Adaptive Networks
The intriguing interplay of the dynamics ON and OF networks

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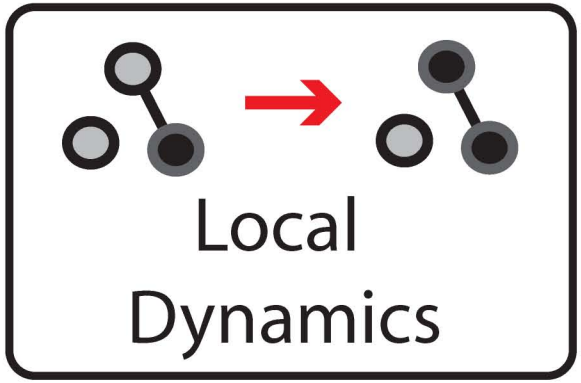
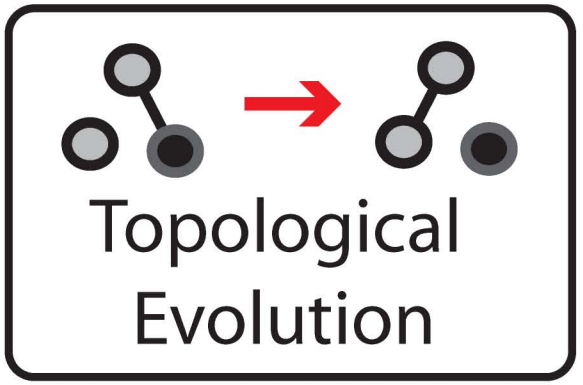
Dynamical Networks have two different faces.

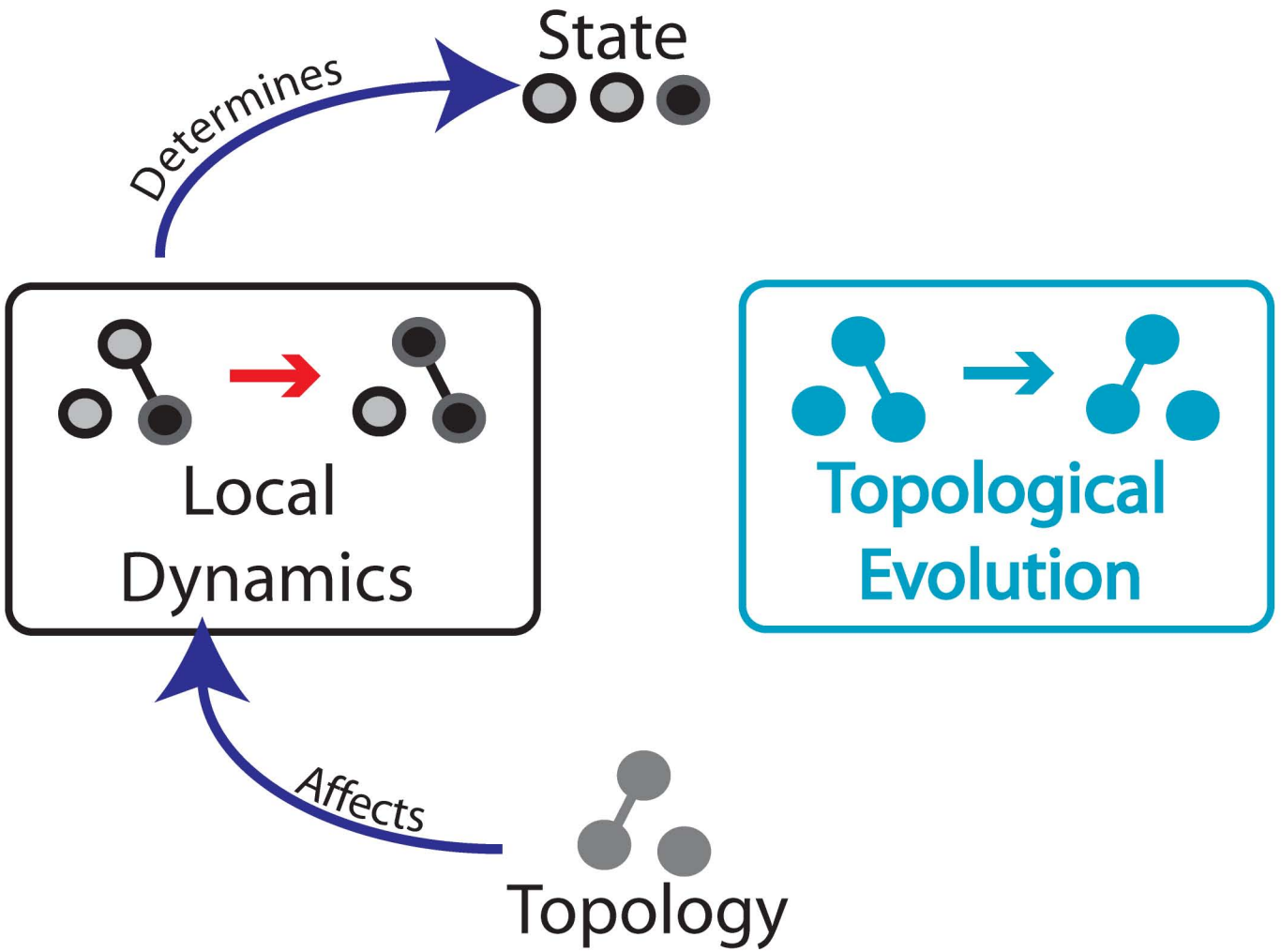
Dynamics ON Networks



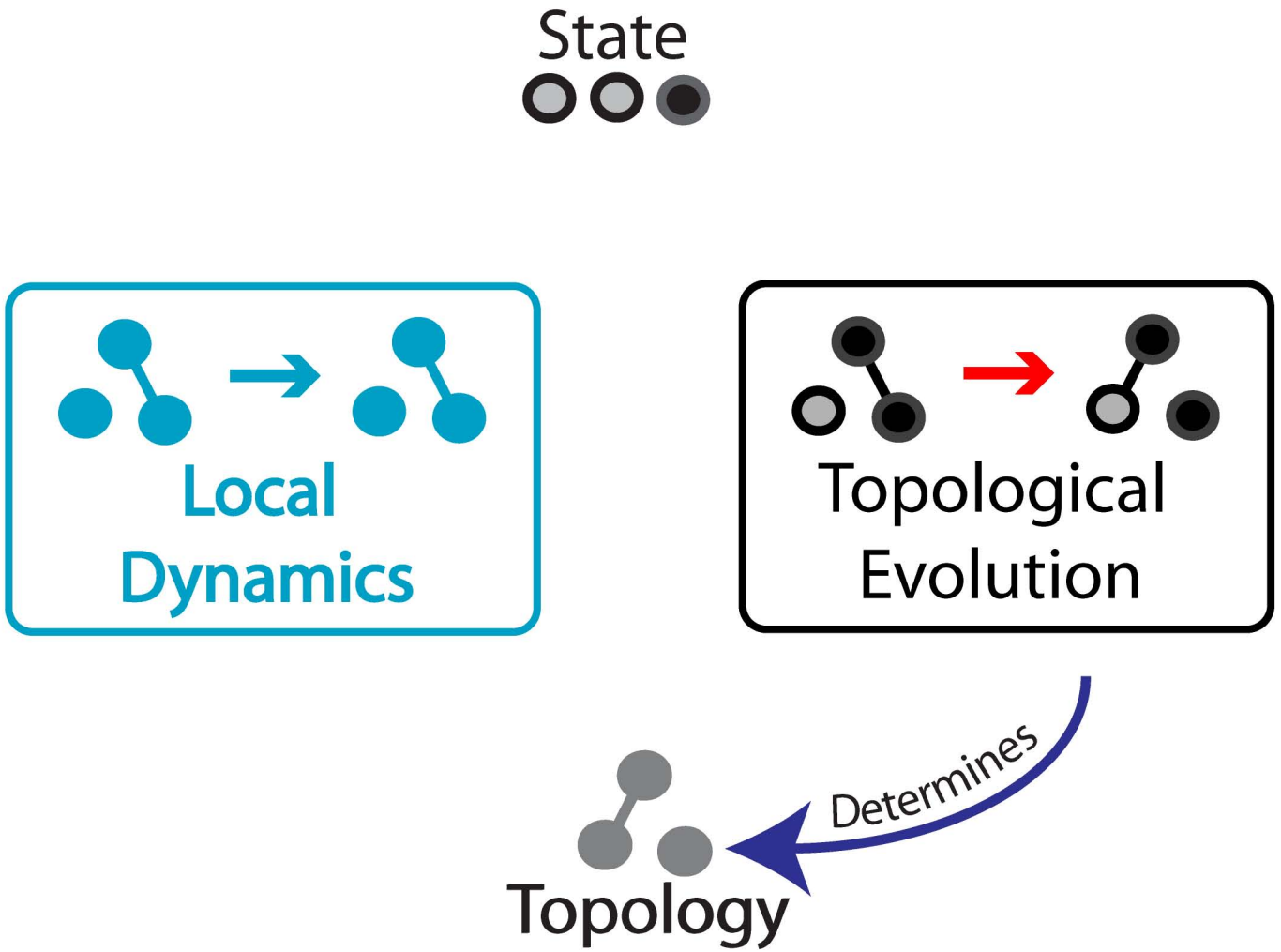
Dynamics OF Networks



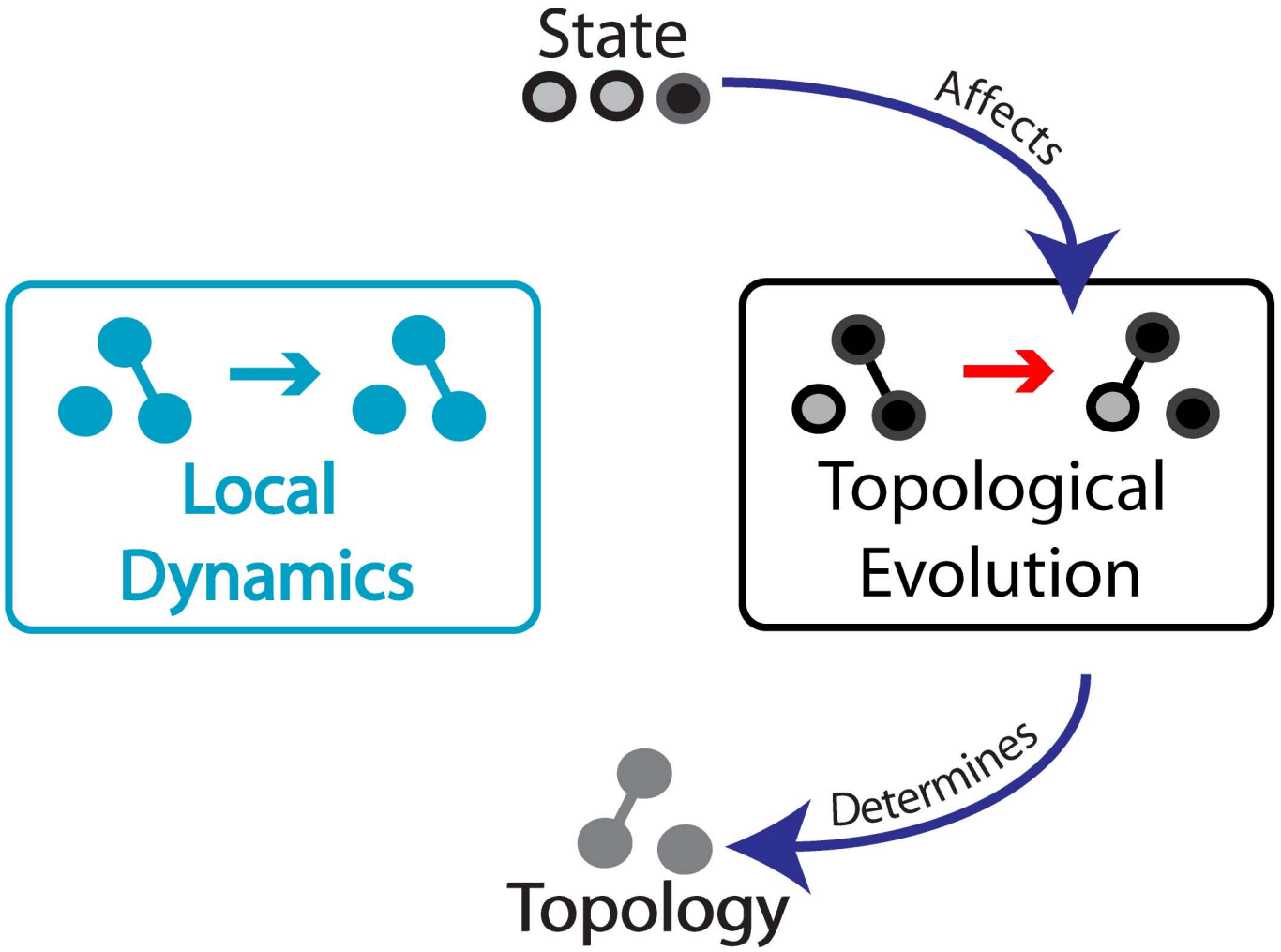
But: Local dynamics depends on topology



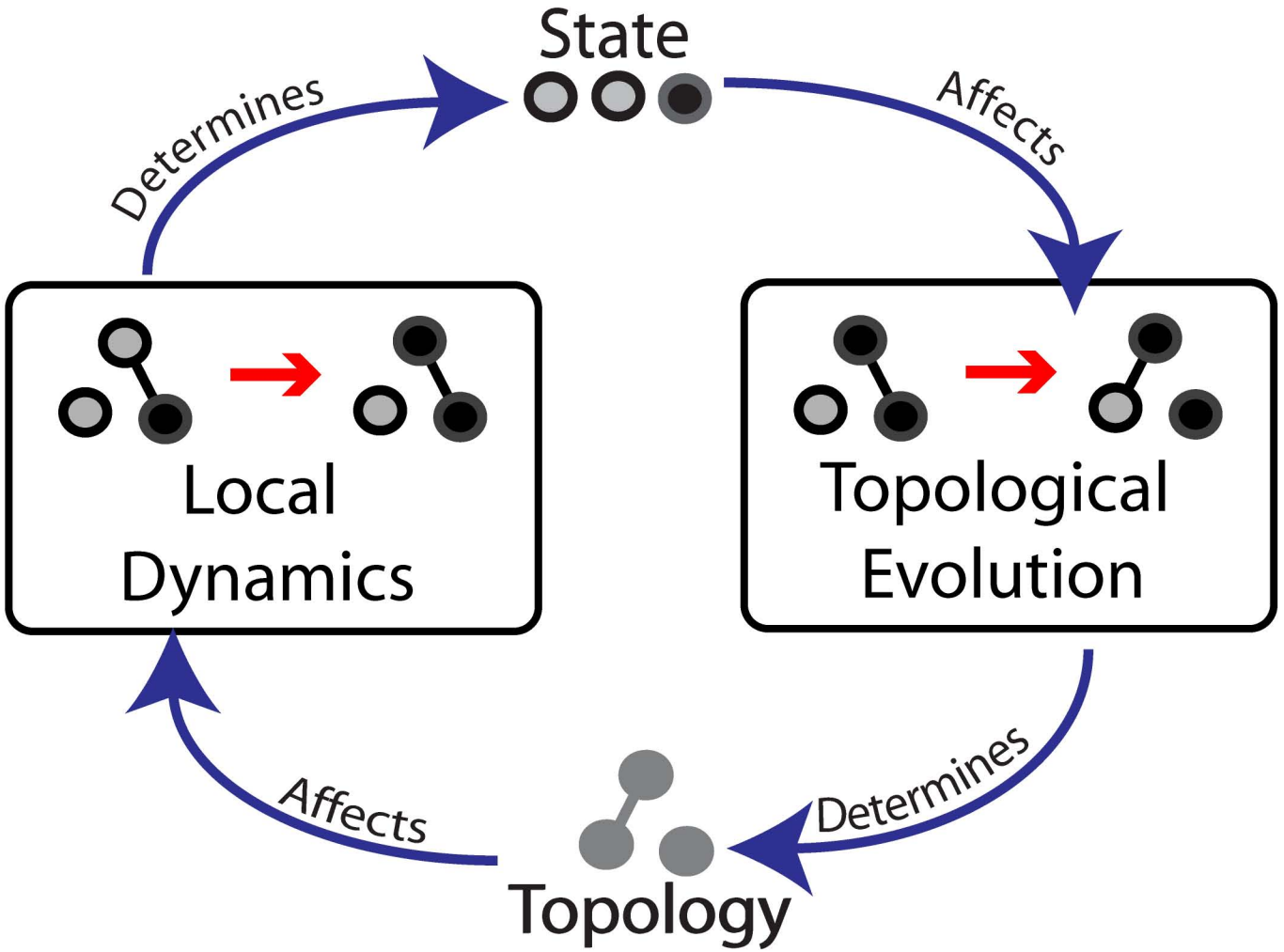
But: Topological Evolution determines topology.



But: Evolution can depend on the state.



Adaptive Networks: dynamics ON and OF the net.



1 + 1 = ?

Adaptive Networks combine
topological evolution and **local dynamics**.

Local Dynamics + Topological Evolution = New Phenomena

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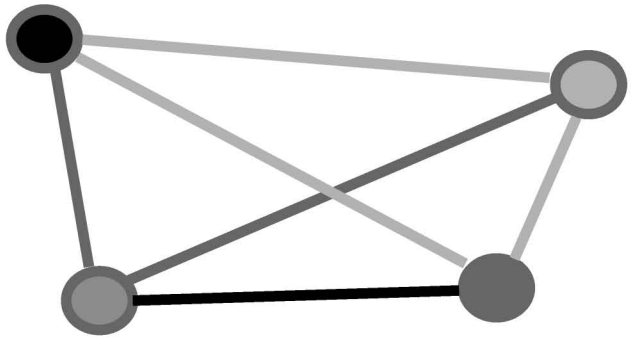
Spontaneous division of labor

Ito and Kaneko, Phys. Rev. Lett. **88**, 028701-4 (2002)

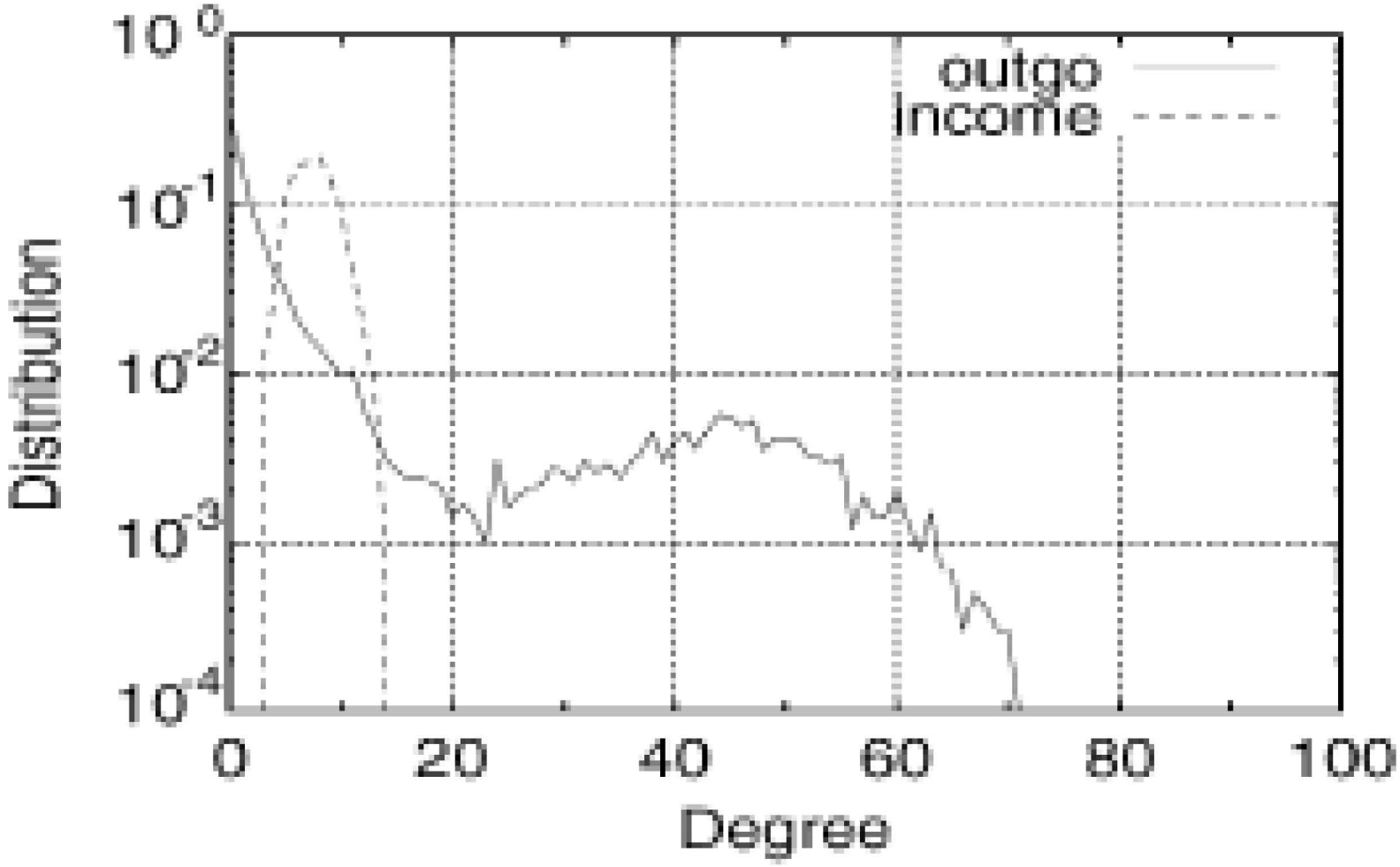
- Consider: ensemble of oscillators (maps)
- Coupling according to a given topology
- Link strength is a continous variable (weighted network)



- Network is initially homogeneous
- **Like-and-Like Rule:** Coupling between nodes in similar states increases.
- But: Incoming coupling strength is normalized to one

Spontaneous division of labor

Ito and Kaneko, Phys. Rev. Lett. **88**, 028701-4 (2002)

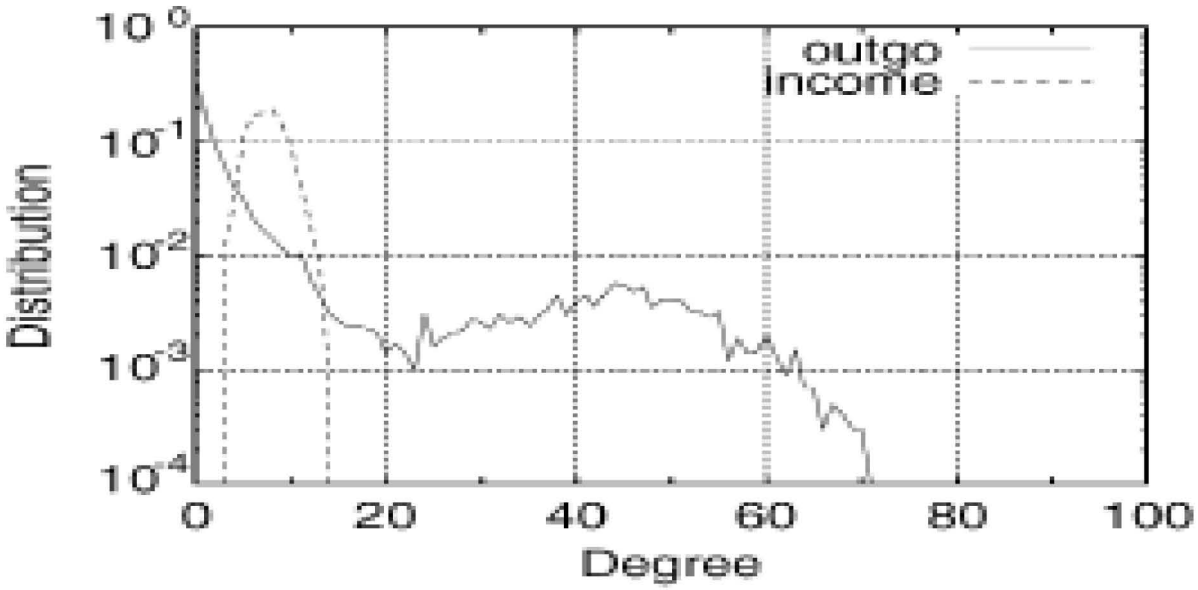


Spontaneous division of labor

Ito and Kaneko, Phys. Rev. Lett. **88**, 028701-4 (2002)

Leaders: Strong Impact on other nodes

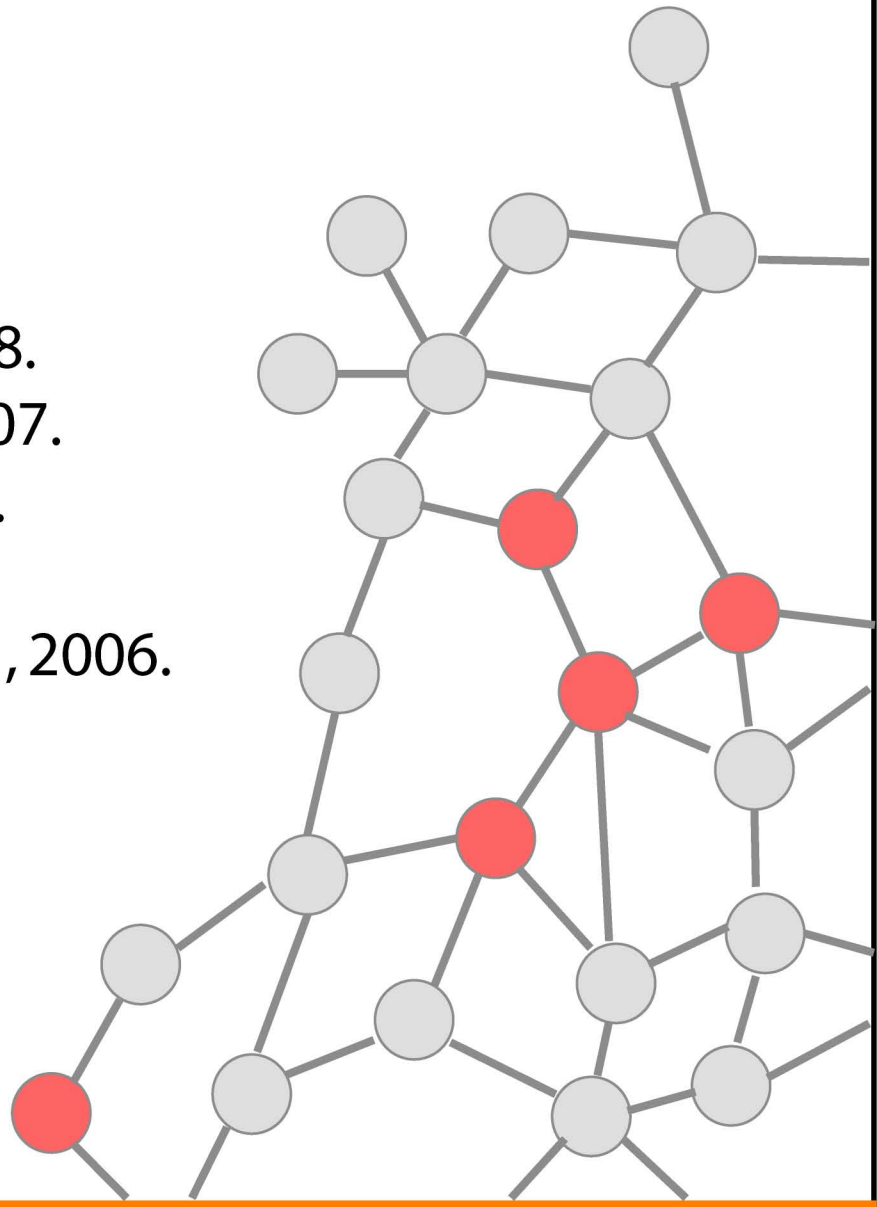
Followers: Only very weak impact on other nodes



While the system never reaches a steady state
leaders remain leaders and followers remain followers
in the long-term behavior

Opinion Formation on adaptive networks

Nardini et al., PRL **100**, 158701, 2008.
Vazquez et al., PRL **100**, 108702, 2008.
Benczik et al., EPL **82**, 480061, 2008.
Kozma and Barrat, PRE **77,** 0161021, 2008.
Holme and Newman, PRE **74**, 056108, 2007.
Gil and Zanette, Phys. Lett. A **35**, 89, 2006.
Ehrhardt et al., PRE **74**, 0361061, 2006.
Grabowski and Kosinski, PRE **73**, 0161351, 2006.



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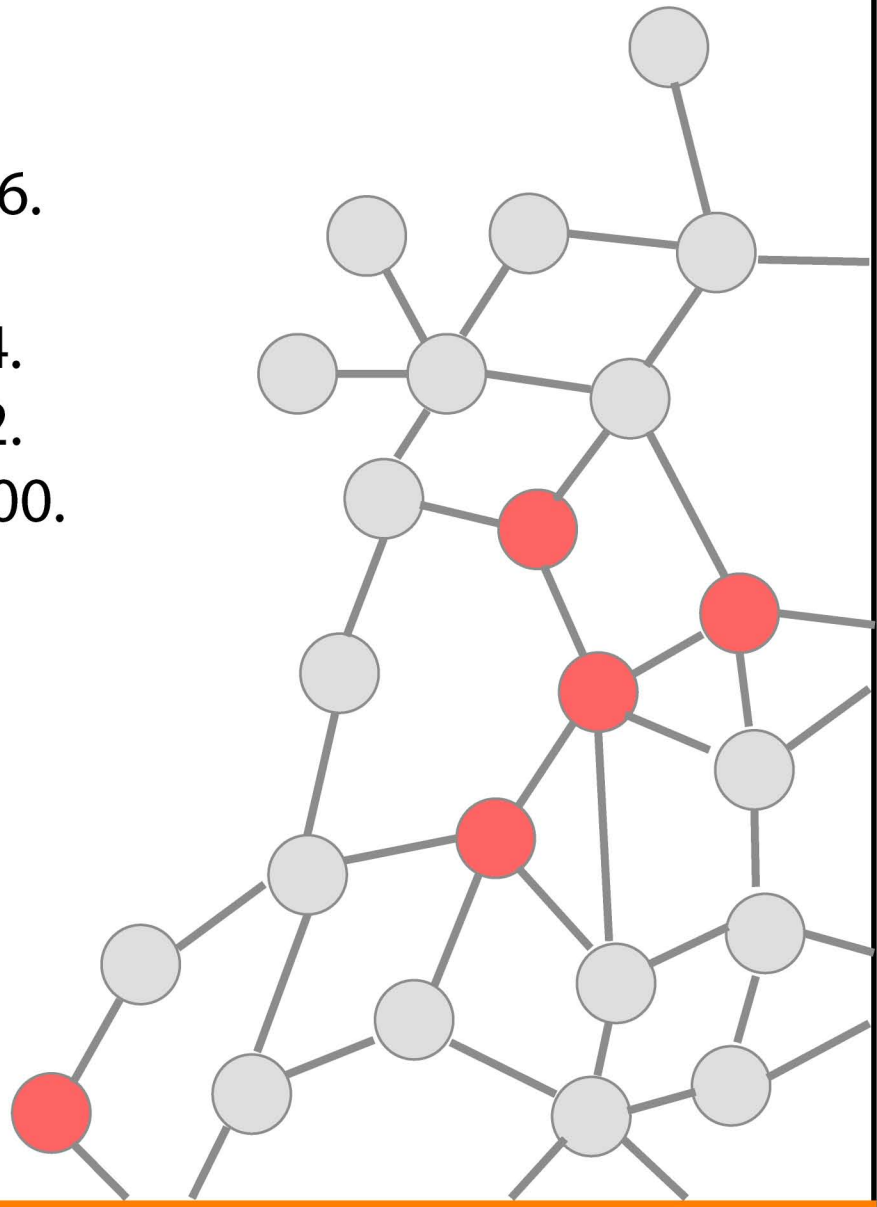
Cooperation on adaptive networks

Pacheco et al., JTB **250**, 723, 2008.
Holme and Ghoshal, PRL **96**, 908701,2006.
Pacheco et al., PRL **97**, 258103, 2006.
Zimmermann et al., PRE **96**, 065102, 2004.
Ebel and Bornholdt, PRE **66**, 056118, 2002.
Skyrms and Pemantle, PNAS **97**, 9340, 2000.



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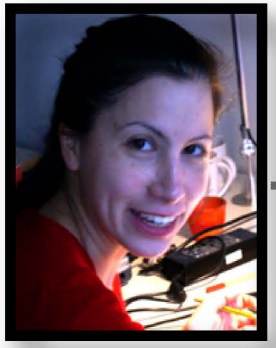
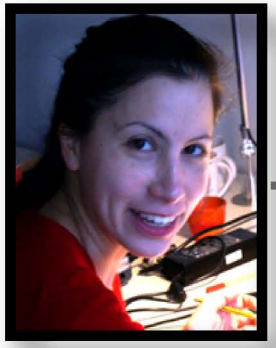
Continuous Snowdrift Game



Ly Do
(Player A)

Thilo Gross
(Player B)

Continuous Snowdrift Game



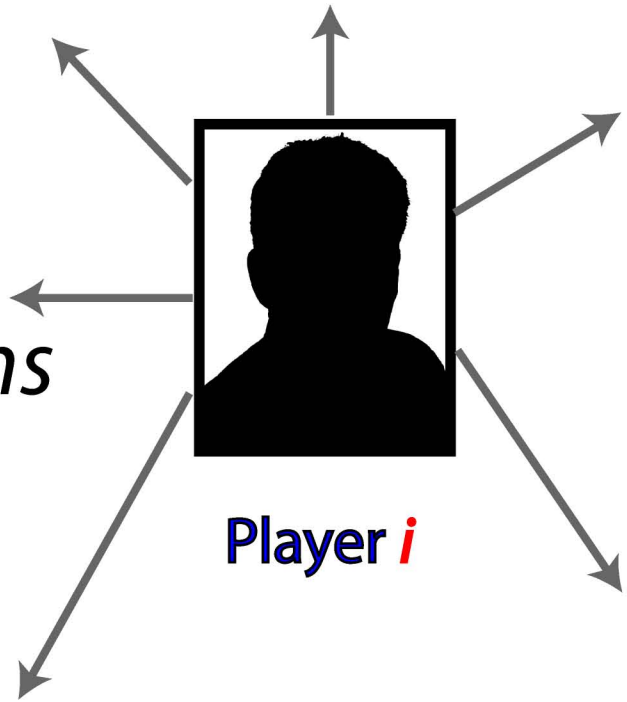
Ly Do
(Player A)

Thilo Gross
(Player B)

$$\text{Payoff A} = \text{Benefit}(e_A + e_B) - \text{Cost}(e_A)$$

Multi-Player Snowdrift Game

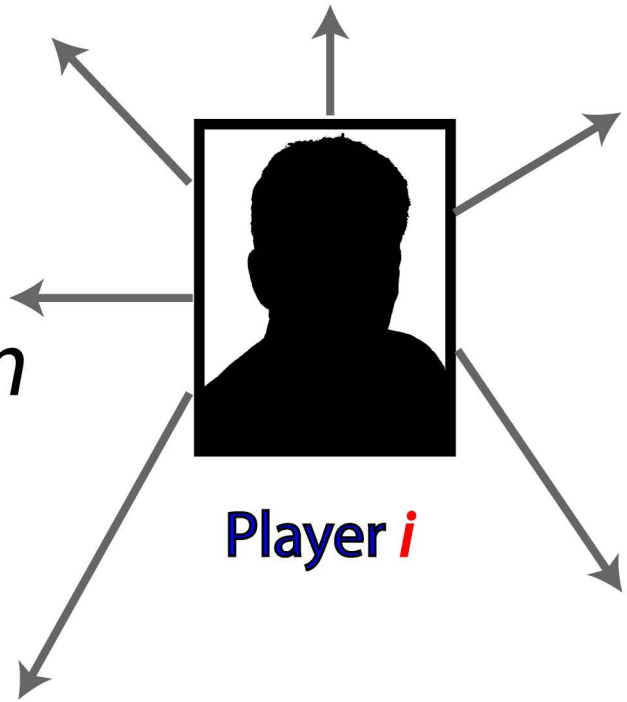
Now:
Many different Collaborations



Multi-Player Snowdrift Game

Benefits add up.

Cost is a function of the sum of investments.

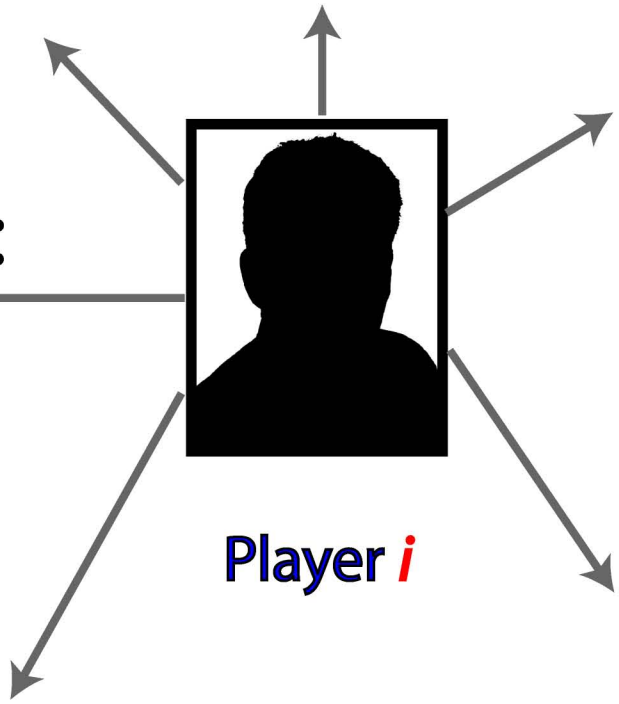


$$p_i = \left(\sum_j b\left(e_{ij} + e_{ji}\right) \right) - c\left(\sum_k e_{ik} \right)$$

Multi-Player Snowdrift Game

Payoff from one investment:

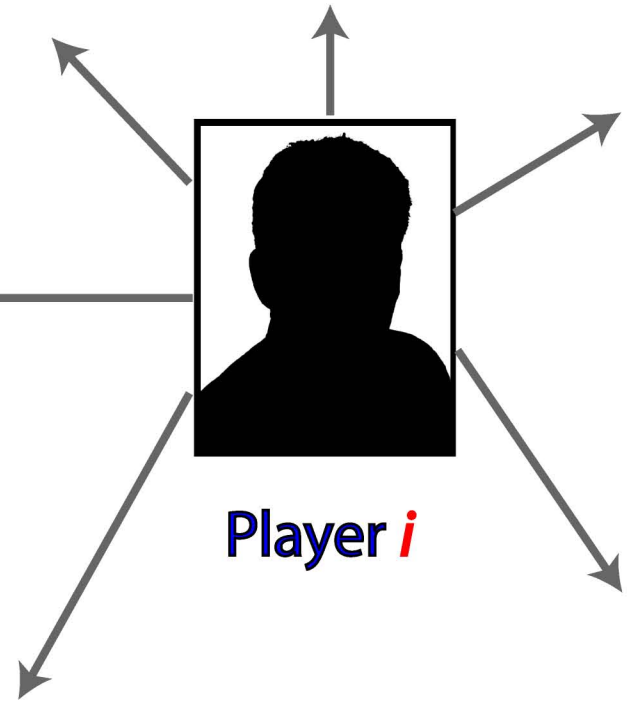
$$p_{ij} = b\left(e_{ij} + e_{ji}\right) - \frac{e_{ij}}{\sum_k e_{ik}} c \left(\sum_k e_{ik}\right)$$



Multiplayer Snowdrift Game

Evolutionary Optimization

$$\frac{\partial}{\partial t} e_{ij} = \frac{\partial}{\partial e_{ij}} \sum_{j \neq i} p_{ij}$$

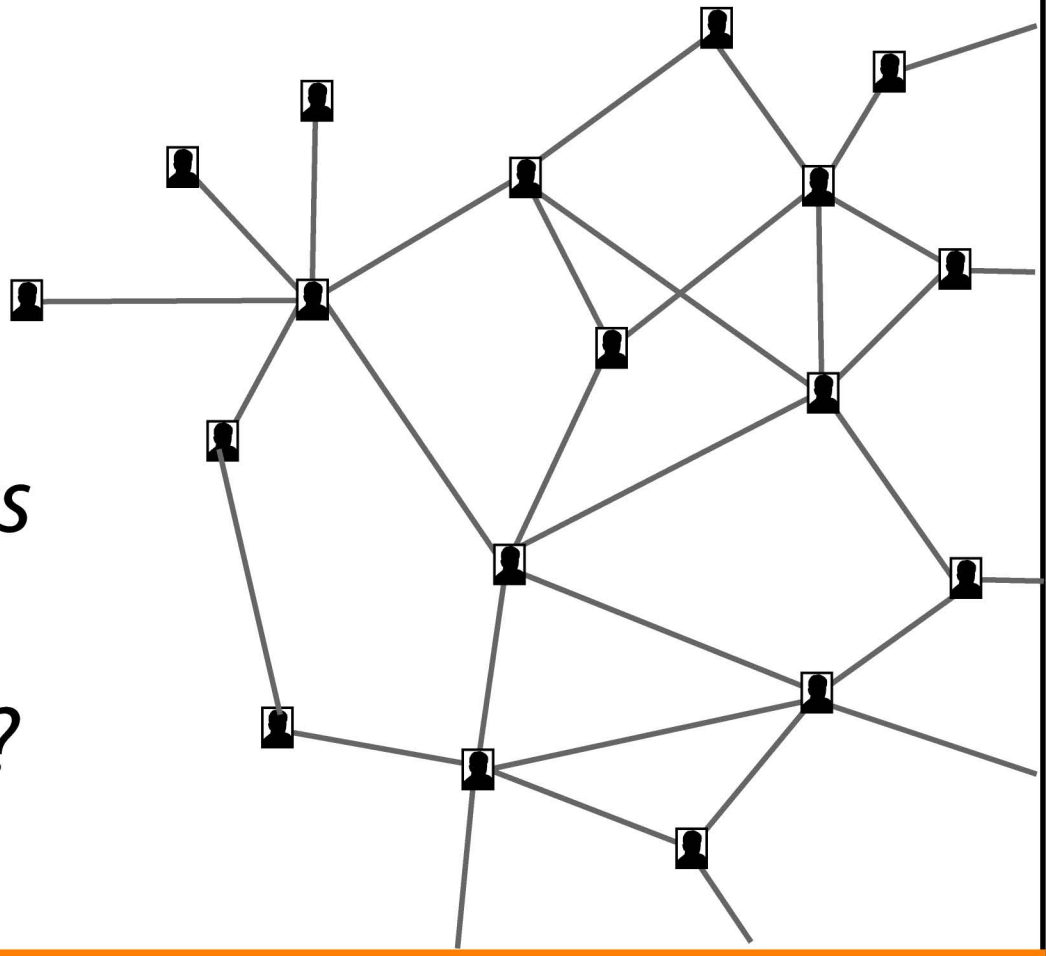


(Players are selfish!)

Multiplayer Snowdrift Game

Number of Players *N*

Number of Links *N(N-1)*

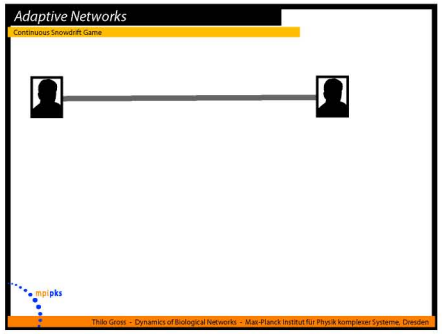


Initially:

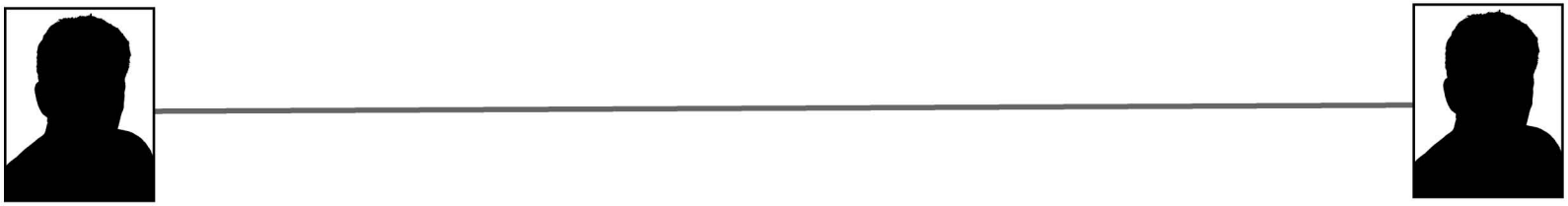
All-to-all, equal investents

Outcome?

Multiplayer Snowdrift Game



Consider a random link ...



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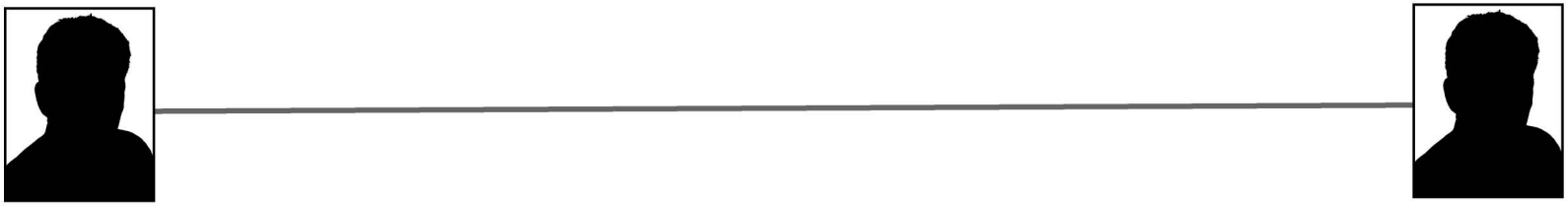

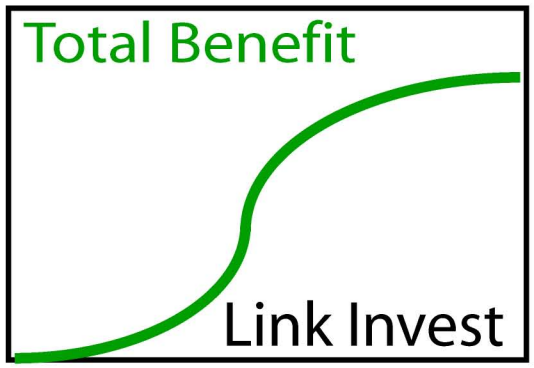

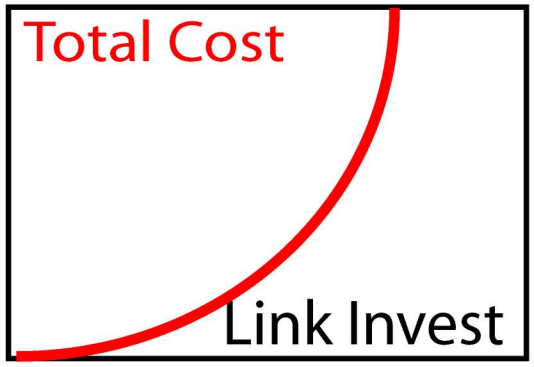

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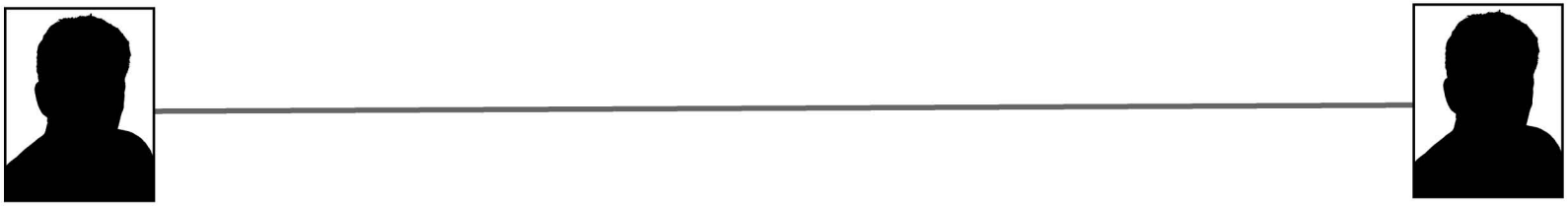

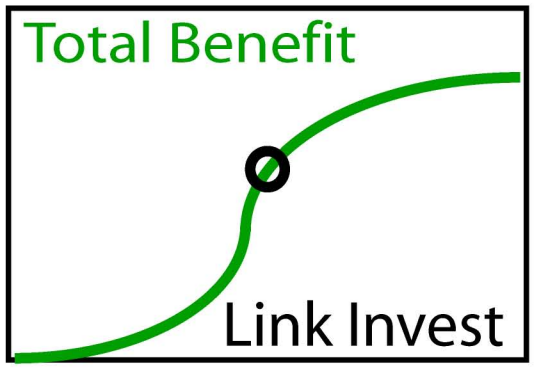

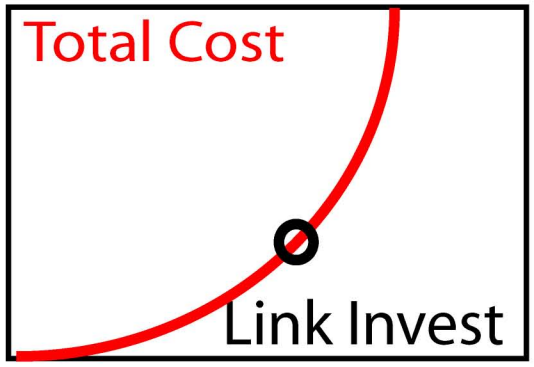

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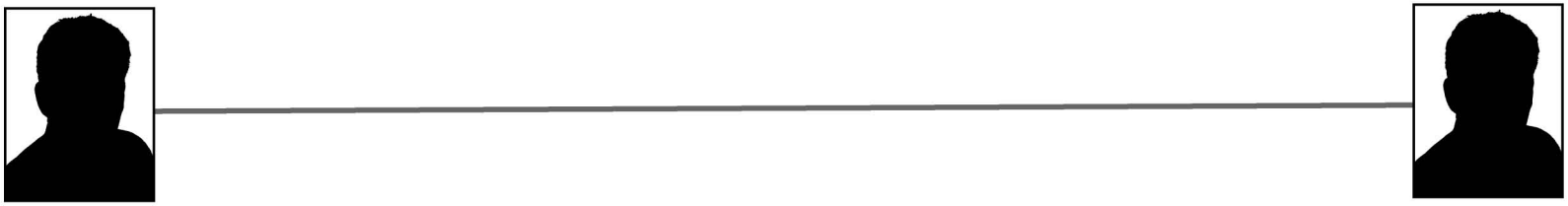
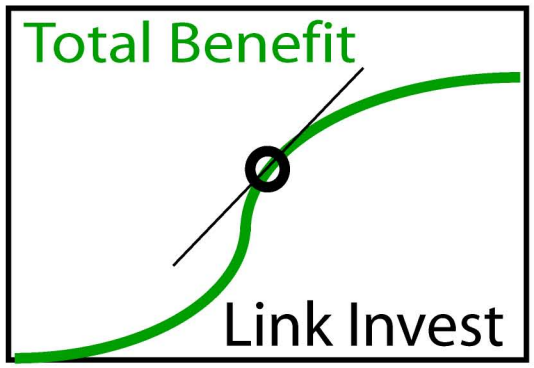
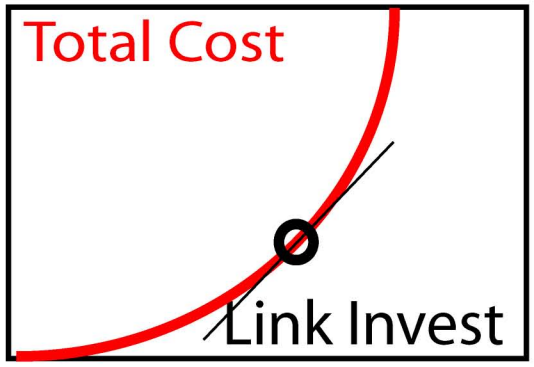


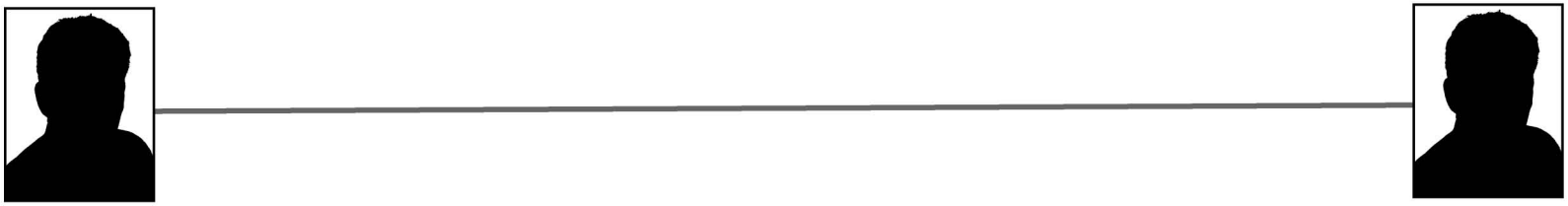


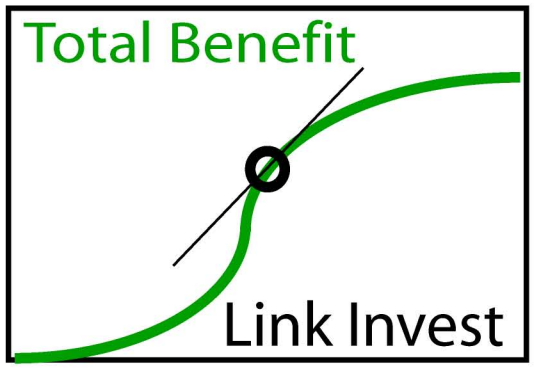


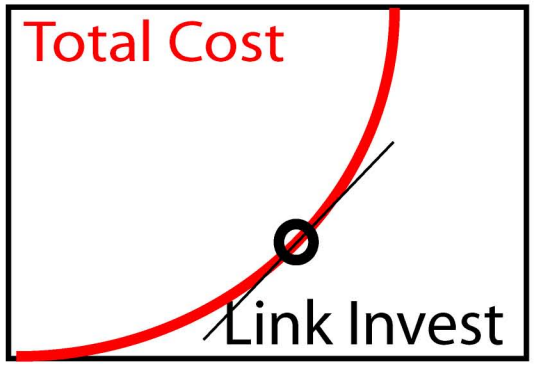
Stationarity Implies:

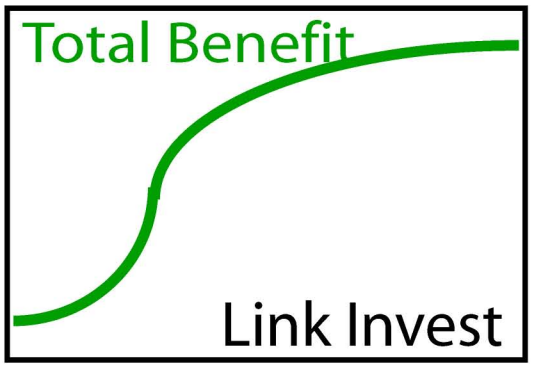
Cost and Benefit have indentical slopes

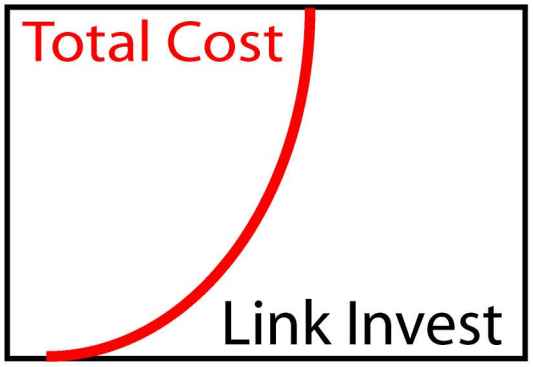
Consider a random link ...











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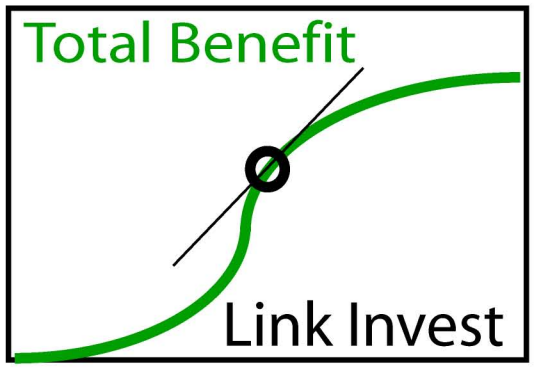
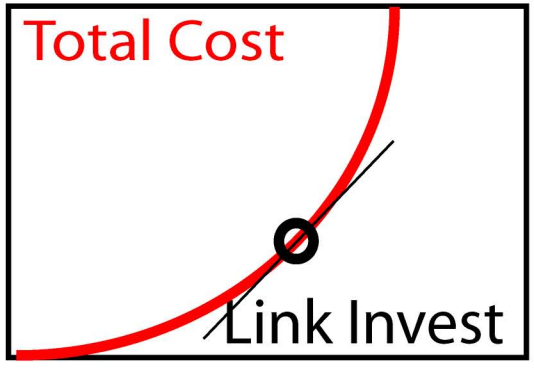
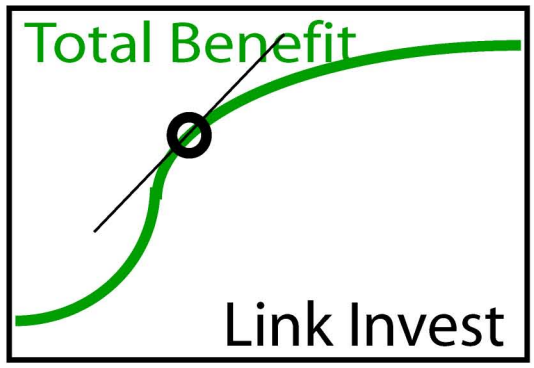
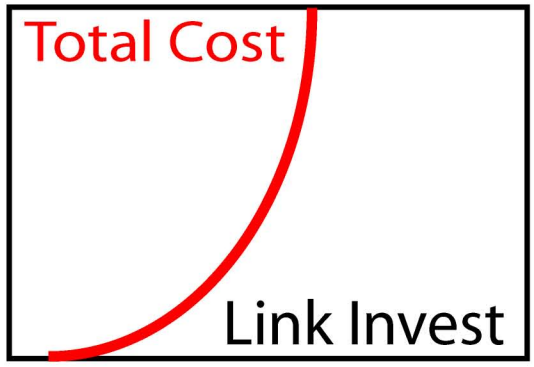
Consider a random link ...

Benefit(Invest1 + Invest 2)









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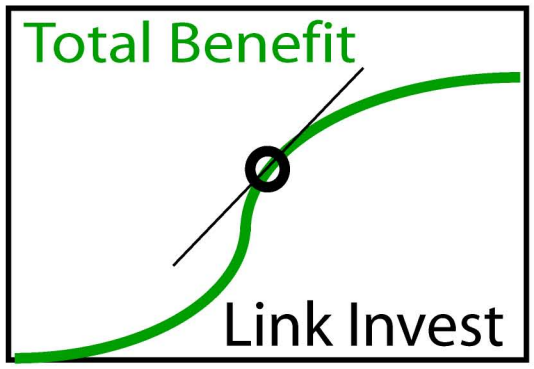
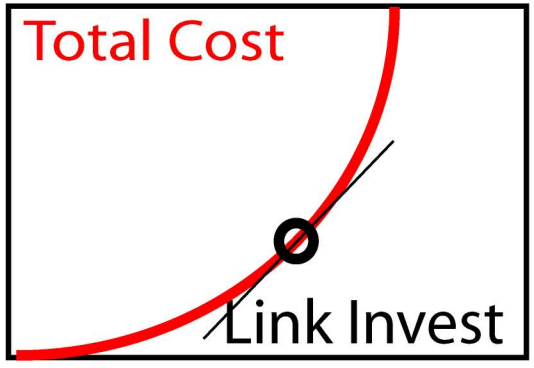
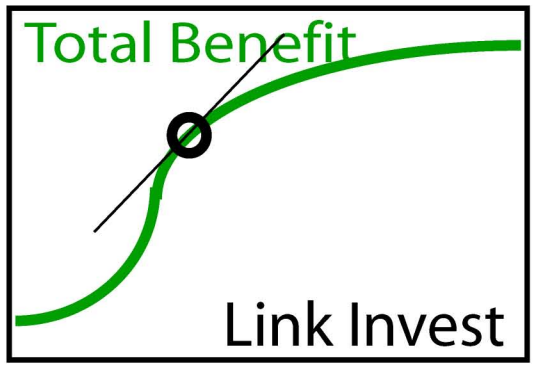
By design: Equal slope on benefit curves

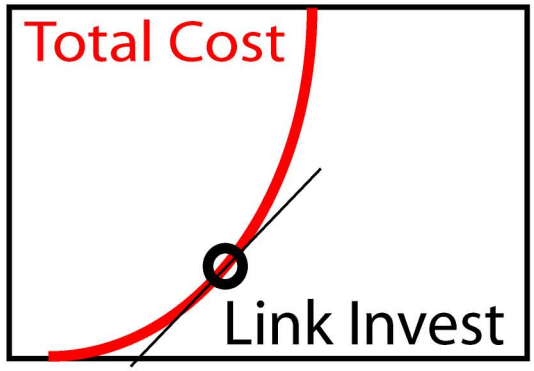
Consider a random link ...

Benefit(Invest1 + Invest 2)









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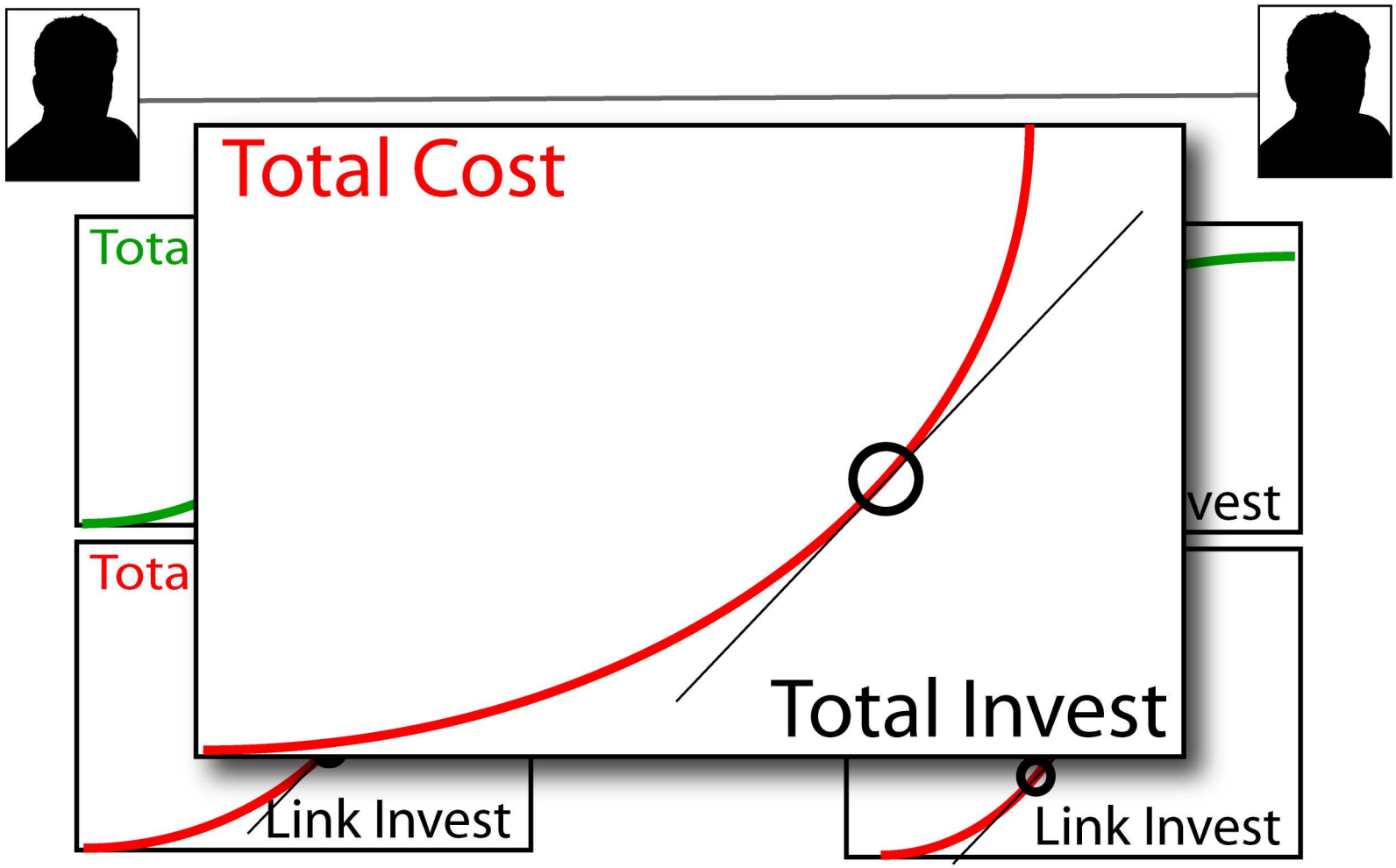
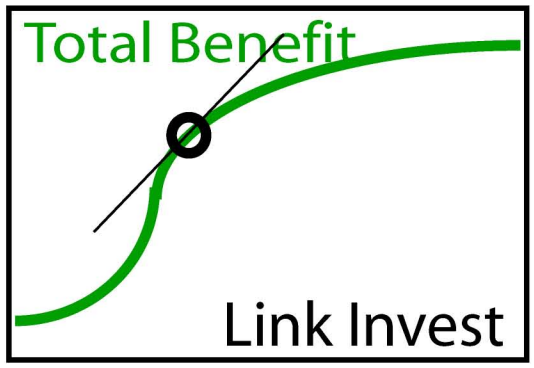
Consider a random link ...

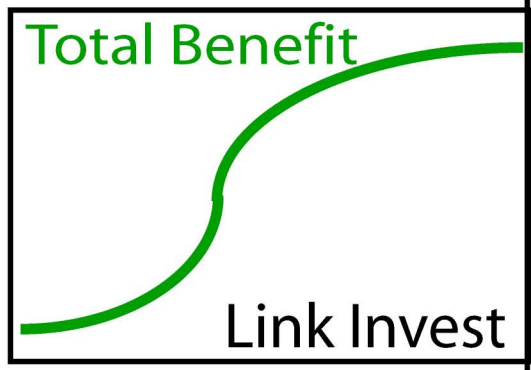


This Implies: **Equal Total Investment**

Consider a random player ...

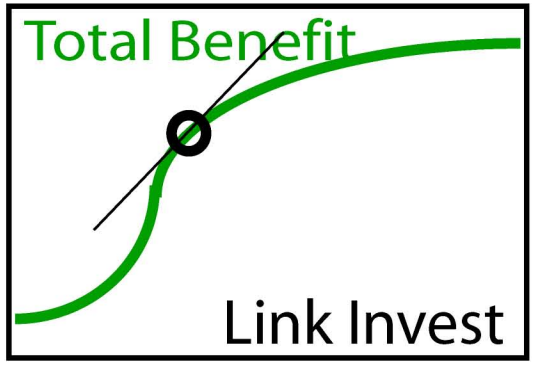
What about the investment in another Link?

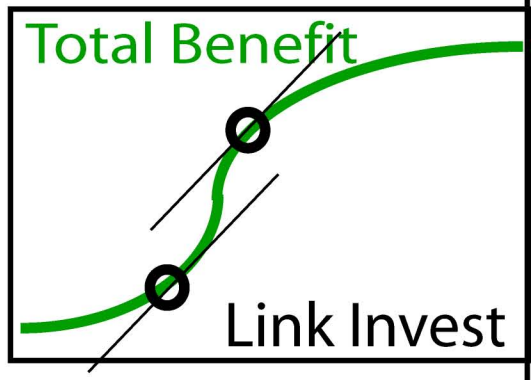




Consider a random player ...

What about the investment in another Link?



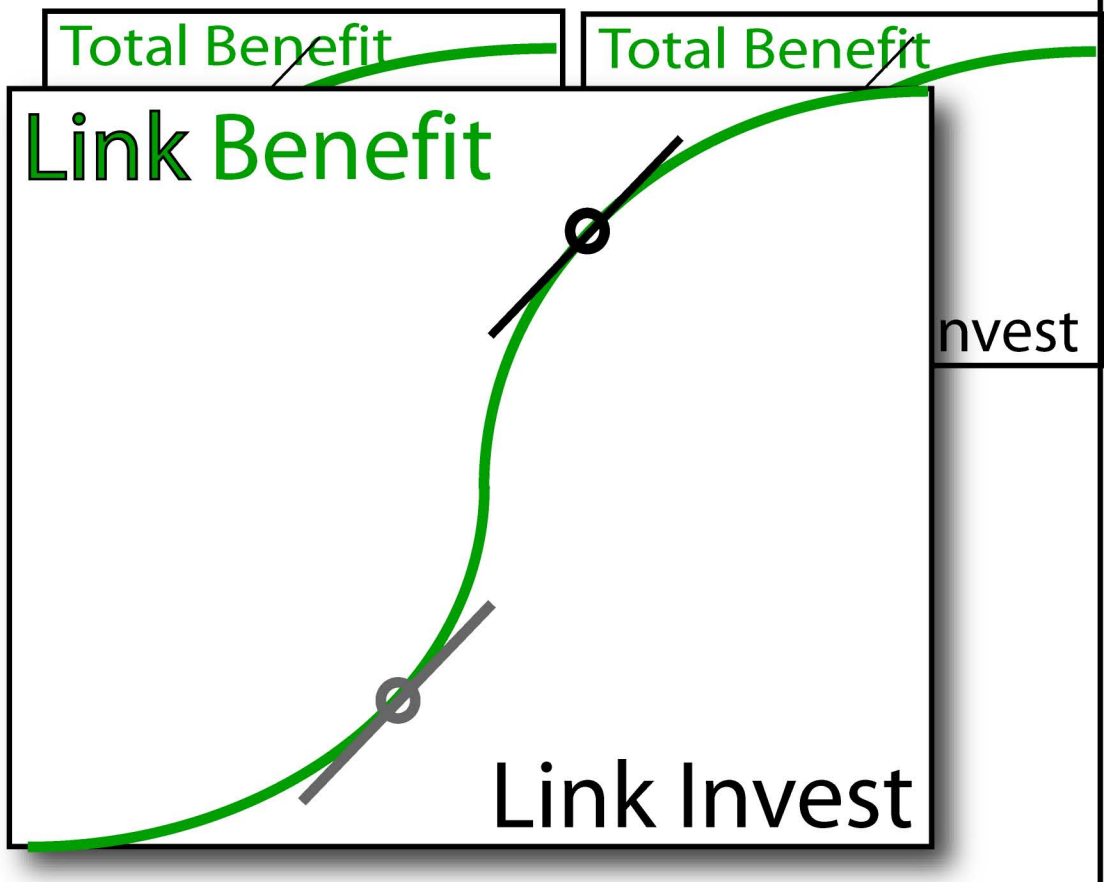


Stationarity implies:
Identical slope of benefit curve

Consider a random player ...

For sigmoidal Functions:

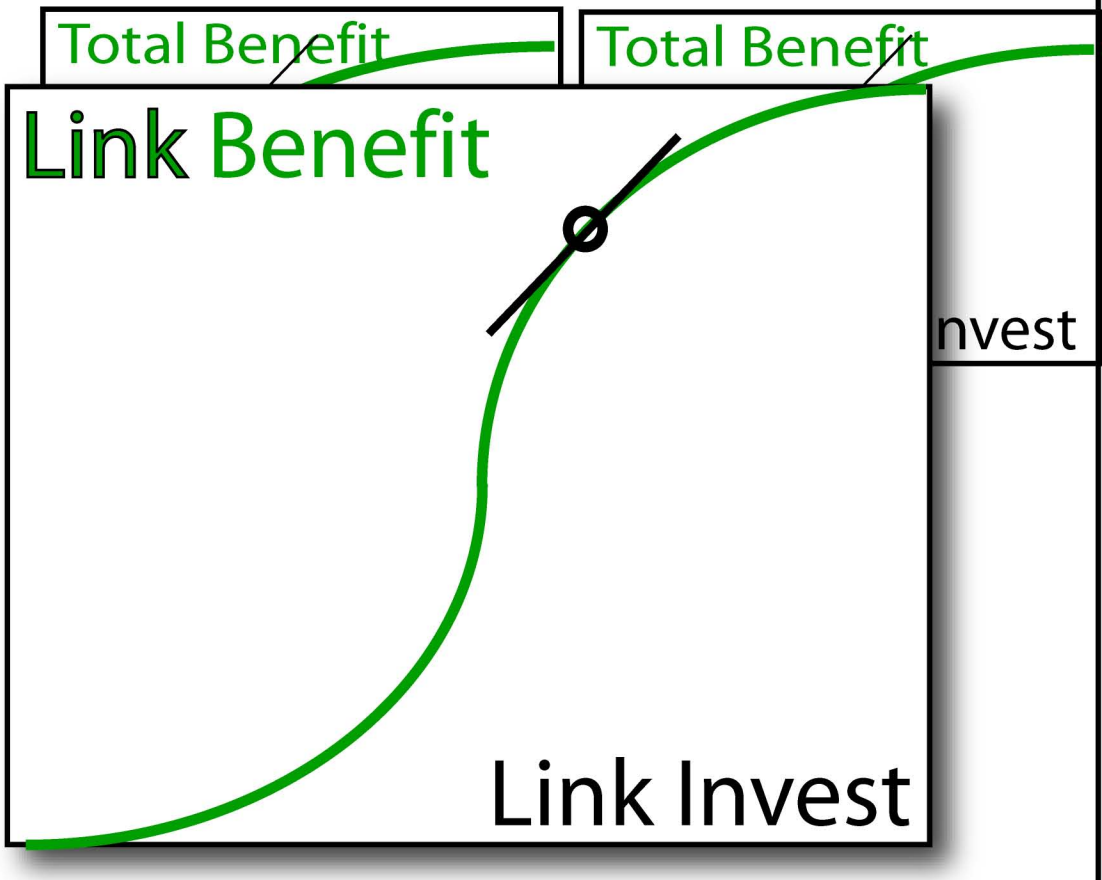
Same slope implies one of two possible link benefits



Consider a random player ...

For sigmoidal Functions:

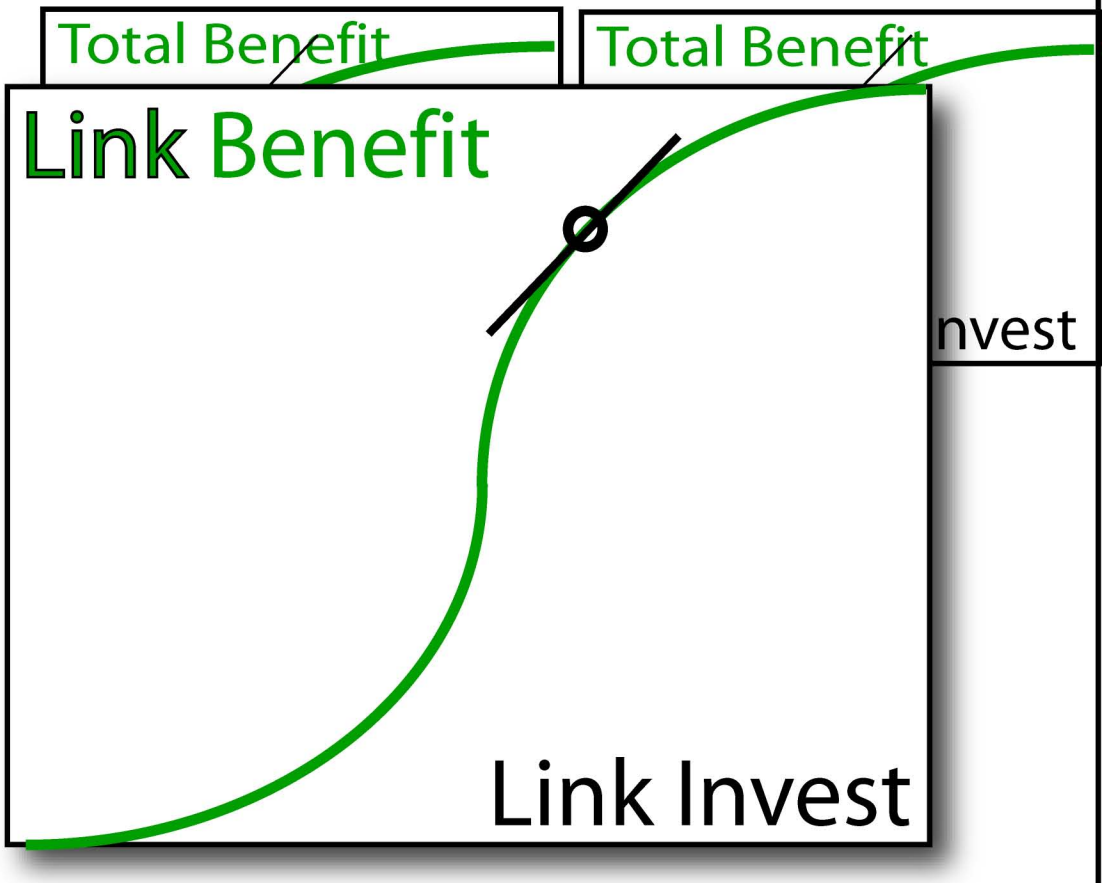
One turns out to be dynamically unstable



Consider a random player ...

For sigmoidal Functions:

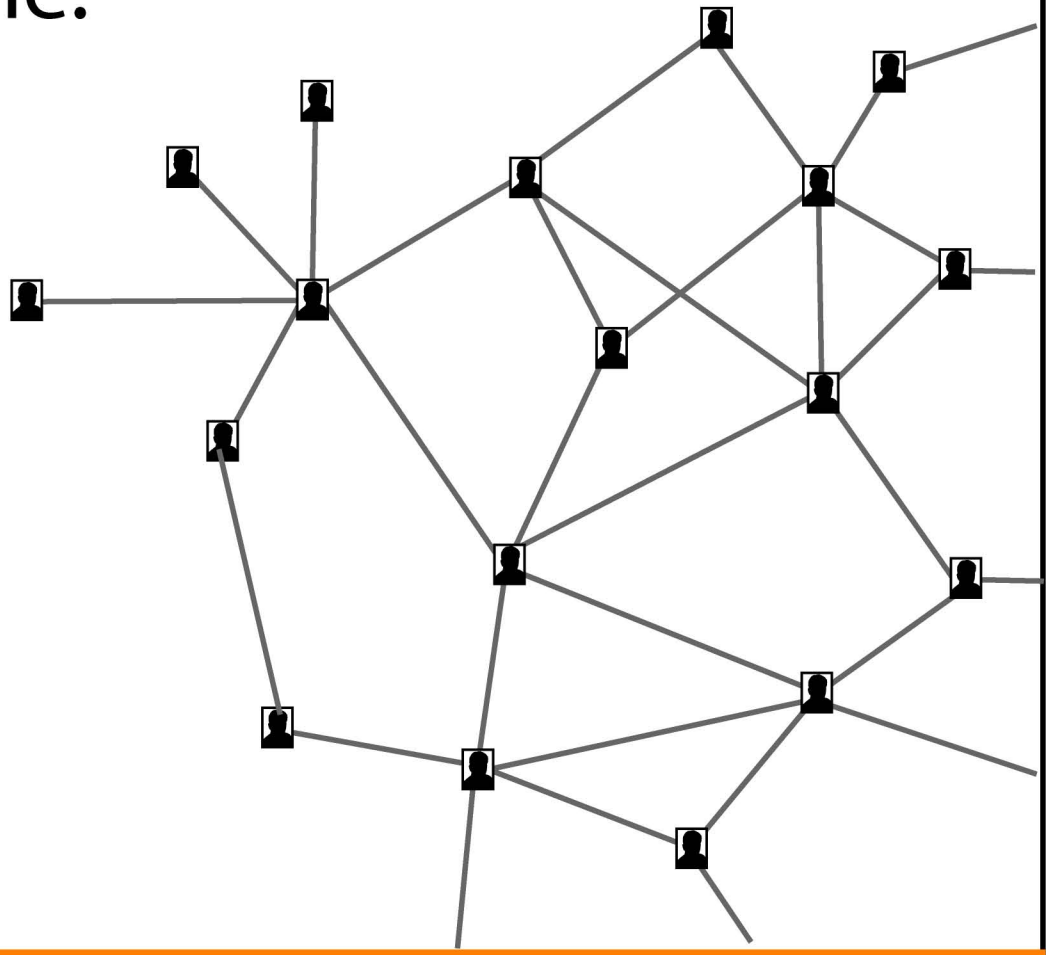
All Links produce the same benefit



In a connected component ...

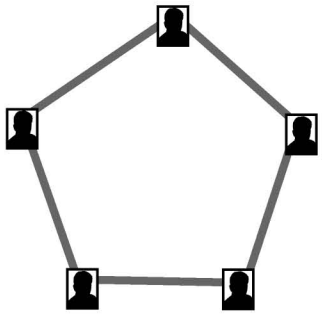
All players invest the same.

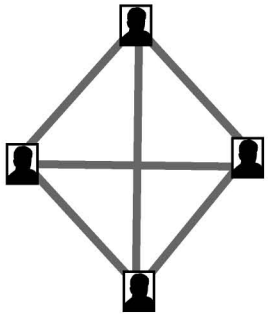
All links produce the same benfit.



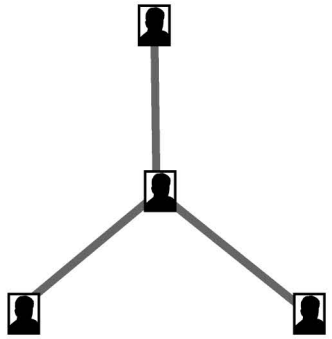
Is this fair ?

Fairness depends on topology





Fair Topologies



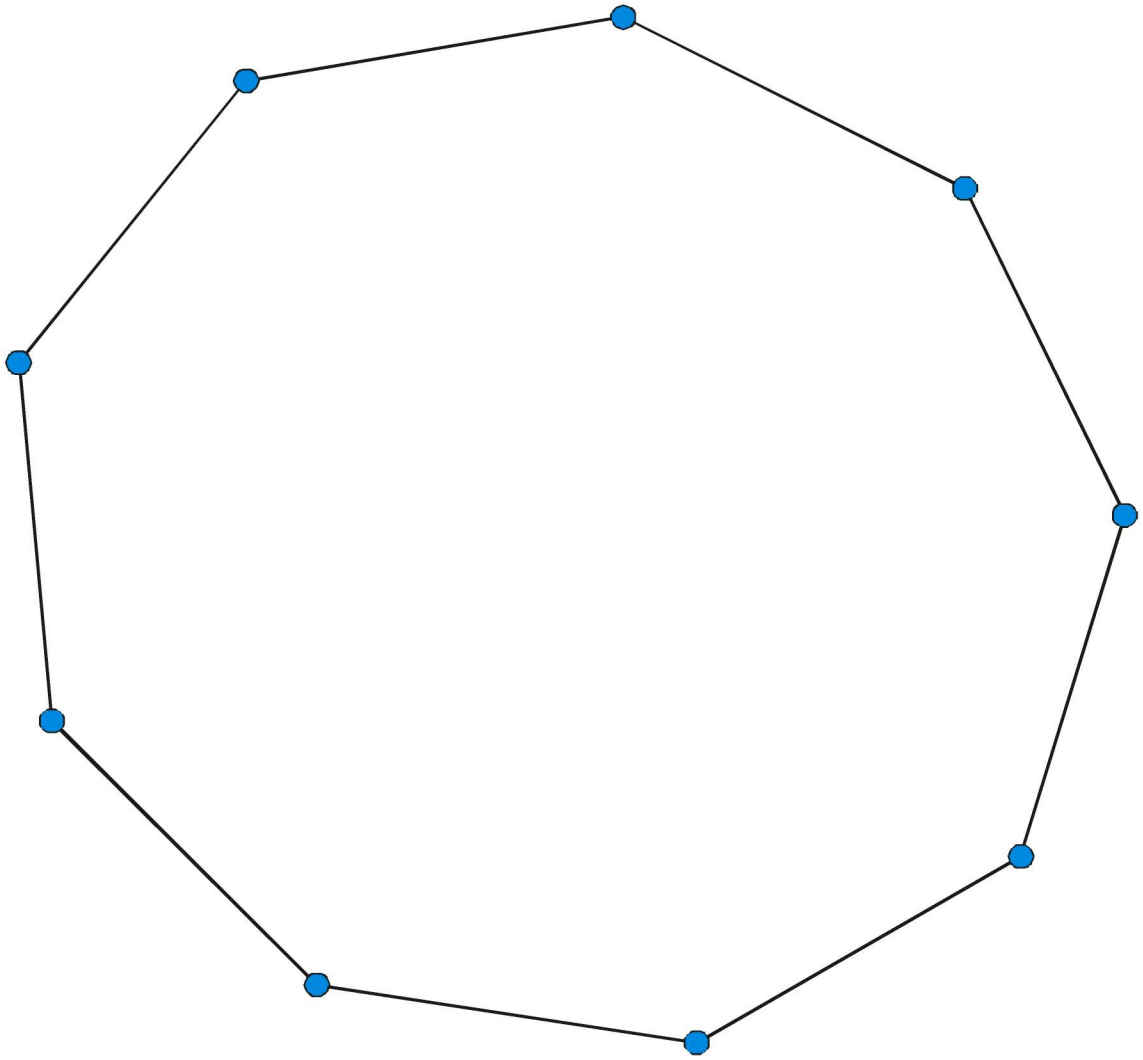
Unfair Topology

Why are links broken?

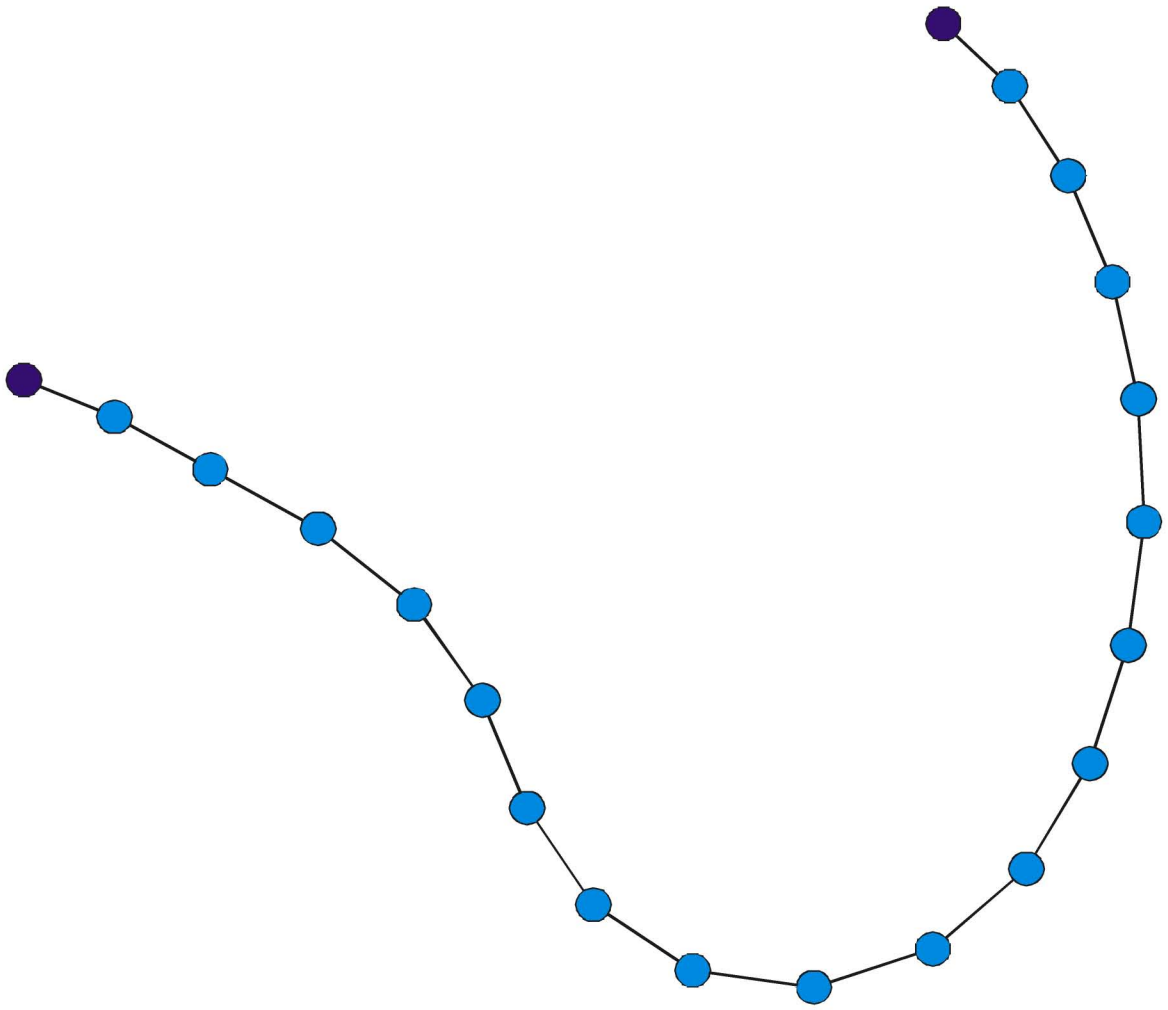
$$\frac{\textit{Investment per Link}}{\textit{Investment per Node}} = \frac{\textit{Number of Nodes in Component}}{\textit{Number of Links in Component}}$$

Cost and Benefit functions determine the **average degree** of the nodes (but not uniquely)

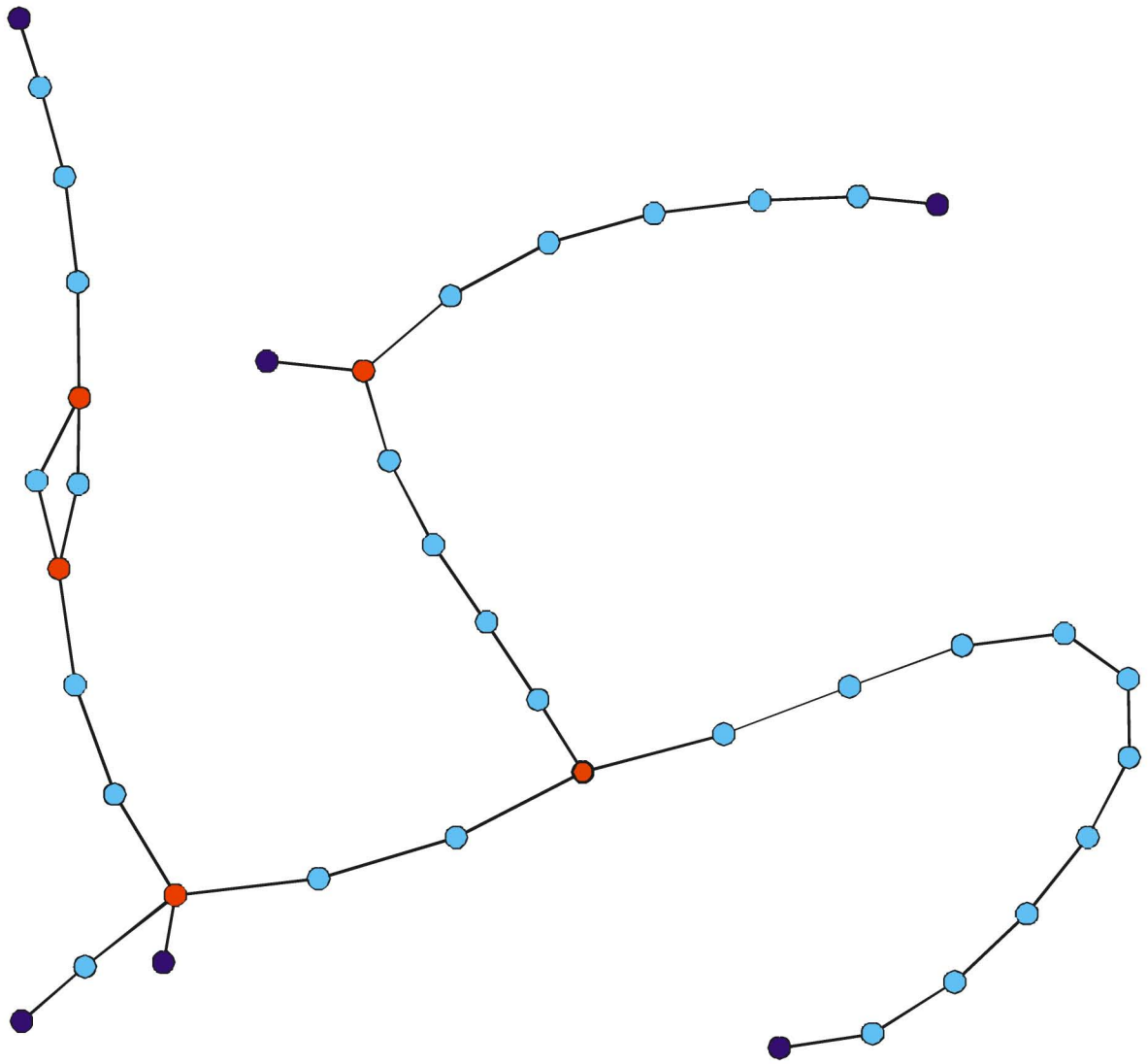
Results: N = 9



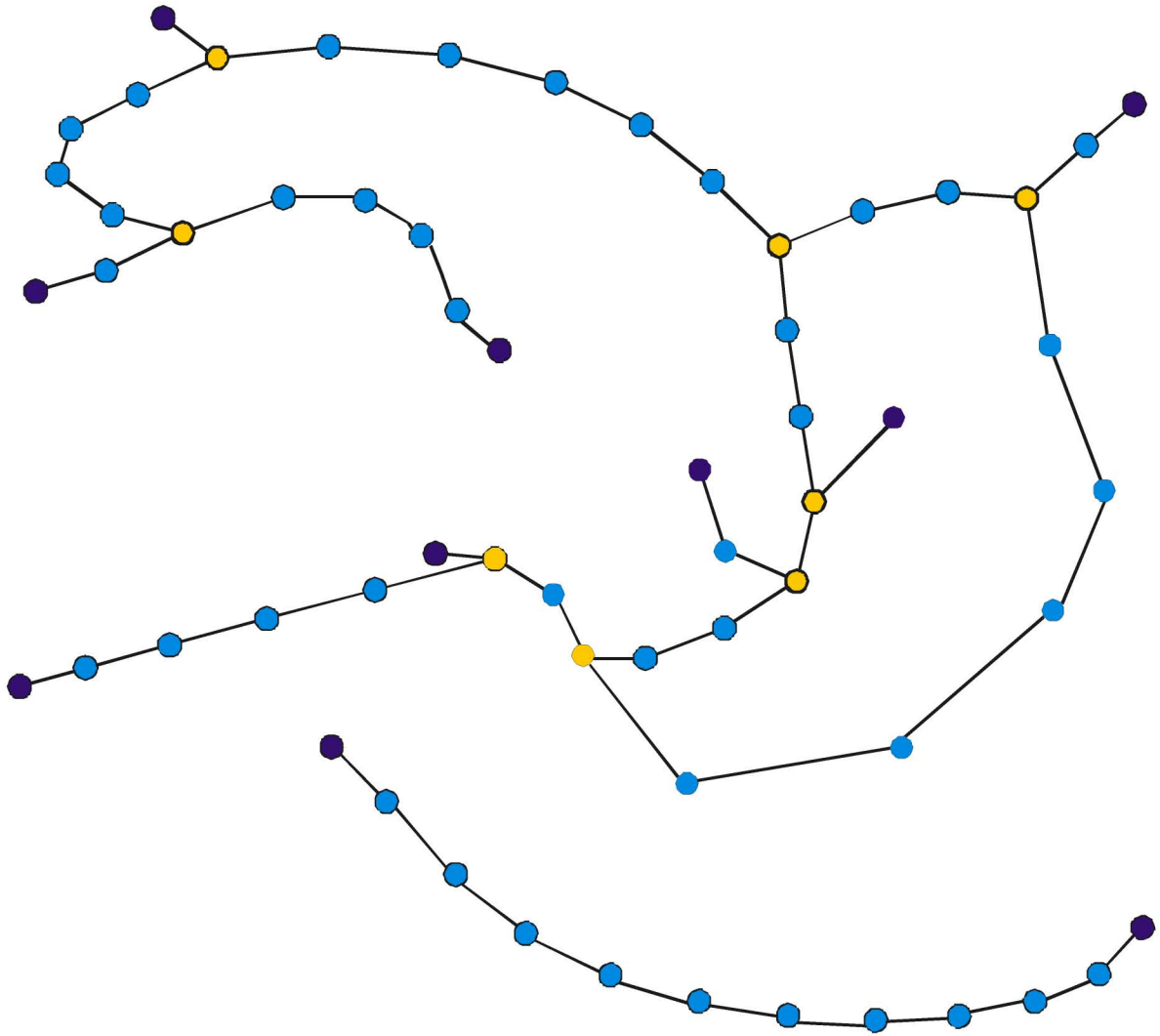
Results: N = 20



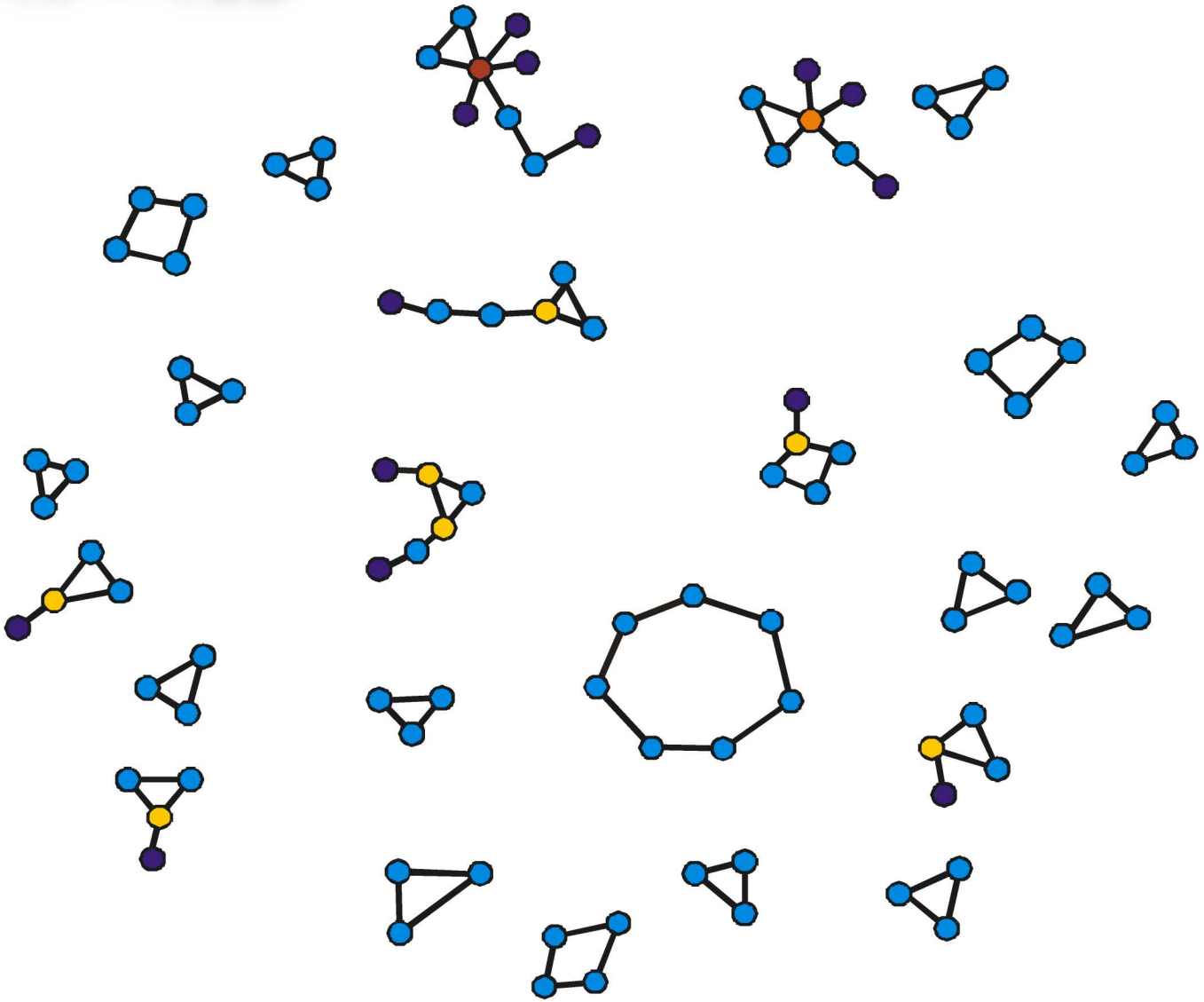
Results: N = 40



Results: N = 60



Results: N = 100



Results: Summary

- Same total investment per link.
- Same total investment per node.
- (Almost) right mean degree.
- Topologies are mostly fair.

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No Leaders? No Followers?

The adaptive Snowdrift game

So far ...

... only topological dynamics.
... no "internal" degrees of freedom.
... not really an adaptive network.

The adaptive Snowdrift game

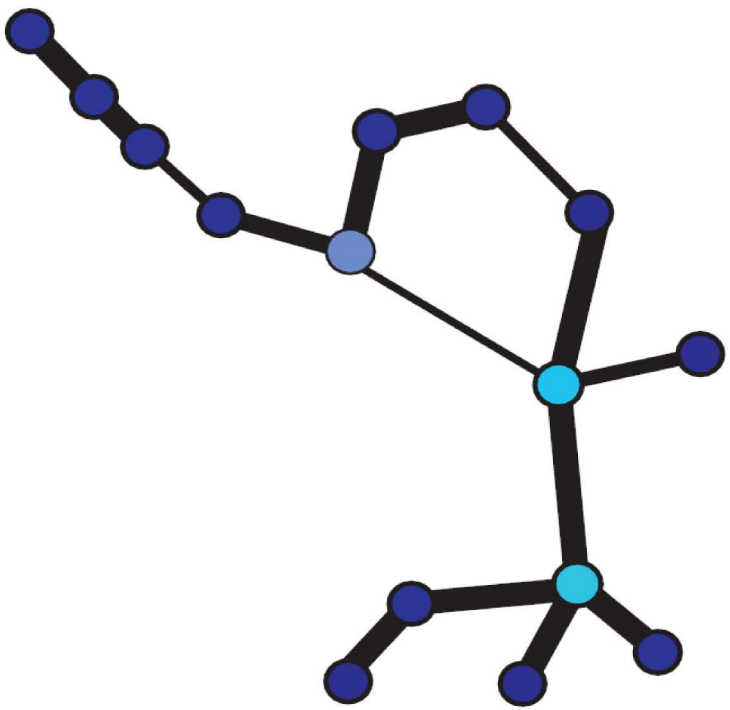
So far ...

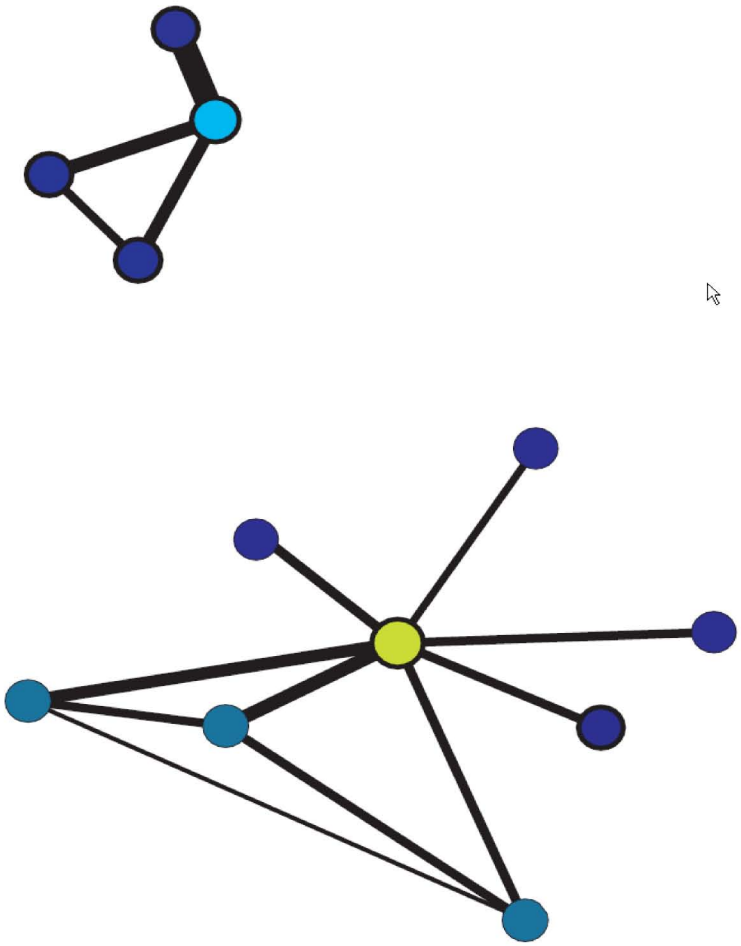
... only topological dynamics.
... no "internal" degrees of freedom.
... not really an adaptive network.

Now ...

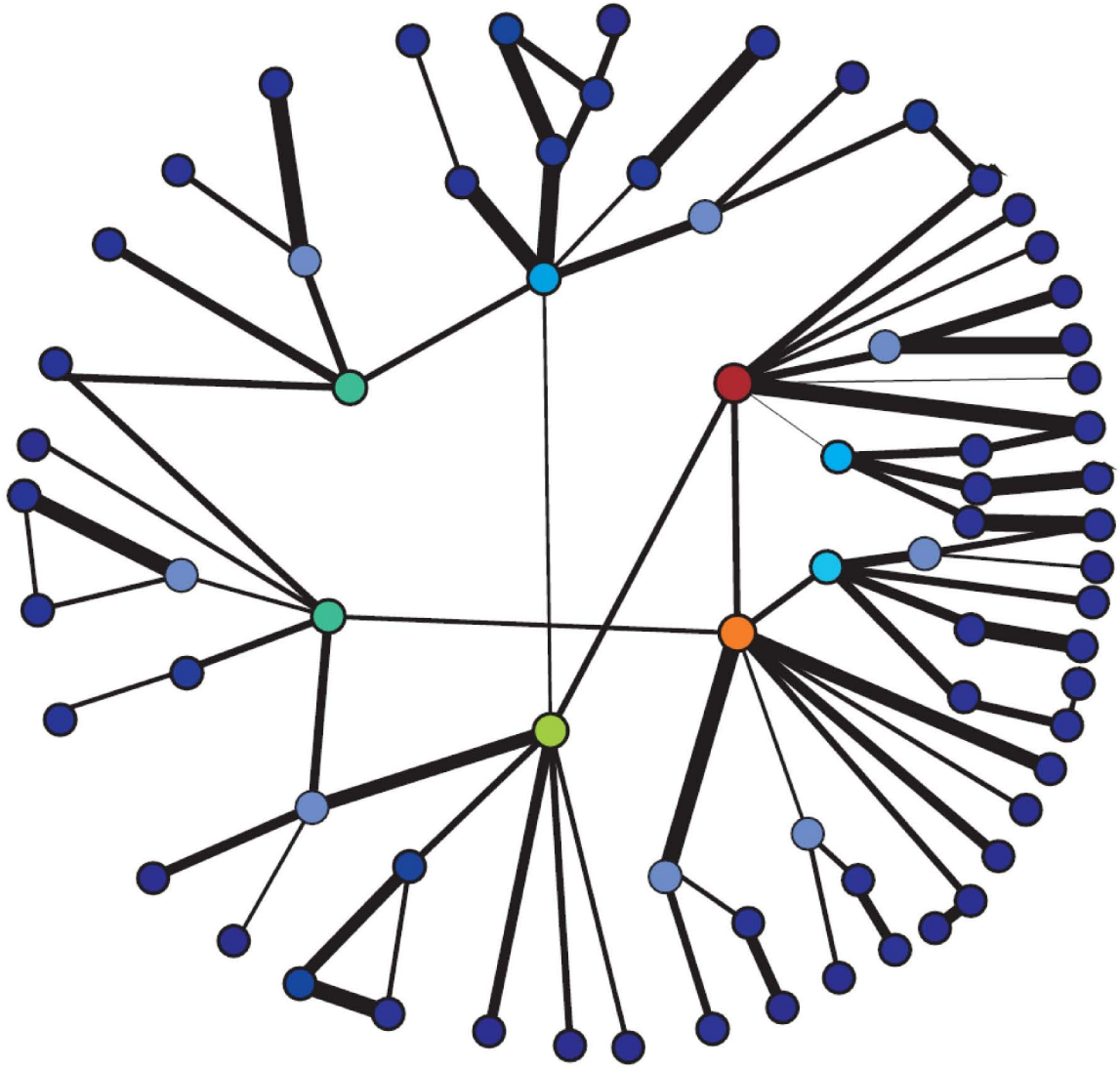
... investment becomes cheaper for successful players.

The adaptive Snowdrift game





The adaptive Snowdrift game



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Conclusion

The snowdirft game generates **relatively fair topologies** in which everybody invests the same and every link generates the same benefit.

Internal degrees of freedom turn the model into an **adaptive network**. It builds-up complex structures at the cost of fairness.

Thank you very much
for your attention