Ariana

Inverse problems in earth observation and cartography

Shape modelling via higher-order active contours and phase fields

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Overview

- Problem: entity extraction from (remote sensing) images.
  - Need for prior 'shape' knowledge.
- Modelling prior shape knowledge: higher-order active contours (HOACs).
  - Two examples: networks (roads) and circles (tree crowns).
  - Difficulties.
- Phase fields:
  - What are they and why use them?
  - Phase field HOACs.
  - Two examples: networks (roads) and circles (tree crowns).
- Future.

Problem: entity extraction

- Ubiquitous in image processing and computer vision:
  - Find in the image the region occupied by particular entities.
    - E.g. for remote sensing: road network, tree crowns,…

Problem formulation

- Calculate a MAP estimate of the region:

$$\hat{R} = \arg\max_{R} P(R|I;K)$$

$$P(R|I;K) \propto P(I|R;K)P(R|K)$$

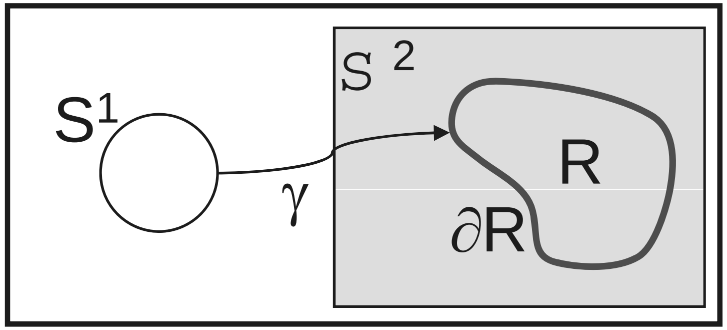
- In practice, minimize an energy:

$$\begin{aligned} E(R;I) &= -\ln P(I|R;K) - \ln P(R|K) \\ &= E_I(I;R) + E_G(R) + \text{const} \end{aligned}$$

$$\hat{R} = \arg\min_{R} E(R;I)$$

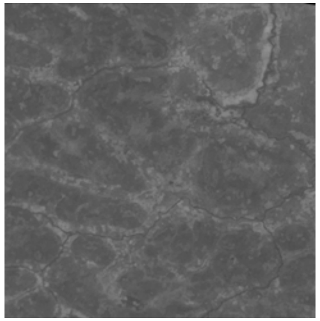
Building E_G: active contours

- A region is represented by its boundary, $\partial R = [\gamma]$, the 'contour'.
- Standard prior energies:
  - Length of ∂R and area of R.
  - Single integrals over the region boundary:
    - Short range dependencies.
    - Boundary smoothness.
  - Insufficient prior knowledge.



Difficulties

- (Remote sensing) images are complex.
- Regions of interest distinguished by their shape.
  - But topology can be non-trivial, and unknown a priori.
- Strong prior information about the region needed, without constraining the topology.



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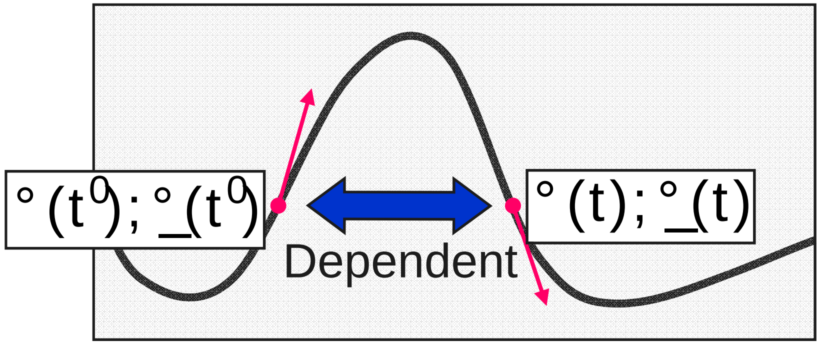
Building a better E_G: higher-order active contours

- Introduce prior knowledge via long-range dependencies between tuples of points.

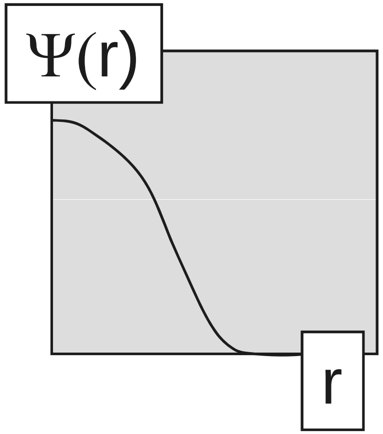


- How? Multiple integrals over the contour.
  - E.g. Euclidean invariant two-point term:

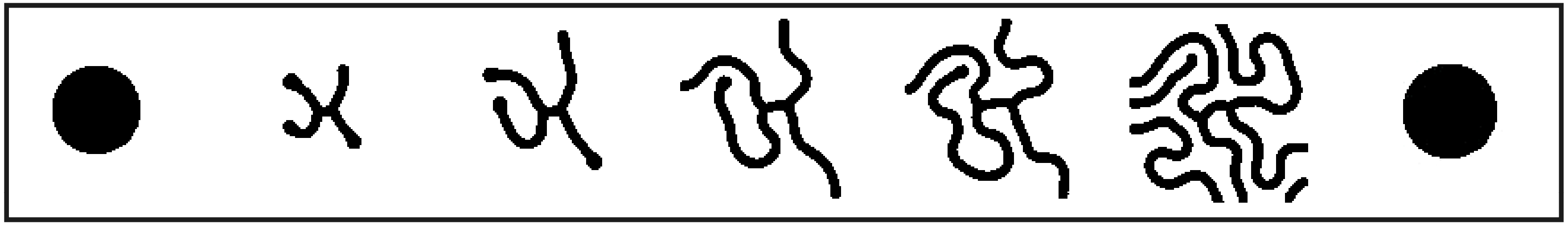
$$E(\gamma) = -\iint dt\, dt'\, \dot{\gamma}(t) \cdot \dot{\gamma}(t')\, \Psi(|\gamma(t) - \gamma(t')|)$$

Prior for networks

$$E_G(R) = \lambda L(\partial R) + \alpha A(R) - \frac{\beta}{2} \iint dt\, dt'\, \dot{\gamma}(t) \cdot \dot{\gamma}(t')\, \Psi(|\gamma(t) - \gamma(t')|)$$

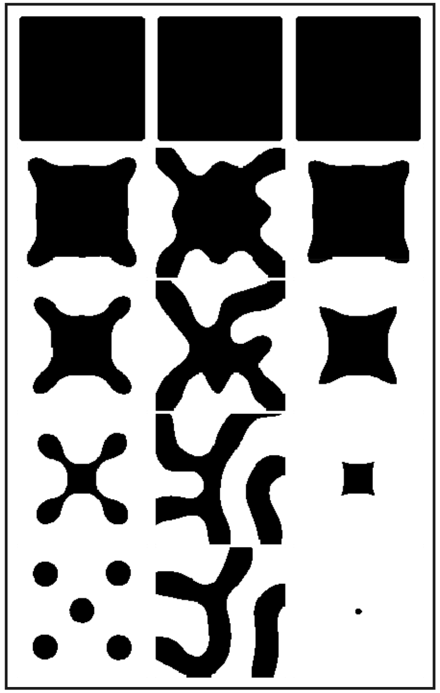


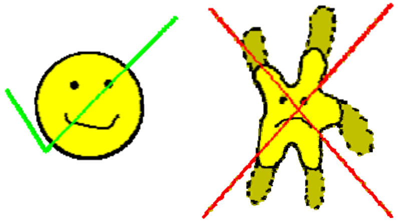
- Gradient descent with this energy.
  - A perturbed circle evolves towards a structure composed of arms joining at junctions.



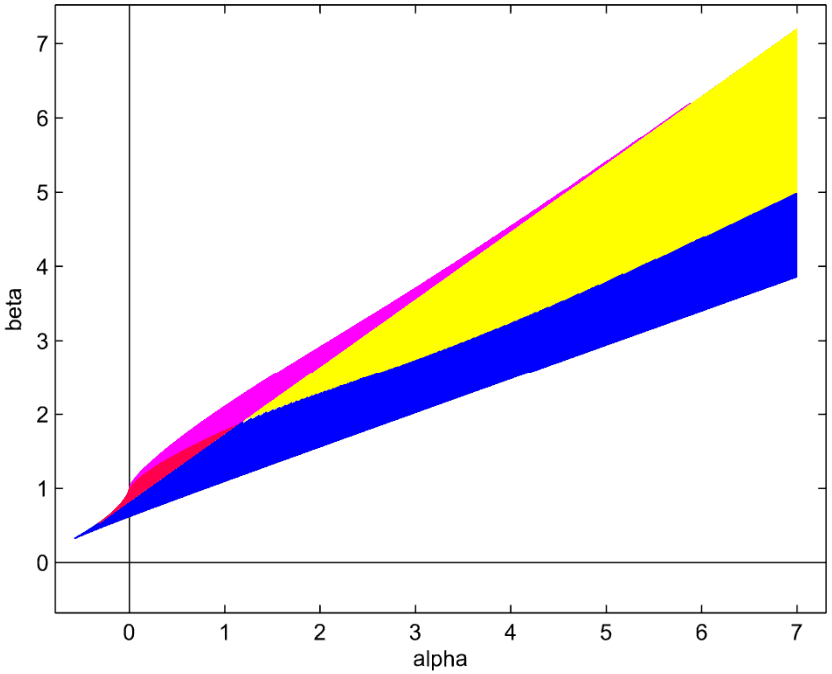
Prior for a gas of circles

- The same energy E_G can model a 'gas of circles' for certain parameter ranges.
  - Which ranges?
- A circle should be a stable configuration of the energy (local minimum).
  - Stability analysis.

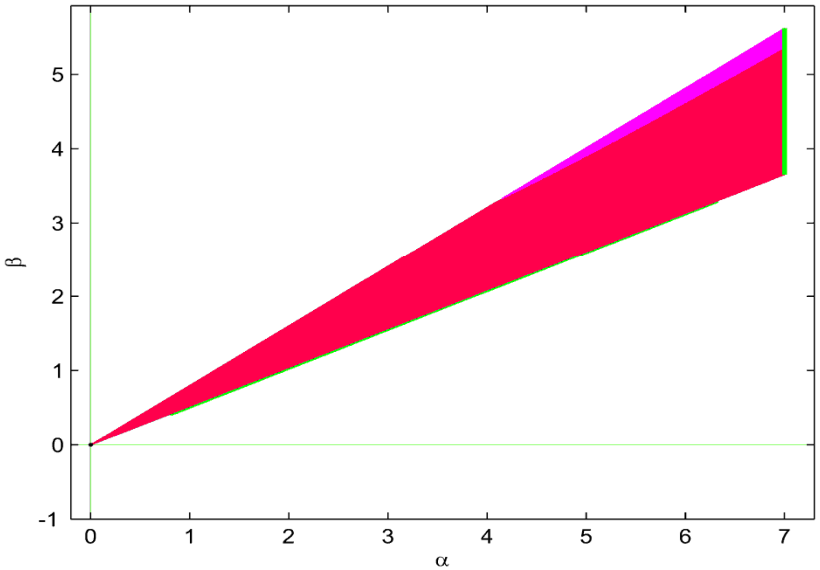




Phase diagrams



Circle



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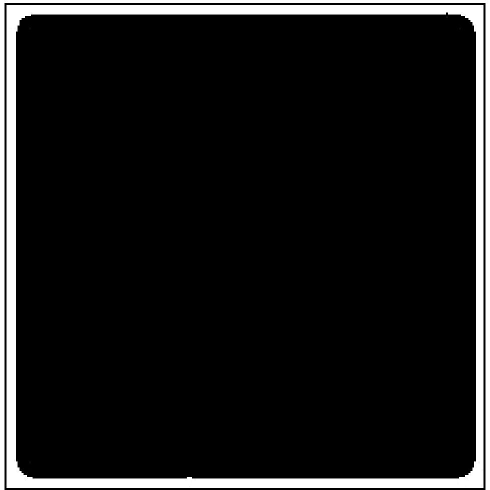
Optimization problem

- Minimize $E(R, I) = E_I(I, R) + E_G(R)$.
- Algorithm: gradient descent using distance function level sets.
- But the gradient of the HOAC term is non-local, and requires:
  - The extraction of the contour;
  - Many integrations around the contour;
  - Velocity extension.

Example I: HOAC results



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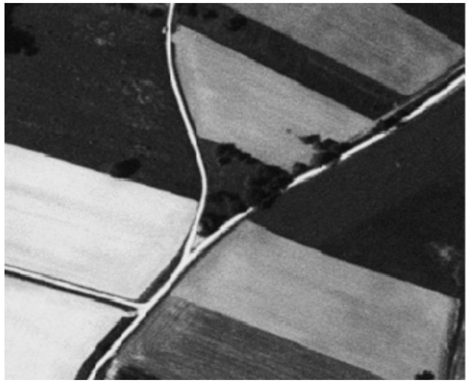

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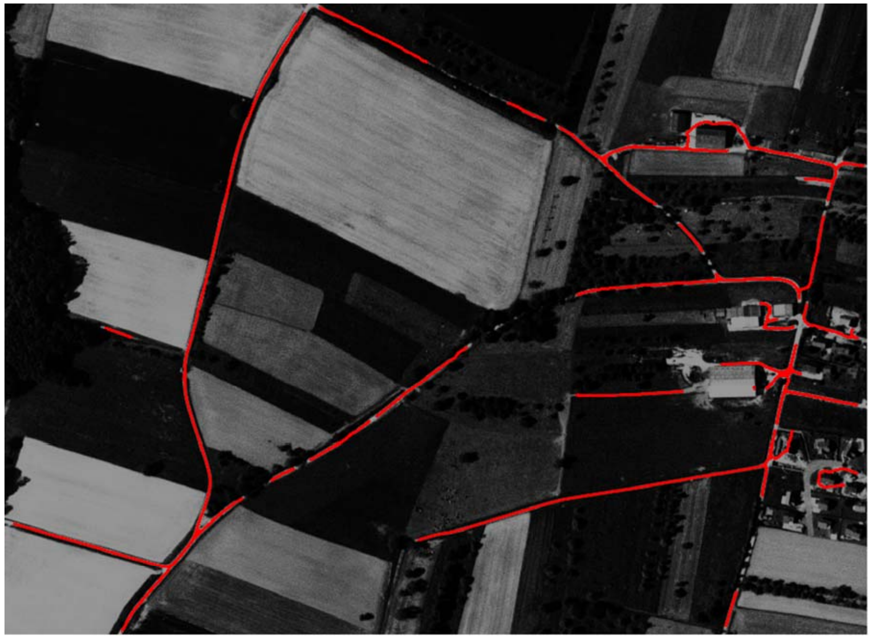
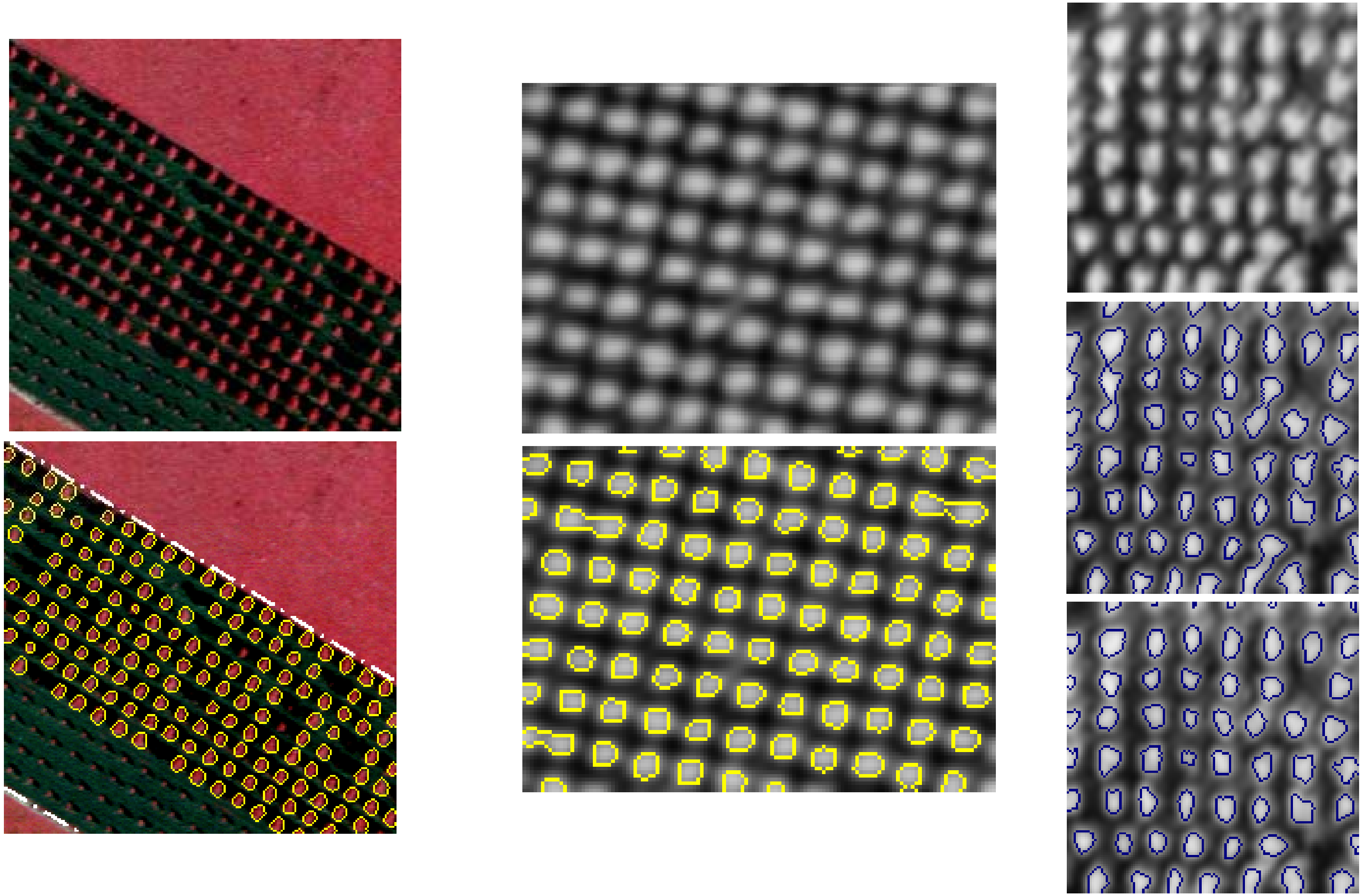
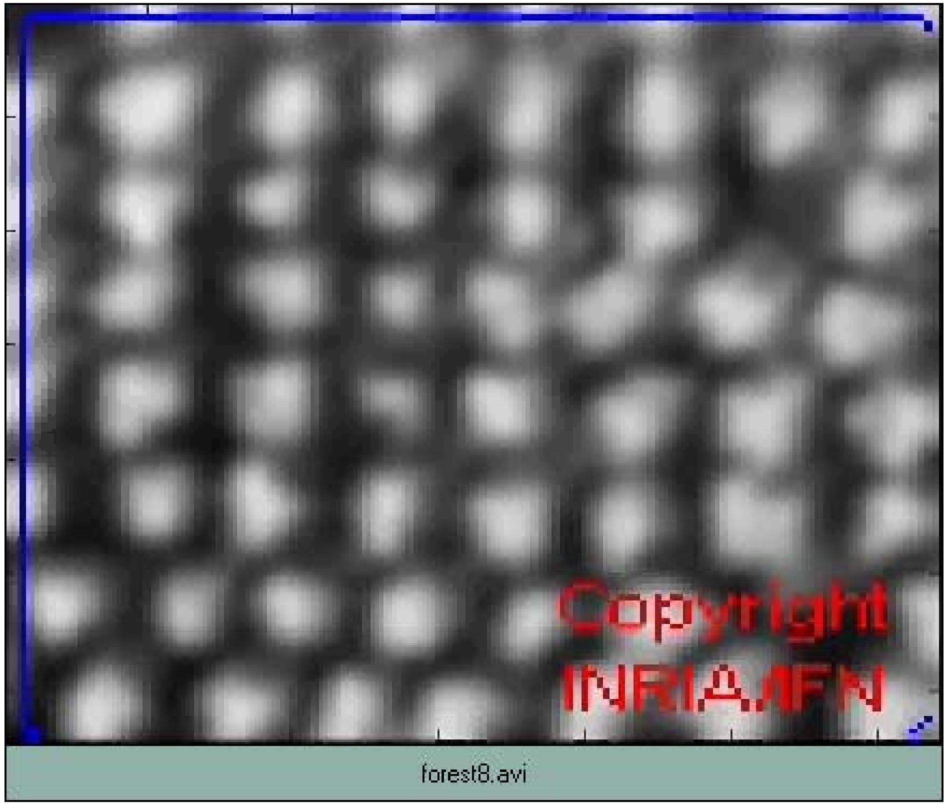
Example I: HOAC results



Example II: HOAC results



Example II: HOAC results



Problems with HOACs

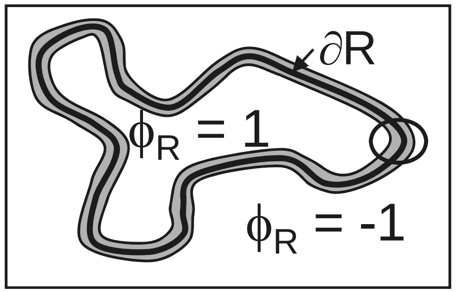
- Modelling:
  - Space of regions is complicated to express in the contour representation.
  - Probabilistic formulation is difficult.
    - Parameter and model learning are hampered.
- Algorithm:
  - Not enough topological freedom.
  - Gradient descent is complex to implement for higher-order terms.
  - Slow.
- Solution: 'phase fields'.

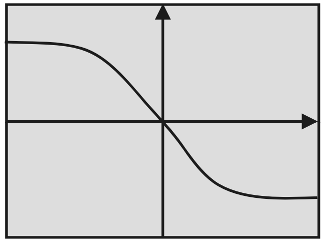
Phase fields

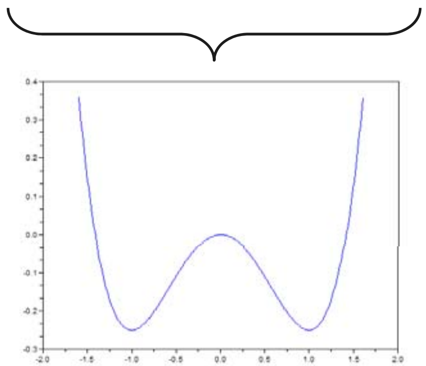
- Phase fields are a level set representation ($\zeta_z(\phi) = \{x : \phi(x) > z\}$), but the functions, ϕ, are unconstrained.
- How do we know we are modelling regions?

$$E_0(\phi) = \int_\Omega d^2x \left\{ \frac{D}{2} \nabla\phi \cdot \nabla\phi + \lambda\left(\frac{1}{4}\phi^4 - \frac{1}{2}\phi^2\right) \right\}$$

$$\phi_R = \arg \min_{\phi : \zeta(\phi) = R} E_0(R)$$







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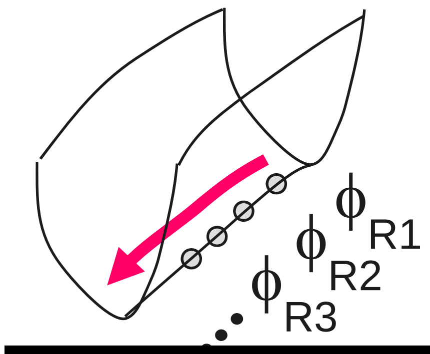
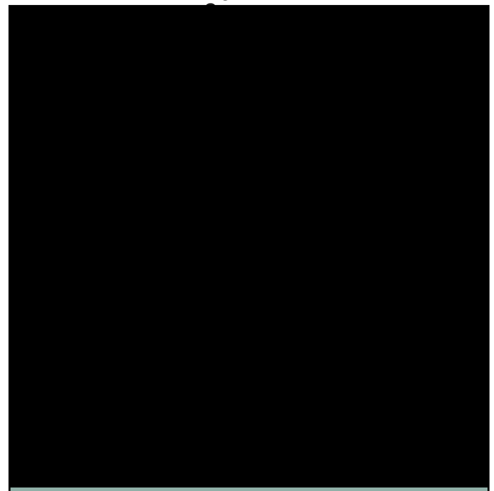
Relation to active contours

- One can show that

$$E_0(Á_R)\ '\ \ ¸_C\, L(@R)$$



- ϕ_R is a minimum for fixed R.
  - Thus gradient descent with E_0 mimics gradient descent with L: 'valley following'.



curveshortening.avi

- Can also add odd potential term to mimic:

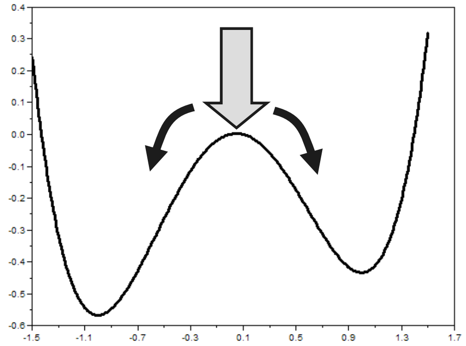
$$E_0(Á_R)\ '\ \ ¸_C\, L(@R) + ®_C\, A(R)$$

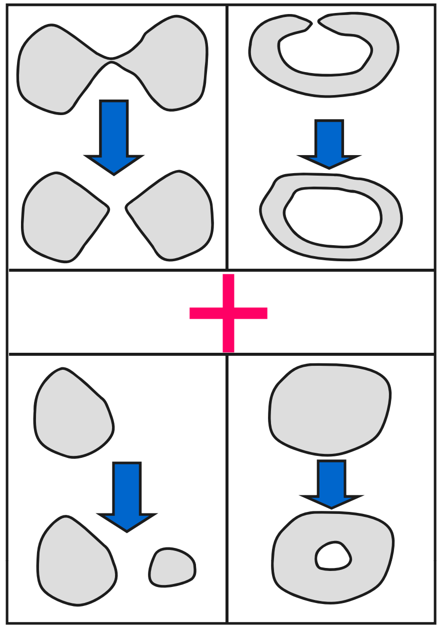
Why use them?

- Complex topologies are easily represented.
- Representation space is linear:
  - ϕ can be expressed, e.g., in wavelet basis for multiscale analysis of shape.
  - Probabilistic formulation (relatively) simple.
- Gradient descent is based solely on the PDE arising from the energy functional:
  - No reinitialization or ad hoc regularization.
  - Implementation is simple and the algorithm is fast.

Why use them?

- Neutral initialization:
  - No initial region.
  - No bias towards “interior” or “exterior”.
- Greater topological freedom:
  - Can change number of connected components and number of handles without splitting or wrapping.





How to write HOACs as phase fields?

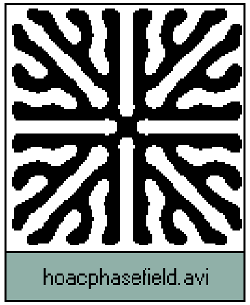
- Use that $\nabla \phi_R$ is zero except near ∂R, where it is proportional to the normal vector.

$$E_Q(R) = -\frac{\beta_C}{2} \iint dt\, dt'\, \dot{\gamma}(t) \cdot G_C(\gamma(t), \gamma(t')) \cdot \dot{\gamma}(t')$$

$$E_{NL}(\phi) = -\frac{\beta}{2} \iint_{\Omega^2} d^2x\, d^2x'\, \nabla\phi(x) \cdot G(x, x') \cdot \nabla\phi(x')$$

- One can show that

$$E_{NL}(\phi_R; \beta; G) \simeq E_Q(R; \beta_C; G)$$



Phase fields : likelihood energies E_l

- $\nabla\phi$: normal vector to the contour.
- $\nabla\phi\cdot\nabla\phi$: boundary indicator.
- $(1 \pm \phi)/2$: characteristic function of the region (+) or its complement (-).
- Using these elements, one can construct the equivalents of active contour and HOAC likelihoods.

Optimization problem

- Minimize

$$
\begin{aligned}
E(R;I) &= E_I(I;R) + E_G(R) \\
&= E_I(I;R) + E_0(R) + E_{NL}(R)
\end{aligned}
$$

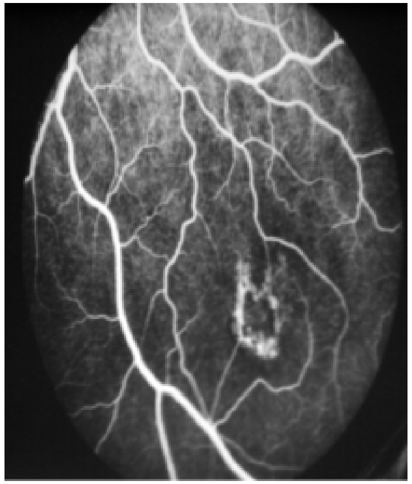
- Algorithm: gradient descent, but…

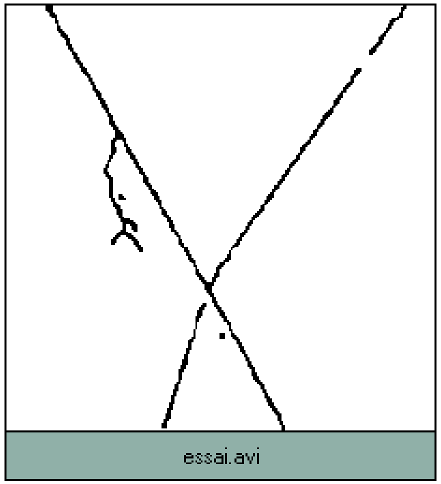
Major advantage of phase fields for HOAC energies

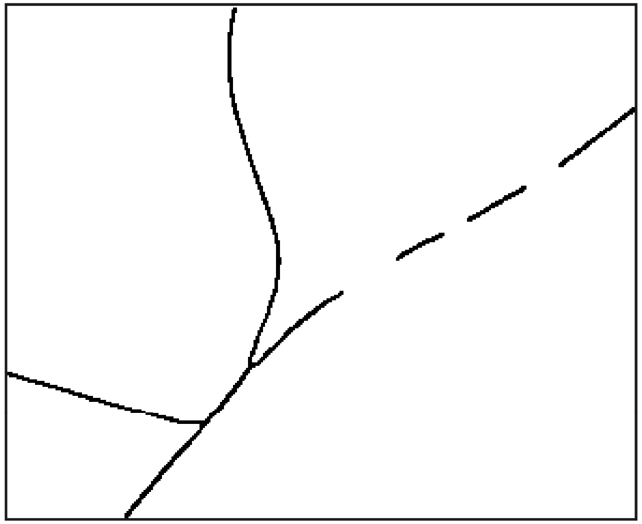
- Whereas, due to the multiple integrals, HOAC terms require
  - Contour extraction, contour integrations, and force extension,
- Phase field HOACs require only a convolution:

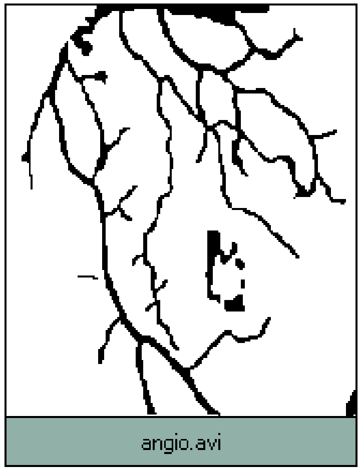
$$\frac{\delta E_{NL}}{\delta \phi(x)} = -\beta \int_{\Omega} d^2x' \, \nabla^2 G(x - x') \, \phi(x')$$

Phase field HOACs: results



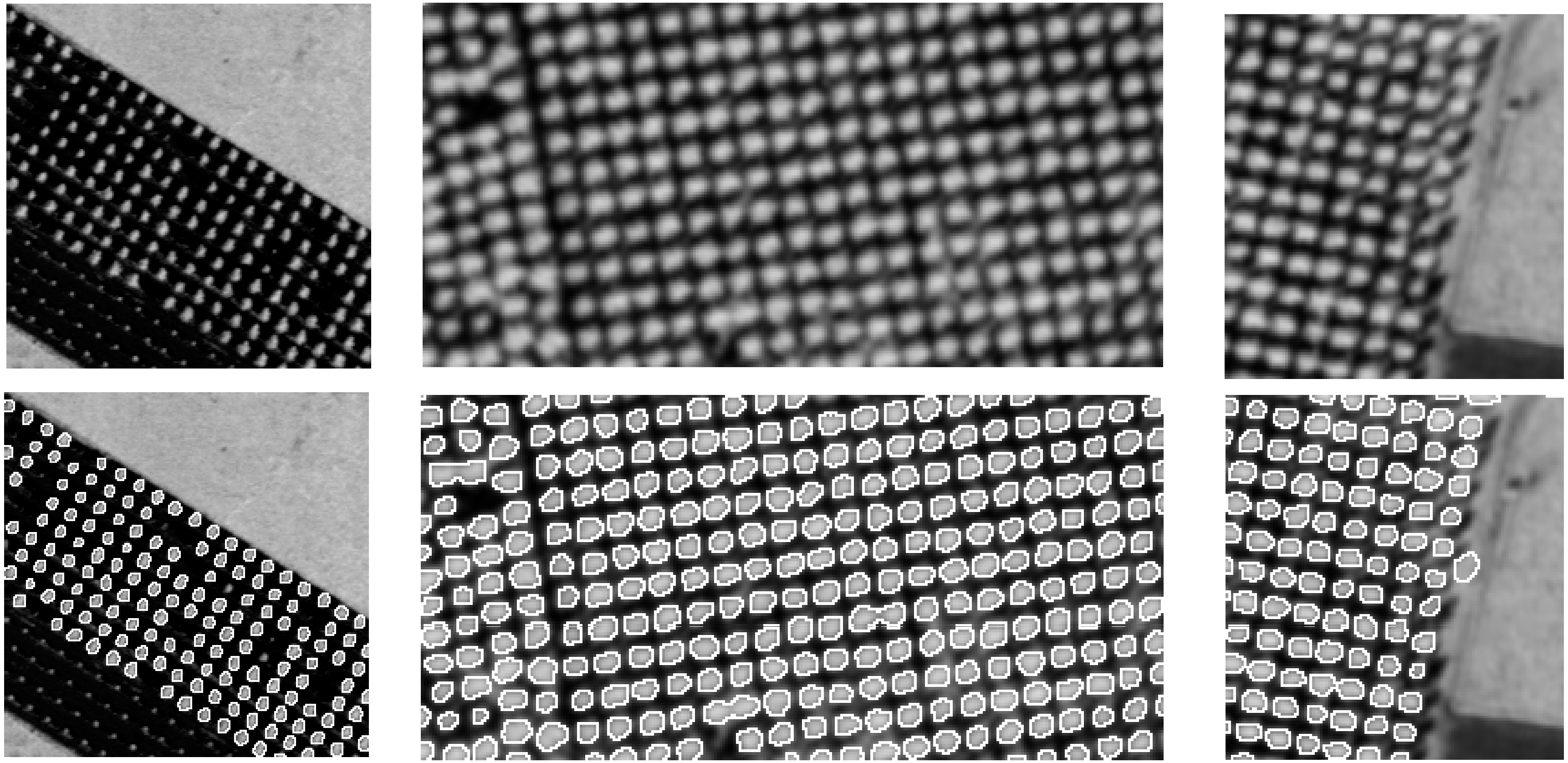




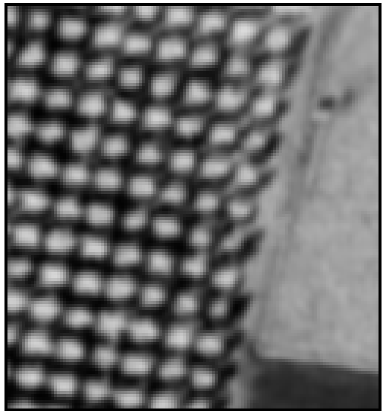


Phase field HOACs: VHR image results

Phase field HOACs: tree crown results



Phase field HOACs: results



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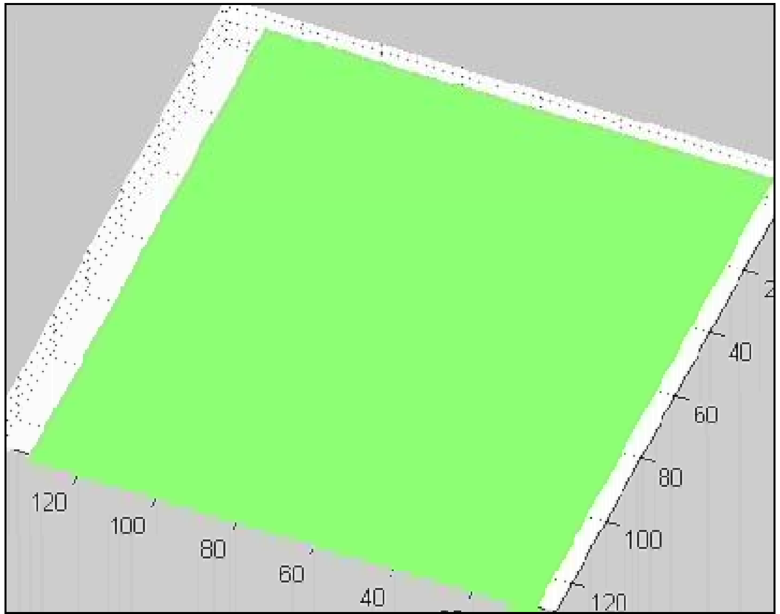


forest42_amin_indeo.avi

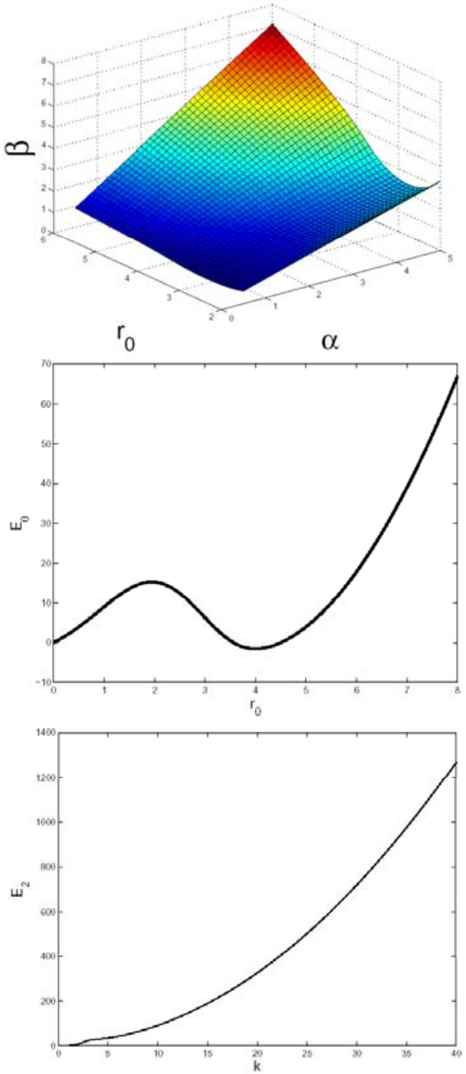
Future

- New prior models
  - Directed and rectilinear networks (rivers, big cities…).
  - ‘Gas of rectangles’;
  - Controlled perturbed circle.
- Multiscale models;
- New algorithms (multiscale, stochastic,…);
- Parameter estimation;
- Higher-dimensions;
- …

Thank you

Stability analysis

$$
\begin{aligned}
E_{C;g}(\gamma_0 + \delta\gamma) = &\left[\lambda_C 2\pi r_0 + \alpha_C \pi r_0^2 - \frac{\beta_C}{2} 2\pi \int_0^{2\pi} F_{00}\, dp \right] \\
&+ a_0 \left[\lambda_C 2\pi + \alpha_C 2\pi r_0 - \frac{\beta_C}{2} 4\pi \int_0^{2\pi} F_{10}\, dp \right] \\
&+ \sum_k |a_k|^2 \Bigg[\lambda_C \pi r_0 k^2 + \alpha_C \pi \\
&\quad - \frac{\beta_C}{2} 4\pi \Bigg\{ 2 \left(\int_0^{2\pi} F_{20}\, dp + \int_0^{2\pi} F_{21} e^{i r_0 k p}\, dp \right) \\
&\quad + k \left(2 i r_0 \int_0^{2\pi} F_{23} e^{i r_0 k p}\, dp \right) \\
&\quad + k^2 \left(r_0^2 \int_0^{2\pi} F_{24} e^{i r_0 k p}\, dp \right) \Bigg\} \Bigg]
\end{aligned}
$$



HOACs: E_I

- Linear term that favours large gradients normal to contour.

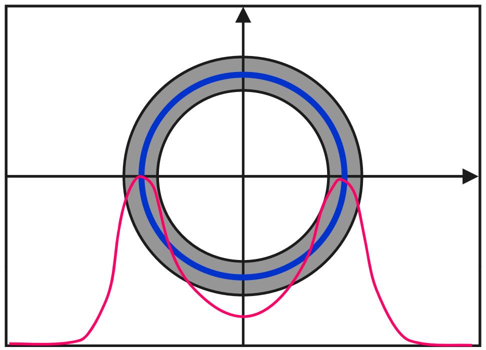
$$E_{I,1}(\gamma) = \int_{S^1} dt\, \dot{\gamma}(t) \times \nabla I(\gamma(t))$$

- Quadratic term that favours pairs of points with tangents and image gradients parallel or anti-parallel.

$$E_{I,2}(\gamma) = -\iint dt\, dt'\, \underbrace{\dot{\gamma}\cdot\dot{\gamma}'}_{\text{tangent vectors}}\, \overbrace{(\nabla I \cdot \nabla I')}^{\text{gradients}}\, \underbrace{\Psi(|\gamma(t) - \gamma(t')|)}_{\text{weighting function}}$$

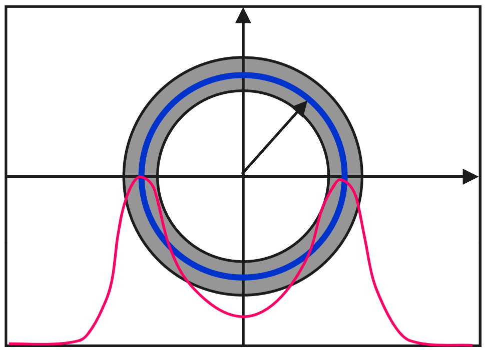
Nonlinearity in the model, not the representation

- Space of regions $\mathcal{S}$ is not a linear space.
- Possibility 1: use representation space isomorphic to $\mathcal{S}$ and put energy/probability on this space.
- Possibility 2: use larger linear space with probability peaked on nonlinear subset isomorphic to $\mathcal{S}$.



Nonlinearity in the model, not the representation: example

- Probability distribution P on S^2.
  - P pushes forward to Q on S^1.
  - If P is strongly peaked at r_0 then $Q(\theta)$ ($P(r_0, \theta)$.
  - Gradient descent with –ln(P) on S^2 mimics gradient descent with -ln(Q) on S^1 ('valley following').



$$P(r;\mu) = d^2x\, e^{i\, E(r;\mu)}$$

$$£ \exp ¡ \left(\frac{r^4}{4} ¡ \frac{r^2}{2}\right)$$

Turing stability

- To avoid decay of interior and exterior, we require stability of functions $\phi_{\pm} = \pm 1$:
  - $\delta^2 E/\delta\phi^2$ must be positive definite ($E = E_0 + E_{NL}$).
- For prior terms, this is diagonal in the Fourier basis:

$$\frac{\delta^2 E}{\delta\phi(k')\delta\phi(k)}(\phi_{\pm}) = \delta(k; k')\left\{ k^2(D - \beta\hat{G}(k)) + 2(\lambda \mp \alpha) \right\}$$

- Gives condition on parameters.
- Better result would be existence and uniqueness of ϕ_R for 'any' R.