Disorder Inequality: A Combinatorial Approach to Nearest Neighbor Search

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Nearest Neighbors: an Example

Input: Set of objects

Task: Preprocess it





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Most similar

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Nearest Neighbors

From computational perspective almost all algorithmic problems in the Web represent some form of nearest neighbor problem:

Search space: object domain $\mathbb{U}$, similarity function σ

Input: database $S = \{p_1, \ldots, p_n\} \subseteq \mathbb{U}$

Query: $q \in \mathbb{U}$

Task: find $\operatorname{argmax} \sigma(p_i, q)$

Contribution

- Combinatorial framework: new approach to data mining problems that does not require triangle inequality
- New algorithms for nearest neighbor search
- Experiments
- Tutorial, website

Outline

1. Motivation
2. Combinatorial Framework
3. New Algorithms
4. Directions for Further Research

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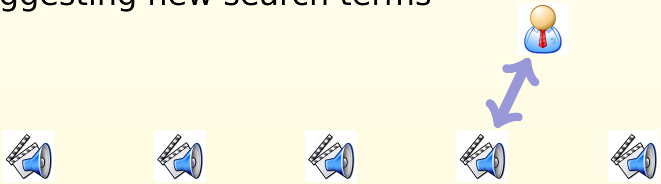
Motivation

Similarity Search for the Web

- Recommendations
- Personalized news aggregation
- Ad targeting
- “Best match” search
  Resume, job, BF/GF, car, apartment
- Co-occurrence similarity
  Suggesting new search terms



Nearest Neighbors: Prior Work

Sphere Rectangle Tree Orchard's Algorithm k-d-B tree Geometric near-neighbor access tree Excluded middle vantage point forest mvp-tree Fixed-height fixed-queries tree AESA Vantage-point tree LAESA R*-tree Burkhard-Keller tree BBD tree Navigating Nets Voronoi tree Balanced aspect ratio tree Metric tree vp^s-tree M-tree Locality-Sensitive Hashing SS-tree R-tree Spatial approximation tree Multi-vantage point tree Bisector tree mb-tree Cover tree Hybrid tree Generalized hyperplane tree Slim tree Spill Tree Fixed queries tree X-tree k-d tree Balltree Quadtree Octree Post-office tree

Challenge: Separation Effect

In theory:

Triangle inequality

Doubling dimension is $o(\log n)$

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Typical web dataset has separation effect

$$\text{For almost all } i, j: \quad 1/2 \le d(p_i, p_j) \le 1$$

Challenge: Separation Effect

In theory:

- Triangle inequality
- Doubling dimension is $o(\log n)$

Typical web dataset has separation effect

$$\text{For almost all } i, j: \quad 1/2 \leq d(p_i, p_j) \leq 1$$

Classic methods fail:

- In general metric space exact problem is intractable
- Branch and bound algorithms visit every object
- Doubling dimension is at least $\log n/2$

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Combinatorial Framework

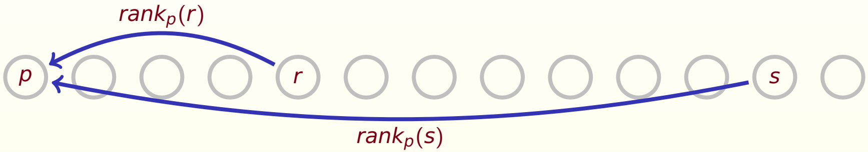
Comparison Oracle

- Dataset $p_1, \ldots, p_n$
- Objects and distance (or similarity) function are NOT given
- Instead, there is a comparison oracle answering queries of the form:

Who is closer to A: B or C?

Disorder Inequality

Sort all objects by their similarity to p:



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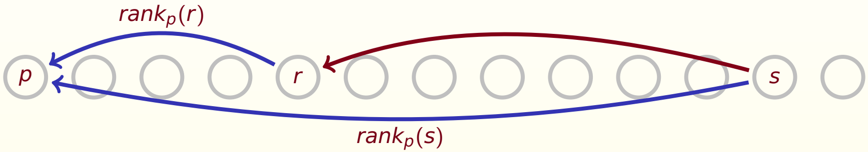
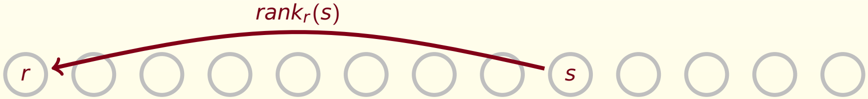


Then by similarity to r:



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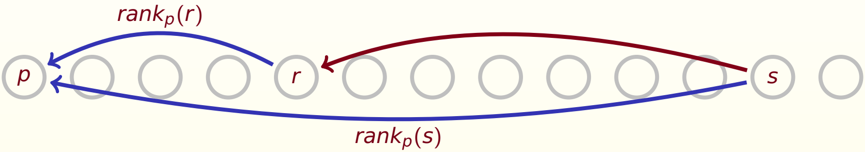

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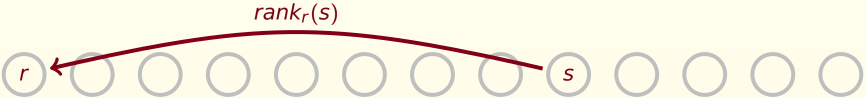

Dataset has **disorder** D if

$$\forall p, r, s: \quad rank_r(s) \leq D(rank_p(r) + rank_p(s))$$

Combinatorial Framework

$$=$$

Comparison oracle

Who is closer to A: B or C?

$$+$$

Disorder inequality

$$rank_r(s) \leq D(rank_p(r) + rank_p(s))$$

Combinatorial Framework: FAQ

- Disorder of a metric space? Disorder of $\mathbb{R}^k$?
- In what cases disorder is relatively small?
- Experimental values of D for some practical datasets?
- Disorder constant vs. other concepts of intrinsic dimension?

Combinatorial Framework: Pro & Contra

Advantages:

- Does not require triangle inequality for distances
- Applicable to any data model and any similarity function
- Require only comparative training information
- Sensitive to “local density” of a dataset

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Limitation: worst-case form of disorder inequality

Disorder vs. Others

- If expansion rate is c, disorder constant is at most c^2
- Doubling dimension and disorder dimension are incomparable
- Disorder inequality implies combinatorial form of “doubling effect”

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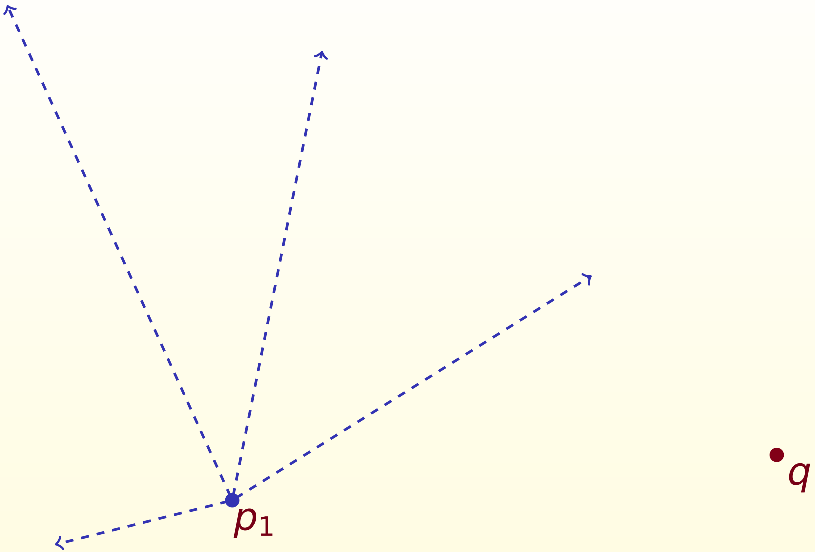
New Algorithms

Ranwalk Informally (1/2)



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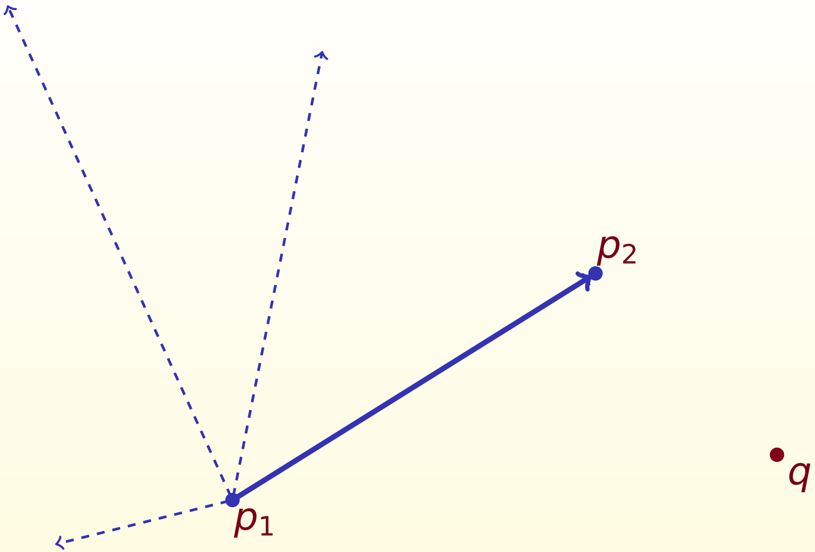

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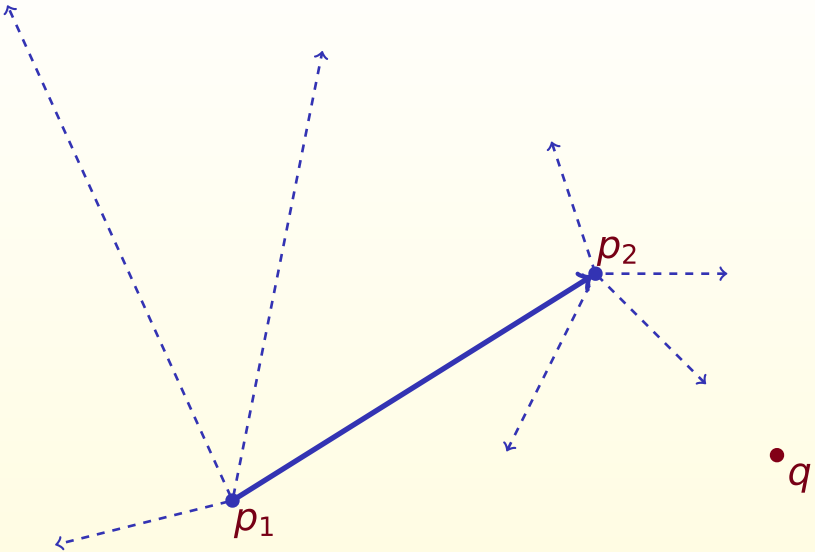

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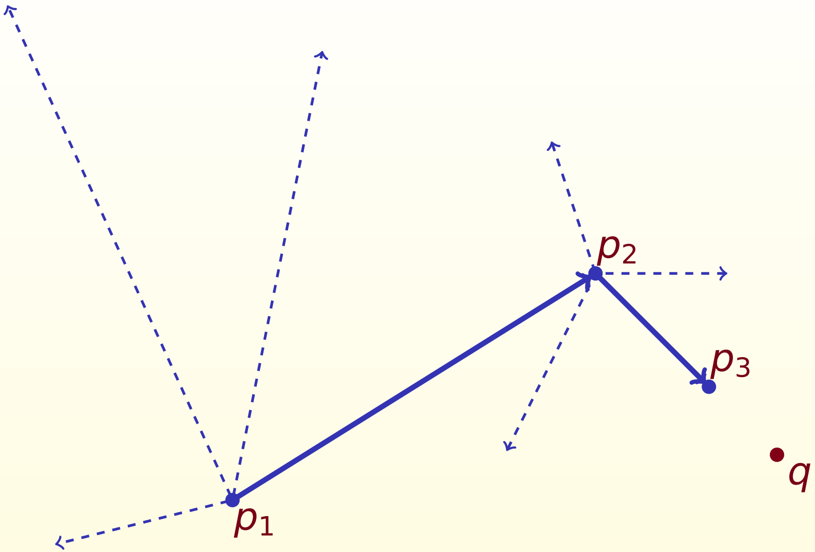

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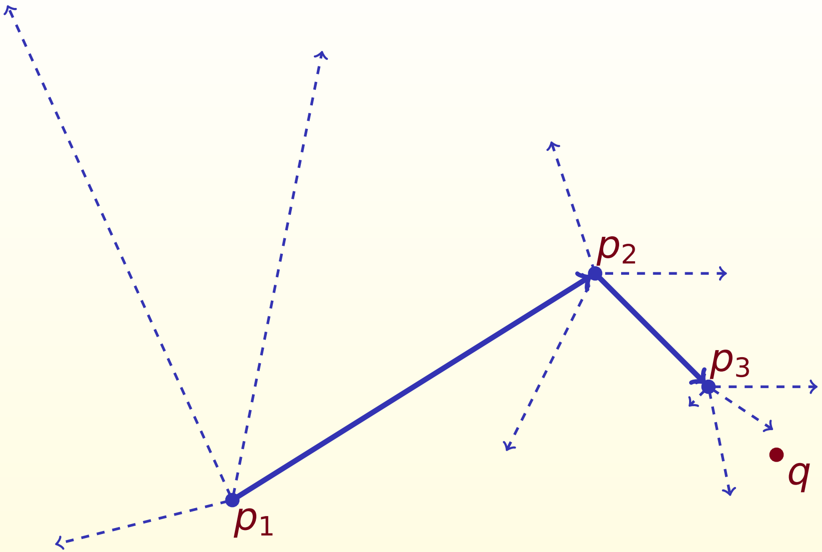

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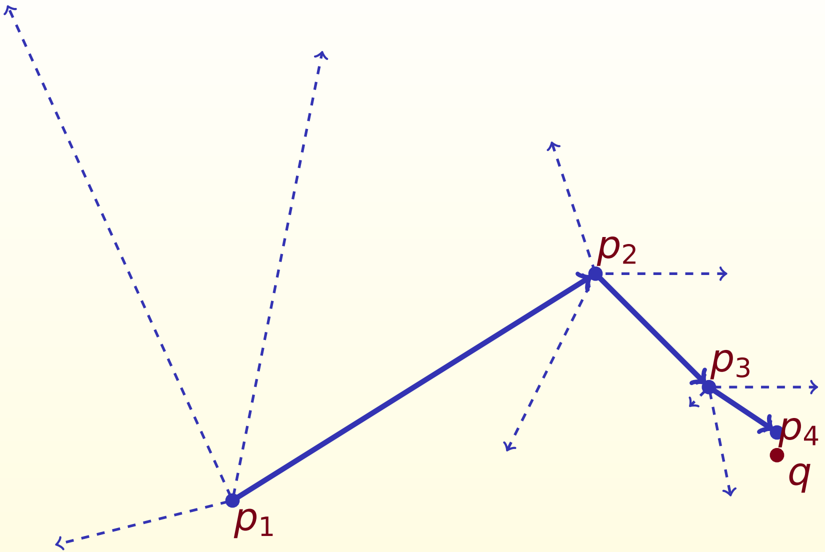

Ranwalk Informally (2/2)

Hierarchical greedy navigation:

1. Start at random city p_1

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3. Among all railway routes from p_2 choose the one going most closely to q, move there (p_3)

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4. Among all bus routes from p_3 choose the one going most closely to q, move there (p_4)

Ranwalk Informally (2/2)

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5. Repeat this $\log n$ times and return the final city

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Transport system: for level k choose c random arcs to $\frac{n}{2^k}$ neighborhood

Ranwalk Algorithm

Preprocessing:

- For every point p in database we sort all other points by their similarity to p

  Data structure: n lists of $n-1$ points each.

Query processing:

1. Step 0: choose a random point p_0 in the database.
2. From $k=1$ to $k=\log n$ do Step k: Choose $D' := 3D(\log\log n + 1)$ random points from $\min(n, \frac{3Dn}{2^k})$-neighborhood of p_{k-1}. Compute similarities of these points w.r.t. q and set p_k to be the most similar one.
3. If $\text{rank}_{p_{\log n}}(q) > D$ go to step 0, otherwise search the whole D^2-neighborhood of $p_{\log n}$ and return the point most similar to q as the final answer.

Analysis of Ranwalk

Assume that database points together with query point $S \cup \{q\}$ satisfy disorder inequality with constant D:

$$\text{rank}_x(y) \leq D(\text{rank}_z(x) + \text{rank}_z(y)).$$

Then Ranwalk algorithm always answers nearest neighbor queries correctly

Resources:

- Preprocessing space: $\mathcal{O}(n^2)$
- Preprocessing time: $\mathcal{O}(n^2 \log n)$
- Expected query time: $\mathcal{O}(D \log n \log \log n + D^2)$

Variation: Arwalk

Arwalk: moving all random choices to preprocessing

Assume that database points together with query point $S \cup \{q\}$ satisfy disorder inequality with constant D

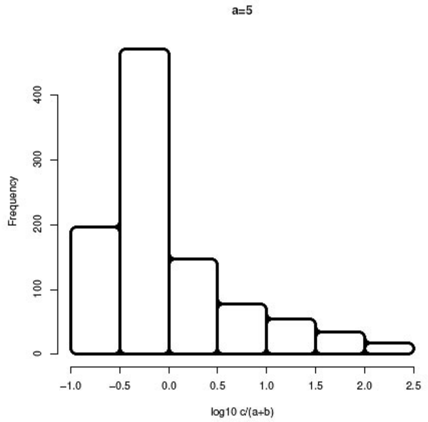
Then for any probability of error δ Arwalk algorithm answers nearest neighbor query within the following constraints:

- Preprocessing space: $\mathcal{O}(nD \log n(\log \log n + \log 1/\delta))$
- Preprocessing time: $\mathcal{O}(n^2 \log n)$
- Query time: $\mathcal{O}(D \log n(\log \log n + \log 1/\delta))$

Experiment

Reuters-RCV1 corpus:

1. Fix range R
2. Choose random $a, b \in [1..R]$
3. Choose random $p \in S$
4. Take r s.t. $rank_p(r) = a$
5. Take s s.t. $rank_r(s) = b$
6. Let $c = rank_p(s)$
7. Return $\frac{c}{a+b}$



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Directions for Further Research

Recent Results

Yury Lifshits and Shengyu Zhang
Similarity Search via Combinatorial Nets

- Better nearest neighbors:
  - Deterministic
  - Preprocessing $poly(D) n \log^2 n$ time
  - Price: search time increases to $D^4 \log n$
- Combinatorial algorithms for other problems:
  - Near duplicates
  - Navigation in a small world
  - Clustering

Future of Combinatorial Framework

- Other problems in combinatorial framework:
  - Low-distortion embeddings
  - Closest pairs
  - Community discovery
  - Linear arrangement
  - Distance labelling
  - Dimensionality reduction
- What if disorder inequality has exceptions, but holds in average?
- Insertions, deletions, changing metric
- Metric regularizations
- Experiments & implementation

Sponsored Links

http://yury.name

http://simsearch.yury.name

Tutorial, bibliography, people, links, open problems

Yury Lifshits and Shengyu Zhang
Similarity Search via Combinatorial Nets
http://yury.name/papers/lifshits2008similarity.pdf

Navin Goyal, Yury Lifshits, Hinrich Schütze
Disorder Inequality: A Combinatorial Approach to Nearest Neighbor Search
http://yury.name/papers/goyal2008disorder.pdf

Benjamin Hoffmann, Yury Lifshits, Dirk Novotka
Maximal Intersection Queries in Randomized Graph Models
http://yury.name/papers/hoffmann2007maximal.pdf

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- Combinatorial framework:
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Thanks for your attention!

Questions?