18.03 Hour Exam II

March 22, 2006

Your Name
Your Recitation Leader's Name
Your Recitation Number and Time

There are five problems. Do all your work on these pages. No calculators or notes may be used. The point value (out of 100) of each problem is marked in the margin. Solutions will be available from the UMO, on the web, and at recitation on Thursday.

Exponential Response Formula:
$x_p = Ae^{rt}/p(r)$ solves $p(D)x = Ae^{rt}$ provided $p(r) \neq 0$.

Resonant Response Formula:
$x_p = Ate^{rt}/p'(r)$ solves $p(D)x = Ae^{rt}$ provided $p(r) = 0$ and $p'(r) \neq 0$.

Exponential Shift Law:
$p(D)(e^{rt}u) = e^{rt}p(D + rI)u$.

Problem	Points
1	
2	
3	
4	
5	
Total	

Do not begin until you are told to do so.

[10] **1. (a)** For what values of k is the equation $\ddot{x} + 6\dot{x} + kx = 0$ underdamped?

[10] **(b)** It is observed that a certain solution to $\ddot{x} + 6\dot{x} + kx = 0$ satisfies $x(t) = 0$ for two values of t differing by $\pi/2$ and nowhere in between. (It may also vanish for other values of t.) What is k?

[10] **2. (a)** For what real (positive or negative) values of k do some solutions of $\ddot{x}+6\dot{x}+kx=0$ grow without bound as $t \to \infty$?

[12] **(b)** What is the solution of $\ddot{x}+6\dot{x}+13x=13$ such that $x(0)=1$ and $\dot{x}(0)=1$?

[10] **3. (a)** What is the amplitude of the sinusoidal solution of $\ddot{x} + 6\dot{x} + 13x = \cos(\omega t)$, as a function of the input circular frequency ω?

[10] **(b)** At what frequency ω is the phase lag of the sinusoidal solution to $\ddot{x} + 6\dot{x} + 13x = \cos(\omega t)$ equal to $90°$?

[12] **4.** **(a)** Find a particular solution of $\ddot{x} + 6\dot{x} + 13x = e^{-t}$.

[12] **(b)** Find a particular solution of $\ddot{x} + 6\dot{x} + 13x = 13t + 19$.

[14] **5.** Find a particular solution of $\ddot{x} + 6\dot{x} + 13x = e^{-3t}\cos(2t)$.