6.002 CIRCUITS AND ELECTRONICS

Energy and Power

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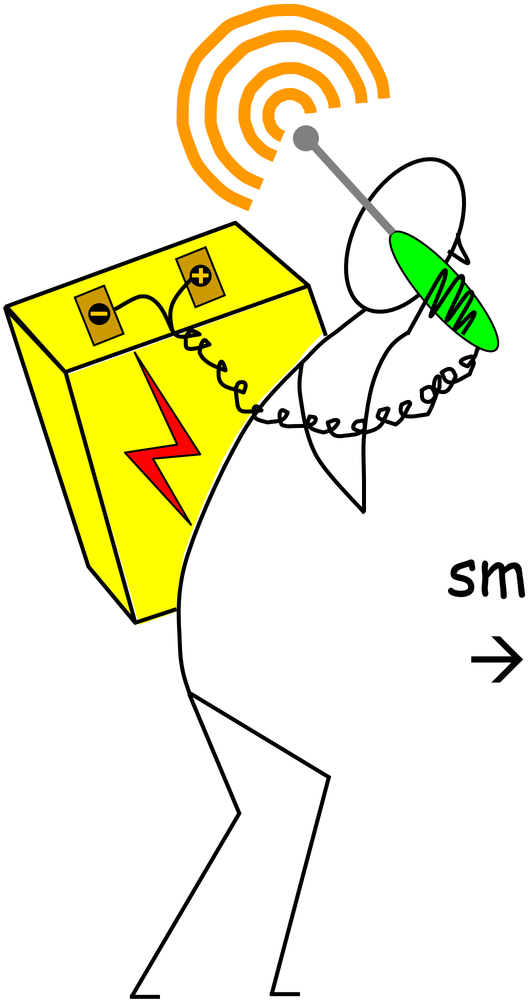
Why worry about energy?



small batteries
→ good

Today:

- How long will the battery last?
  in standby mode
  in active use
- Will the chip overheat and self-destruct?

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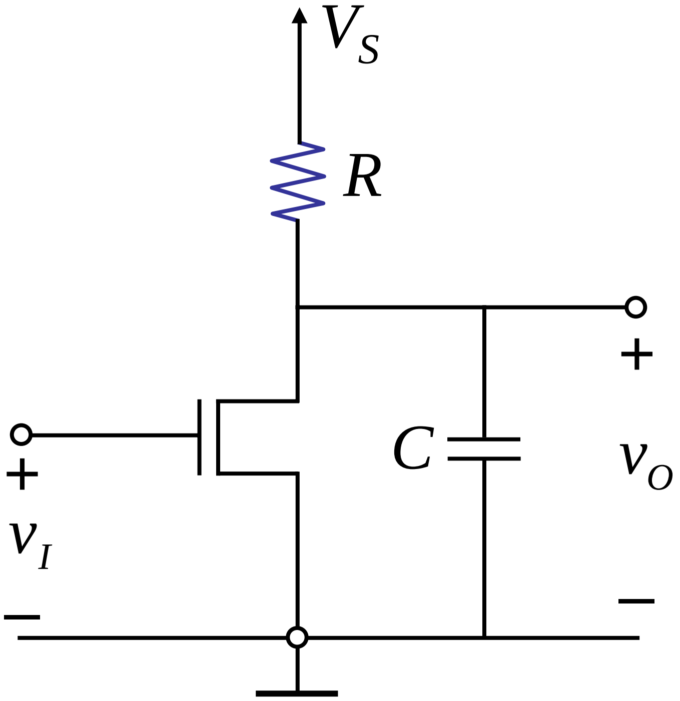
Look at energy dissipation in MOSFET gates



C: wiring capacitance and
C_{GS} of following gate

Let us determine
standby power
active use power

Let's work out a few related examples first.

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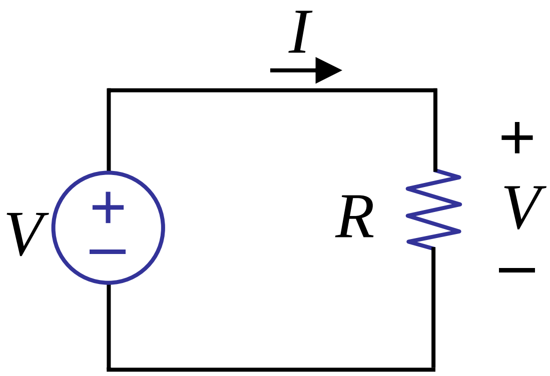
Example 1:



Power $$P = VI = \frac{V^2}{R}$$

Energy dissipated in time T

$$E = VIT$$

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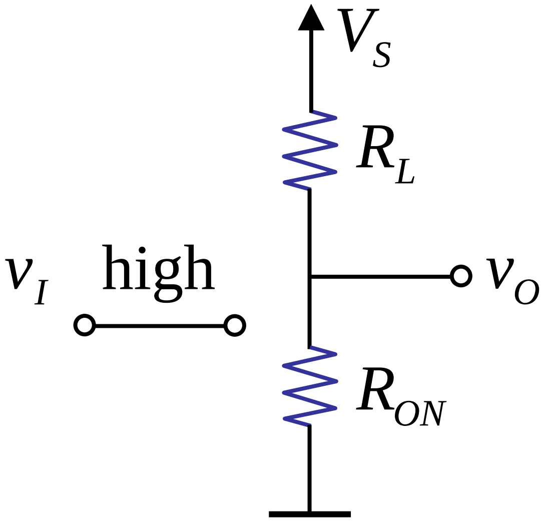
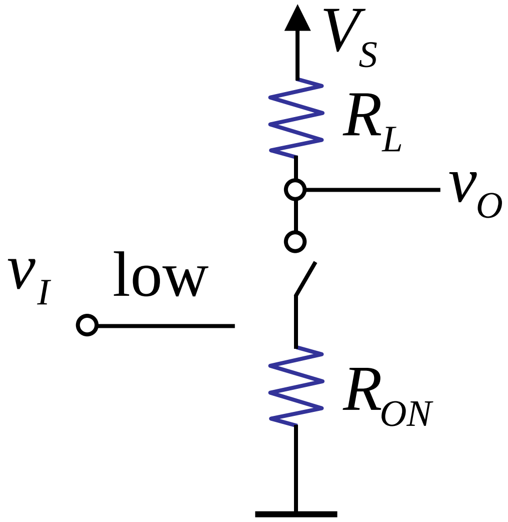
Example 1:

for our gate





$$P = \frac{V_S^{\ 2}}{R_L + R_{ON}}$$ ★

$$P = 0$$

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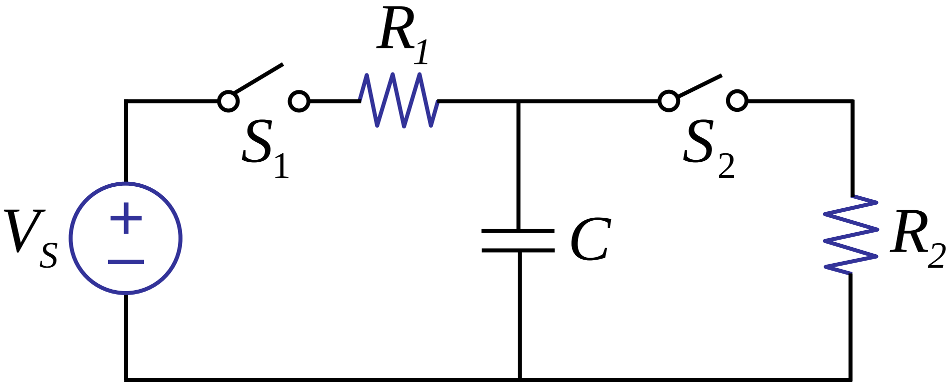
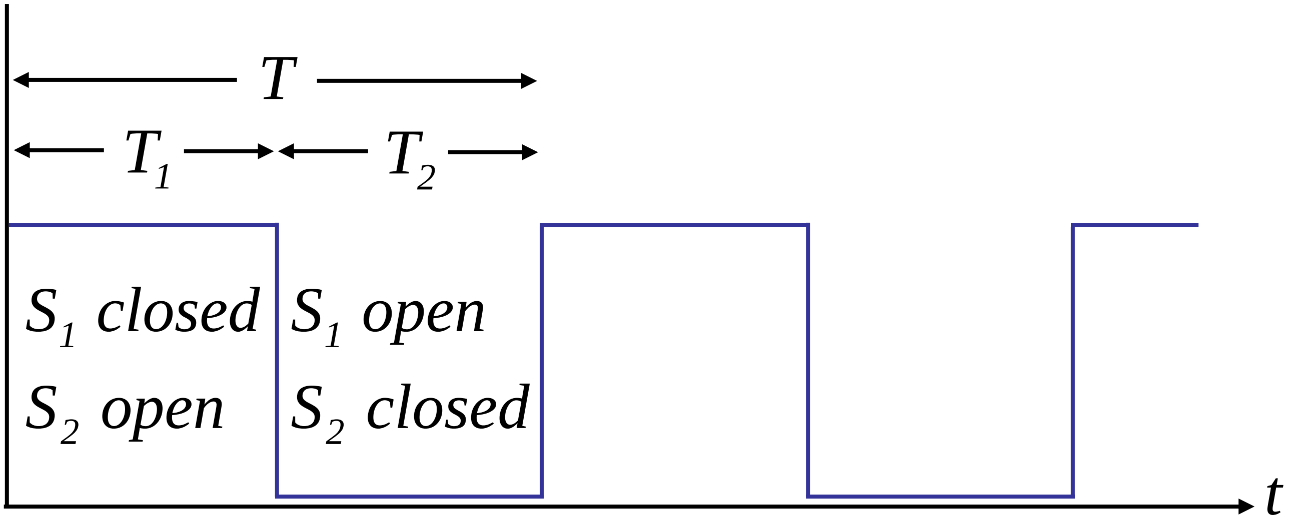
Example 2:

Consider





Find energy dissipated in each cycle.

Find average power $\overline{P}$.

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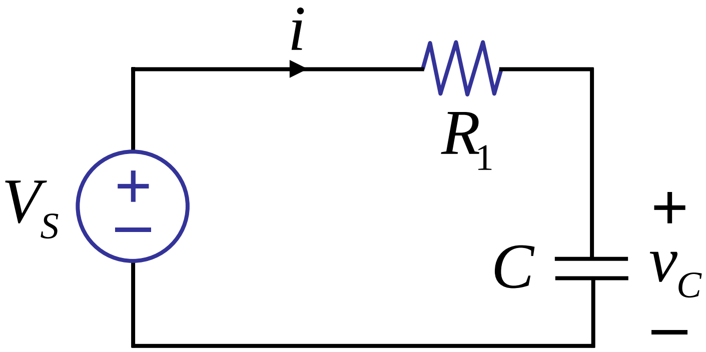
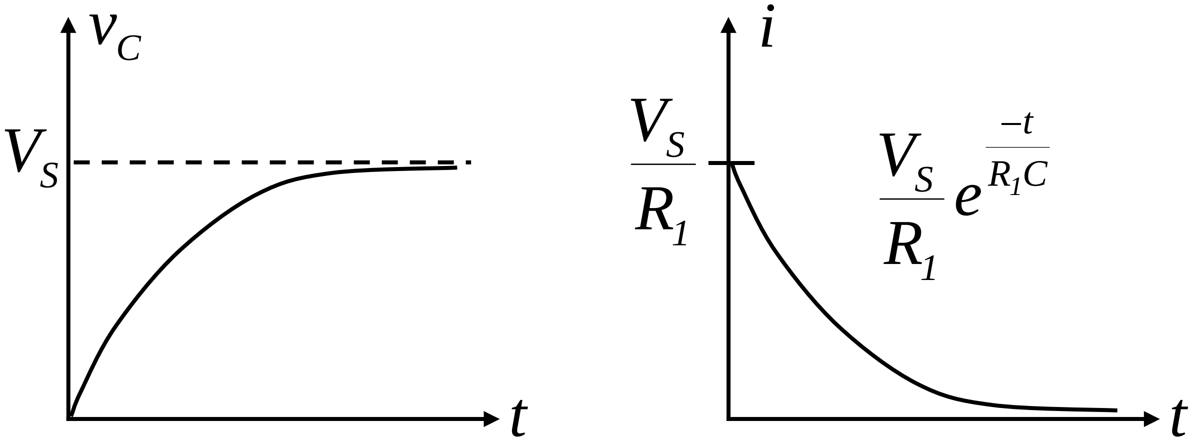
T_1 : S_1 closed, S_2 open



assume
$v_C = 0$ at $t = 0$



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Total energy provided by source during T_1

$$E = \int_0^{T_1} V_S i \, dt$$

$$= \int_0^{T_1} \frac{V_S^{\,2}}{R_1} e^{\frac{-t}{R_1 C}} dt$$

$$= -\frac{V_S^{\,2}}{\cancel{R_1}} \cancel{R_1} C \, e^{\frac{-t}{R_1 C}} \Big|_0^{T_1}$$

$$= C V_S^{\,2} \left(1 - e^{\frac{-T_1}{R_1 C}} \right)$$

$$\approx C V_S^{\,2} \quad \textit{if} \quad T_1 >> R_1 C$$

I.e., if we wait long enough

$$\left.\begin{aligned} &\frac{1}{2} C V_S^{\,2} \ \textit{stored on } C, \\ E_1 = &\frac{1}{2} C V_S^{\,2} \ \textit{dissipated in } R_1 \end{aligned}\right\} \text{Independent of R!}$$

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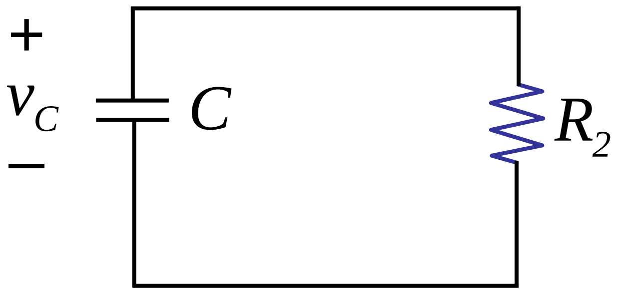
T_2: S_2 closed, S_1 open



Initially, $v_C = V_S$ (recall $T_1 >> R_1C$)

So, initially,

energy stored in capacitor $= \frac{1}{2}CV_S{}^2$

Assume $T_2 >> R_2C$

So, capacitor discharges ~fully in T_2

So, energy dissipated in R_2 during T_2

$$E_2 = \frac{1}{2}CV_S{}^2$$

E_1, E_2 independent of R_2!

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Putting the two together:

Energy dissipated in each cycle

$$E = E_1 + E_2$$

$$= \frac{1}{2} C {V_S}^2 + \frac{1}{2} C {V_S}^2$$

$$E = C {V_S}^2 \quad \textit{energy dissipated in charging \& discharging } C$$

Assumes C charges and discharges fully.

Average power

$$\overline{P} = \frac{E}{T}$$

$$= \frac{C {V_S}^2}{T}$$

$$= C {V_S}^2 f$$

★★

frequency $f = \frac{1}{T}$

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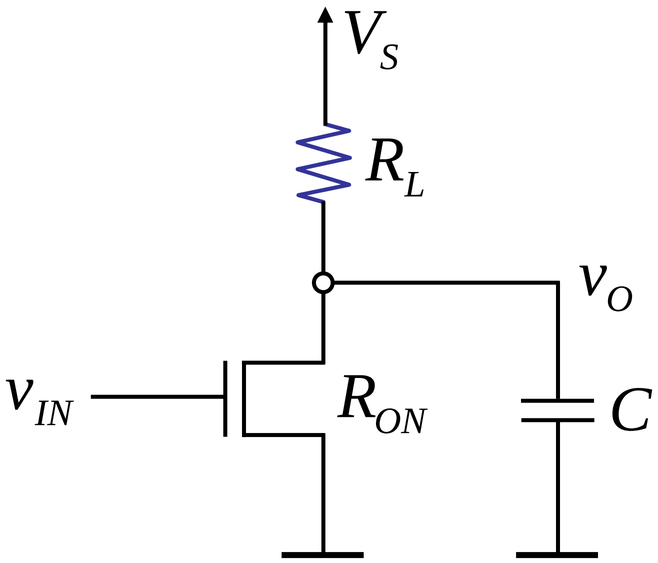
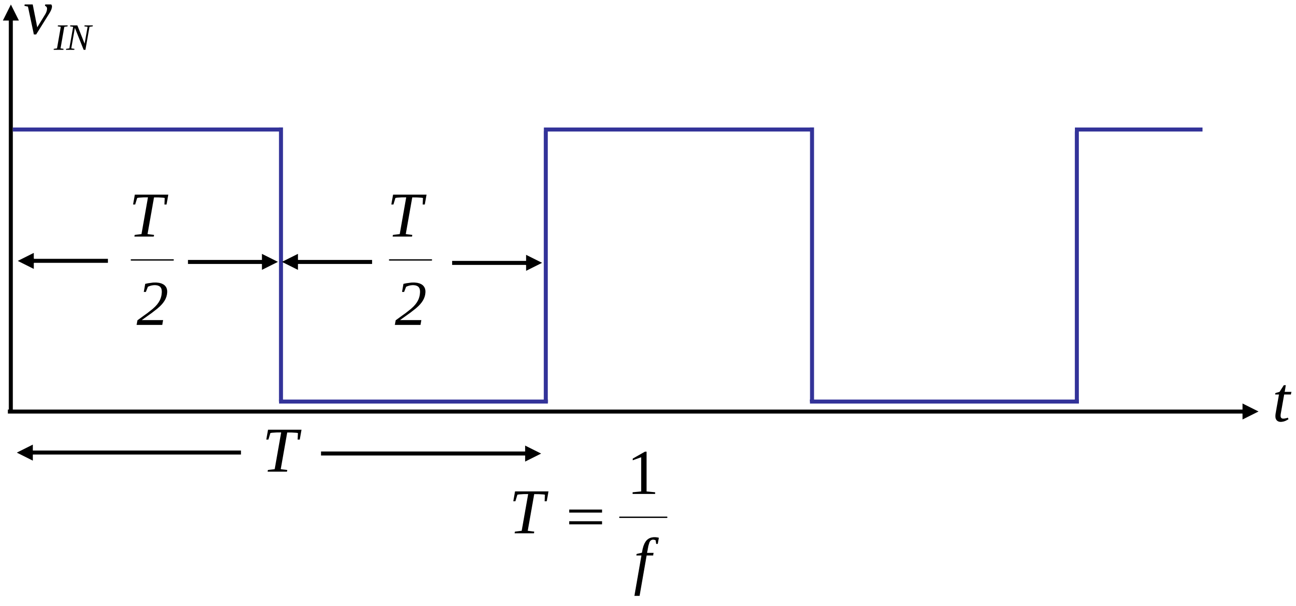
Back to our inverter —



What is $\overline{P}$ for the following input?



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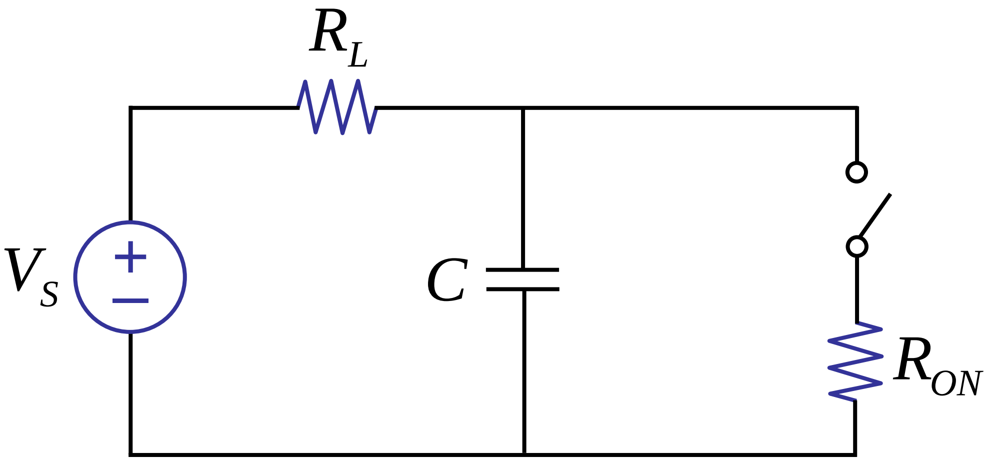
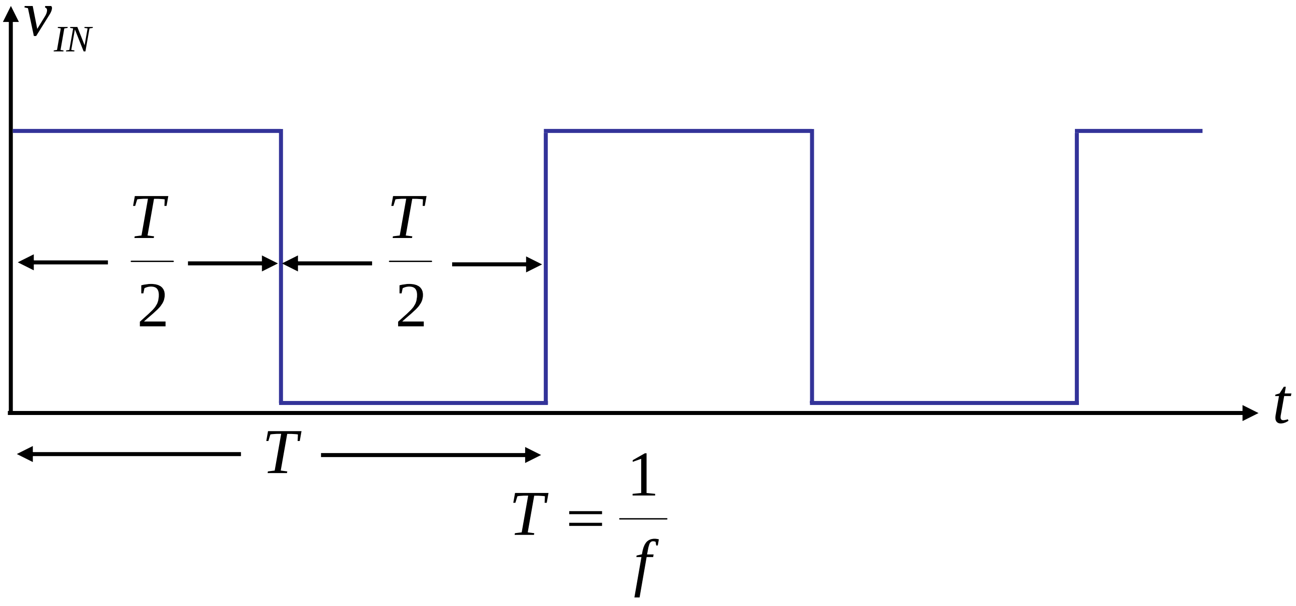
Equivalent Circuit



What is $\overline{P}$ for the following input?



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What is $\overline{P}$ for gate?

We can show (see section 12.2 of A & L)

$$\overline{P} = \frac{V_S^{\,2}}{2(R_L + R_{ON})} + C V_S^{\,2} f \, \frac{R_L^{\,2}}{(R_L + R_{ON})^2}$$

when $R_L >> R_{ON}$

$$\overline{P} = \frac{V_S^{\,2}}{2R_L} + C V_S^{\,2} f$$

remember ★

remember ★★

$\overline{P}_{STATIC}$

independent of f.
MOSFET ON half
the time.

$\overline{P}_{DYNAMIC}$

related to switching
capacitor

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What is $\overline{P}$ for gate?

when $R_L >> R_{ON}$

$$\overline{P} = \frac{{V_S}^2}{2R_L} + C{V_S}^2 f$$

In standby mode, half the gates in a chip can be assumed to be on. So $\overline{P}_{STATIC}$ per gate is still $\frac{V_S^2}{2R_L}$.

Relates to standby power.

In standby mode, $f \rightarrow 0$, so dynamic power is 0

Some numbers…

a chip with 10^6 gates clocking at 100 MHZ

$C = 1\,fF$

$R_L = 10\,k\Omega$

$f = 100 \times 10^6$

$V_S = 5V$

$$\overline{P} = 10^6 \left[\frac{25}{2 \times 10^4} + 10^{-15} \times 25 \times 100 \times 10^6 \right]$$

$$= 10^6 \left[1.25\ \text{milliwatts} + 2.5\ \text{microwatts} \right]$$

problem!

$1.25KW!$

must get rid of this

next lecture

$2.5W$

not bad

$\alpha V_S{}^2$

αf

reduce V_S

$5V \rightarrow 1V$

$2.5W \rightarrow 150\,mW$

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