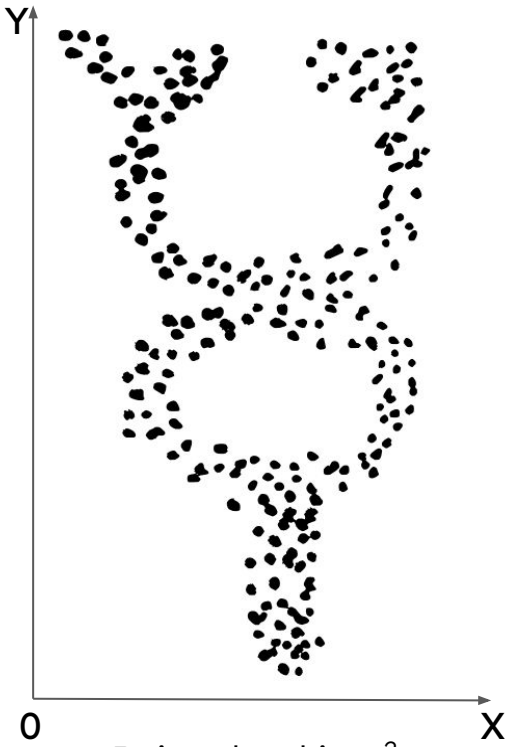
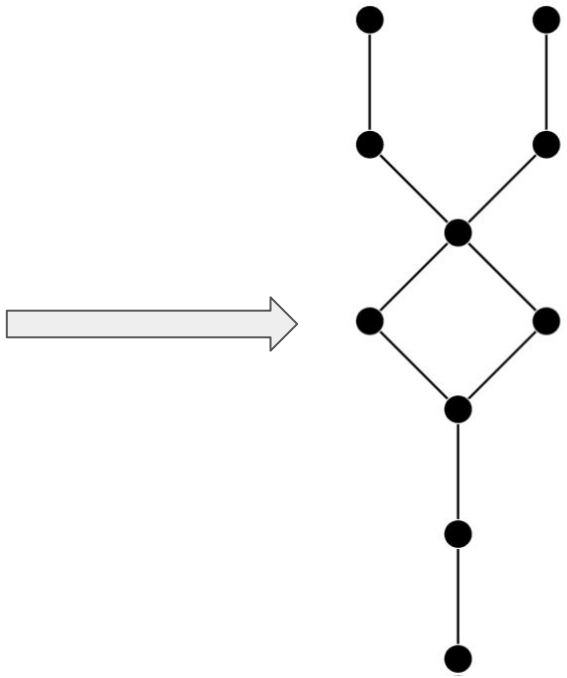
Towards Testing the Significance of Branching Points and Cycles in Mapper Graphs

Patrik Zajec, Primož Škraba and Dunja Mladenić

Topological structure of a point cloud

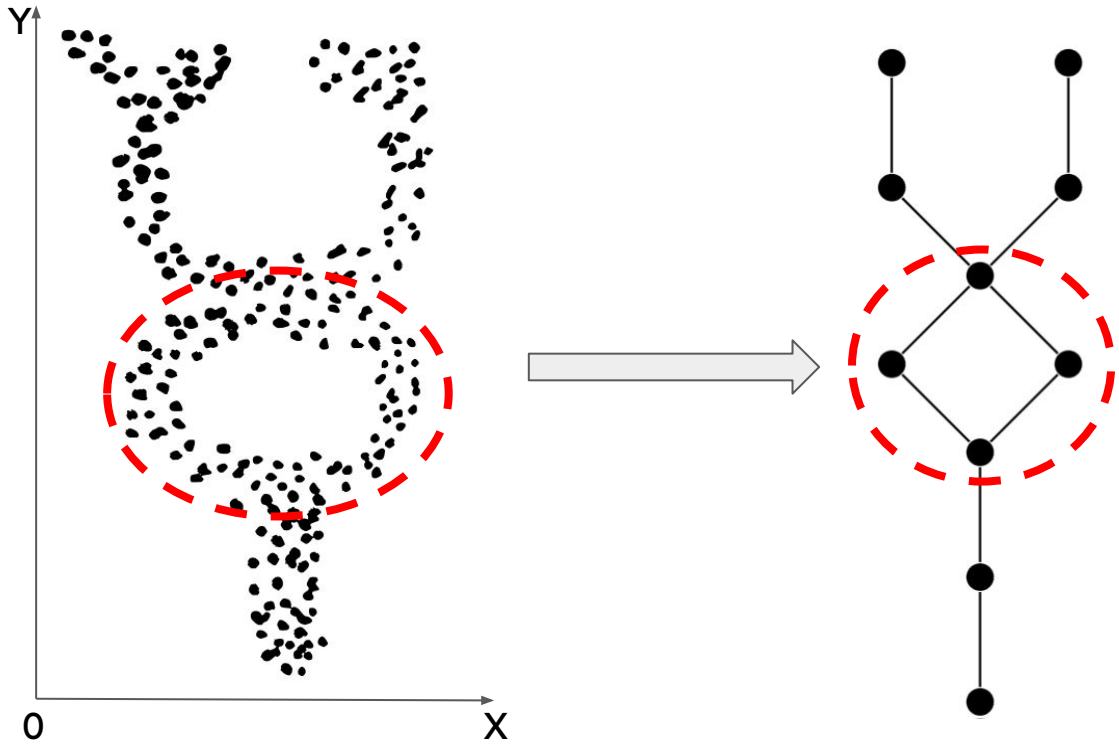


Point cloud in $\mathbb{R}^2$

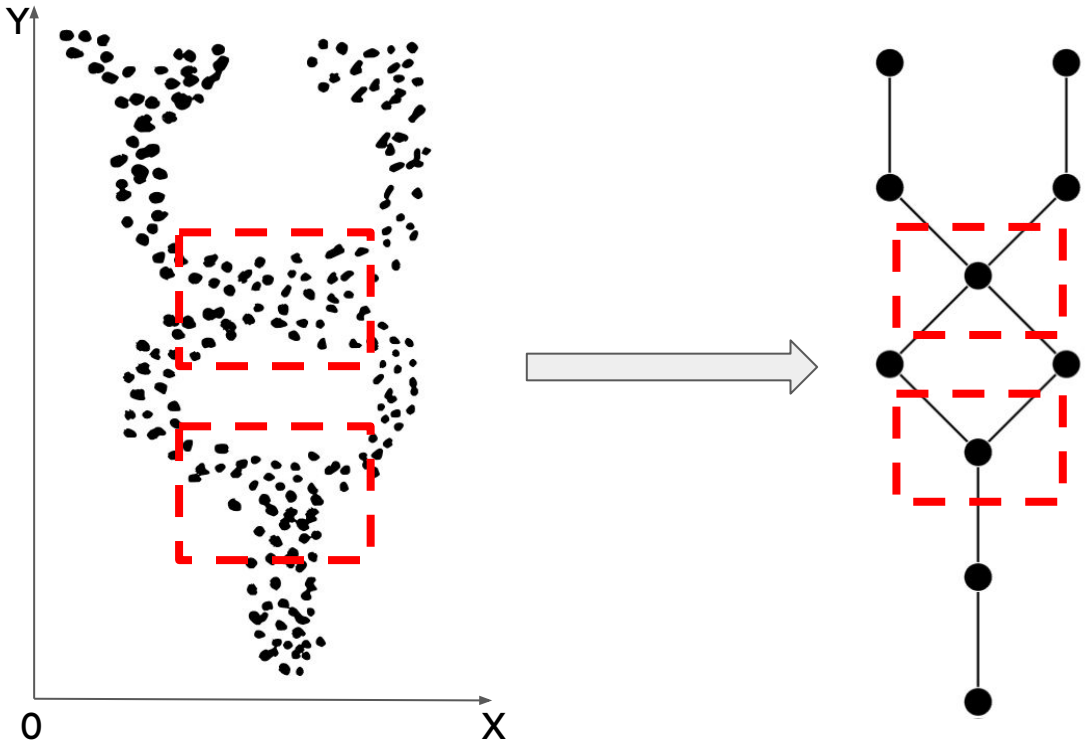


Graph capturing its topological structure

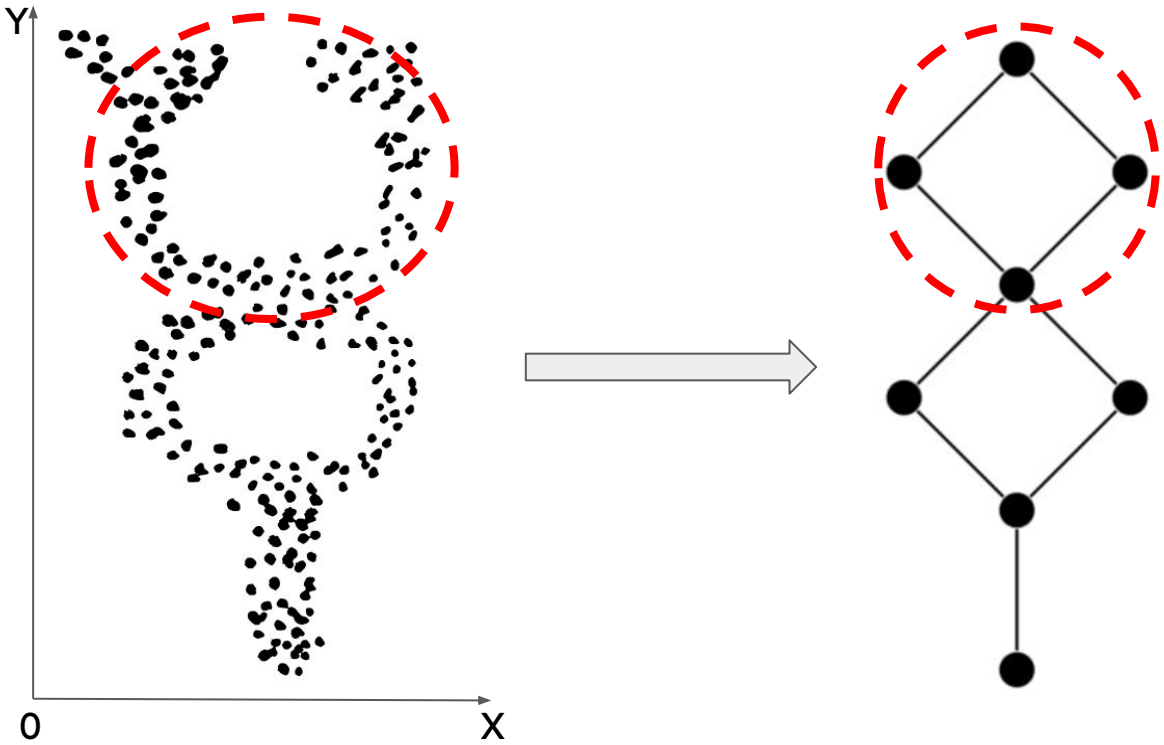
Focus on **cycles** and branching points



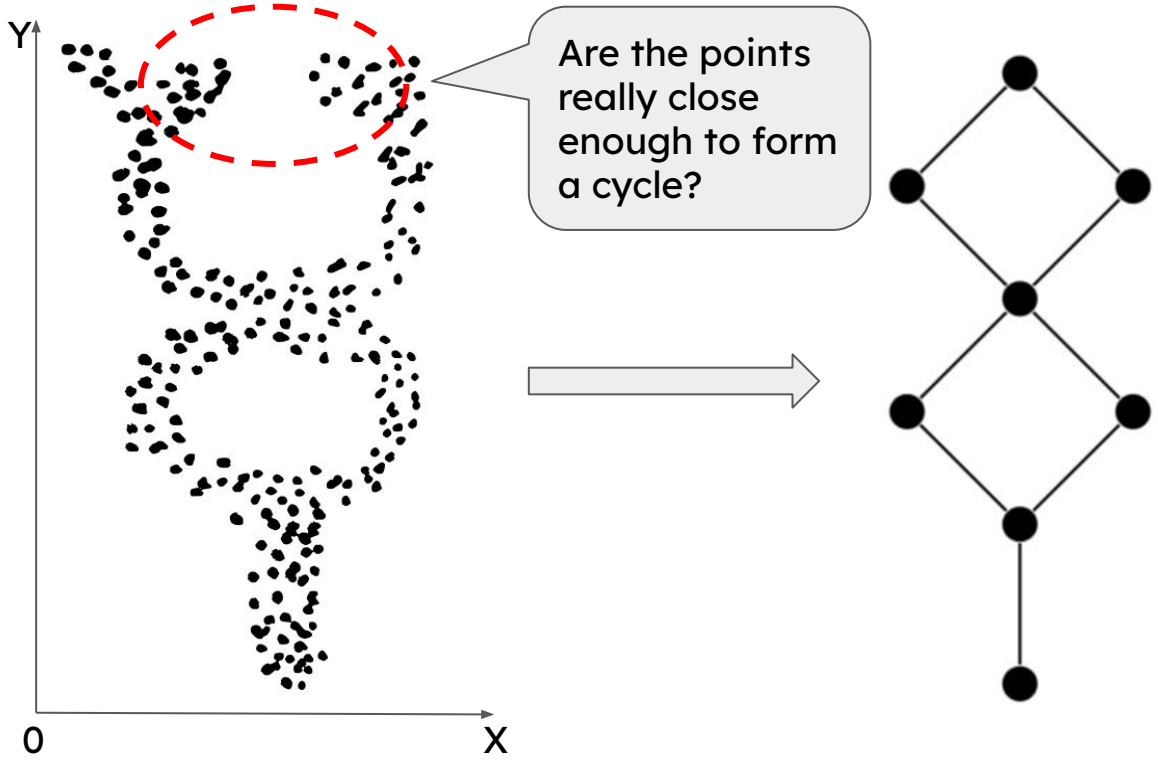
Focus on cycles and **branching points**



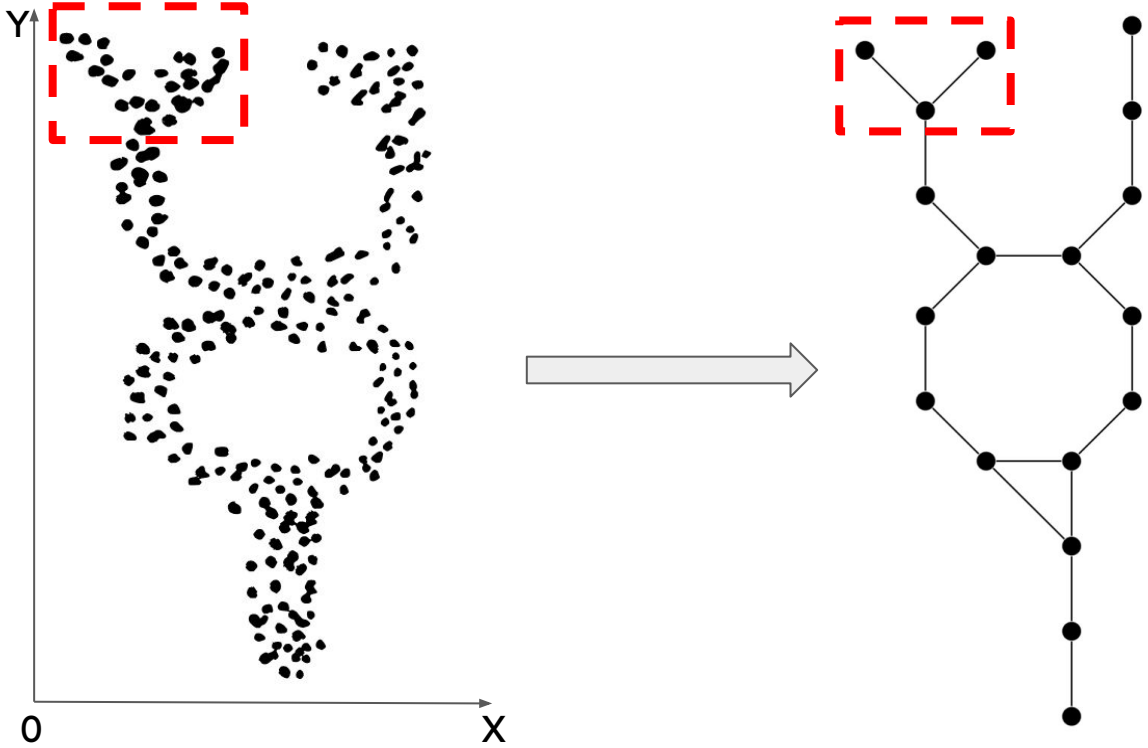
Other possible graphs - additional cycles



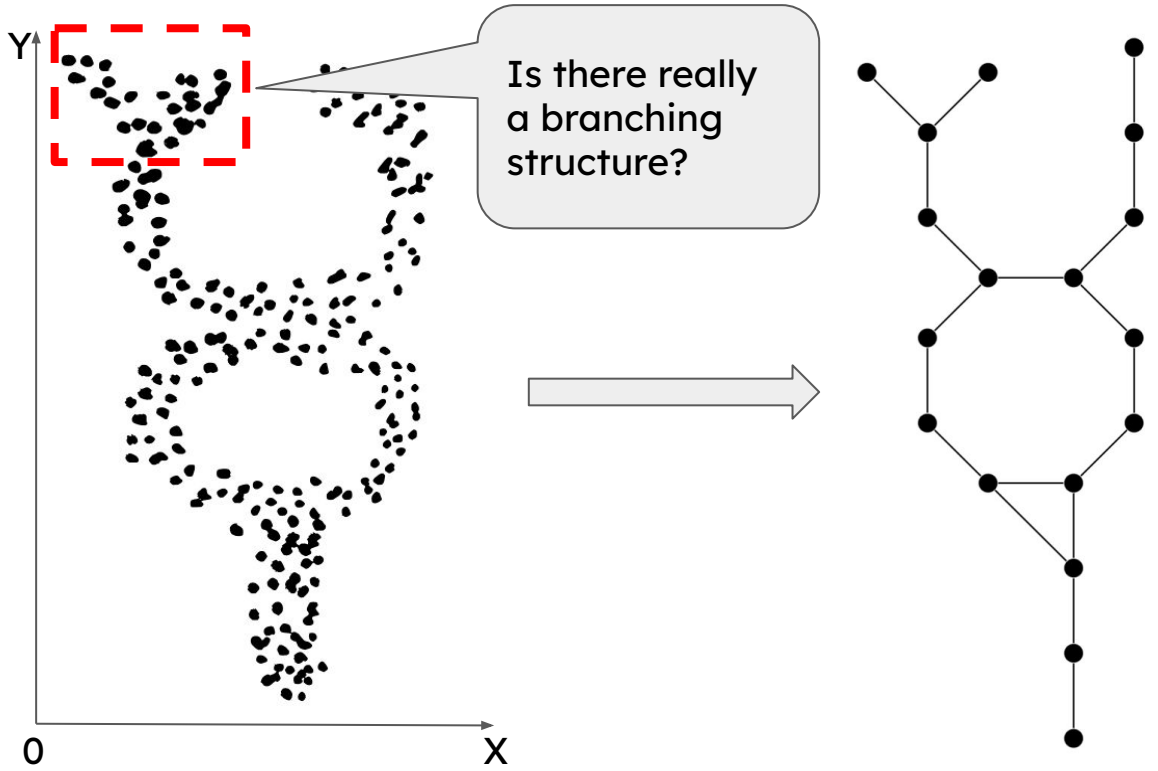
Other possible graphs - additional cycles



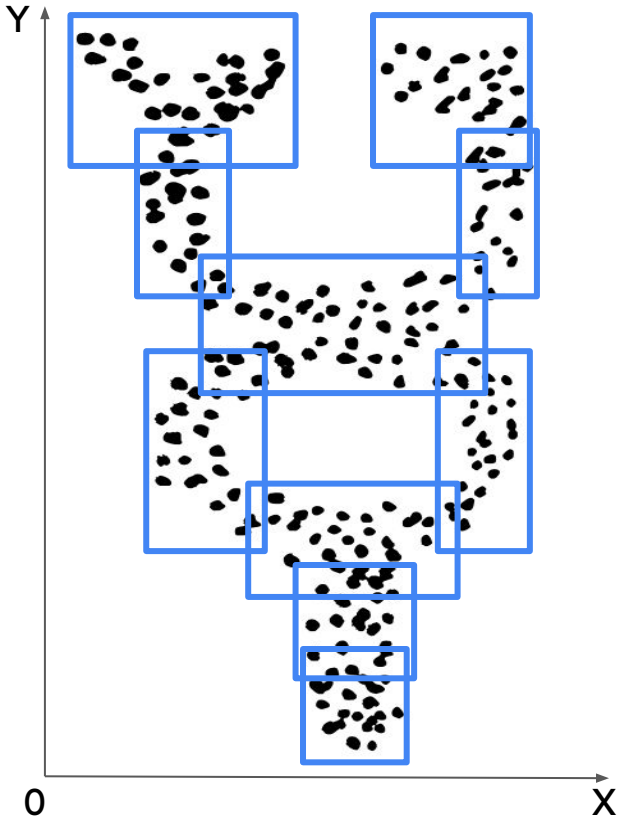
Other possible graphs - additional branching points



Other possible graphs - additional branching points



Point cloud to graph using mapper algorithm

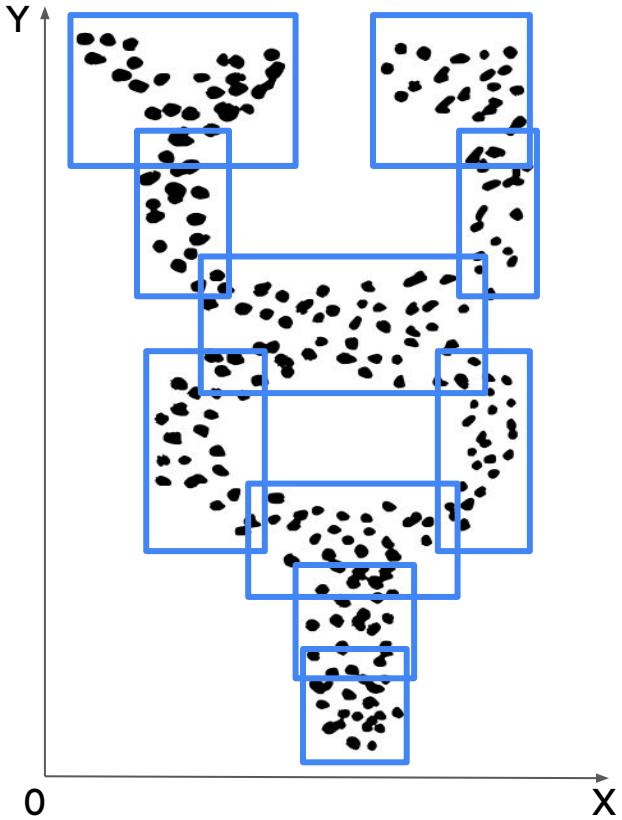


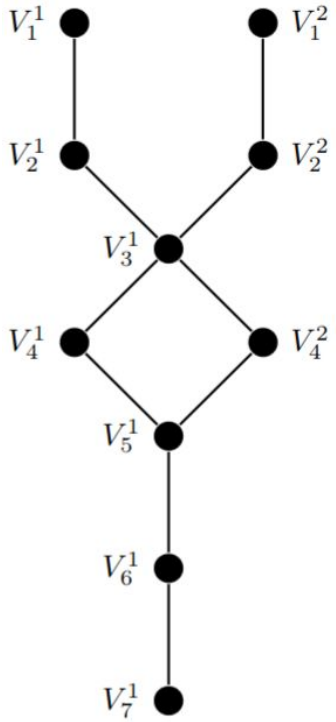
1. Split the points into overlapping subsets. Each subset is represented as a vertex in the graph.

   Function φ maps each vertex to its subset of points.

Gurjeet Singh, Facundo Memoli, and Gunnar Carlsson. 2007. Topological Methods for the Analysis of High Dimensional Data Sets and 3D Object Recognition. In Eurographics Symposium on Point-Based Graphics. M. Botsch, R. Pajarola, B. Chen, and M. Zwicker, editors. The Eurographics Association. isbn: 978-3-905673-51-7. doi: 10.2312/SPBG/SPBG07/091-100.

Point cloud to graph using mapper algorithm





2. Two vertices are connected by an edge if their sets of points overlap:

$$\varphi(u) \cap \varphi(v) \neq \varnothing.$$

Problem setting

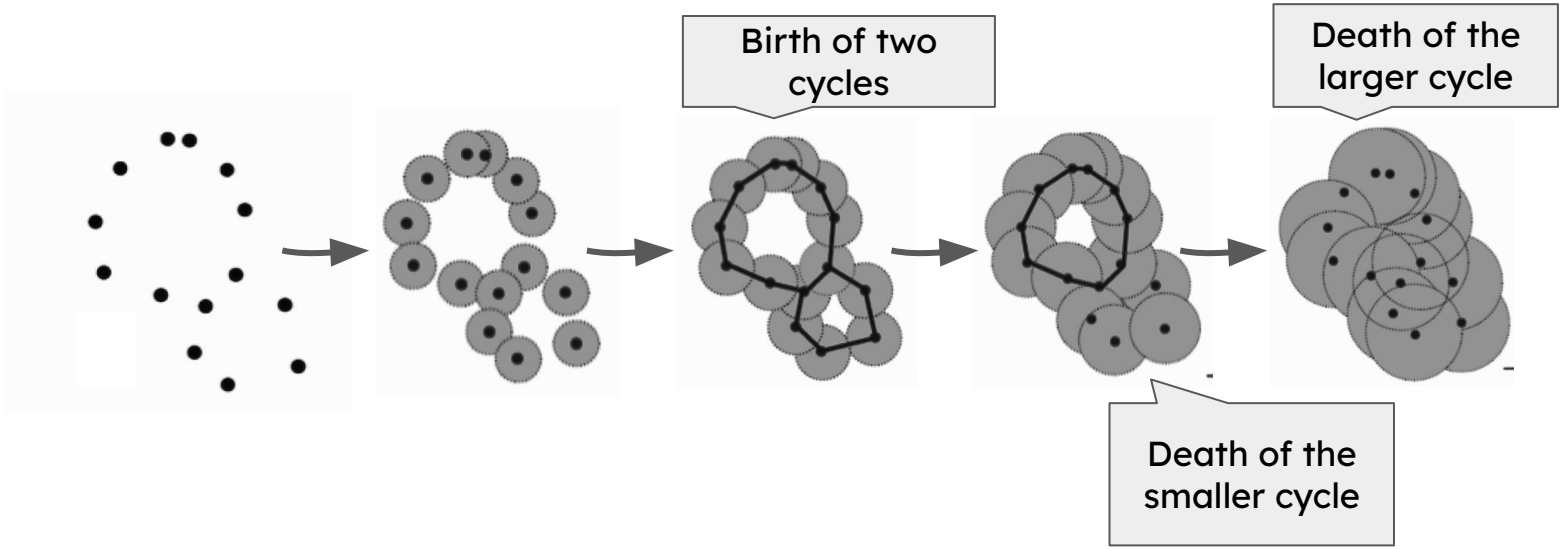
Given:

- a point cloud $P = \{p \in \mathbb{R}^n\}$,
- a graph $G = (V, E)$ and a function φ mapping each vertex v to $S \subset P$.

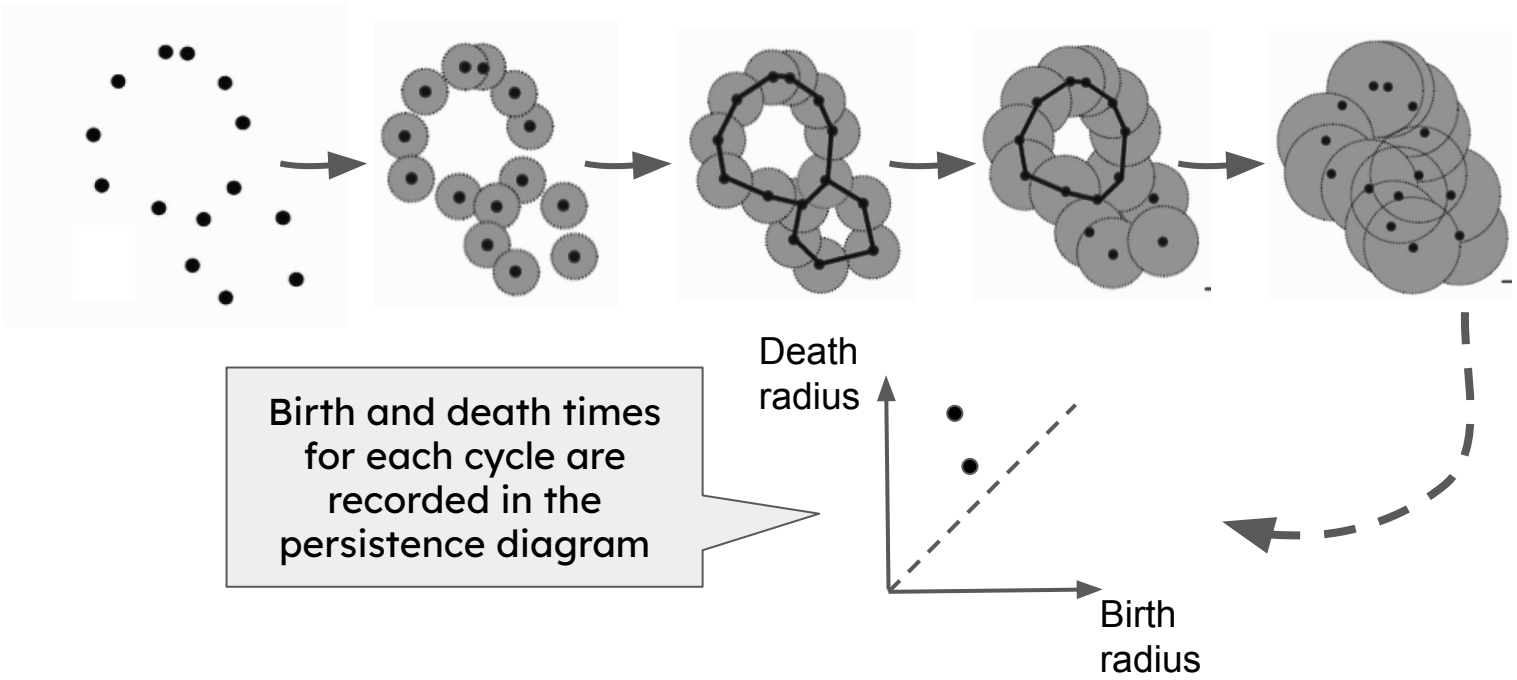
Determine:

- for each simple cycle $\{v_1, \ldots, v_k\}$ whether $\bigcup_{i=1\ldots k} \varphi(v_i)$ has a significant cyclic structure,
- for each branching point v (vertex of degree ≥ 3) whether $\bigcup_{w \in N(v)} \varphi(w)$ has a significant branching structure.

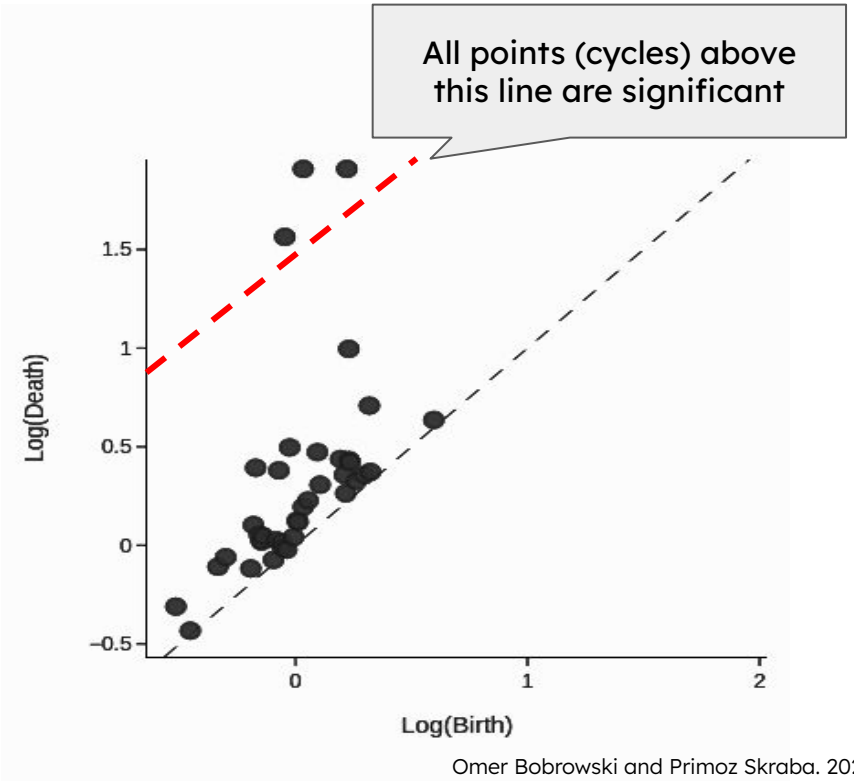
Checking cycles using (persistent) homology



Checking cycles using (persistent) homology



Significance testing of cycles

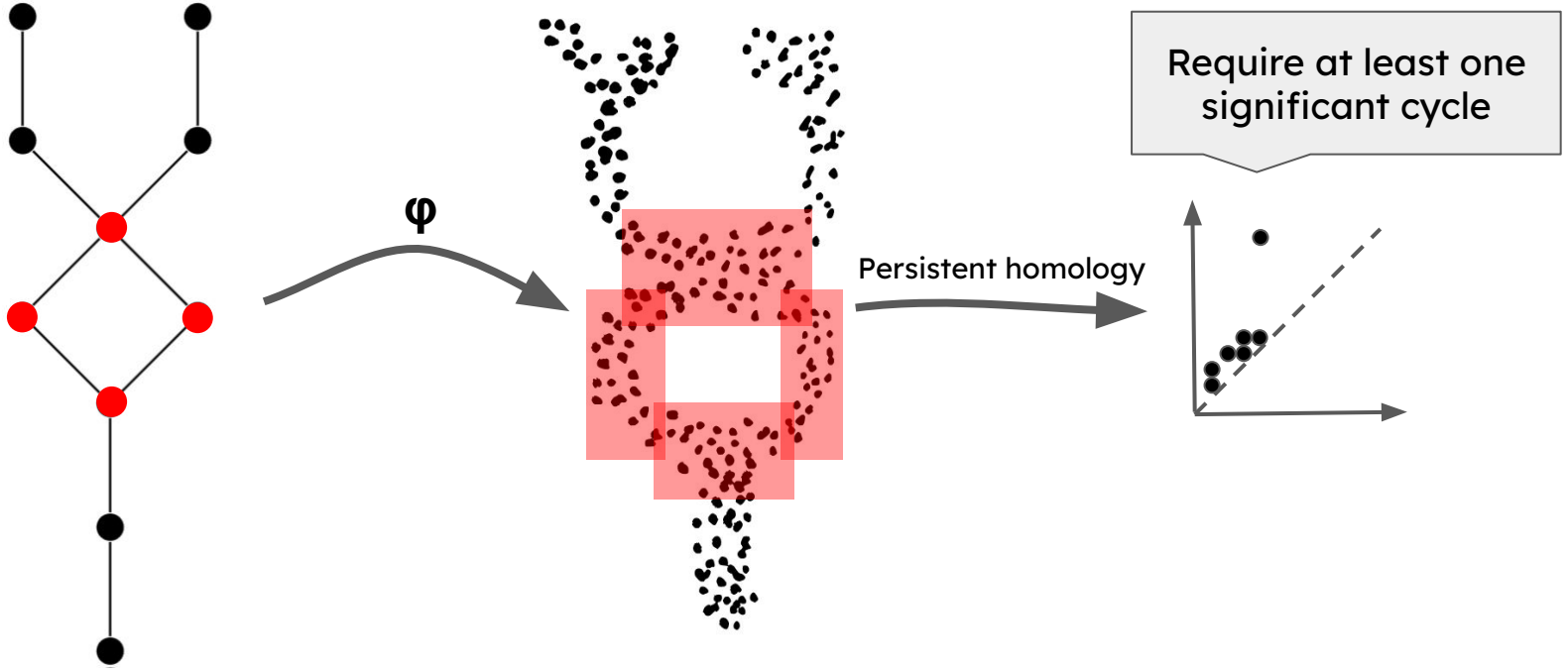


Which points (cycles) appear due to randomness and noise in the data and which are statistically significant?

(Bobrowski and Škraba, 2023) proposed first test that allows us to test each point for significance.

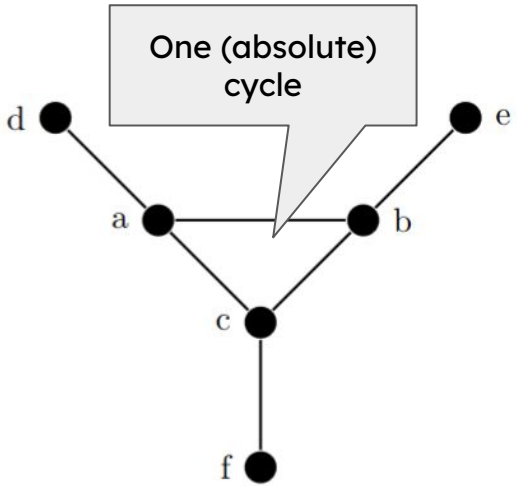
Omer Bobrowski and Primoz Skraba. 2023. A universal null-distribution for topological data analysis. Scientific Reports, 13, 1, (July 2023), 12274. doi: 10.1038/s41598-023-37842-2

Testing simple cycles

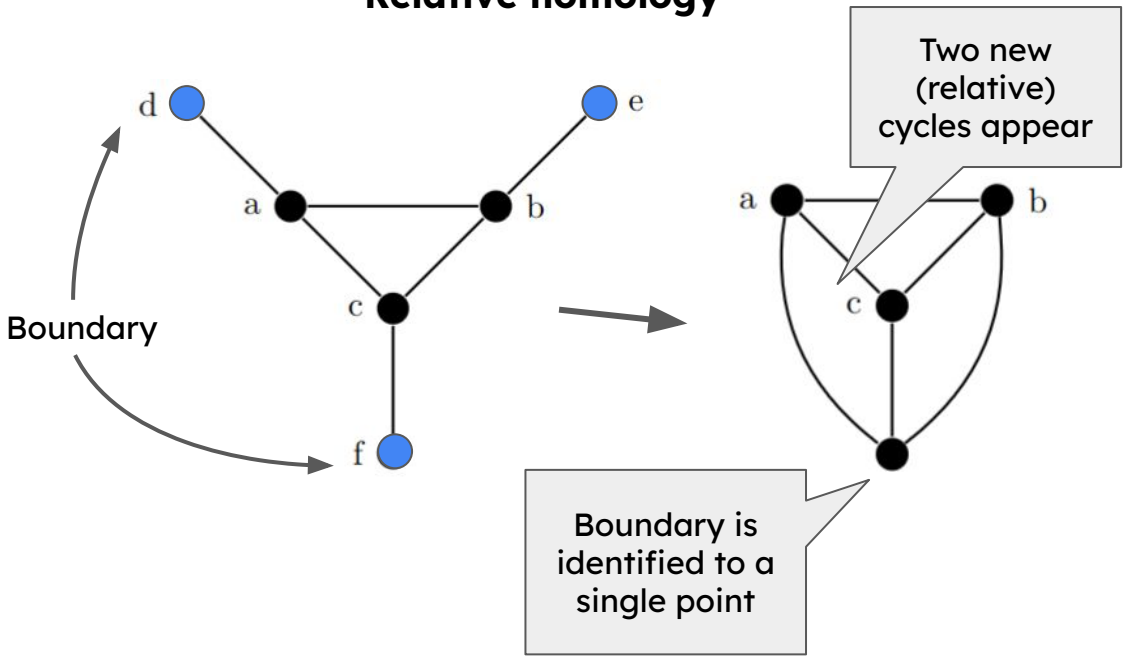


Checking branching points with (persistent) relative homology

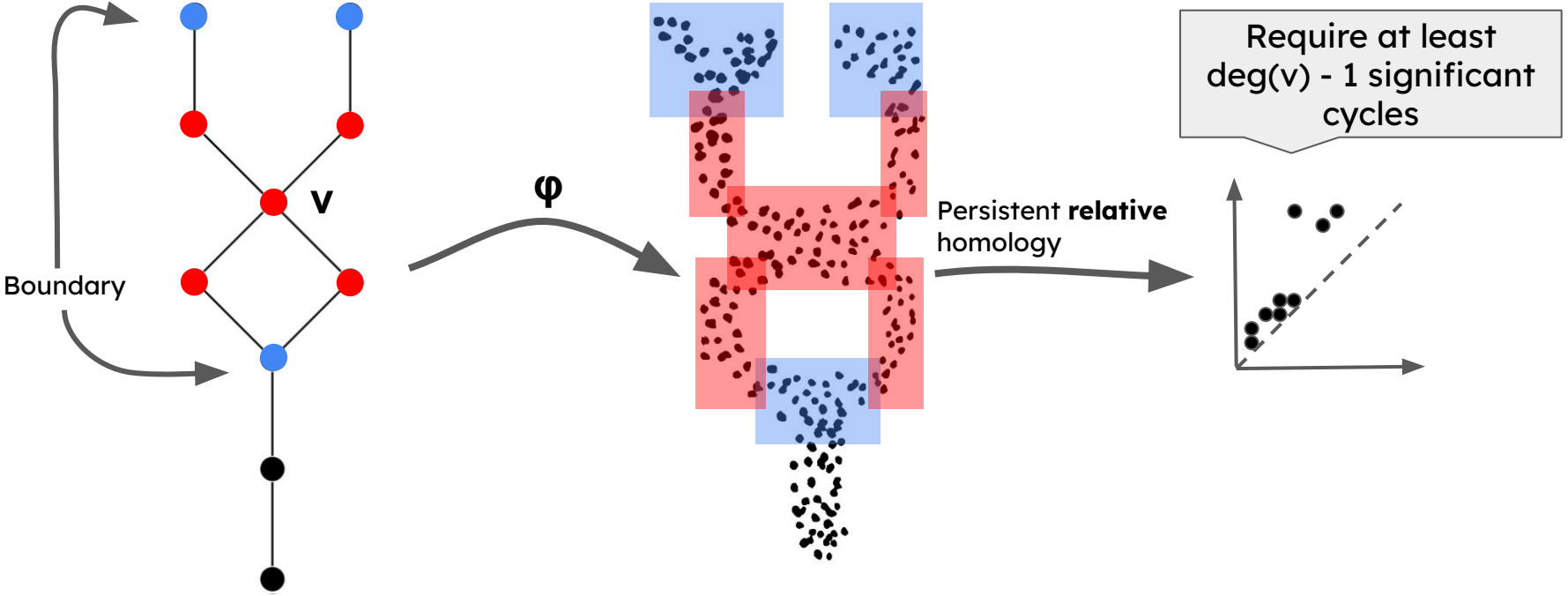
Absolute homology



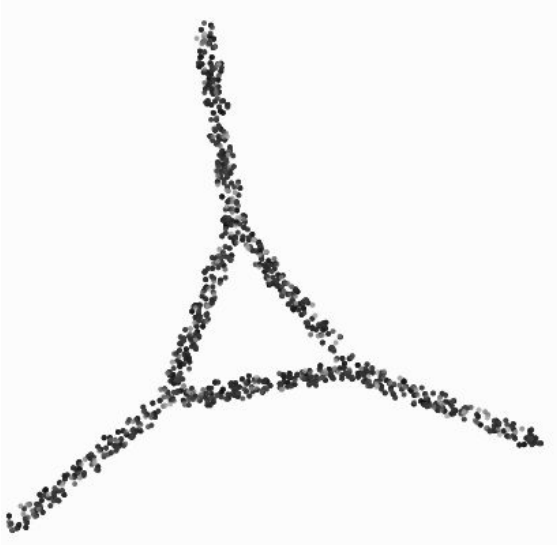
Relative homology



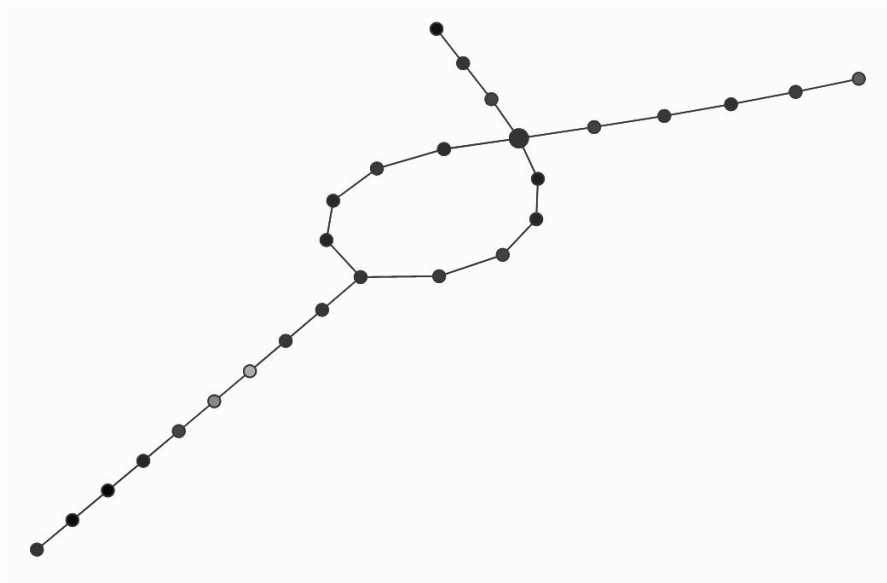
Testing branching points



Experiment 1: Y-shaped point cloud

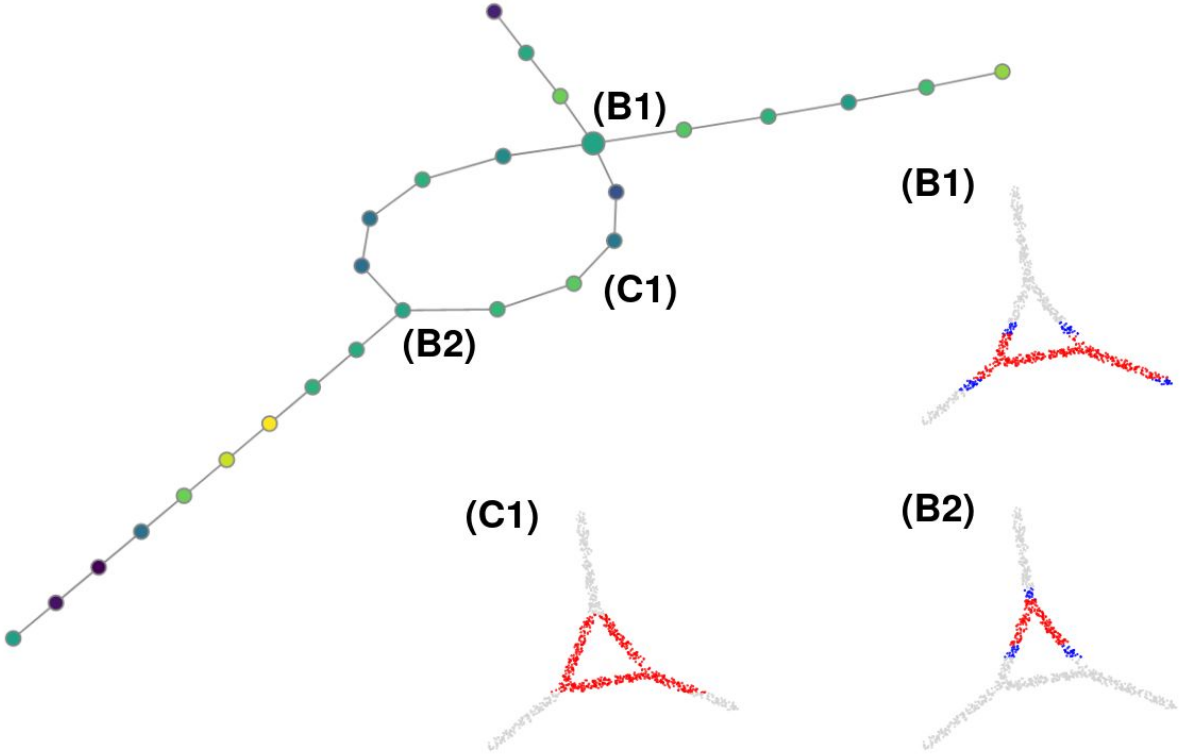


Point cloud

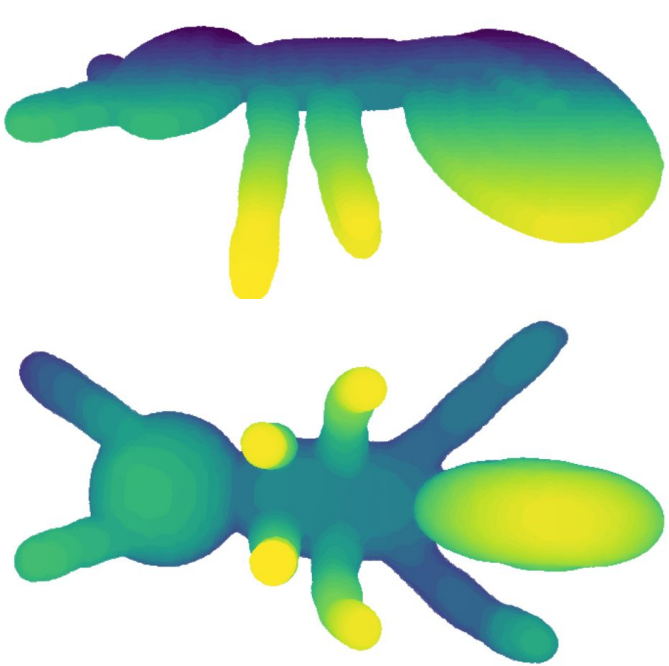


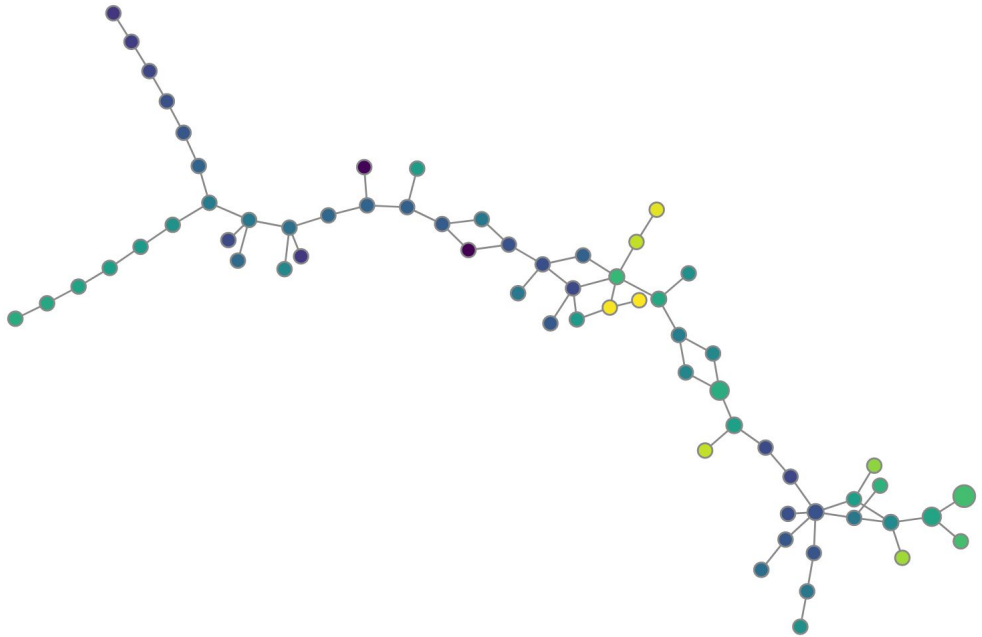
Graph constructed by mapper algorithm

Experiment 1 - Mapper graph



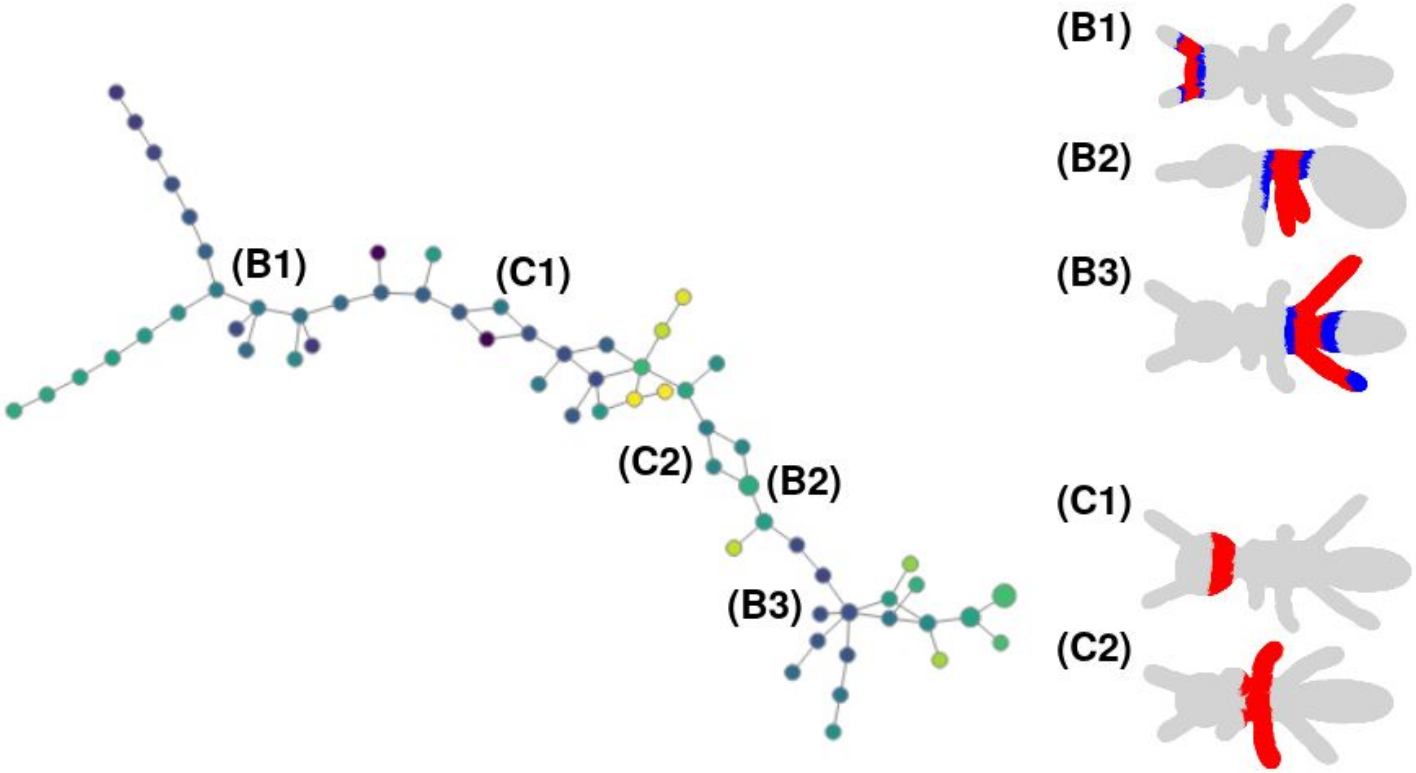
Experiment 2: 3D ant surface



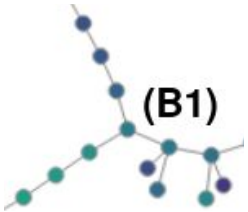


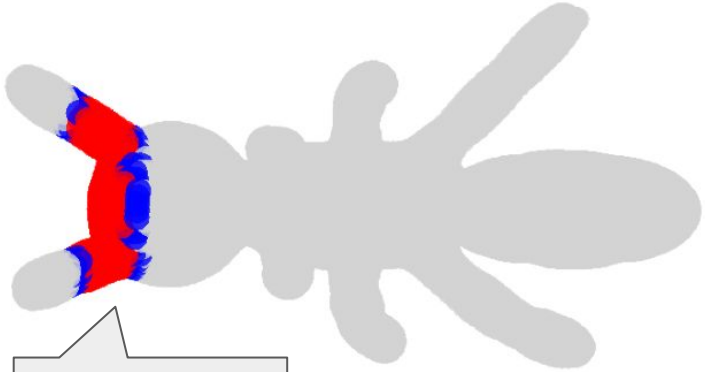
Xiaobai Chen, Aleksey Golovinskiy, and Thomas Funkhouser. 2009. A benchmark for 3d mesh segmentation. ACM Trans. Graph., 28, 3, Article 73, (July 2009), 12 pages. doi: 10.1145/1531326.1531379

Experiment 2 - Mapper graph

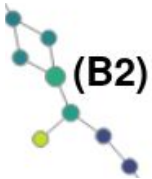


Experiment 2 - Branching points

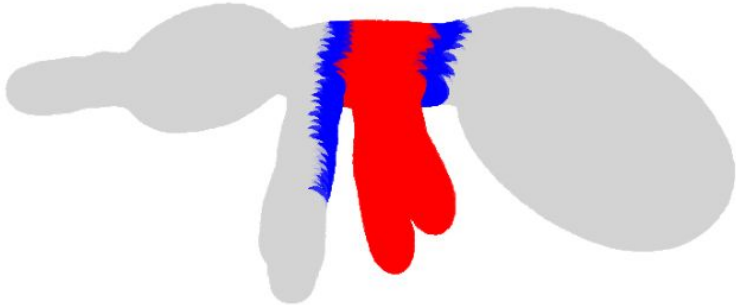




Recognized as a branching point of degree 3.

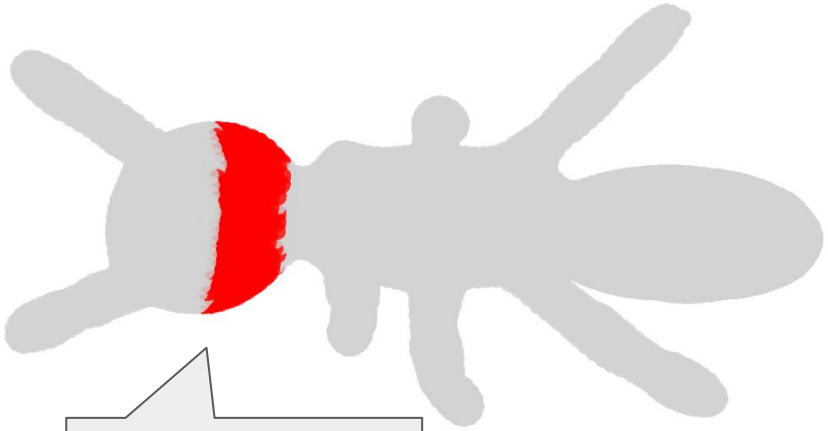


Only one relative cycle. Not a branching point.



Experiment 2 - Cycles

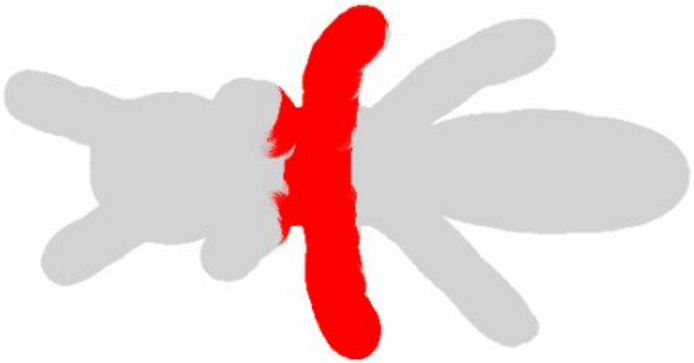
(C1)



Wraps around ant's hollow head. Recognized as a cycle.

(C2)

No significant cycle detected.



Future work

- **Approach**: new approach for testing the branching structure, better strategy for selection of boundary points.
- **Experiments:** more experiments on complex, high-dimensional point clouds (with known and unknown structure).

Thank you for your attention!