Hierarchical Model-Based Reinforcement Learning: R-MAX + MAXQ

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Outline

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Introduction

Problem Learn behaviors in unknown environments

Criterion Minimize number of suboptimal actions taken

Idea 1 Model-Based Reinforcement Learning

- Probabilistic finite-time convergence
- Efficient use of sample data
- Robust exploration using model uncertainty

Idea 2 Hierarchical Reinforcement Learning

- Intuitive approach to scaling to large problems
- Decomposition of tasks into subtasks

Our Contribution

Integration of model-based and hierarchical RL for fully stochastic, finite problems

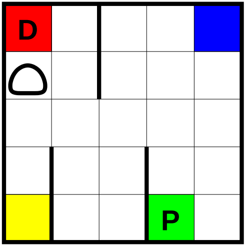
The Taxi Domain

State Variables

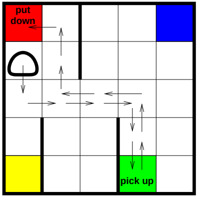
- *x* coordinate
- *y* coordinate
- Passenger location
  (at 1 of 4 landmarks or in the taxi)
- Destination location
  (at 1 of 4 landmarks)

Actions

North, South, East, West, PickUp, PutDown



The Taxi Hierarchy



Optimal policy

- Navigate to the passenger
- Pick up the passenger
- Navigate to the destination
- Put down the passenger

Composite actions

- Set of child actions A^i
- Set of terminal states $T^i \subseteq S$
- Goal rewards $\tilde{R}^i : T^i \to \mathbb{R}$

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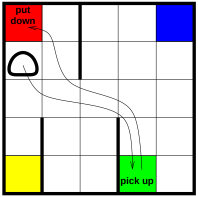

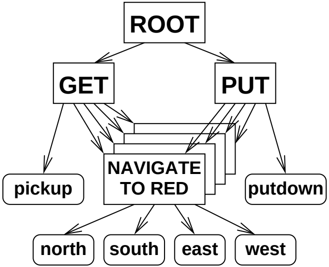

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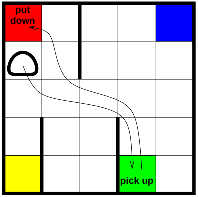

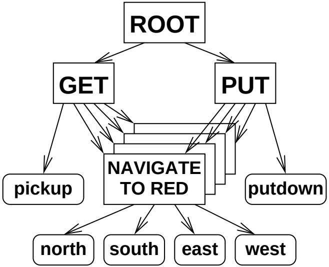

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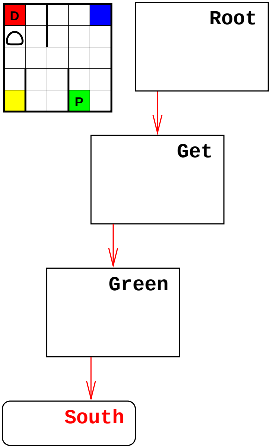
MAXQ Decomposition of the Value Function

Decompose value function

- $V^i(s) = \max_a Q^i(s, a)$
  Total expected reward (for action i)
- $Q^i(s, a) = V^a(s) + C^i(s, a)$
  Reward if i executes a first
- $C^i(s, a) = E_{k,s'}\left[\gamma^k V^i(s')\right]$
  Reward i expects after executing a



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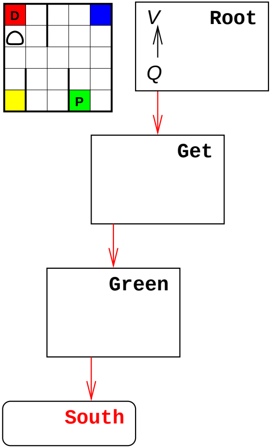
$V^{\text{Root}}($  $) \quad = \quad Q^{\text{Root}}($  $, \texttt{Get})$



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V^{\texttt{Root}}(\;) &= Q^{\texttt{Root}}(\;, \texttt{Get}) \\
&= V^{\texttt{Get}}(\;) + C^{\texttt{Root}}(\;, \texttt{Get})
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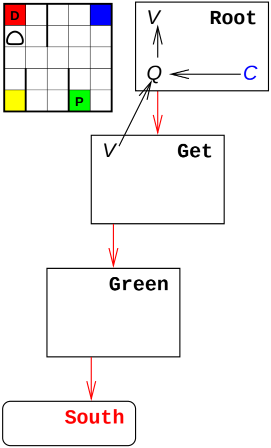
$$



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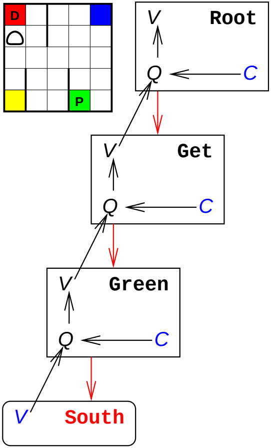


$$
\begin{aligned}
V^{\texttt{Root}}(\,) &= Q^{\texttt{Root}}(\,, \texttt{Get}) \\
&= V^{\texttt{Get}}(\,) + C^{\texttt{Root}}(\,, \texttt{Get}) \\
&= V^{\texttt{South}}(\,) + C^{\texttt{Green}}(\,, \texttt{South}) \\
&\quad + C^{\texttt{Get}}(\,, \texttt{Green}) + C^{\texttt{Root}}(\,, \texttt{Get})
\end{aligned}
$$

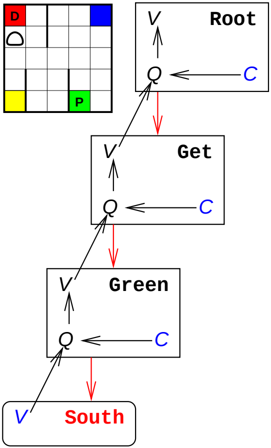
The MAXQ-Q Algorithm

Overview of MAXQ-Q

- Learn V for each primitive action
- Learn C for each composite action
- Use Q-learning-like update rules

Properties of MAXQ-Q

- Facilitates state abstraction:
  Different representation for each C, V
- Learning proceeds bottom-up
- Parameters tuned for each action
- Asymptotic convergence



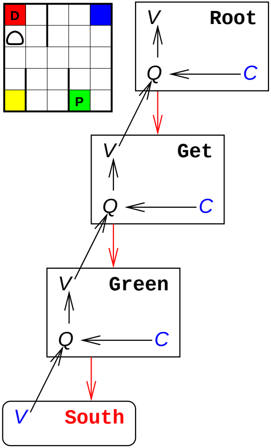
Model Decomposition

MAXQ Model Decomposition

1. Learn models of primitive actions
2. Plan using existing models
3. Compute abstract model
4. Apply induction

$$R^a(s) = R^{\pi^a(s)}(s) + \sum_{s' \in S \setminus T^a} P^{\pi^a(s)}(s, s') R^a(s')$$

$$P^a(s, x) = P^{\pi^a(s)}(s, x) + \sum_{s' \in S \setminus T^a} P^{\pi^a(s)}(s, s') P^a(s', x)$$



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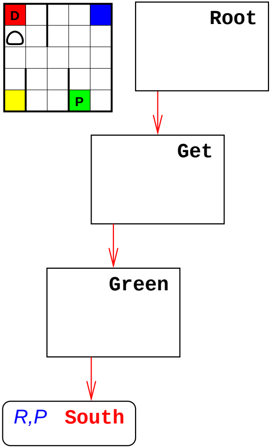

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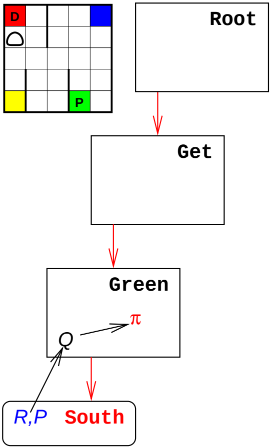

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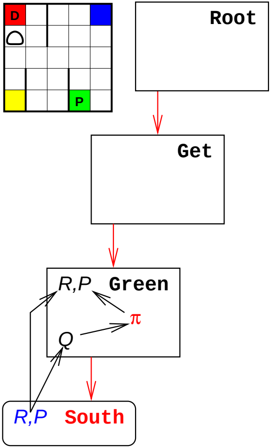

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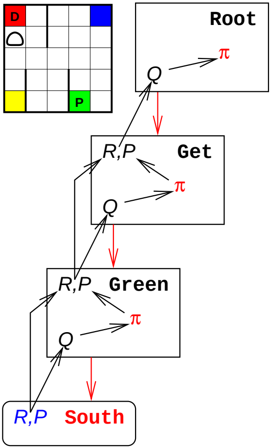

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1. Learning with Hierarchies of Models
   - Learning in Structured Environments
   - MAXQ Decomposition

2. The R-MAXQ Algorithm
   - R-MAX Exploration
   - Results

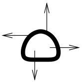
R-MAX Models of Primitive Actions

Maximum-likelihood estimation, given sufficient data

$$R^a(s) = \frac{\text{total reward}}{\text{\# of transitions}} \qquad P^a(s, s') = \frac{\text{\# of transitions to } s'}{\text{\# of transitions}}$$

Optimistic models, given insufficient data

$$R^a(s) = V^{\max} \qquad P^a(s, s') = 0$$



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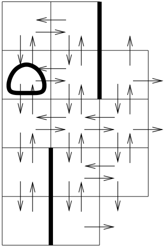

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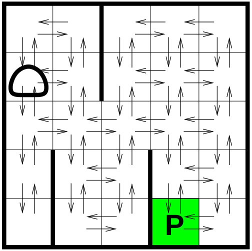

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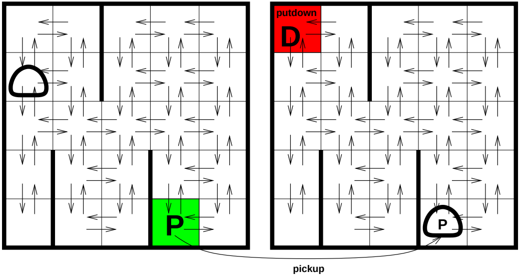
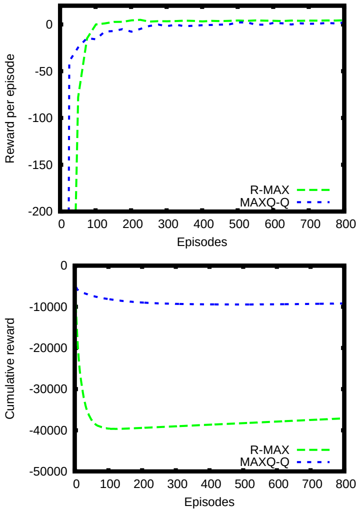


The R-MAX Algorithm

Procedure for each time step

1. Update model
2. Compute value function
3. Choose greedy action

- Thorough exploration due to initial optimism
- Very large negative rewards in exploratory episodes
- High-quality policy after initial exploration

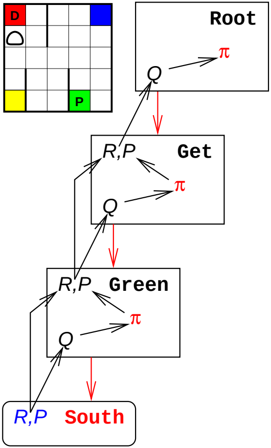


The R-MAXQ Algorithm

Procedure for each time step

1. Update R-MAX primitive models
2. Compute MAXQ composite models
3. Resume executing hierarchical policy

- Propagates optimism up hierarchy
- Memoizes models across time steps
- Employs prioritized sweeping



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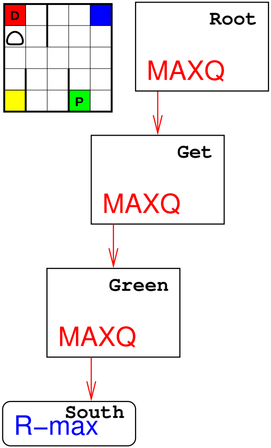

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Experimental Setup

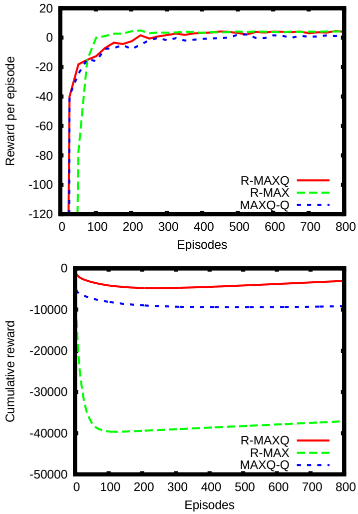
- Environment: stochastic Taxi
- MAXQ-Q
  - Replication of Dietterich's original algorithm
  - Boltzmann exploration
  - Parameters from Dietterich's implementation
- R-MAX primitive models
  - Each state-action optimistic until sample size $m = 5$
  - Planning with value iteration until $\epsilon = 0.001$
- State abstraction:

MAXQ-Q: All of Dietterich's abstractions

R-MAX: *Max-node irrelevance* for each primitive model
Example:
`South` ignores `Passenger` and `Destination`

R-MAXQ: Also *max-node irrelevance* for abstract models
Example: `Get` ignores `Destination`

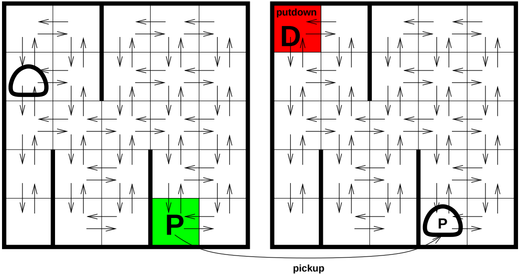
Empirical Results



- R-MAXQ learning curve dominates MAXQ-Q curve
- R-MAXQ converges to same asymptote as R-MAX
- R-MAXQ avoids most of the costly exploration of R-MAX

Eager Exploration Versus Lazy Exploration

- R-MAX experiments with `pickup` and `putdown` at all 50 states reachable from the initial state.
- R-MAXQ attempts `pickup` (`putdown`) at only 5 (4) reachable states in `Get` (`Put`).
- R-MAXQ never attempts `putdown` outside the four landmark locations.



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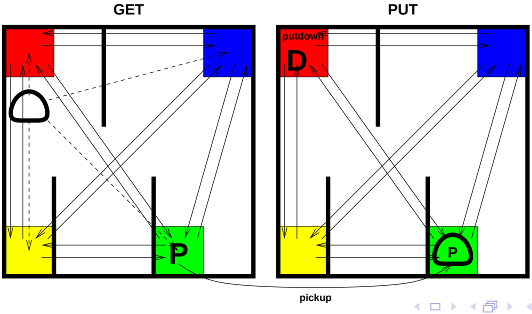

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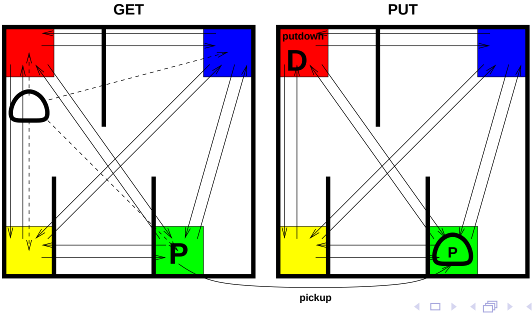
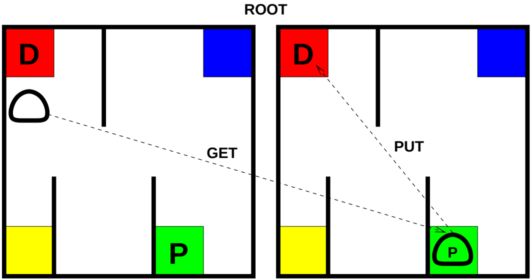


The Role of Hierarchy

- Improve computational complexity (already known)
- Decompose tasks into smaller subtasks
  - Fewer primitive actions per subtask
  - Explicit state abstraction at lower levels
  - Smaller "completion sets" of reachable states at higher levels (related to *result distribution irrelevance*)



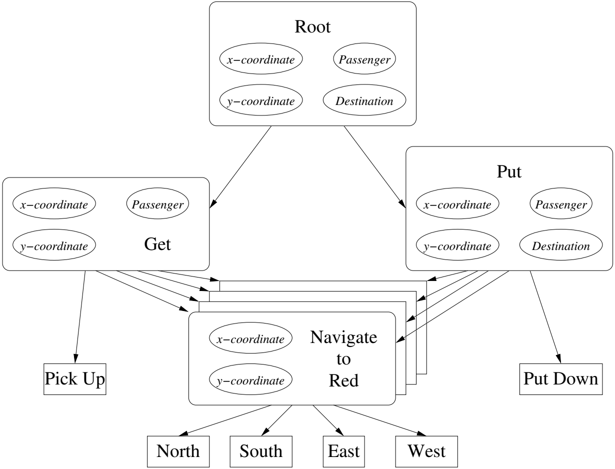
Summary

- R-MAXQ combines R-MAX's robust exploration with MAXQ's incorporation of hierarchical domain knowledge.
- With regard to sample complexity, a primary role of hierarchy may be to constrain unnecessary exploration.

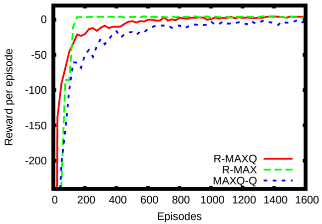
Future Work

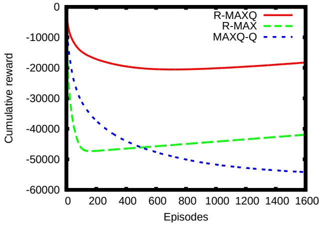
- Application to larger, even continuous, domains
- Guidelines for the design or discovery of hierarchies

The Abstract Taxi Hierarchy



Empirical Results Without State Abstraction





- R-MAX performs slightly worse.
  - Navigational actions require 16 times as much data, since they no longer ignore passenger location and destination.
  - `Pickup` requires 4 times as much data, since it no longer ignores passenger destination.
- R-MAXQ still benefits from never executing `putdown` outside of the four landmark locations.
- MAXQ-Q performs poorly without state abstraction.

The Sample Complexity of R-MAXQ I

- For the same threshold amount of experience per state-action, R-MAXQ will spend no more time exploring that R-MAX.
- However, the threshold required to ensure a given level of near-optimality may be exponentionally worse in the height of the hierarchy.
- These (weak) guarantees make no assumptions about the quality of the hierarchy! (In the same way that the R-MAX guarantees make no assumptions about the policy used to transform a bound on model error into a bound on value function error.)

The Sample Complexity of R-MAXQ II

Theorem

If m samples of each state-action guarantee that R-MAX converges to an ϵ-optimal policy with probability $1-\delta$, then $m' = O\left(m\left(\frac{TL}{1-\delta}\right)^{2h}\right)$ samples of each primitive state-action suffice for R-MAXQ to converge to a recursively ϵ-optimal policy with probability $1-\delta$.

- L is $O\left(\frac{\log \epsilon}{1-\gamma}\right)$
- T is the maximum number of reachable terminal states for any composite action
- h is the height of the hierarchy