

# Addressing the validation problem for social simulations: the Adversarial Scheduling Approach

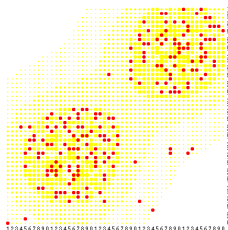
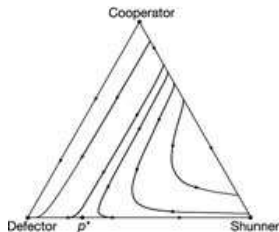
Author [Gabriel Istrate](#), eAustria Research Institute, Timișoara, Romania. email: [gabrielistrate@acm.org](mailto:gabrielistrate@acm.org)

Conference International Workshop on Challenges and Visions in the Social Sciences





# Two sides of the same coin



- Share family of similar **interaction-based** models (Blume and Durlauf)
- Agents located at the vertices of a graph.
- They have a **state**.



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- Example from EGT: equilibrium selection.
- Peyton-Young: adding continuous noise to best-response dynamics can select (risk dominant) equilibria.

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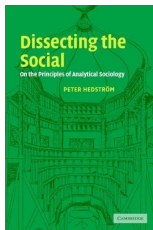
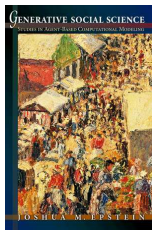
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- Synchronous vs asynchronous (Huberman & Glance)
- Want something more principled.

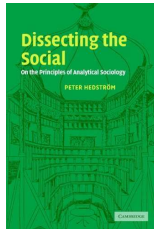
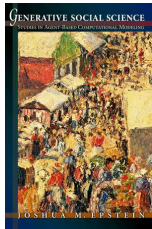
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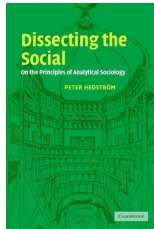
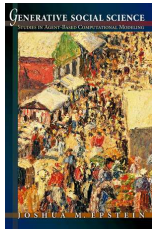
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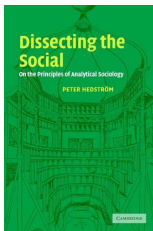
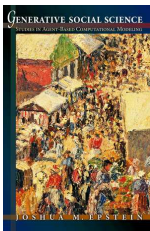
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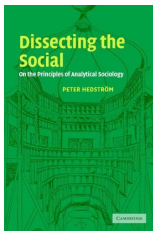
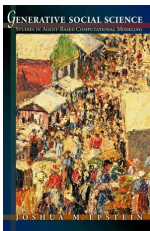
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- Formal logic in social science: not that popular. Temporal logic (Elster), logic in organization theory (Péli, Hannan).

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- Strategy used in Sugarscape (Axtell-Epstein), Brownian agents (Schweitzer), varieties of emergence (Gilbert).
- Provides insight on ingredients of mechanism-based explanations.

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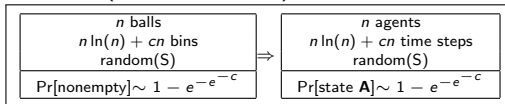
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**Coupon Collector Lemma.**
- Adversarial Scheduling: **fair scheduler**. One that touches all nodes.
- Second stylized fact: **covering law**. Mapping from one domain (balls and bins) to agents.



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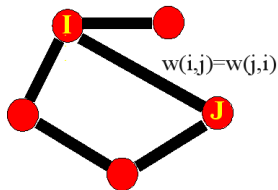
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  - Scheduling effectively introduces coupling in the system.

## Third version: adding interaction

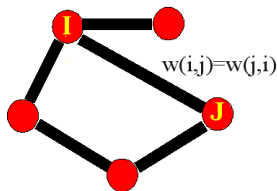


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A	(a,a)	(c,d)
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- $A$  is a *risk-dominant* equilibrium.
- $p^\beta(x_i \rightarrow z | \bar{x}) \sim e^{\beta \cdot \nu_i(z, \bar{x}_{-i})}$ , where  $\nu_i(z, \bar{x}_{-i})$ , the payoff of the  $i$ 'th agent should he play strategy  $z$  while the others' profile remains the same is given by 
$$\nu_i(z, \bar{x}_{-i}) = \sum_{(i,j) \in E} w_{ij} m_{z,x_j}.$$

# Peyton-Young's result

## Definition

Consider a Markov process  $P^0$  defined on a finite state space  $\Omega$ . For each  $\epsilon > 0$ , define a Markov process  $P^\epsilon$  on  $\Omega$ .  $P^\epsilon$  is a **regular perturbed Markov process** if all of the following conditions hold.

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- If  $P_{xy}^0 > 0$  then there exists  $r(m) > 0$ , the *resistance of transition  $m = (x \rightarrow y)$* , such that as  $\epsilon \rightarrow 0$ ,  $P_{xy}^\epsilon = \Theta(\epsilon^{r(m)})$ .

Let  $\mu^\epsilon$  be the (unique) stationary distribution of  $P^\epsilon$ . A state  $S$  is a *stochastically stable strategy* if  $\underline{\lim}_{\epsilon \rightarrow 0} \mu^\epsilon(S) > 0$ .

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- Theorem (P.Y.) Under **random scheduling A** only stochastically stable state.

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- Markov chain on state space  $V^{\{A,B\}} \times V$ .
- **Theorem [GMR08]**: *Under RW scheduling the set  $S_0 = \{(A, x) | x \in V\}$  is the set of stochastically stable states.*

# Proof idea

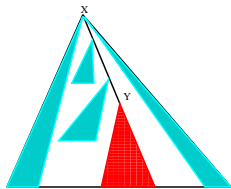
## Definition

A *tree rooted at node  $j$*  is a set  $T$  of edges such that for any state  $w \neq j$  there exists a unique (directed) path from  $w$  to  $j$ . The *resistance of a rooted tree  $T$*  is the sum of resistances of all edges in  $T$ .

## Proposition

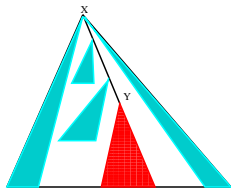
Let  $P^\epsilon$  be a regular perturbed Markov process, and for each  $\epsilon > 0$  let  $\mu^\epsilon$  be the unique stationary distribution of  $P^\epsilon$ . Then  $\lim_{\epsilon \rightarrow 0} \mu^\epsilon = \mu^0$  exists, and  $\mu^0$  is a stationary distribution of  $P^0$ . The *stochastically stable states* are precisely *those states  $z$  such that there exists a tree rooted at  $z$  of minimal resistance (among all rooted trees)*.

## Proof idea (II)



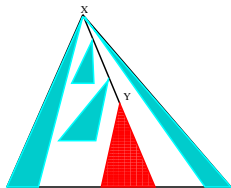
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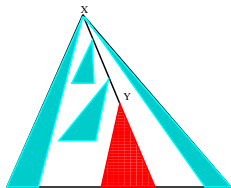
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- "Reverse" path from  $X$  to  $Y$ . Transform subtrees of  $T$  (four cases).

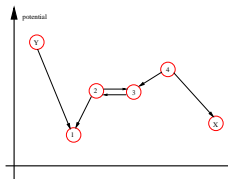
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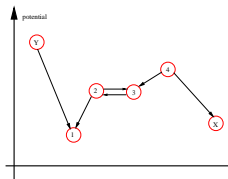


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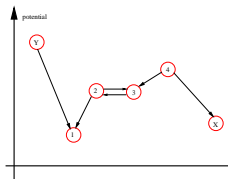
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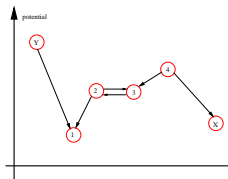
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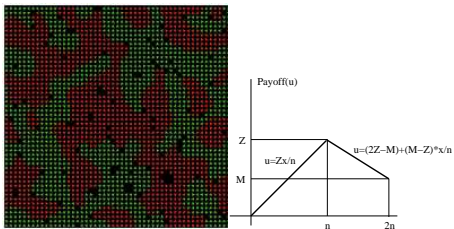
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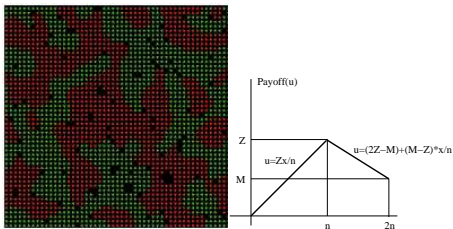
Potential game & . . .
$\text{fair}(S) \ \& \ \text{nonadaptive}(S) \gg (\forall x) A(x) \text{ only stable state}$

# Application: Schelling's Segregation Model



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Peyton-Young (1-D), Zhang, Pollicott & Weiss (2-D).

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- Similar idea: *Schelling's Segregation Model*. Peyton-Young (1-D), Zhang, Pollicott & Weiss (2-D).
- Scheduler: **(Markovian contagion)**  
To each pair of vertices  $e$  we associate a probability distribution  $D_e$  on  $V \times V$ . We then choose the next scheduled pair according to the following process: If  $t_i$  is the pair scheduled at stage  $i$ , we choose  $t_{i+1}$ , the next scheduled pair, by sampling from  $D_{t_i}$ .

# Application: Schelling's Segregation Model (II)

- **Agents' utility function:**  $u_i(\cdot) = rw(\cdot) + \epsilon$ , where  $r$  is a positive constant, and  $w(x)$  is defined as the difference between the number of neighbours of  $x$  having the same color and the number of neighbors of  $x$  having the opposite color. Further assumption: same constant  $r$ , but possibly different constants  $\epsilon$ .
- Update:

$$Pr[\text{switch}] = \frac{e^{\beta[u_1(\cdot|\text{switch})+u_2(\cdot|\text{switch})]}}{e^{\beta[u_1(\cdot|\text{switch})+u_2(\cdot|\text{switch})]} + e^{\beta[u_1(\cdot|\text{not switch})+u_2(\cdot|\text{not switch})]}}$$

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- Several details specific to Schelling's SM. Structure of stochastically stable states.



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where  $e(S', S)$  is the number of edges with one endpoint in  $S'$  and the other in  $S$ , and  $\deg(i)$  is the degree of vertex  $i$ . A graph  $G$  is  **$(r, k)$ -close-knit** if every vertex is part of a  $r$ -close-knit set  $S$ , with  $|S| = k$ .

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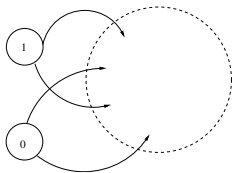
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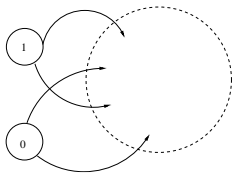
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## Extension: work in progress



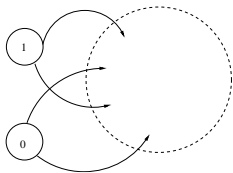
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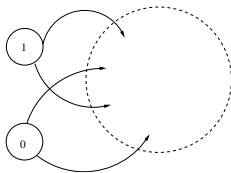
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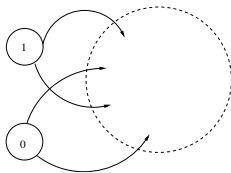
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- Markovian model:  $V^{\{A,B\}} \times \{0,1\}$ .
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