# Combinatorial Prediction Markets 

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## Buy Low, Sell High

Will price

## "Pays \$1 if <br> Obama wins"

 rise or fall?
(All are "gambling" "prediction" "info")

Today's Current Event Prices
65\% Obama next US president 15-22\% Bird Flu confirmed in US by 2009 6-10\% 9.0 Richter Earthquake by 2009 40-60\% Yahoo CEO Yang resigns by 2009
3-15\% US war act on N. Korea by 4/2009 20-21\% Bin Laden caught by 4/2009 40-46\% US or Israel air strike on Iran by 4/2009 28-30\% US max tax rate > 40\% in 2010 21-40\% Any nation drop Euro by 2011 20-28\% China war act on Taiwan by 2011 19-29\% Google Lunar Prize won by 2013

\% I.E.M. beat presidential election polls 451/596 (Berg et al '01)
me NFL, beat ave., rank 7 vs. 39 of 1947 (Pennock et al '04)

- Vs. Public Experts
a Racetrack odds beat weighed track experts (Figlewski '79)
- If anything, track odds weigh experts too much!
© OJ futures improve weather forecast (Roll '84)
6 Stocks beat Challenger panel (Maloney \& Mulherin '03)
mas demand markets beat experts (Spencer '04)

6. Econ stat markets beat experts 2/3 (Wolfers \& Zitzewitz '04)

- Vs. Private Experts
a HP market beat official forecast $6 / 8$ (Plott ${ }^{\prime} 00$ )
m Eli Lily markets beat official 6/9 (Servan-Schreiber '05)
microsoft project markets beat managers (Proebsting '05)
(xPree beat corp error, 3.5 vs $6.6 \%$


## T1 Markets



| Item | $\mathbf{1 9 8 8}$ | $\mathbf{1 9 9 2}$ | $\mathbf{1 9 9 6}$ | $\mathbf{2 0 0 0}$ | $\mathbf{2 0 0 4}$ | All |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| \# big <br> polls | 59 | 151 | 157 | 229 | 368 | 964 |
| Poll <br> "wins" | 25 | 43 | 21 | 56 | 110 | 255 |
| Market <br> "wins" | 34 | 108 | 136 | 173 | 258 | 709 |
| \% <br> Market | $58 \%$ | $72 \%$ | $87 \%$ | $76 \%$ | $70 \%$ | $74 \%$ |
| P-value | 0.148 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

"Prediction Market Accuracy in the Long Run" Joyce Berg, Forrest Nelson and Thomas Rietz, Jan. 2008.


Focus on a Security of Interestea

## The Fuss:

Sort Ma Analysts often use prices from various markets as indicators of potential events. Theuse of petroleum futures Iransac contract prices by analysts of the Middle East is a classic example: The Policy Analysis Market (PAM) refines this

```
russ mil:
``` approach by trading futures contracts that deal with underlying fundamentals of relevance to the Middle East. Market Initially, PAM will focus on the economic, civil, and military futures of Egypt, Jordan, Iran, Iraq, Israel, Saudi Arabia, Syria, and Turkey and the impact of U.S. involvement with each.
[Click here forea summary of PAM futures contracts]
\({ }^{1} \mathrm{Nec} 40 \mathrm{~T}\)

The contracts fraded on PAM will be based on objective data and observable events. These contracts will be valuable because traders who are registered with PAM will use their money to acquire contracts. A PAM trader TYum who believes that the price of a specific futures contract under-predicts the future status of the issue on which it is en res resented by PAM contra affect civil stability in the country and thei\# country's military. The trading process ap sam contracts. Such combinations represeni
ILu\$104
\(\qquad\)
\(\qquad\) The PAM trading interface presents AA context of the region using a trading lan be active and accessible 24/7 and shou
\(\qquad\)

Cummulative number of companies that have implemented an internal prediction market (lower bound estimate)

A NEW YORK TIMES BUSINESS BESTSELLER
"As entertaining and thought-provoking as The Tipping Point by Malcolm Gladwell. . . . The Wisdom of Crouds ranges far and wide." -The Beston Globe
THE WISDOM OF CROWDS JAMES SUROWIECKI



\section*{Internal Applications}
* Sales - HP, Google, Nokia, XPree, O'Reilly, Best Buy
* Deadlines - Siemens, Microsoft, Misys
* Pick Project - Qualcomm, GE, Lily, Pfizer, Intercontinental Hotels
- Unknown - Novartis, GSK, Motorola, ArcelorMittal, Corning, Dentsu, Masterfoods, Thomson, Yahoo, Abbott, Chrysler, Edmunds, InfoWorld, FritoLay, Erickson, IHG, NBC, HVG, RAND, SAIC, SCA, TNT, Cisco, General Mills, Swisscom

- Forecasting Institution Goal: \({ }^{4}\) Given same participants, resources, topic a Want most accurate institution forecasts
- Separate question: who let participate?
man limit who can trade in market
- Markets have low penalty for add fools and Hope: get more info from amateurs?

\section*{Advantages}
* Numerically precise
- Consistent across many issues
- Frequently updated
- Hard to manipulate
- Need not say who how expert when
- At least as accurate as alternatives

\section*{Ad Agency Decision Markets}


\section*{Corporate Applications}

E[ Revenue | Switch ad agency? ]
E[ Revenue | Raise price 10\%? ]
E[ Project done date \| Drop feature? ]
E[ Project done date | Add personnel? ]
E[ Stock price \| Fire CEO? ]
E[ Stock price | Acquire firm X? ]

\section*{Decision Market Requirements}
- Legal permission
- Outcome
a Measured
saggregate-enough
sa Linear-enough
a Conditional-enough
6 Decision
a Distinct options
a Important enough
as Enough influence
- Public credibility
- Traders
m Enough informed
a Decision-insiders
a Enough incentives
a Anonymity
- Prices

6 Intermediate-enough
an Can show enough

\section*{US President Decision Markets}


Expected level of debt increase


Expected level of troop in Iraq
Depending on the next president party


\section*{Remake CEO Oversight For \$1M!!}
- E[stock|fire CEO?] for all Fortune 500
- Subsidize cash trading, where legal
- Expect tons business press, CEOs look at Manipulating CEOs add liquidity
- Track firms follow advice, vs. not m Statistically signif. diff. in few years
- Sue boards that ignore advice w/o reason
a Shy boards then defer to market advice!!

\section*{Combo Betting \\ \begin{tabular}{|c|c|c|}
\hline Win & Place & Show \\
\hline Not & Not & Not \\
\hline
\end{tabular}


\section*{Sport Finals Tickets}
\begin{tabular}{|l|l|l|l|l|l|l|l|l|l|}
\hline \begin{tabular}{l} 
UEFA \\
EURO \\
2008
\end{tabular} & Austria & Croatia & Czech & Germany & Poland & Portugal & Switzerl. & Turkey \\
\hline France & & & & & & & & \\
\hline Greece & & \begin{tabular}{l} 
Greece v. \\
Croatia
\end{tabular} & \multicolumn{4}{|c|}{ Ticket if Greece in Finals } & \\
\hline Italy & & & & & & & & \\
\hline Netherl. & & & & & & & & \\
\hline Romania & & & & & & & & \\
\hline Russia & & & & & & & & \\
\hline Spain & & & & \begin{tabular}{c} 
Actual \\
Game
\end{tabular} & & & & \\
\hline Sweden & & & & & & & & \\
\hline
\end{tabular}

\section*{PAM Scenario}


\section*{Return to Form}

\section*{Execute a Trade}

If US military involvement in Saudi Arabia in \(3^{\text {rd }}\) Quarter 2003 is not between 105 and 125, this trade is null and void. Otherwise, if Iraq civil stability in \(4^{\text {th }}\) Quarter 2003 is below 85 , then I will receive \(\$ 1.43\), but if it is not below 85 , I will pay \(\$ 2.04\).

Abort trade if price has changed \(\square \quad\) Execute

\section*{Some Consensus Mechanisms}
* Competitive Forecasting - like survey a Formulas define consensus \& score
- Continuous Double Auction a make or take offers to buy or sell
* Call Auction - match accumulated offers
- Market maker - always small spread

\section*{Old Tech Meets New}

6 To gain info, elicit probs \(\mathbf{p}=\left\{p_{i}\right\}_{i}, E_{p}[x \mid A]\) (Verify state i later, N/Q = people/questions)
* Old tech ( \(\sim 1950+\) ): Proper Scoring Rules N/Q © 1: works well, N/Q © 1: hard to combine
* New tech ( \(\sim 1990+\) ): Info/Predict Markets N/Q © 1: works well, N/Q © 1: thin markets
* The best of both: Market Scoring Rules


\section*{Opinion Pool "Impossibile"}
- Task: pool \(T(A)\) from opinions \(p^{1}(A), p^{2}(A), \ldots\)
- Any 2 of IPP, MP, EB \(\mathcal{C}\) dictator \(\left(T=p^{d}\right)\) ! IPP = if A,B indep. in all \(p^{n}\), are indep. in T \(E B=\) commutes: pool, update on info MP = commutes: pool, coarsen states \(\boldsymbol{m} \cdot \mathrm{Cl}\) field)
(MP © \(\mathrm{T}=\boldsymbol{\bullet}_{\mathrm{n}=0} \mathrm{w}_{\mathrm{n}} \mathrm{p}^{\mathrm{n}}\), with \(\mathrm{w}_{\mathrm{n}}\) indep. of A )
* Really want pool via belief origin theory , General solution: let traders figure it out?


\section*{Quantal Response Modularity}
- Noisy choice: prob(act) \(\exp (\lambda *\) payoff \()\)
- When apply to a log MSR, get user reports (= new prices) independent of the last price:
\[
\begin{aligned}
& \text { rationality liquidity }
\end{aligned}
\]
- Simplifies inferences about beliefs from acts
- Ignores that harder to make complex changes

\section*{Laboratory Tests}
3) Joint work with John Ledyard (Caltech), Takashi Ishida (Net Exchange)
- Trained in 3var session, return for 8var
- Metric: Kulback-Leibler \(\Sigma_{i} q_{i} \log \left(p_{i} / q_{i}\right)\) distance from market prices to Bayesian beliefs given all group info

\section*{Environments: Goals, Training}
* Want in Environment:
a Many variables, few directly related
a Few people, each not see all variables
a Can compute rational group estimates
a Explainable, fast, neutral
* Training Environment:
\& 3 binary variables \(X, Y, Z, \quad 2^{3}=8\) combos
m \(\mathrm{P}(\mathrm{X}=0)=.3, \mathrm{P}(\mathrm{X}=\mathrm{Y})=.2, \mathrm{P}(\mathrm{Z}=1)=.5\)
a 3 people, see 10 cases of: \(A B, B C, A C\)
a Random map XYZ to ABC
(Actually:
\begin{tabular}{|cccc}
\hline Case & A & Z & B \\
\(\mathbf{1}\) & 1 & - & \(\mathbf{C}\) \\
2 & 1 & - & 0 \\
3 & 1 & - & 0 \\
4 & 1 & - & 0 \\
5 & 1 & - & 0 \\
6 & 1 & - & 1 \\
7 & 1 & - & 1 \\
8 & 1 & - & 0 \\
9 & 1 & - & 0 \\
10 & 0 & - & 0 \\
OS & & & \\
Sum: & 9 & - & 3 \\
Same & A & B & C \\
A & -- & -- & 4 \\
B & -- & -- & -- \\
C & -- & -- & -- \\
\hline
\end{tabular}

\section*{Experiment Environment}
- 8 binary vars: STUVWXYZ
- \(2^{8}=256\) combinations
- \(20 \%=P(S=0)=P(S=T)\) \(=P(\mathrm{~T}=\mathrm{U})=\mathrm{P}(\mathrm{U}=\mathrm{V})=\ldots\)
\(=P(X=Y)=P(Y=Z)\)
- 6 people, each see 10 cases: ABCD, EFGH, ABEF, CDGH, ACEG, BDFH
- random map STUVWXYZ to ABCDEFGH
(Really:
\begin{tabular}{|lllllllll|}
\hline Case & A & B & C & D & E & F & G & H \\
\(\mathbf{1}\) & 0 & 1 & 0 & 1 & - & - & - & - \\
2 & 1 & 0 & 0 & 1 & - & - & - & - \\
3 & 0 & 0 & 1 & 1 & - & - & - & - \\
4 & 1 & 0 & 1 & 1 & - & - & - & - \\
5 & 0 & 1 & 1 & 1 & - & - & - & - \\
\(\mathbf{6}\) & 1 & 0 & 0 & 1 & - & - & - & - \\
7 & 0 & 1 & 1 & 1 & - & - & - & - \\
\(\mathbf{8}\) & 1 & 0 & 0 & 1 & - & - & - & - \\
9 & 1 & 0 & 0 & 1 & - & - & - & - \\
10 & 1 & 0 & 0 & 1 & - & - & - & - \\
Sum & 6 & 3 & 4 & 10 & - & - & - & - \\
Same A & B & C & D & E & F & G & H \\
A & -- & 1 & 2 & 6 & -- & -- & -- & -- \\
B & -- & -- & 7 & 3 & -- & -- & -- & -- \\
C & -- & -- & -- & 4 & -- & -- & -- & -- \\
D & -- & -- & -- & -- & -- & -- & -- & -- \\
\(\cdots\) & & & & & & & & \\
\hline
\end{tabular}

\section*{MSR Info vs. Time - 3 Variables}


\section*{MSR Info vs. Time - 255 prices}


\section*{Combinatorial Lab Experiments}
- 7 indep. prices from 3 folks in 4 min.
a Simple Double Auction < Scoring Rule ~ Opinion Pool ~ Combinatorial Call < Market Scoring Rule
* 255 indep. prices from 6 folks in 4 min.
s Combinatorial Call ~ Simple Double Auction ~ Scoring Rule < Opinion Pool ~ Market Scoring Rule

\section*{Combo Market Maker Best of 5 Mechs}

3 subjects, 7 prices, 5 minutes


6 subjects, 256 prices, 5 minutes

\(O\)
Compute Tasks
- Represent p(s), \$(s,i)
- Add/settle var, Add/take \$
- Browse \(\mathrm{E}[\mathrm{x} \mid \mathrm{A}]\) \& \(\mathrm{E}[\$ \mid \mathrm{A}]\)
a \& history of changes
- For each \(\mathrm{E}[\mathrm{x} \mid \mathrm{A}]\), show max/min/indifferent \$ edits
- Allow edit of many \(\mathrm{E}[\mathrm{x} \mid \mathrm{A}]\)
m Update \(\$(\mathrm{~s}, \mathrm{i})=\mathrm{b}^{*} \log (\mathrm{p}(\mathrm{s}))\)
a Avoid money pump errors
A. Edit structure?

\section*{How Close Markov Nets?}
\(\checkmark\) Have no forseen error alg. \(*\) But can distribute computation?
\(\checkmark\) Ways to browse E[x|A]
\(\checkmark\) Can allow edit if vars in same clique
 : How support other edits?
- Need good \$(s) repr. to support:
\({ }_{3}\) For i take \(\$\), max edit, must find \(\min _{s} \$(s, i)\)
U Update alg without forseen min \$ error
* How efficiently store histories?
* How allow structure changes?

\title{
Typical Problems In Field Now
}
- Laws on gambling, insider trading
- "Moral" \& "Culture" concerns
- Not really want to know
- Hard to find precise related events
- Little participation for cheap
- Not enough events to validate, learn
* Awkward interfaces
\({ }^{3}\) Self-defeating prophecies
a Decision selection bias
s Price manipulation
m Inform enemies Bozos
a Share less info
: Combinatorics
m Moral hazard
an Alarm public
a Embezzle

\section*{Simple Manipulation Model}

Market maker
\(P=E[v \mid u+x+z] \quad \max _{z} E[z(v-P)+w P]\)
nimamsat
\(\max _{x, \sigma_{\varepsilon}^{2}}^{\text {Informed trader }} E[x(v-P) \mid v+\varepsilon, w+\delta]-c\left(\sigma_{\varepsilon}^{2}\right)\)
\(\begin{array}{lll}v \sim N\left(\bar{v}, \sigma_{v}^{2}\right) & w \sim N\left(\bar{w}, \sigma_{w}^{2}\right) & \begin{array}{c}\text { Noise trader }\end{array}\left(\bar{u}, \sigma_{u}^{2}\right) \\ \varepsilon \sim N\left(\bar{\varepsilon}, \sigma_{\varepsilon}^{2}\right) & \delta \sim N\left(\bar{\delta}, \sigma_{\delta}^{2}\right) & c^{\prime}<0, c^{\prime \prime}>0\end{array}\)
\(E\left[(v-P)^{2}\right]=\frac{\sigma_{v}^{2}}{2} \frac{\sigma_{v}^{2}+2 \sigma_{\varepsilon}^{2}}{\sigma_{v}^{2}+\sigma_{\varepsilon}^{2}}\)

\section*{\(\frac{\partial E\left[(v-P)^{2}\right]}{\partial\left[\sigma_{w}^{2} \text { or } \sigma_{u}^{2}\right]}<0\)}

\section*{Lab Data}

Average End of Period Prices


Hanson, Oprea, Porter JEBO, 2005
\begin{tabular}{|l|l|}
\(\cdots \circ \cdots\) & Non Manipulation \\
\(\longrightarrow-\) & Manipulation \\
\(\square \_\)True Value \\
\hline
\end{tabular}
- 12 subjects, value \(=0,40,100\)
- Each clue like "Not 100".
- 6 manipulators, get bonus for higher price
- Manipulators bid higher
- Others accept lower
- Prices no less accurate

Baseline

- 8 traders, Value \(=0,100\)
- Each Prob \((\) Clue \(=\mathrm{V})=2 / 3\)
- 4 manipulators, bonus for price to hidden target 0,100

Forecast Preferences

- 5 judges see prices, predict
- Manipulators bid toward target
- Prices and judges predictions no less accurate
R. Oprea, D. Porter, C. Hibbert, R. Hanson, D.Tila 2006

\section*{A Scaleable Implementation}
- Overlapping variable patches
- A simple MSR per patch
- If consistent, is Markov network
 a Var independent of rest given neighbors
- Allow trade if all vars in same patch
- Arbitrage overlapping patches a Sure to eventually agree, robust to gaming

\section*{Arbitraging Patches}
\[
A \quad B \quad X
\]


\section*{Arbitraging Patches Continued} A B X


\section*{But Arbitrage Is Not Modular}

1. Everyone agrees on prices
2. Expert on A gets new info, trades
3. Arbitrage updates all prices
4. Expert on H has no new info, but must trade to restore old info!

\section*{Democracy}

Law


\section*{Vote On Values But Bet On Beliefs}
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