Entropy for Time Series Forecasting

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Takens' Delay Embedding Theorem

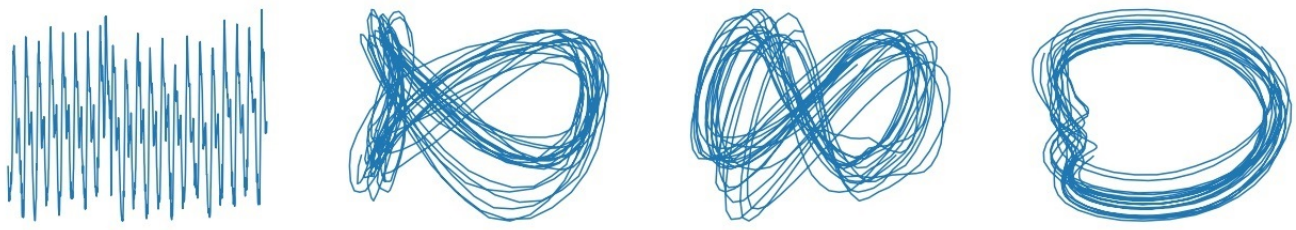
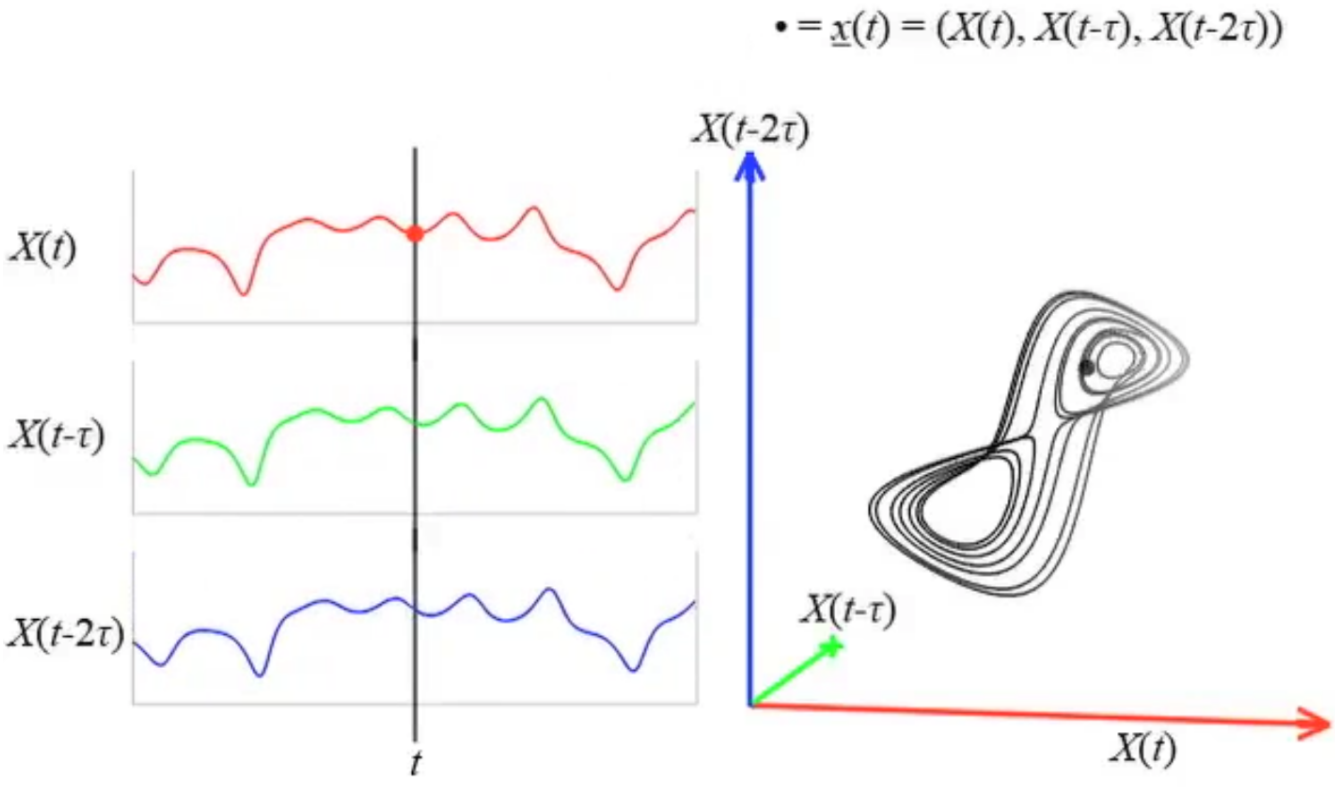


Figure: This plot is a geometrical representation of Takens' Embedding to reconstruct the time series whose sample is given of the left figure. The main idea is to add K delays to the initial time series y and lift it into a state space Y_K of K time delays. Then, to obtain the 3-dimensional attractor one can do dimensionality reduction using Principal Component Analysis method.

How to obtain the atractor



Entropy

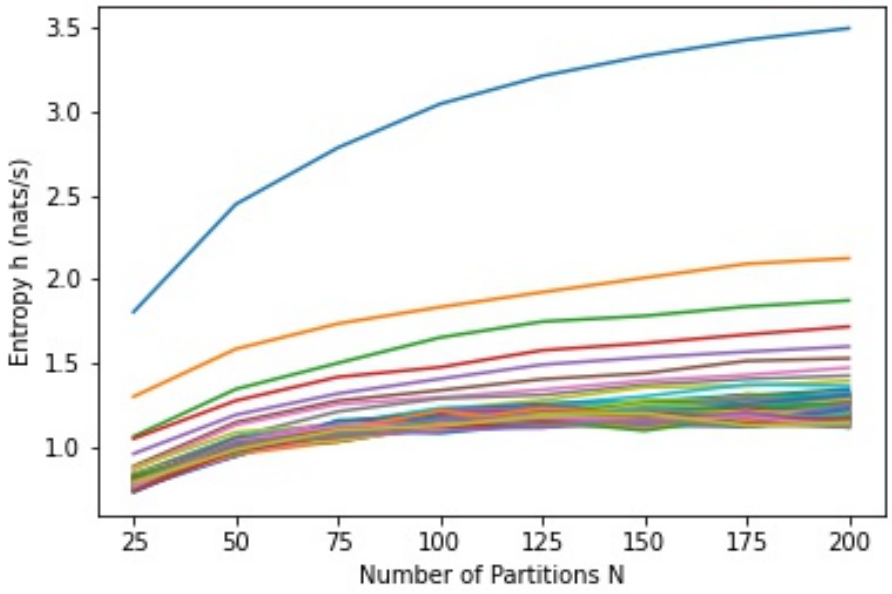


Figure: Entropy is a scientific concept that mostly relates with the (dis)order or uncertainty of a system. Applied to the realm of Information Theory, it can be understood at the average level of information that is intrinsic to the variable that describes the system. This image shows the entropy resulting from clustering the state space Y_k into N Voronoi Cells.

Conditional Entropy

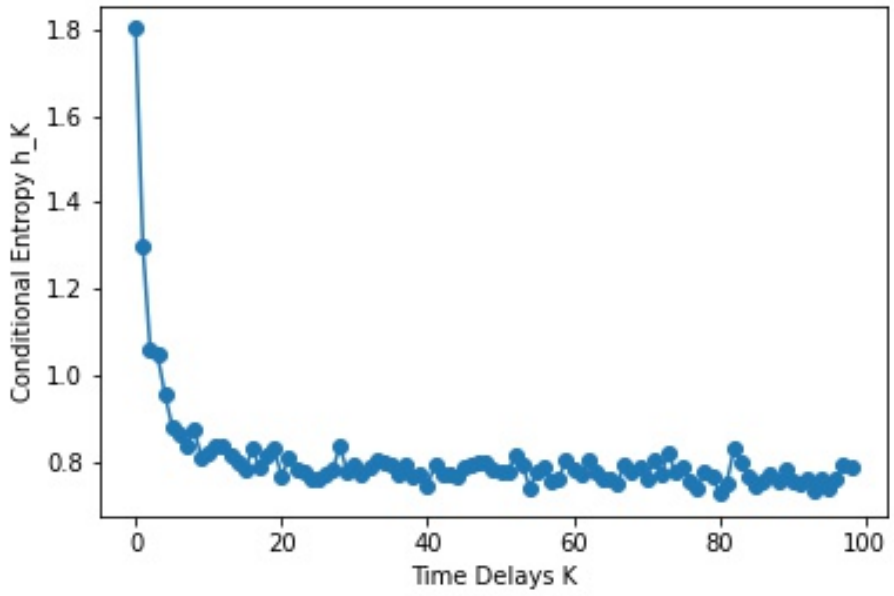


Figure: Considering an ordered sequence of random variable $(y_1, \ldots, y_k)$, the conditional entropy is a measure of the amount of information needed to describe y_k given that we have available information about the previous steps $y_1, \ldots, y_{k-1}$. Here we can see the decaying of conditional entropy as the number of time delays increases. As a means of attaining the day and night cycle dynamics of water, I chose 24 timesteps to use in the LSTM.

LSTM

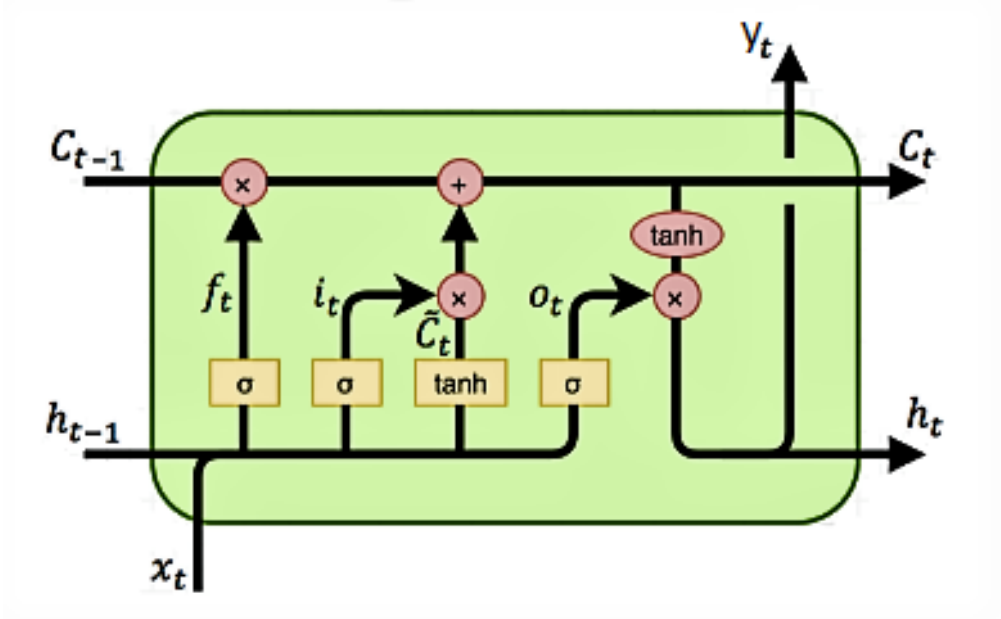


Figure: Long Short Term Memory (LSTM) Networks are a special type of Recurrent Neural Networks (RNN) which rely on gated cells that control the flow of information by choosing what elements of the sequence are passed on to the next module.

Model Predictions

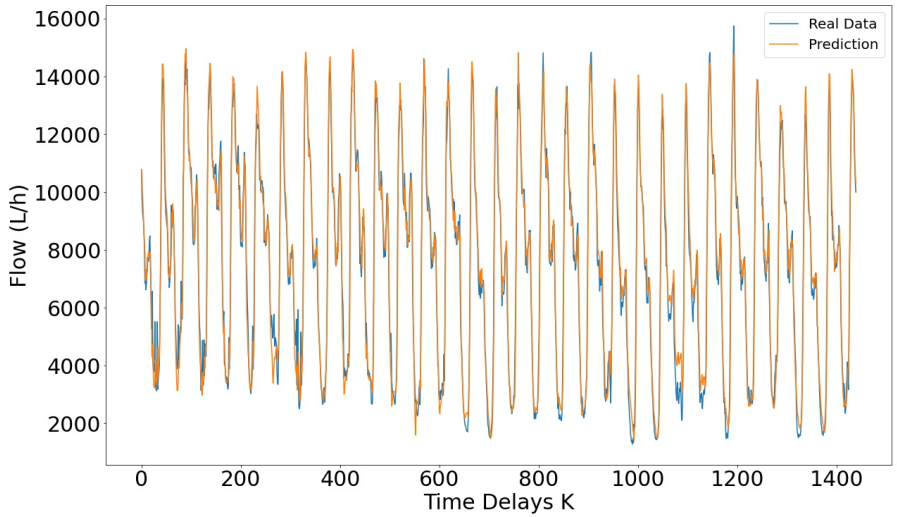


Figure: These predictions were obtained by applying the previously theoretic method of calculating timesteps which were then used for helping the LSTM recognize patters in data.

7 Days Sample and Forecasting

Figure: On top one can view a random 7 day sample and on the bottom the obtained forecasting