Generalized Models of Dynastic Cycles

Thilo Gross

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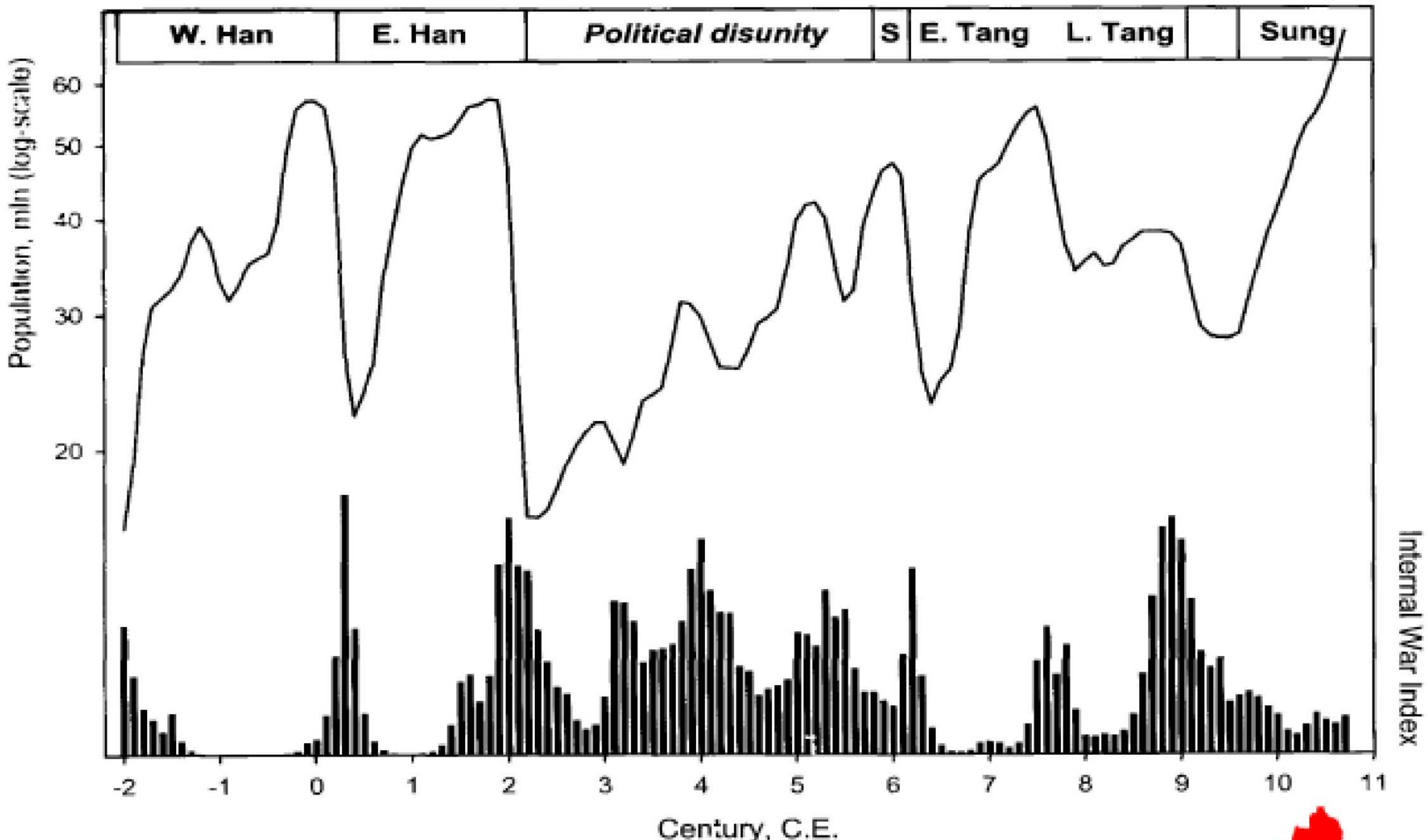
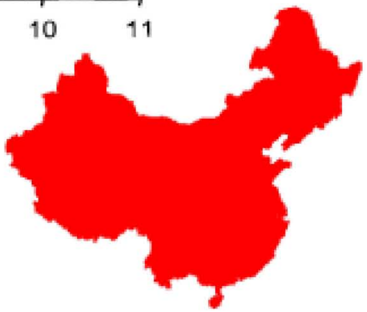
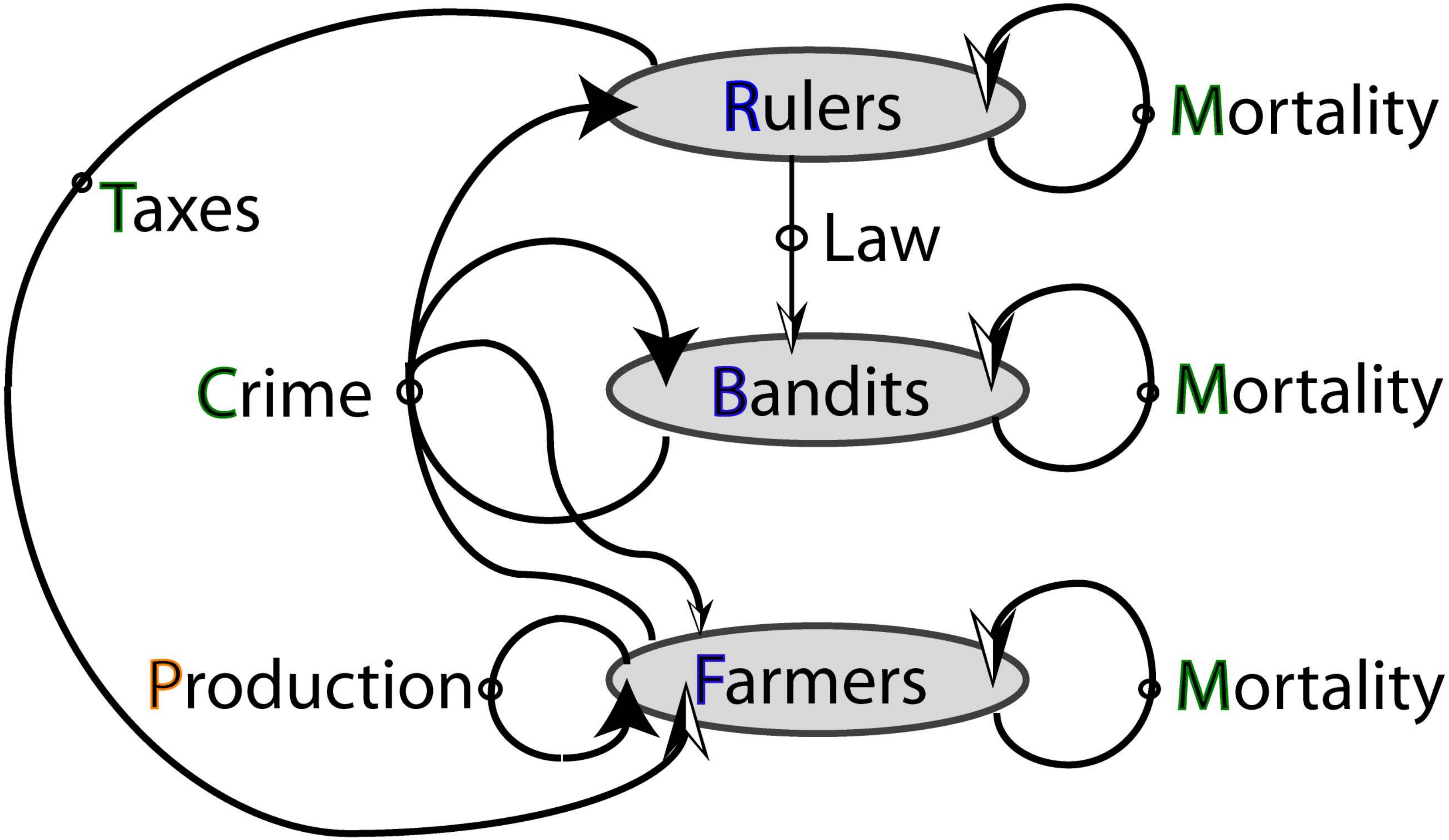
The Dynastic Cycle





Peter Turchin: *Historical Dynamics, 2003*

Dynastic Cycle



Chu and Lee, *J. Pop. Econ* (1994)

The Challenge:

Study large, heterogeneous networks ...

... based on limited information ...

... to extract qualitatitve information ....

... on the long-term dynamics.

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Generalized Models

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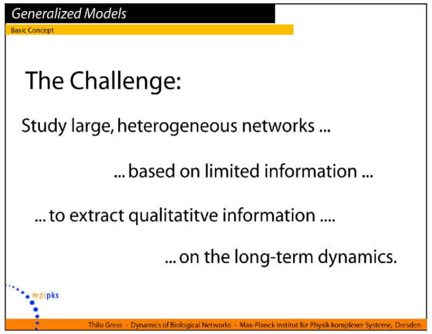
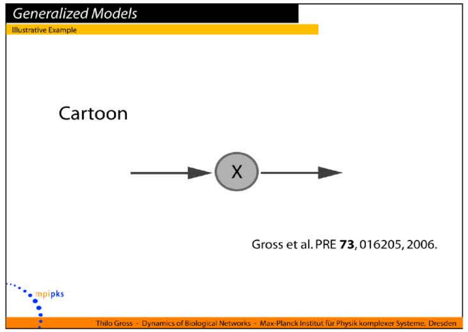
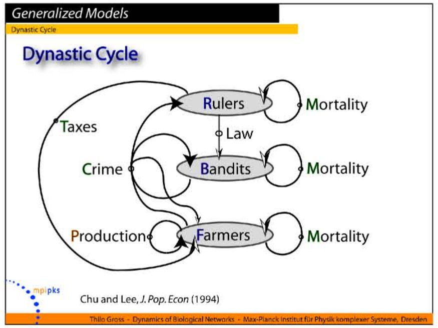
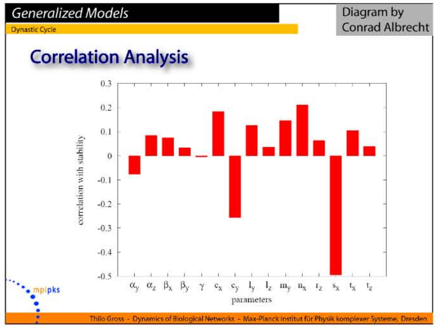
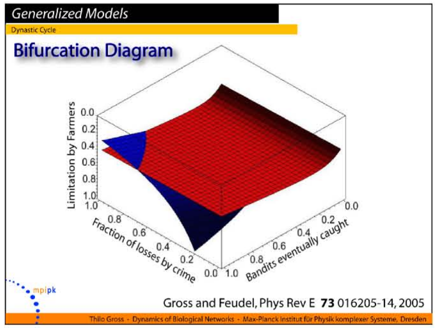
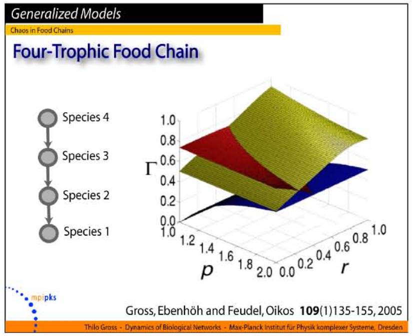


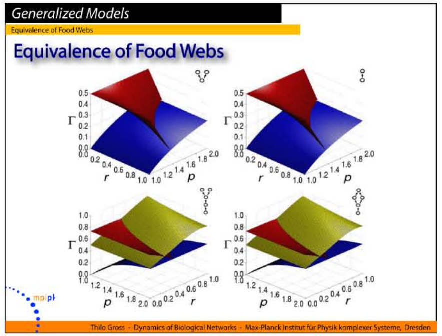


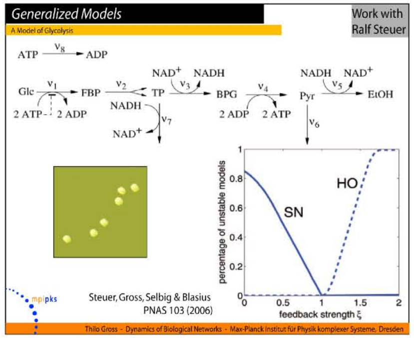


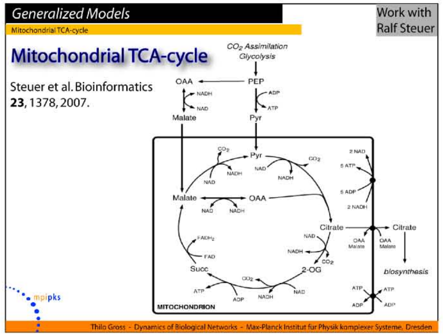


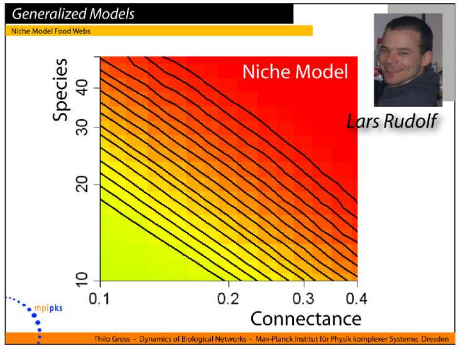




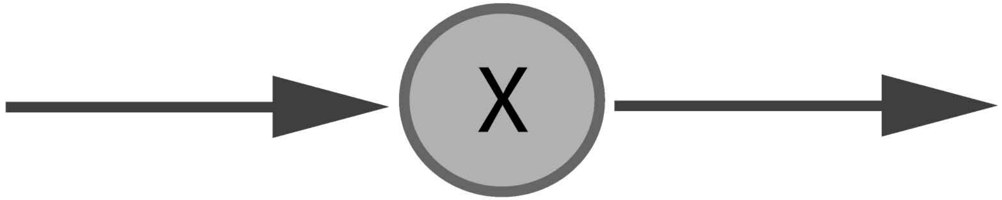






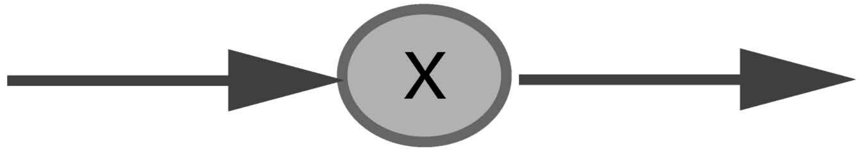


Cartoon



Gross et al. PRE **73**, 016205, 2006.

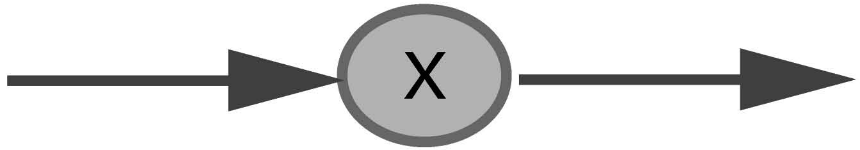
Cartoon



Differential Equation

$$\dot{X} = P(X) - L(X)$$

Cartoon



Differential Equation

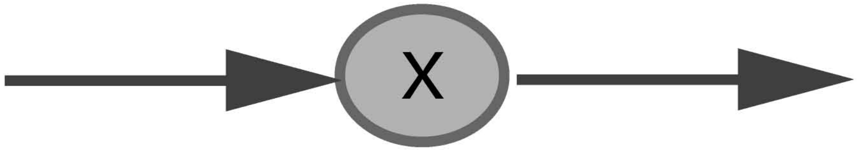
$$\dot{X} = P(X) - L(X)$$

Assume: Steady State

$$X^*, P^* = P(X^*), L^* = L(X^*)$$

Illustrative Example

Cartoon



Differential Equation

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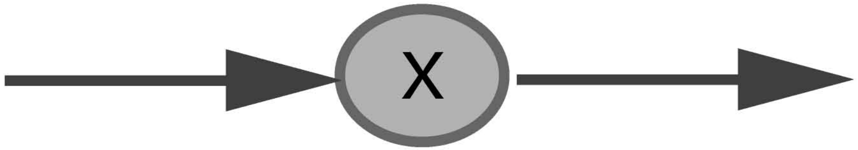
Define: Normalized Variables

$$x = \frac{X}{X^*}, p(x) = \frac{P(X)}{P^*}, l(x) = \frac{L(X)}{L^*}$$

Cartoon



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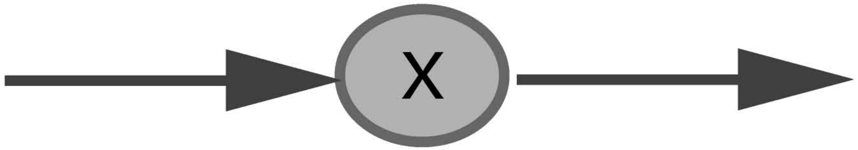
Normalized System

$$\dot{x} = \frac{P^*}{X^*}p(x) - \frac{L^*}{X^*}l(x)$$

Cartoon



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$$\dot{x} = \frac{P^*}{X^*} p(x) - \frac{L^*}{X^*} l(x)$$

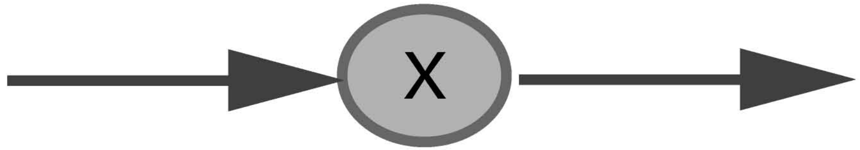
Identify: Parameters

$$\alpha = \frac{P^*}{X^*} = \frac{L^*}{X^*}$$

Generalized Models

Illustrative Example

Cartoon



Differential Equation

$$\dot{X} = P(X) - L(X)$$

Normalized System

$$\dot{x} = \alpha(p(x) - l(x))$$

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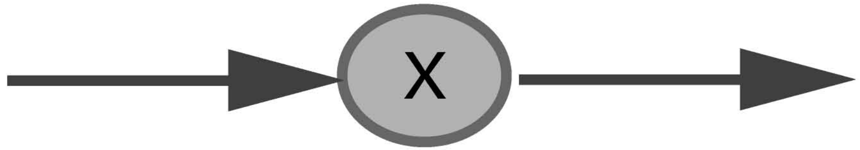
Identify: Parameters

$$\alpha = \frac{P^*}{X^*} = \frac{L^*}{X^*}$$

Compute: Jacobian

$$\mathbf{J} = \alpha(p'(1) - l'(1))$$

Cartoon



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Identify: Parameters

$$\alpha = \frac{P^*}{X^*} = \frac{L^*}{X^*}$$

$$\phi = p'(1)$$

$$\lambda = l'(1)$$

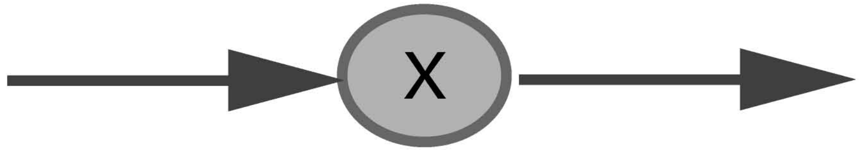
Compute: Jacobian

$$\mathbf{J} = \alpha(\phi - \lambda)$$

Generalized Models

Illustrative Example

Cartoon



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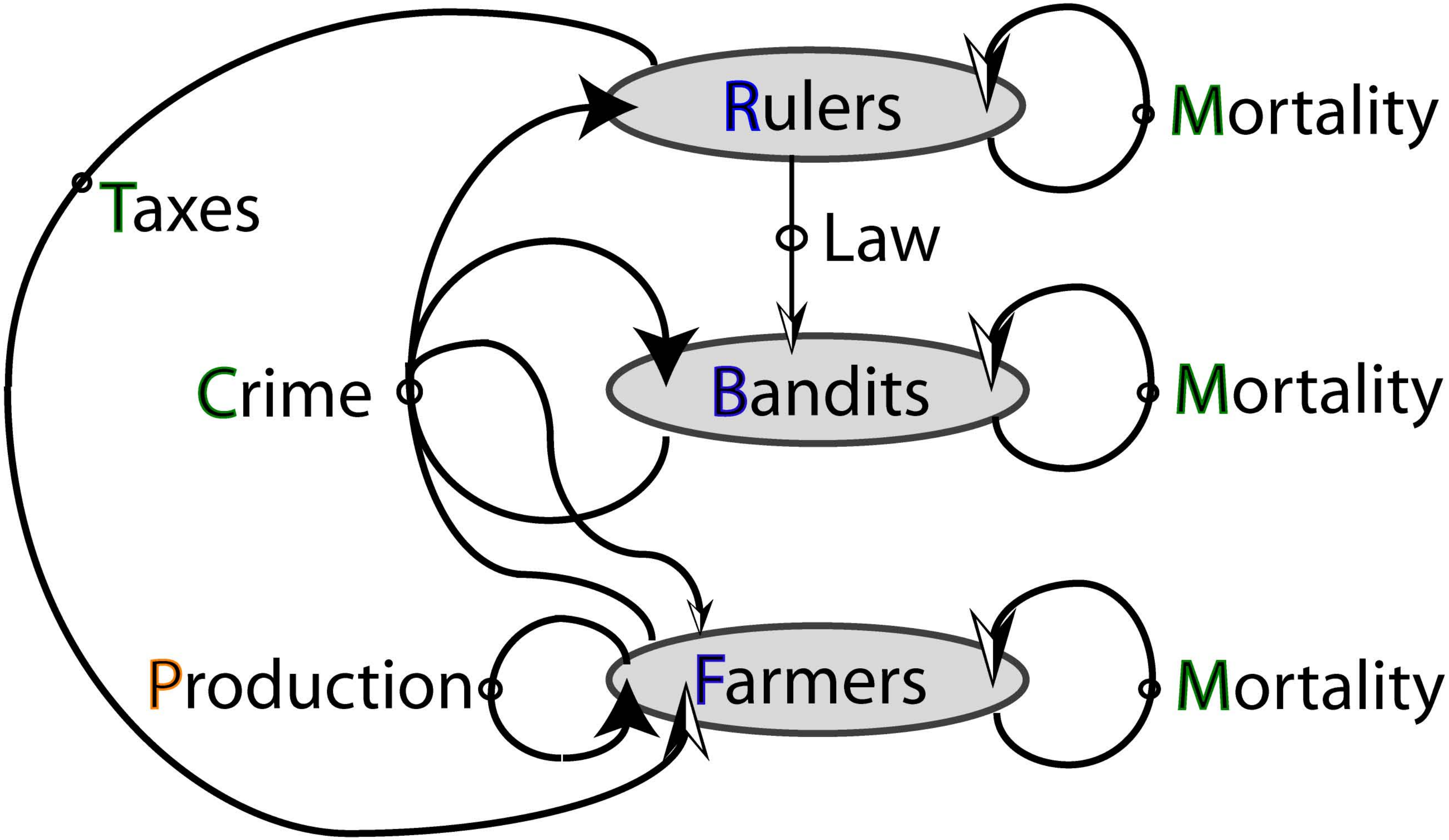
Dynastic Cycle



Chu and Lee, *J. Pop. Econ* (1994)

The Dynastic Cycle

$$\dot{F} = P(F) - C(F,B) - T(F,R) - M(F)$$
$$\dot{B} = C(F,B) - L(R,B) - M(B)$$
$$\dot{R} = C(F,B) - M(R)$$

A **generalized model** of the dynastic cycle

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Gross and Feudel, Phys Rev E **73** 016205-14, 2005

The Dynastic Cycle

$$\dot{F} = P(F) - C(F,B) - T(F,R) - M(F)$$
$$\dot{B} = C(F,B) - L(R,B) - M(B)$$
$$\dot{R} = C(F,B) - M(R)$$

Assume that there is a steady state, then we can define

$$f := \frac{F}{F^*}, \quad p(f) := \frac{P(F)}{P(F^*)}, \quad \ldots$$

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Gross and Feudel, Phys Rev E **73** 016205-14, 2005

The Dynastic Cycle

$$
\begin{aligned}
\dot{f} &= \frac{P^*}{F^*}p(f) - \frac{C^*}{F^*}c(f,b) - \frac{T^*}{F^*}t(f,r) - \frac{M^*}{F^*}m(f) \\
\dot{b} &= \frac{C^*}{B^*}c(f,b) - \frac{L^*}{B^*}l(r,b) - \frac{M^*}{B^*}m(b) \\
\dot{r} &= \frac{C^*}{R^*}c(f,b) - \frac{E^*}{R^*}e(r)
\end{aligned}
$$

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Gross and Feudel, Phys Rev E **73** 016205-14, 2005

The Dynastic Cycle

$$\dot{f} = \alpha_f(p - (\beta c - (1-\beta)t)\rho - (1-\rho)m)$$
$$\dot{b} = \alpha_b(c - \gamma l - (1-\gamma)m)$$
$$\dot{r} = \alpha_r(c - m)$$

We have defined the scale parameters

$\alpha_f = \frac{P^*}{F^*}$ inverse life expectancy of farmers

$\gamma = \frac{1}{\alpha_b}\frac{L^*}{B^*}$ fraction of bandits that get eventually caught

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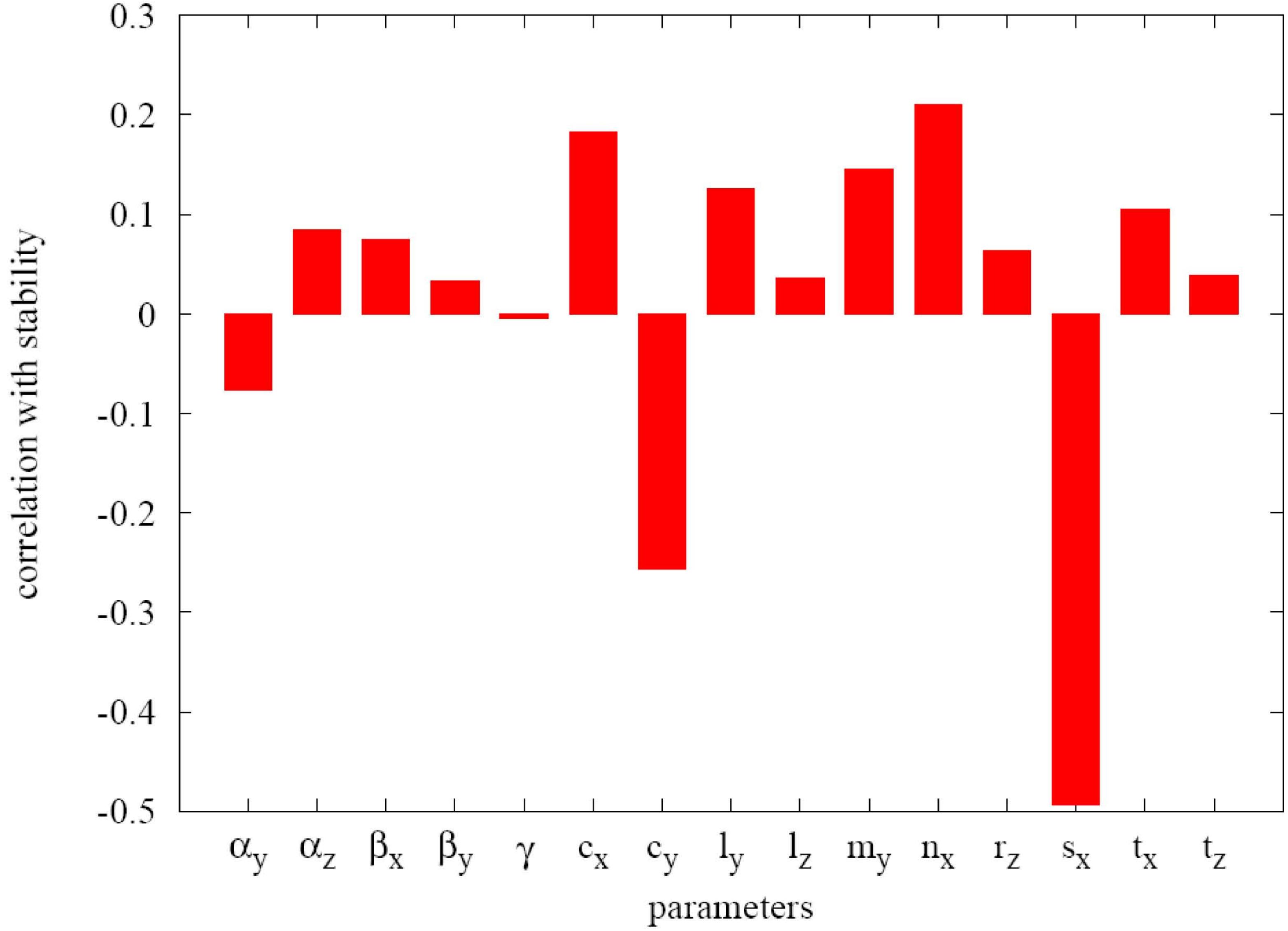
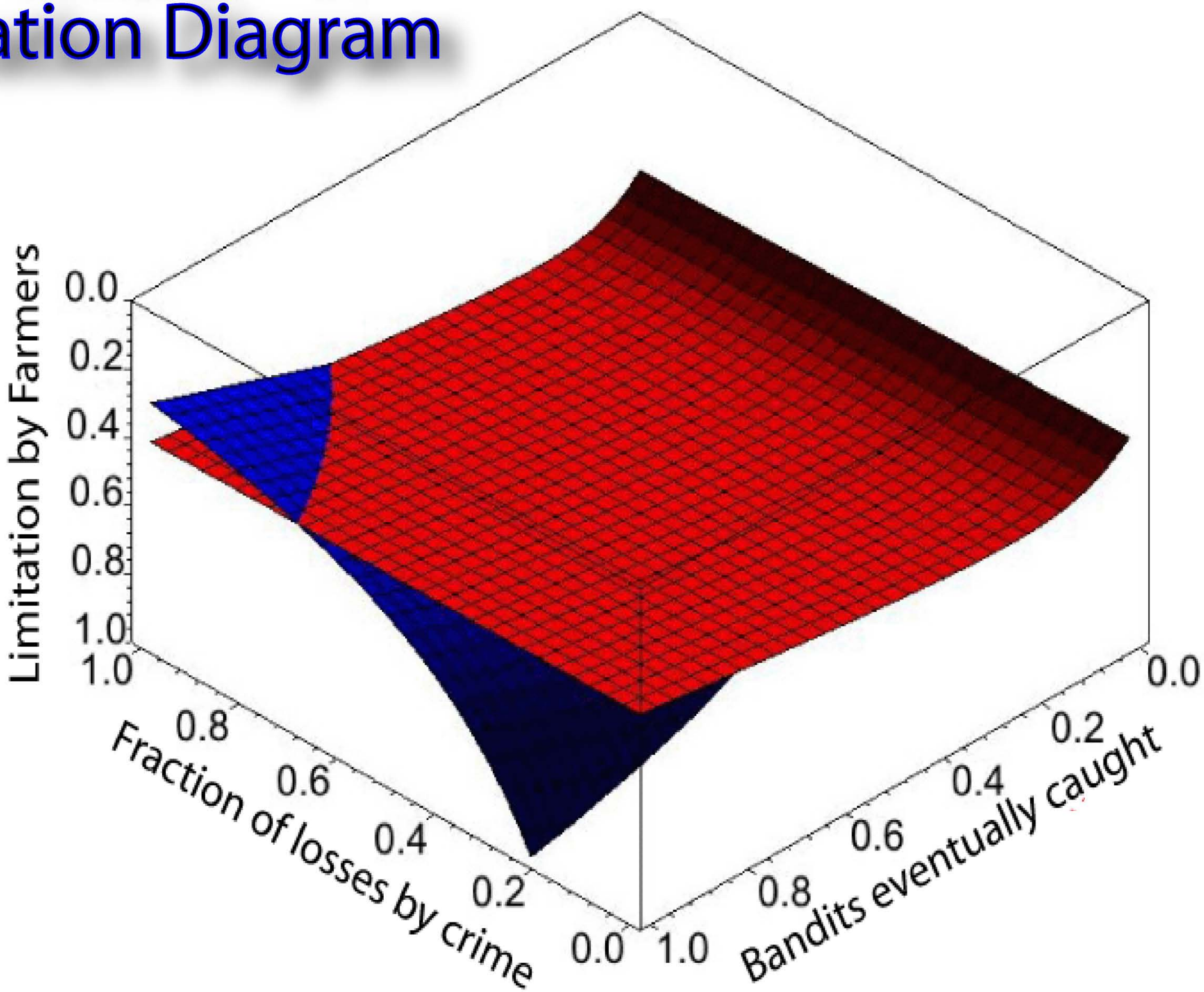
Gross and Feudel, Phys Rev E **73** 016205-14, 2005

Diagram by Conrad Albrecht

Correlation Analysis



Bifurcation Diagram



Gross and Feudel, Phys Rev E **73** 016205-14, 2005

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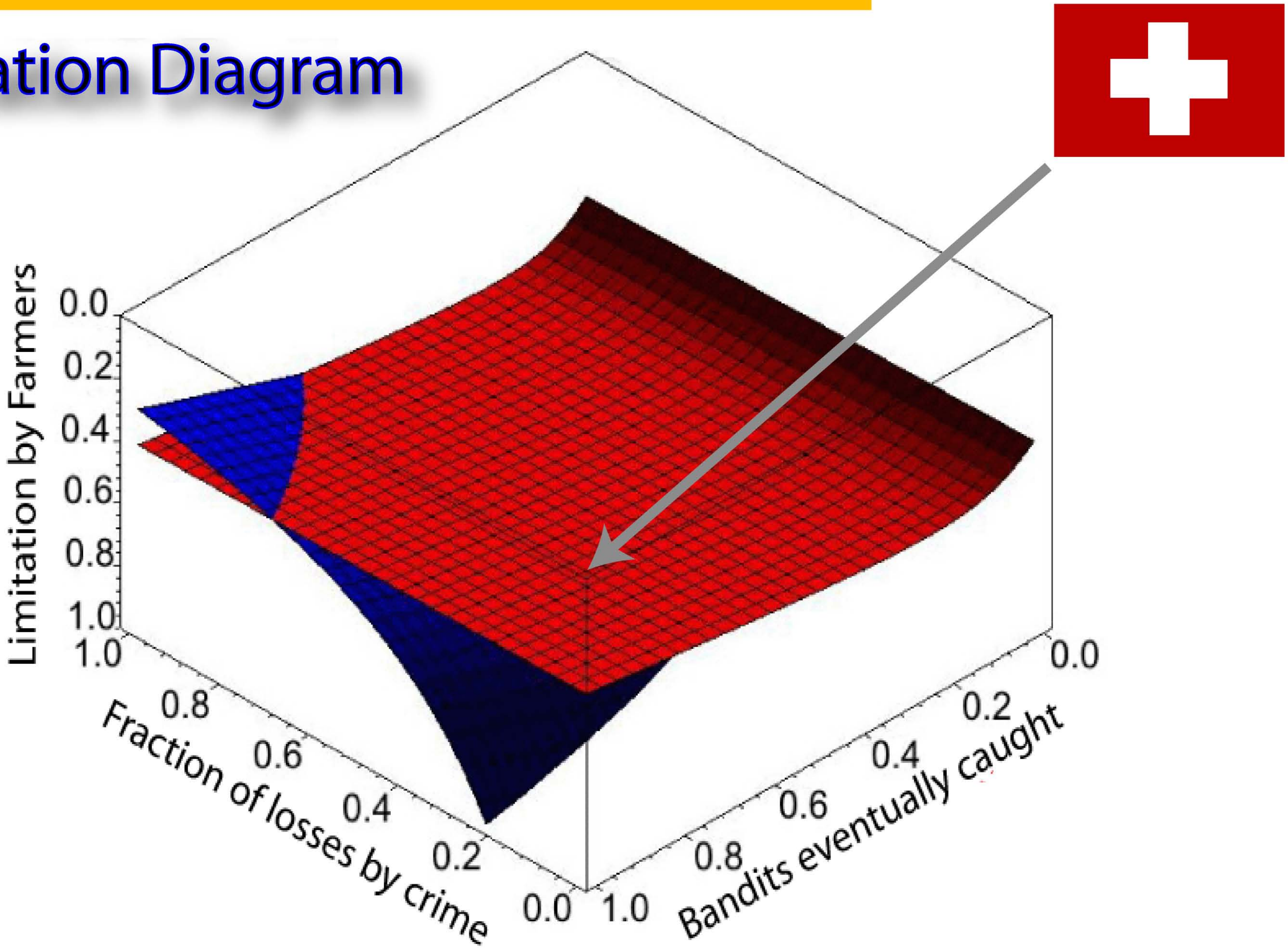
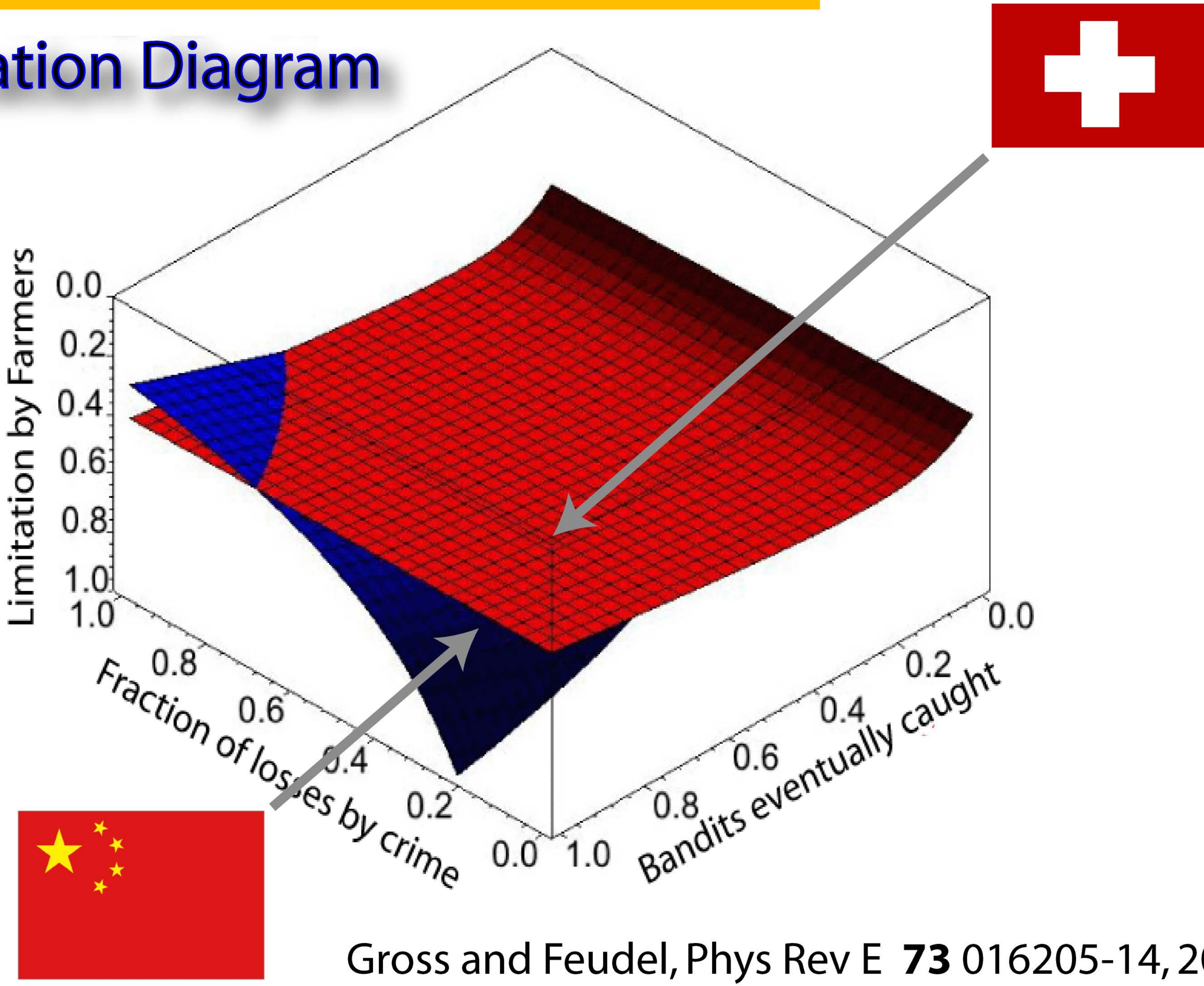


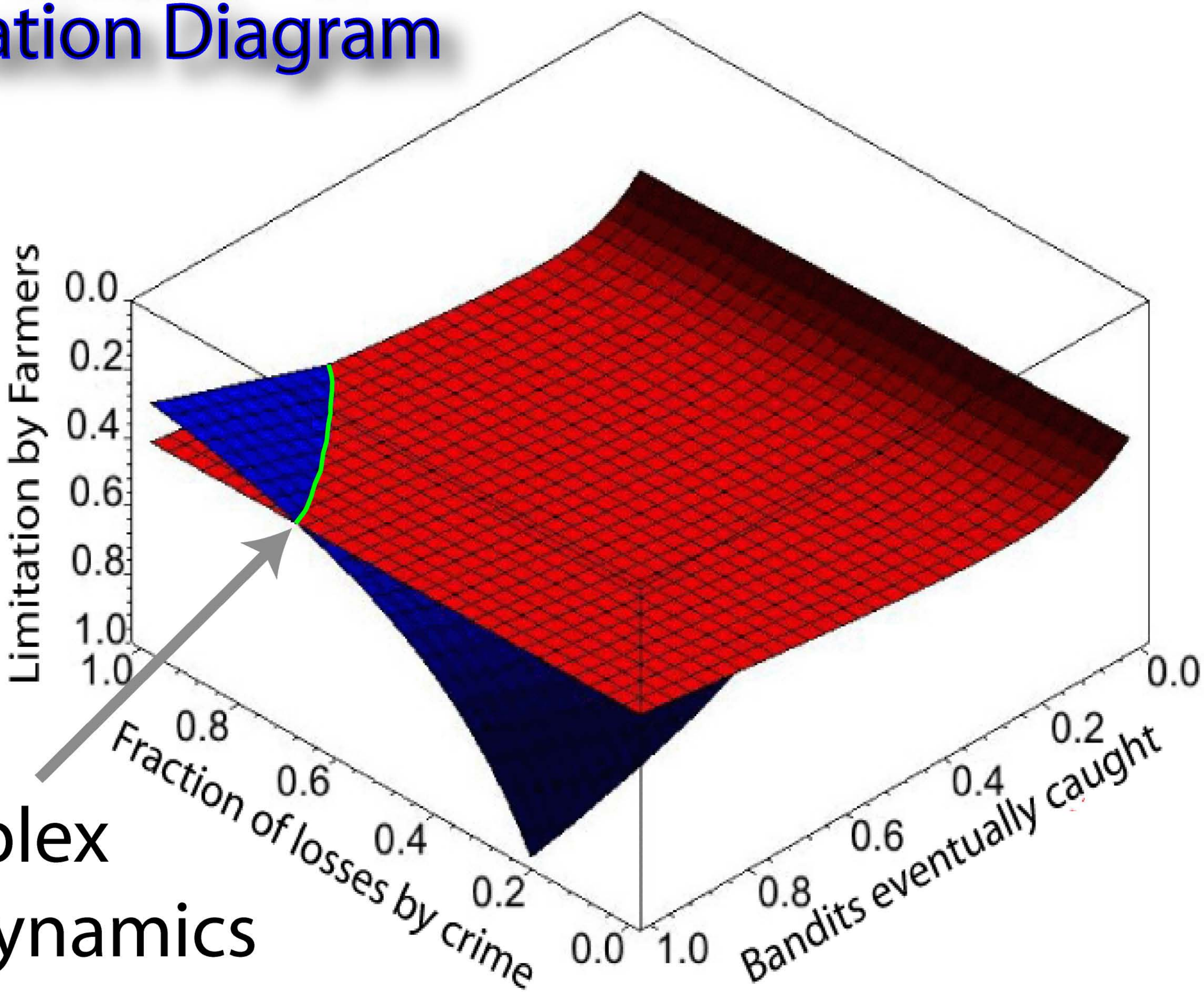
Gross and Feudel, Phys Rev E **73** 016205-14, 2005

Bifurcation Diagram



Gross and Feudel, Phys Rev E **73** 016205-14, 2005

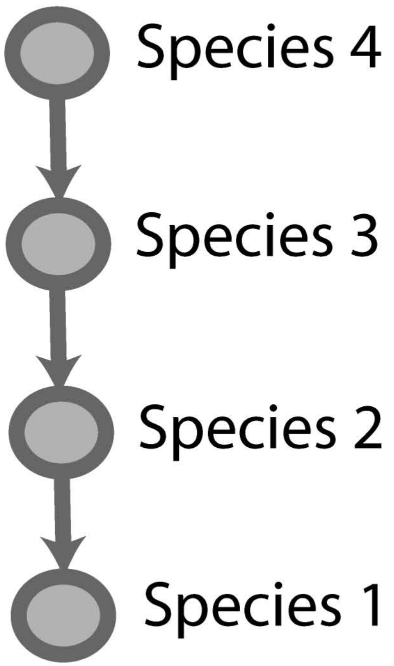
Bifurcation Diagram

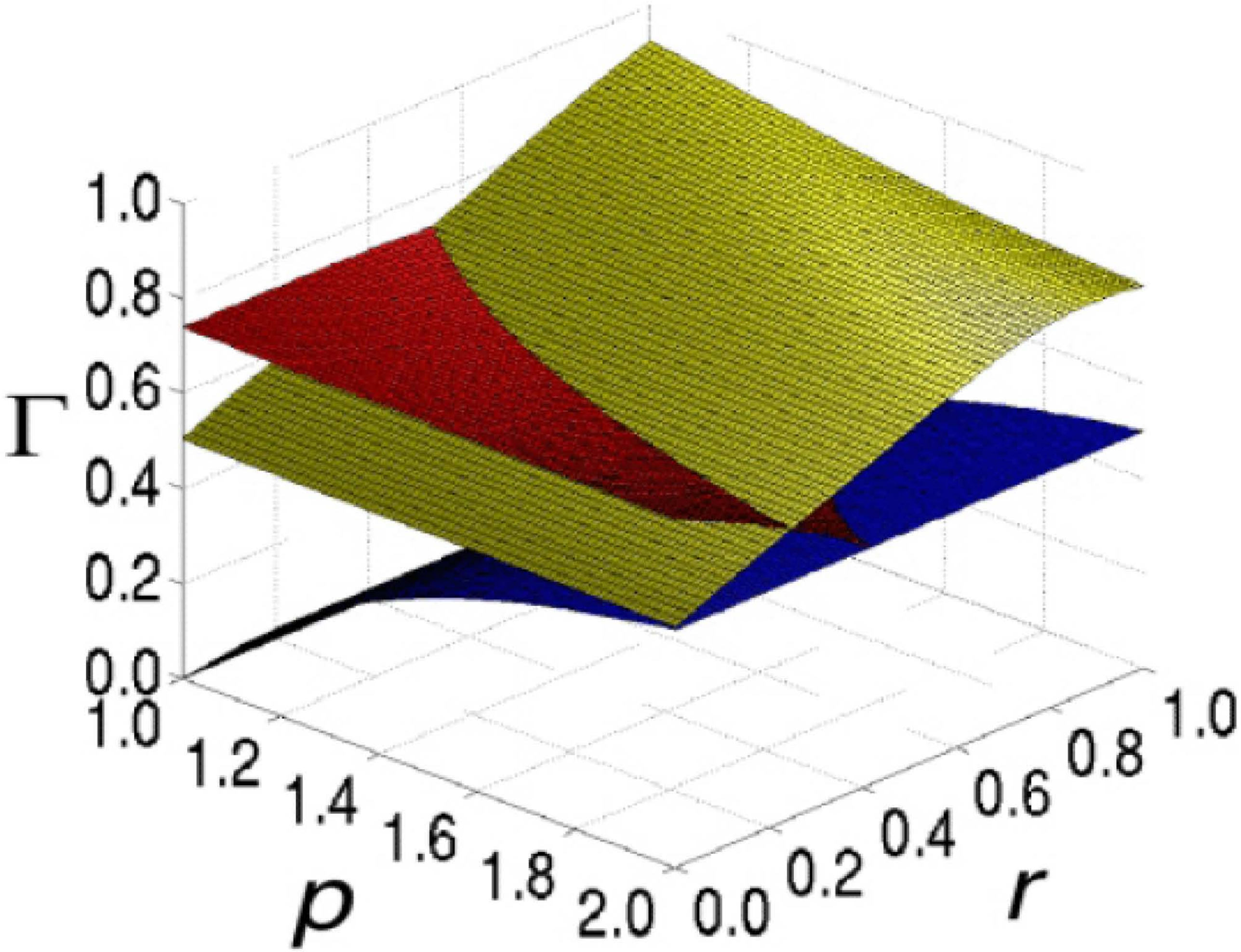


Complex Dynamics

Gross and Feudel, Phys Rev E **73** 016205-14, 2005

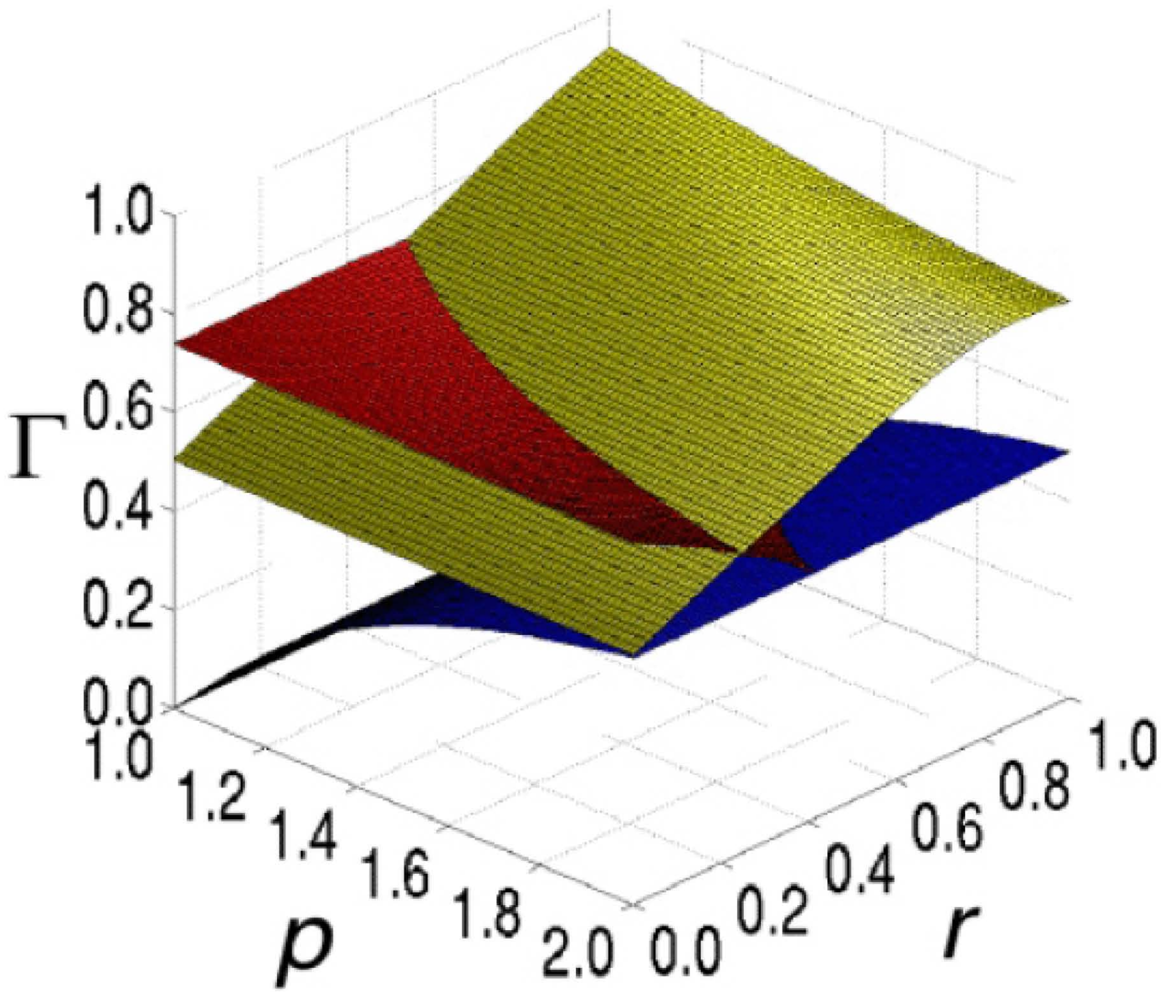
Four-Trophic Food Chain

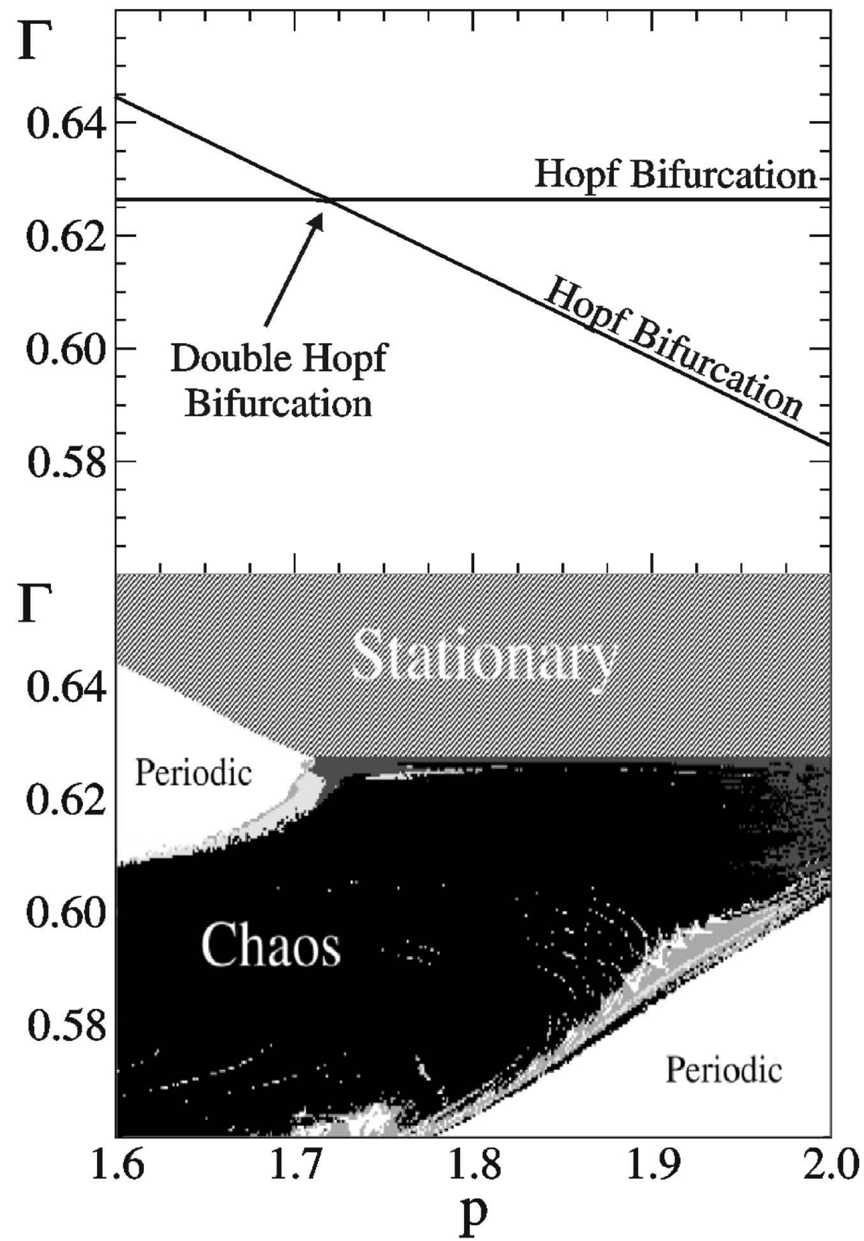




Gross, Ebenhöh and Feudel, Oikos **109**(1)135-155, 2005

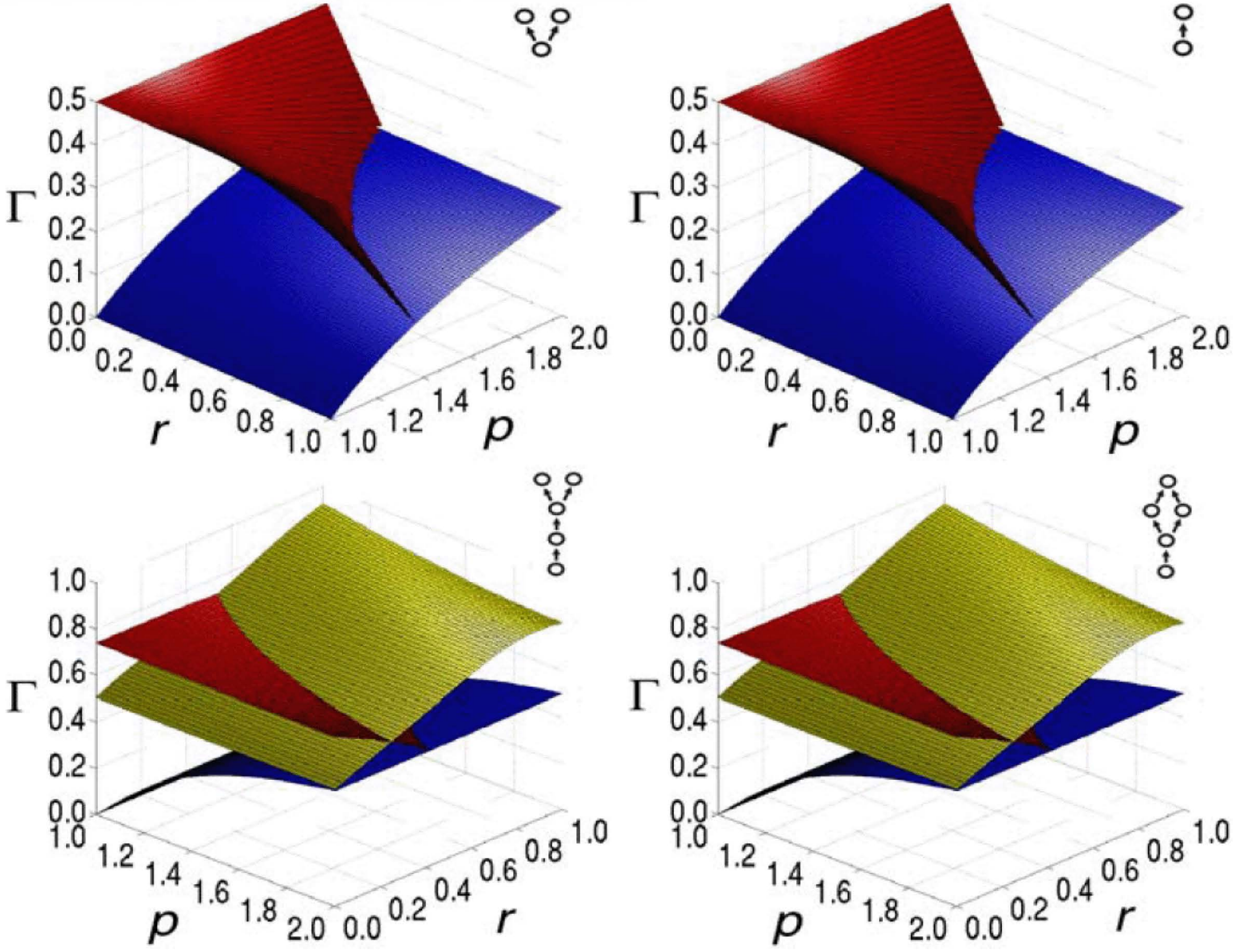
Chaotic Dynamics





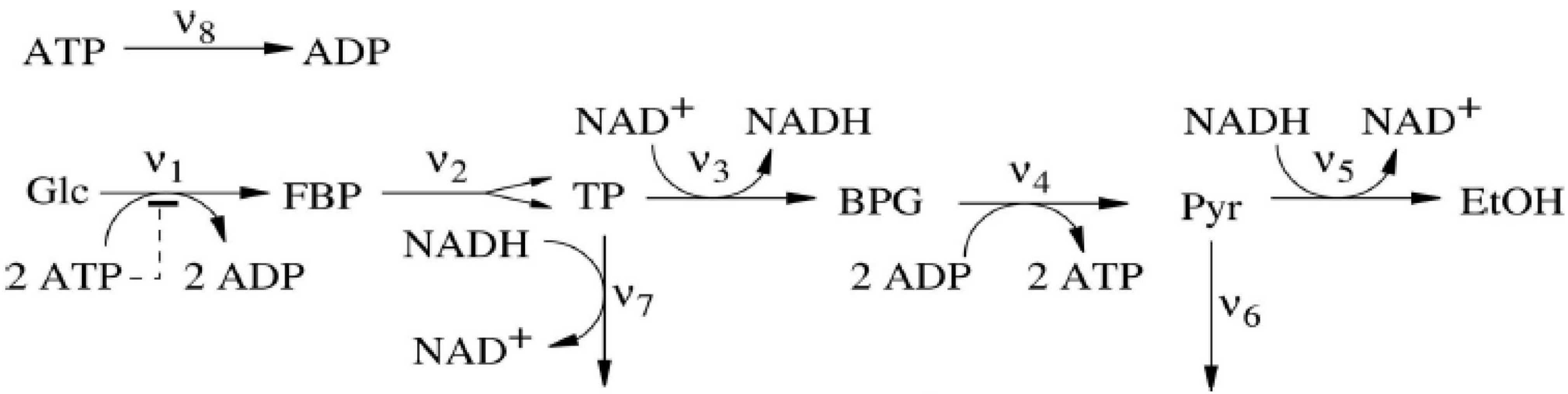
Gross, Ebenhöh and Feudel, Oikos **109**(1)135-155, 2005

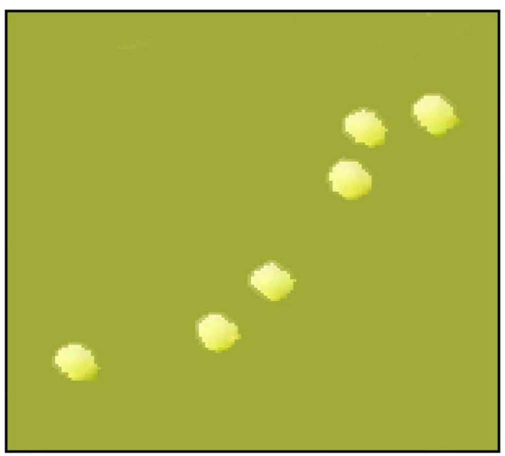
Equivalence of Food Webs

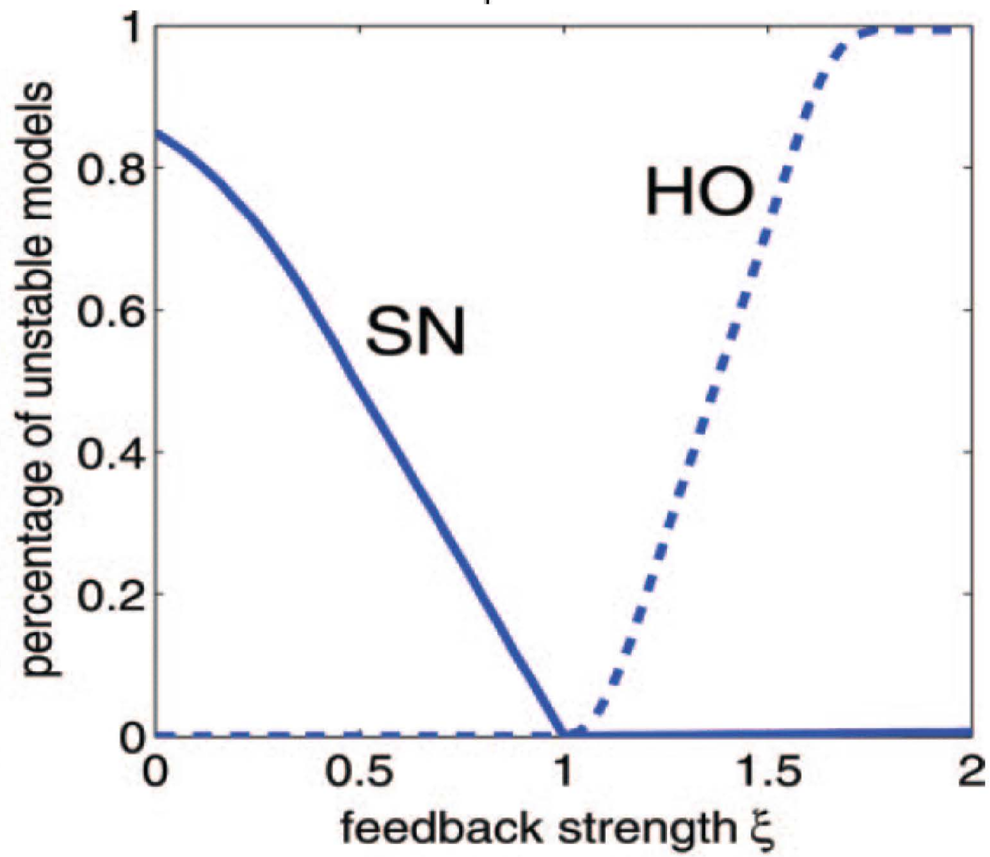


Generalized Models

A Model of Glycolysis





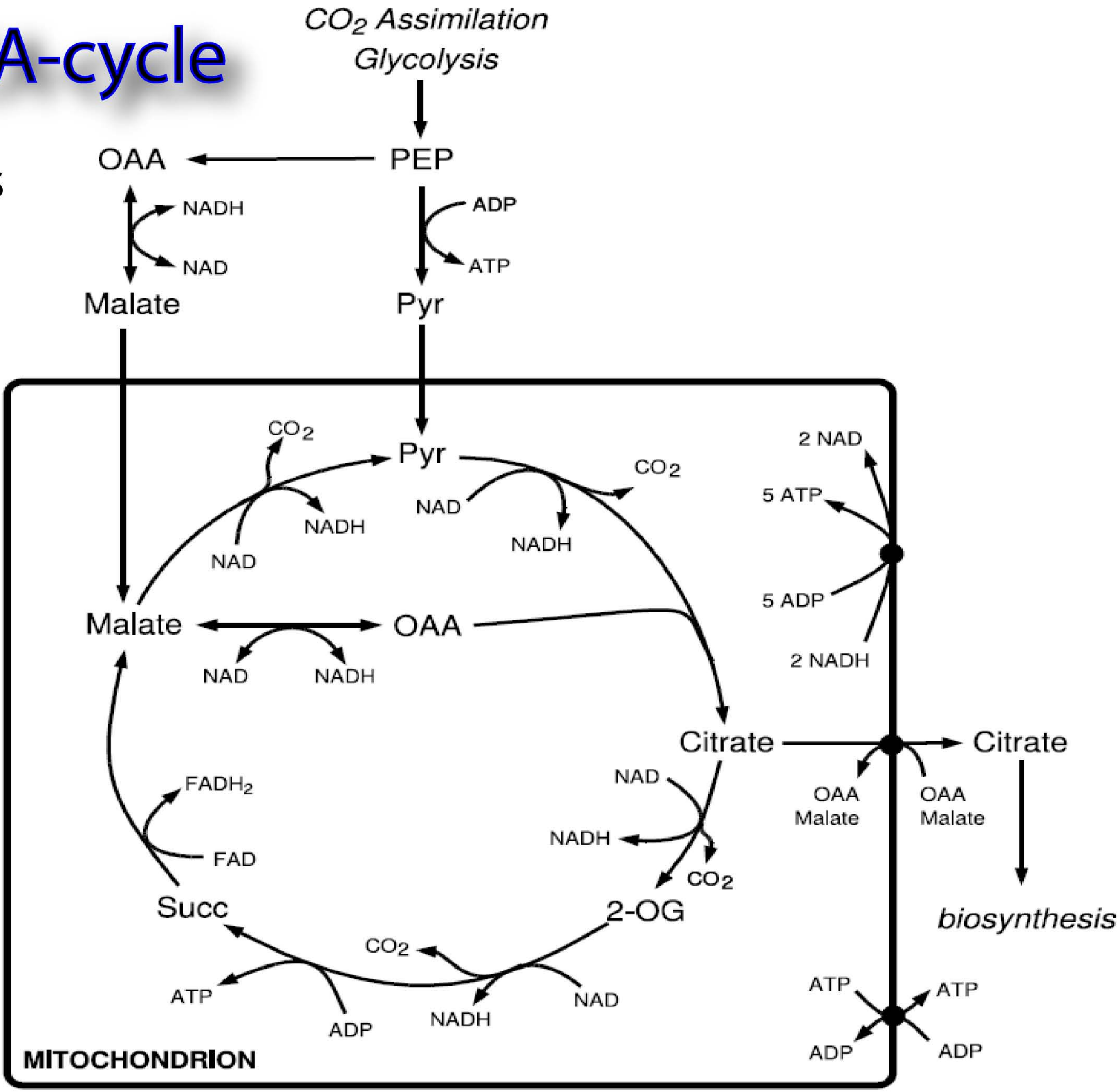


Steuer, Gross, Selbig & Blasius
PNAS 103 (2006)

Work with
Ralf Steuer

Mitochondrial TCA-cycle

Steuer et al. Bioinformatics **23**, 1378, 2007.



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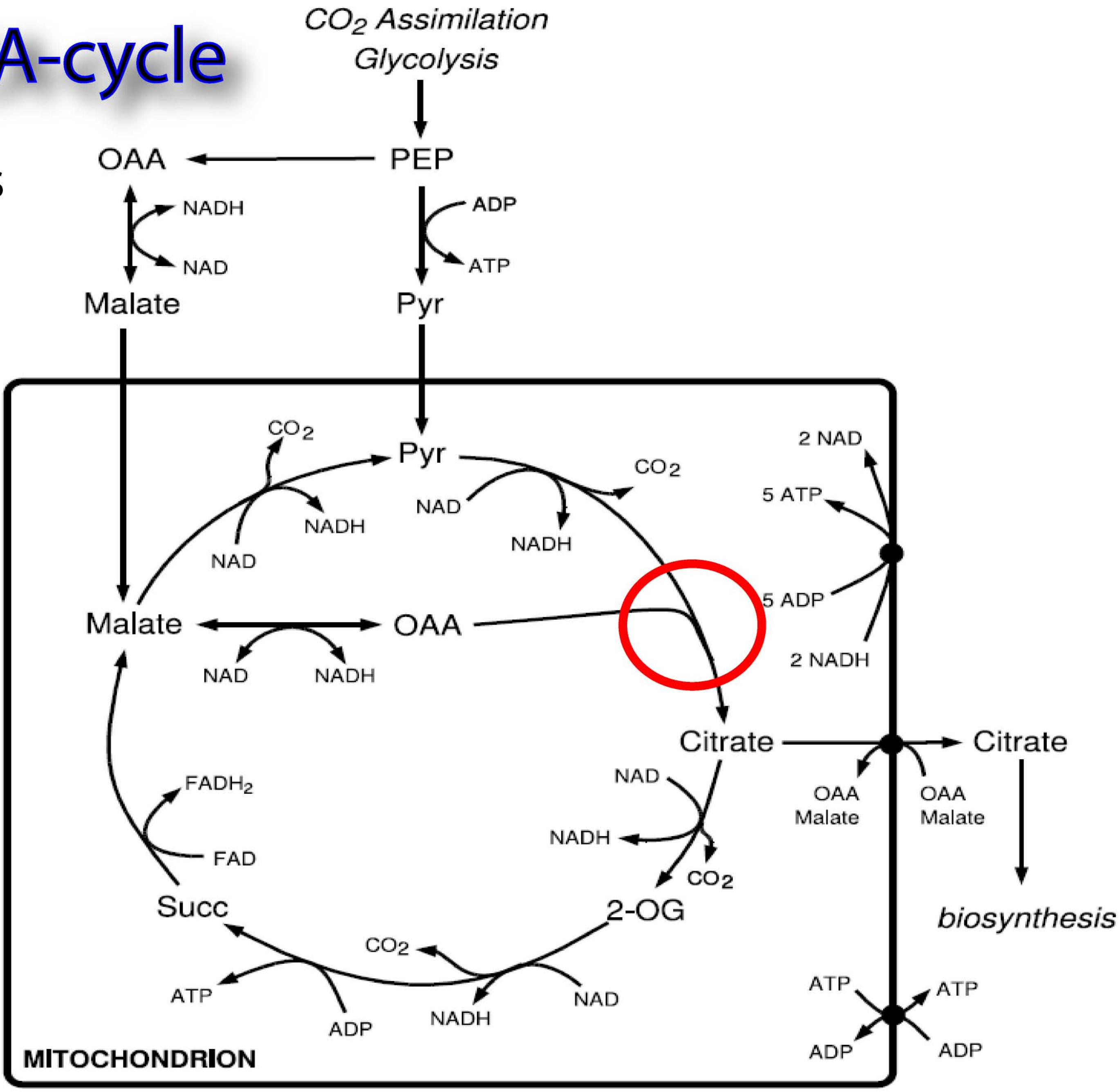
Steuer et al. Bioinformatics **23**, 1378, 2007.

No stable stationary states for mass action kinetics



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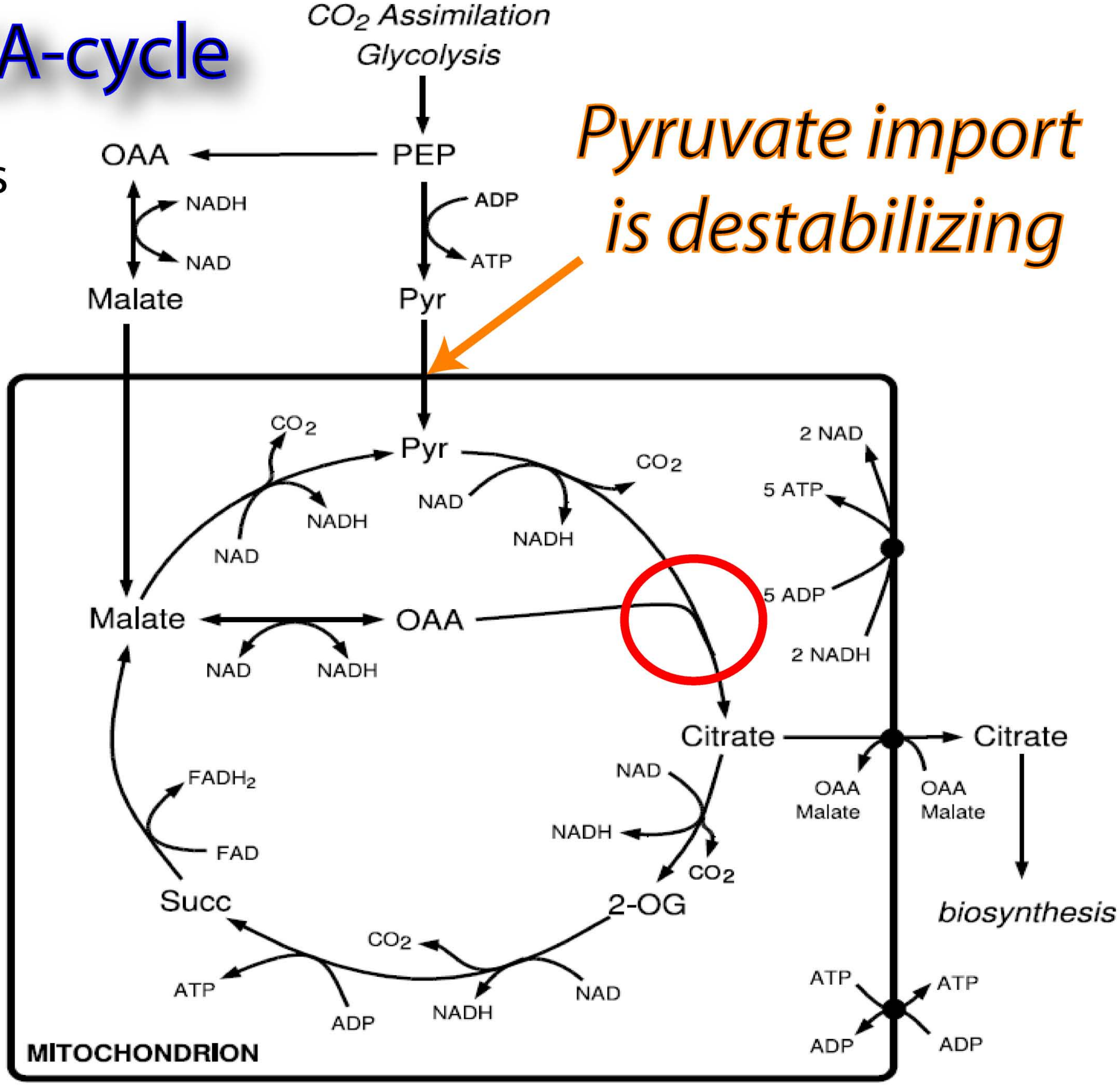
Mitochondrial TCA-cycle

Steuer et al. Bioinformatics **23**, 1378, 2007.

No stable stationary states for mass action kinetics

Pyruvate import is destabilizing



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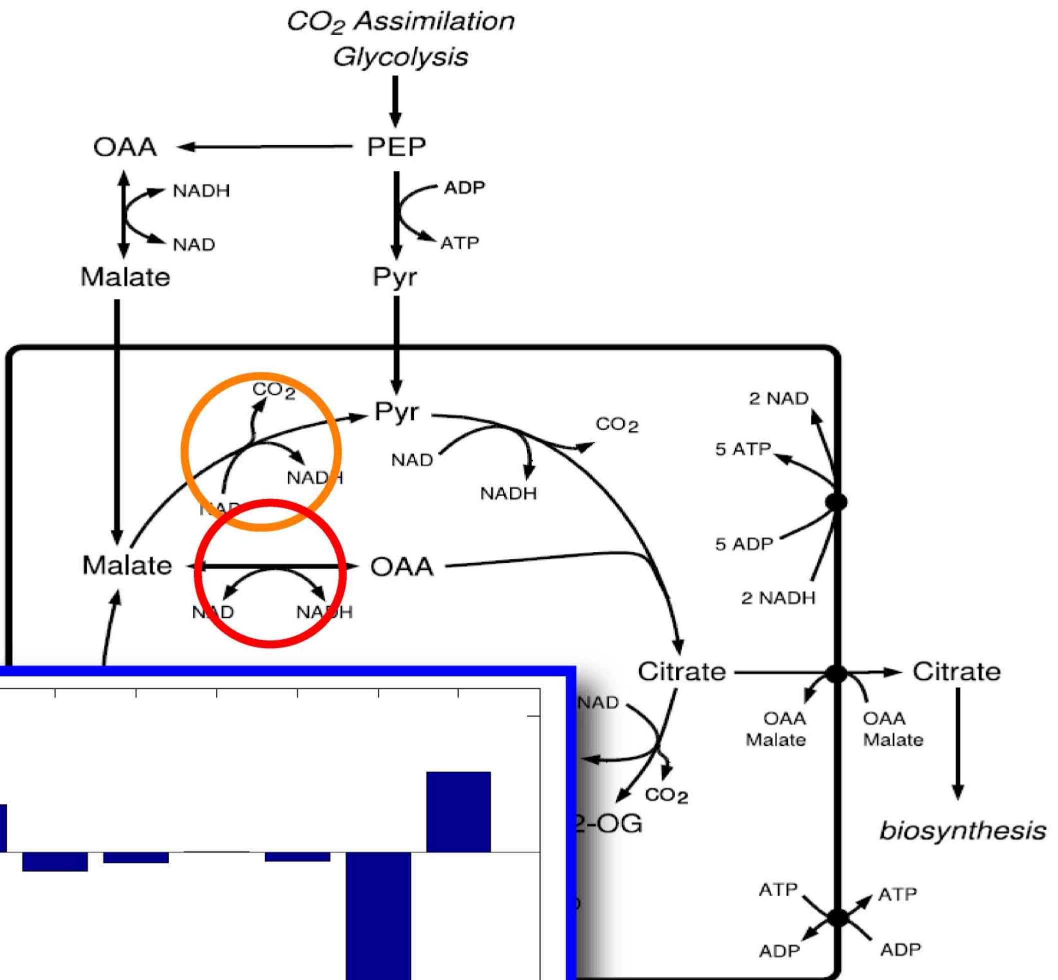
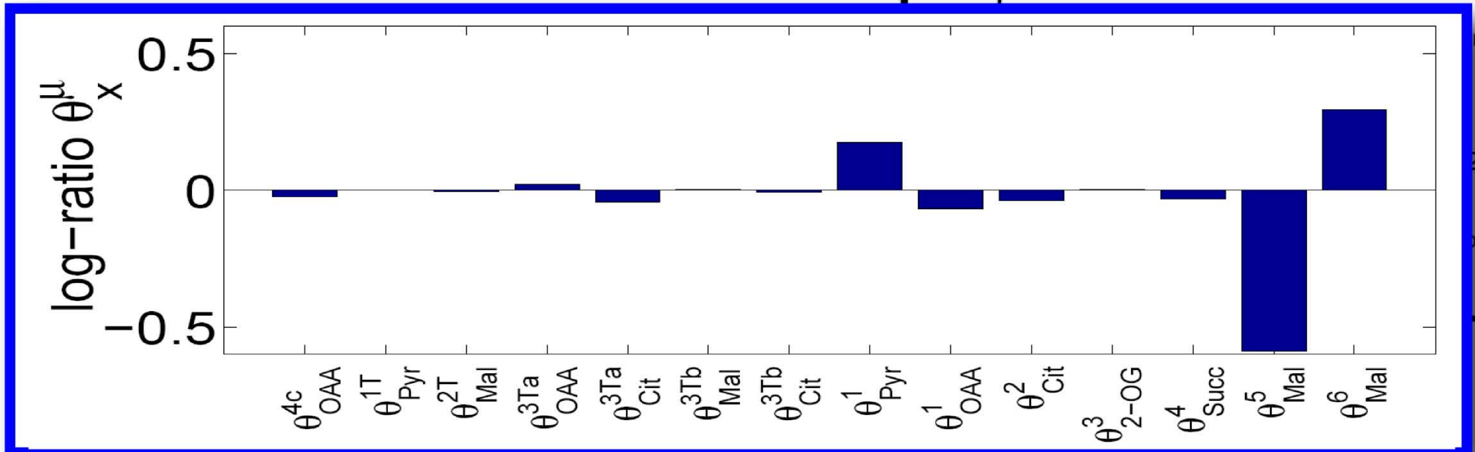
Weak Saturation of NAD-malic reaction



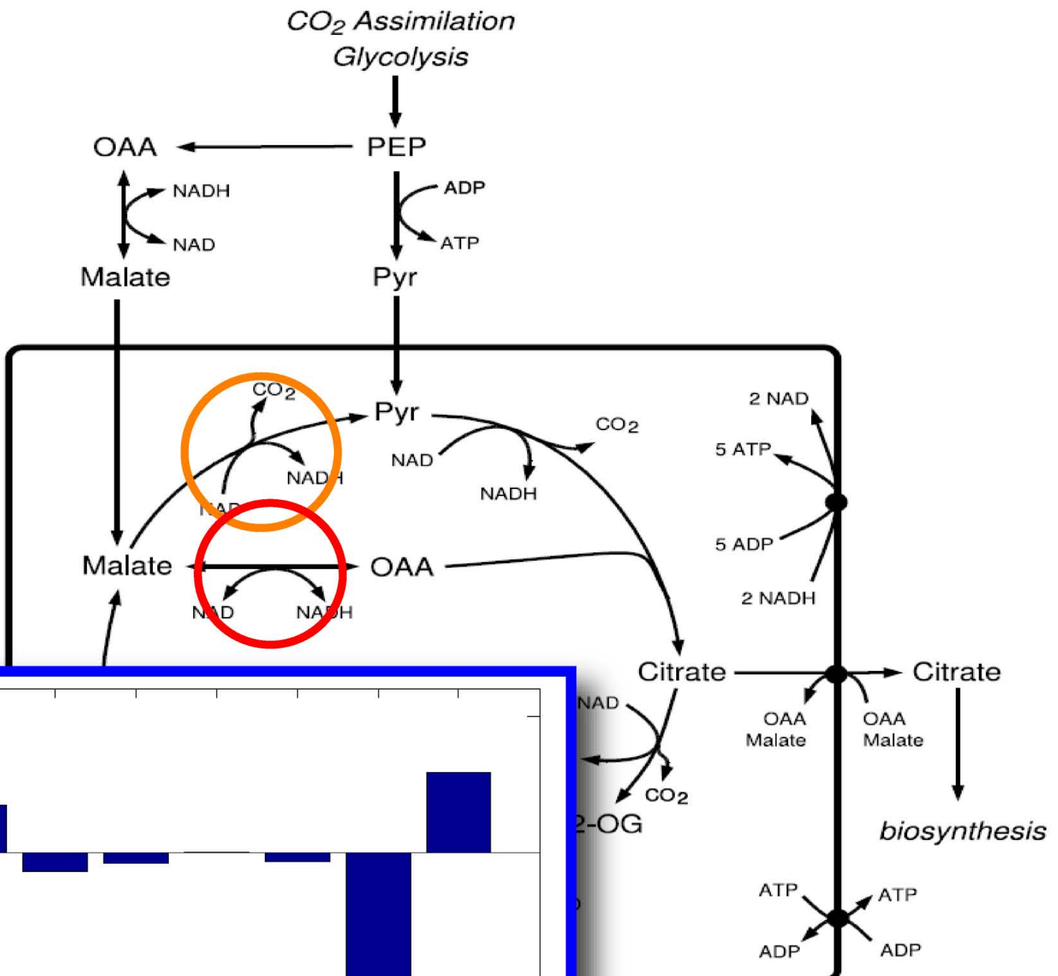


Strong saturation of malatedehydogenase

Work with Ralf Steuer

Mitochondrial TCA-cycle

Steuer et al. Bioinformatics **23**, 1378, 2007.



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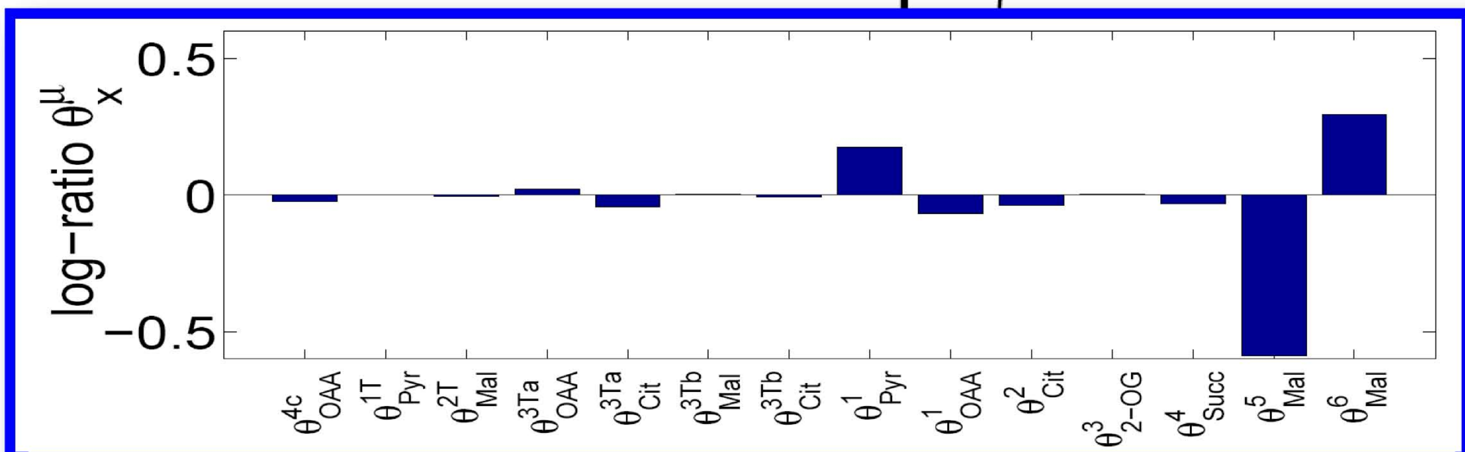

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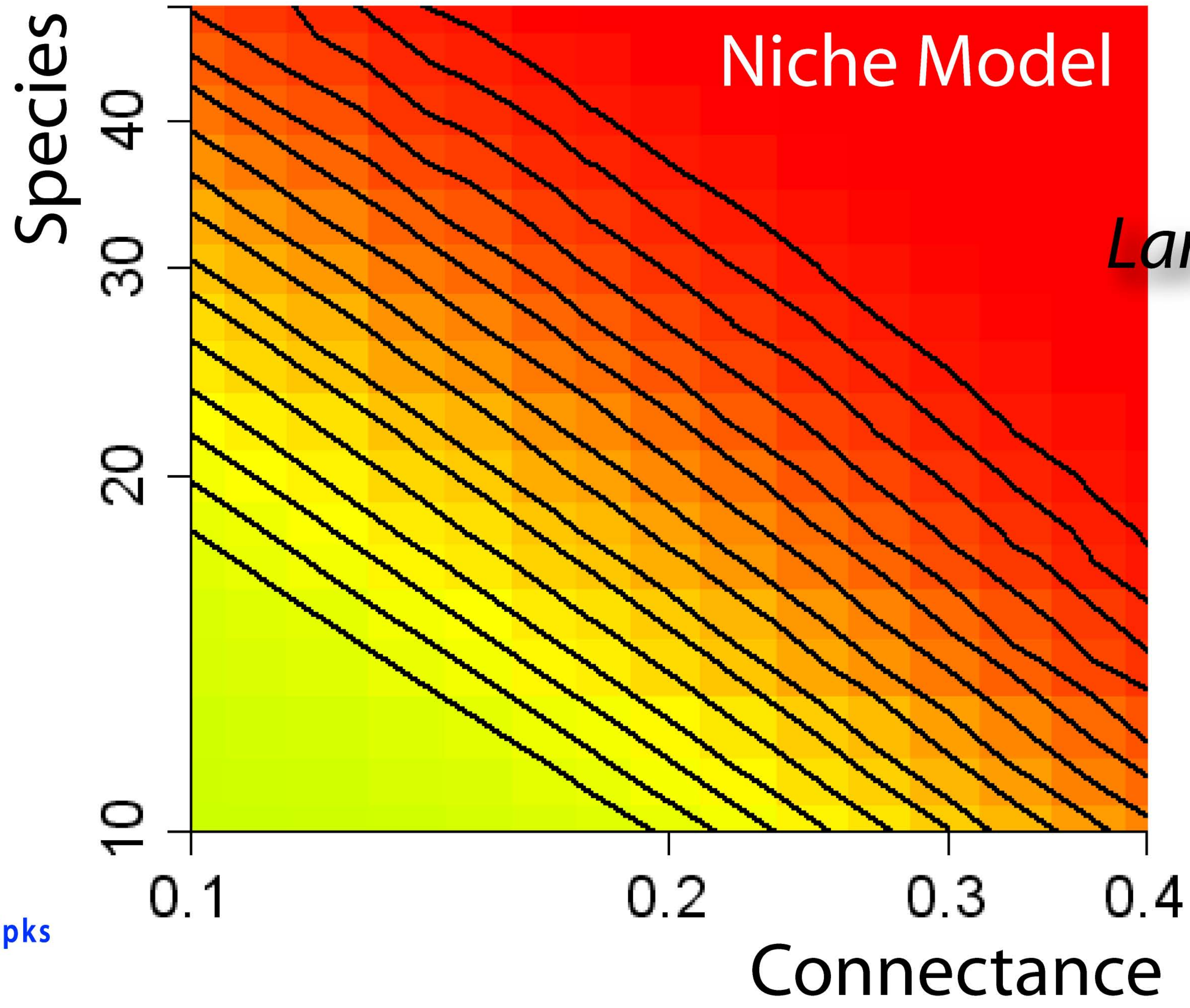
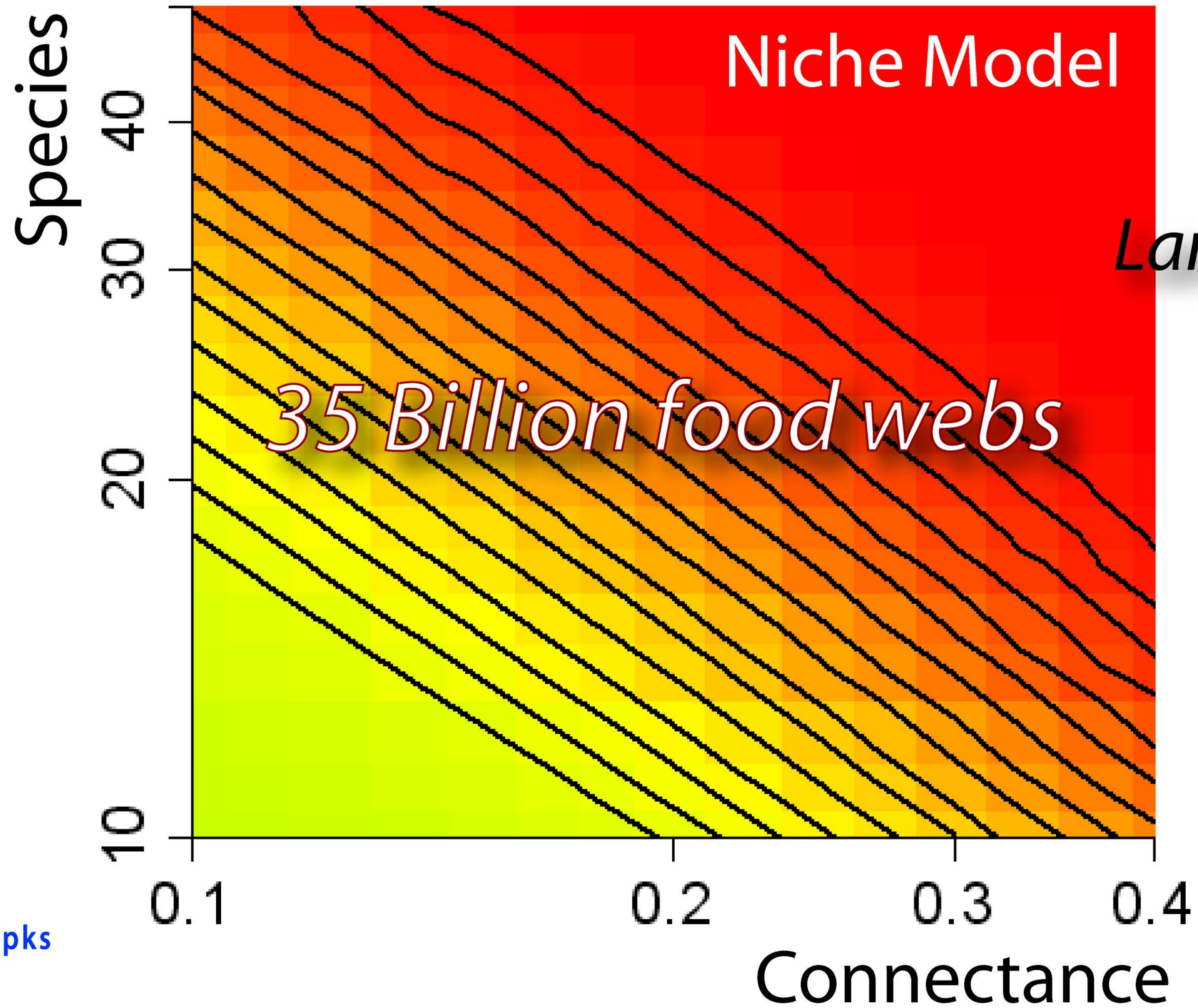
Strong saturation of malatedehydogenase

Niche Model Food Webs



Lars Rudolf


Niche Model
35 Billion food webs
Species
40
30
20
10
0.1
0.2
0.3
0.4
Connectance

Lars Rudolf

Conclusion

Generalized modeling should be used
as a *high-troughput screening tool* before
conventional modeling is attempted.

It is particularly useful if
qualitative informationon
on the *local dynamics* is desired.

Thank you very much
for your attention