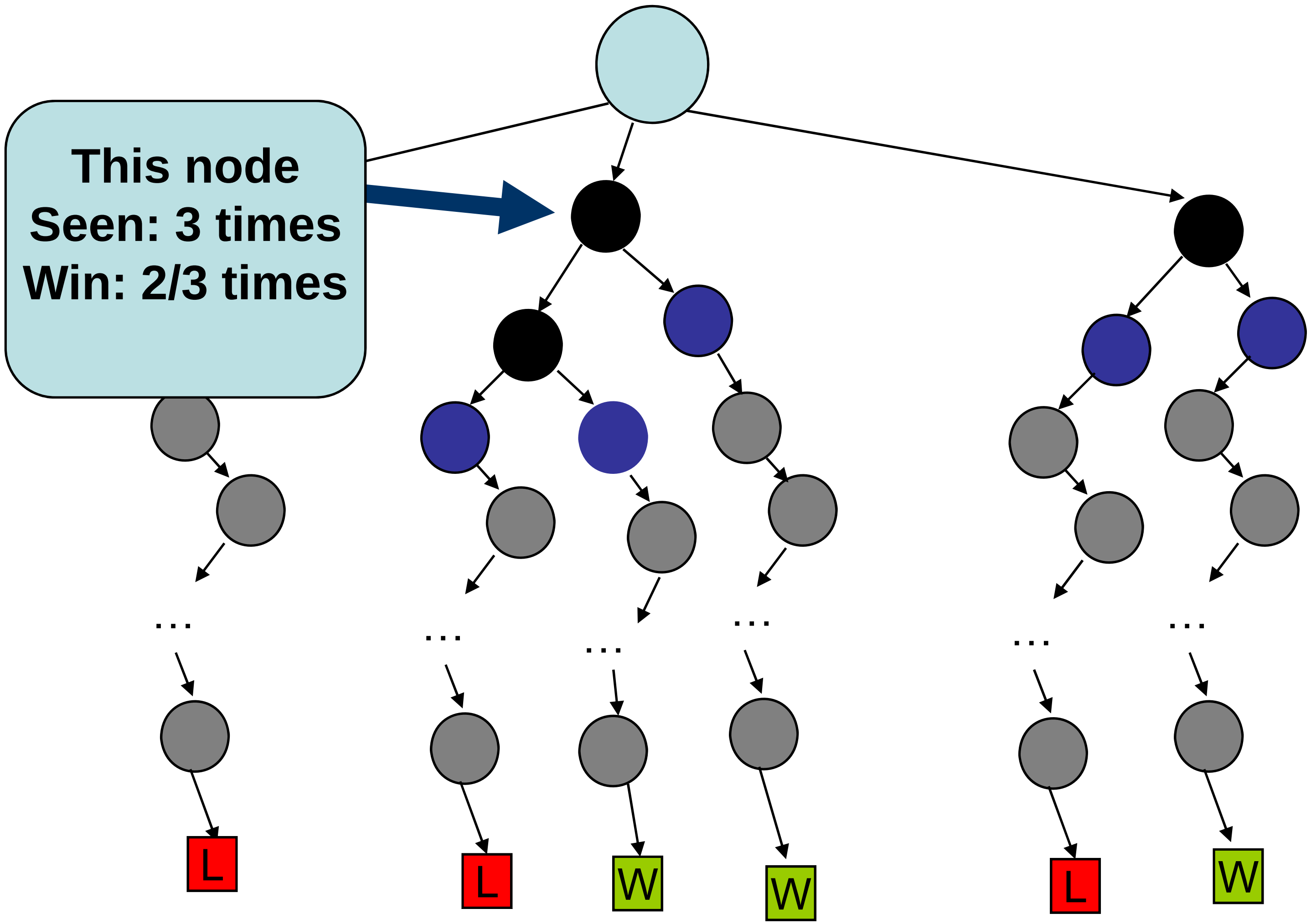
Adaptive Monte Carlo Go



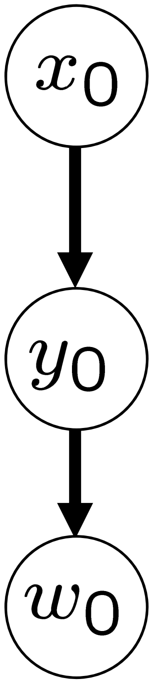
Adaptive Monte Carlo Go



Adaptive Monte Carlo Go



Bayesian Model For Policy



Prior: $p(x_0) = \mathcal{N}(x_0; \mu, \sigma^2)$

$$p(y_t|x_t) = \mathcal{N}(y_t; x_t, \gamma^2)$$

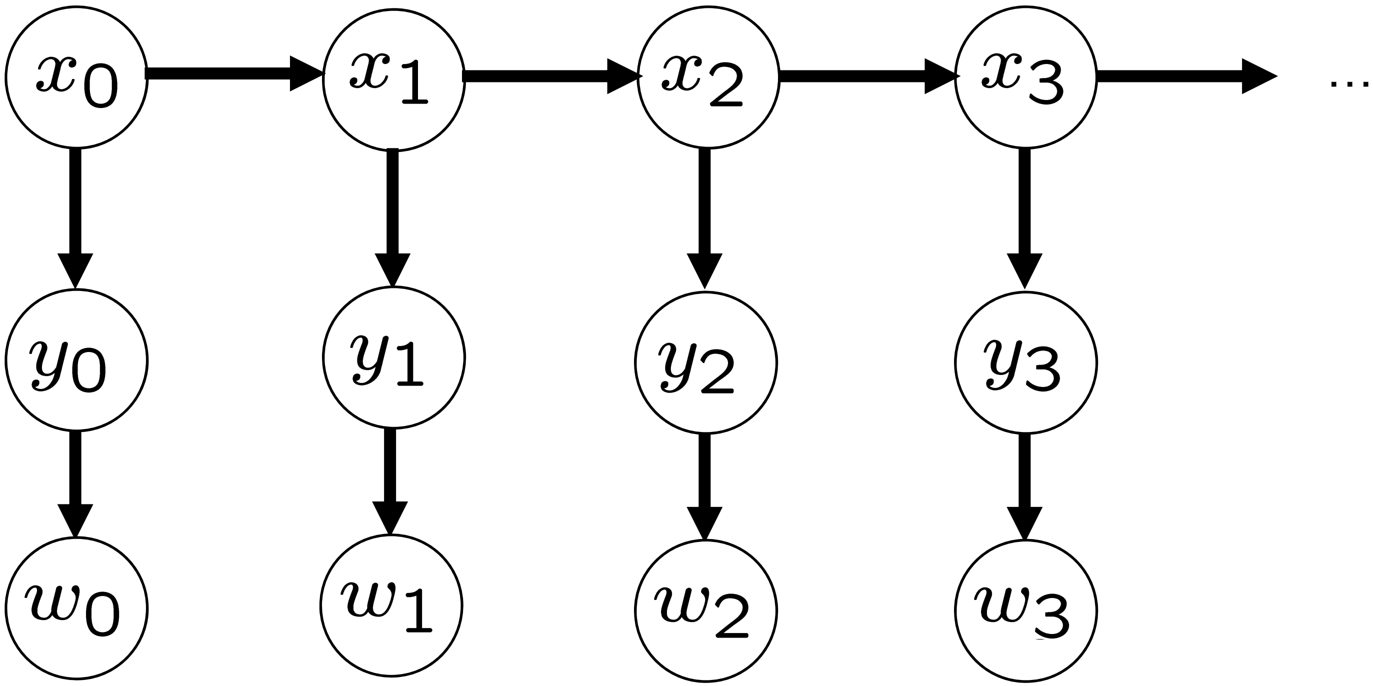
Moment Match Gaussian marginal

Result of Rollout = WIN (TRUE | FALSE)

$$p(w_i|y_i) = \mathbb{I}((y_i > 0) \wedge w_i) + \mathbb{I}((y_i < 0) \wedge \neg w_i)$$

‘Bayesian’ Adaptive MC

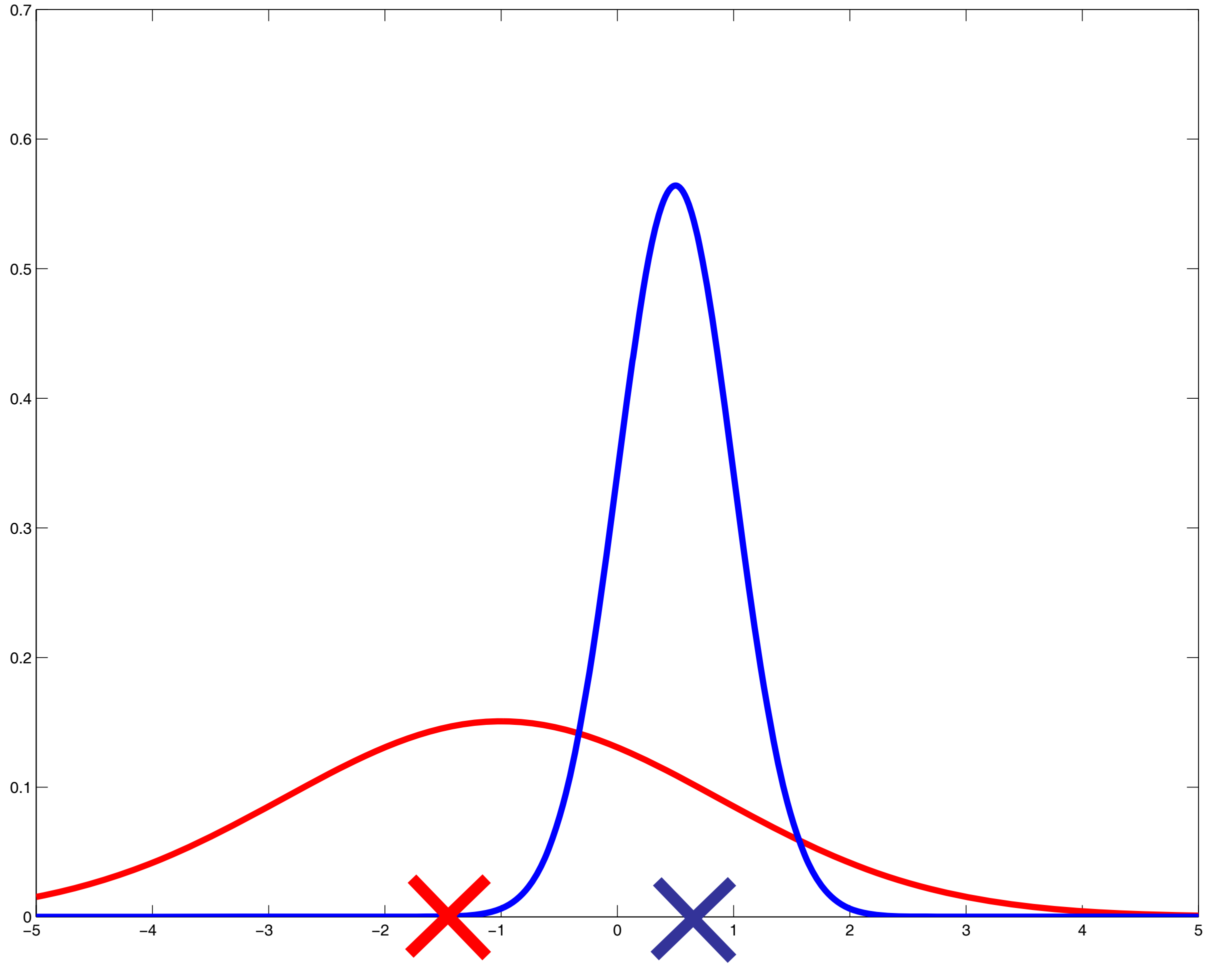
$$p(x_t|x_{t-1}) = \mathcal{N}(x_t; x_{t-1}, \tau^2)$$



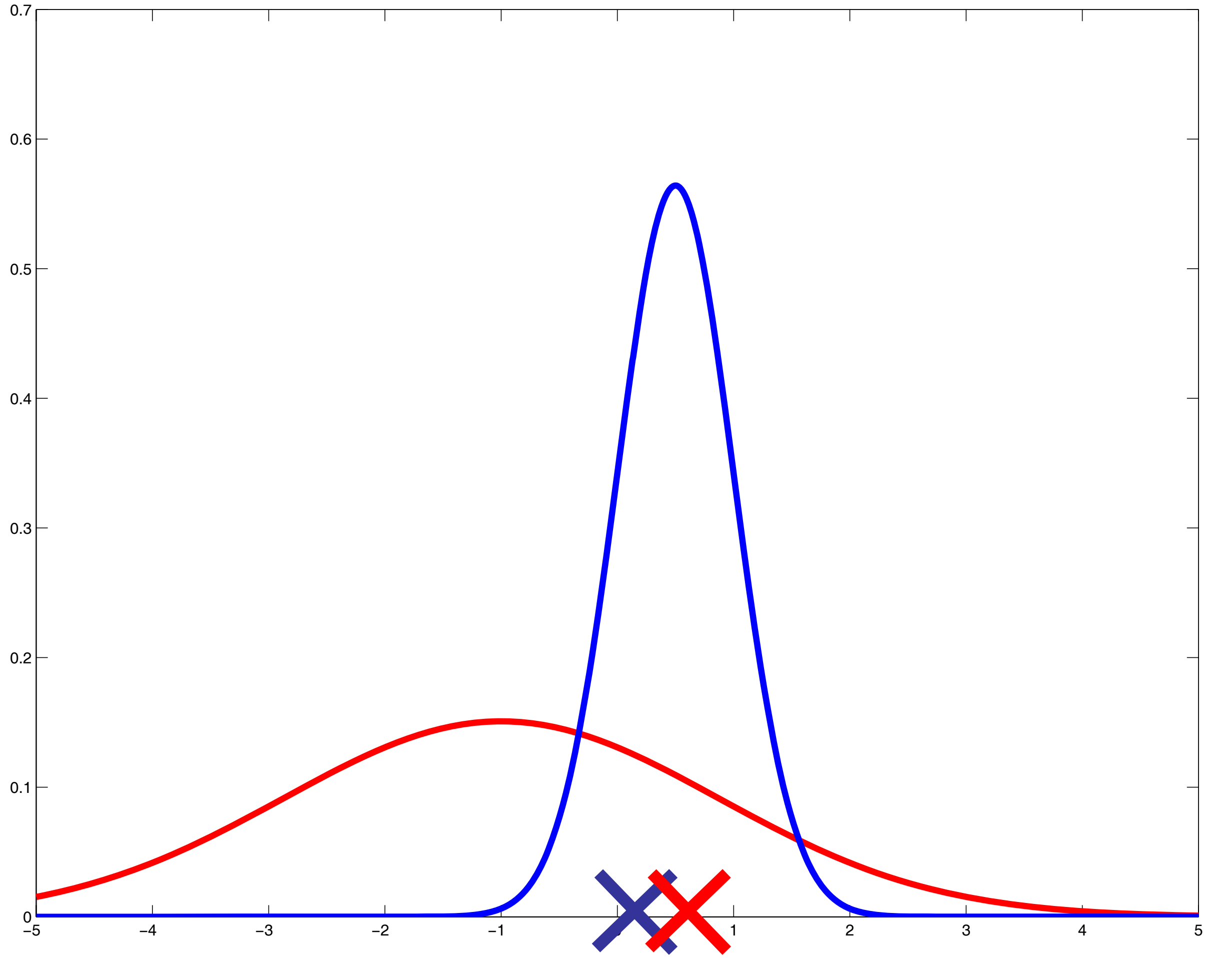
‘Bayesian’ Adaptive MC (BMC)

- Policy:
  - Sample from the distribution for each available move.
  - Pick best.
- Exploitation vs Exploration
  - Automatically adapted.

Exploitation vs Exploration



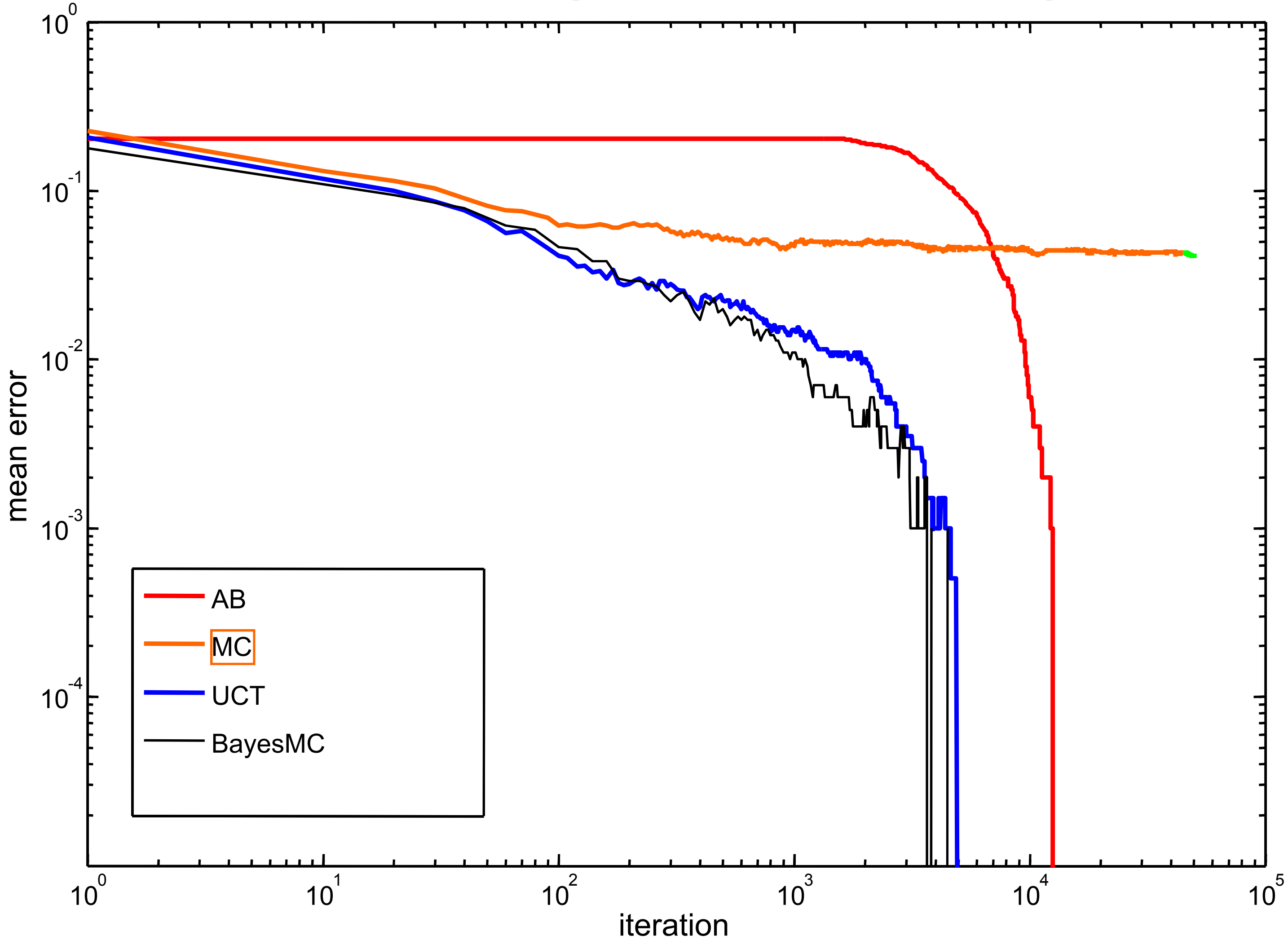
Exploitation vs Exploration



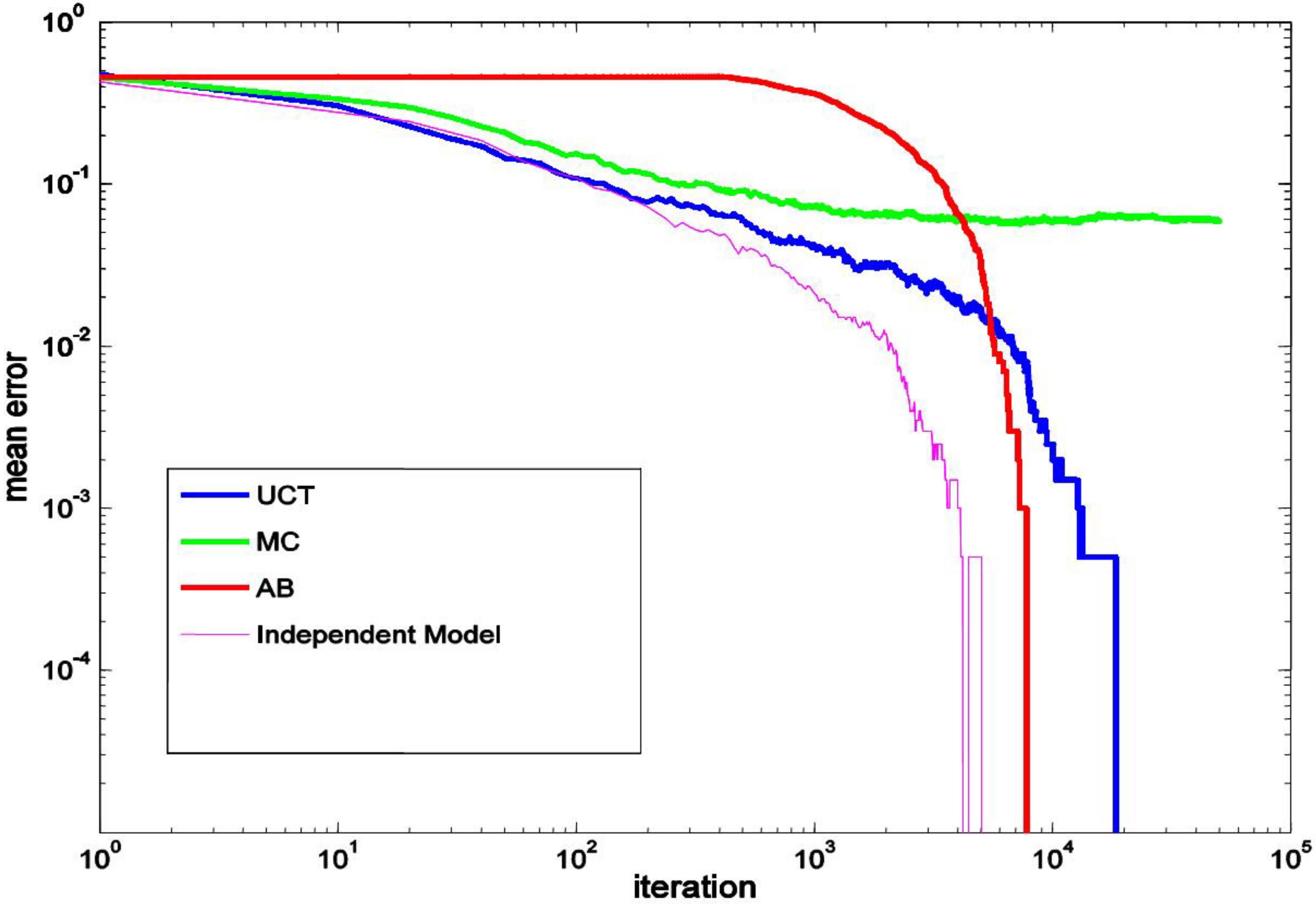
P-Game Trees

- Moves have numerical values
  - MAX moves drawn uniformly from [0,1]
  - MIN moves drawn uniformly from [-1,0]
- Value of leaf is sum of moves from root.
- If leaf value > 0 then win for MAX.
- If leaf value < 0 then loss for MAX.
- Assign win/loss to all nodes via Minimax.
- Qualitatively more like real Go game tree.
- Can simulate the addition of domain knowledge.

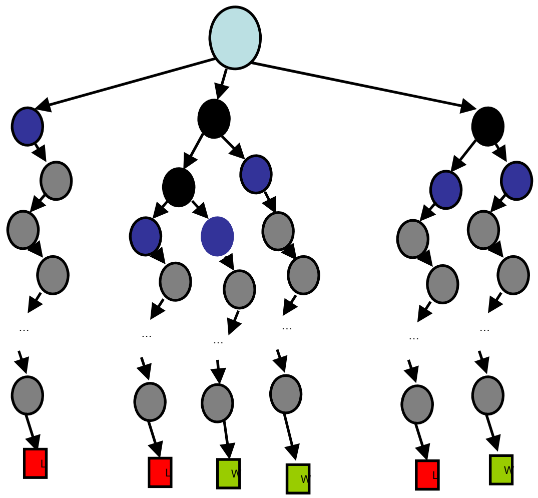
Monte Carlo Planning On P-Game Trees (B=2, D=20)



MC Planning on P-Game Trees (B=7,D=7)



Conclusions



- Areas addressed:
  - Move Prediction
  - Territory Prediction
  - Monte Carlo Go
- Probabilities good for modelling uncertainty in Go.
- Go is a good test bed for machine learning.
  - Wide range of sub-tasks.
  - Complex game yet simple rules.
  - Loads of training data.
  - Humans play the game well.