Broken symmetries in financial markets

Damien Challet

Institute for Scientific Interchange, Turin

Exchange goods without a market

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- Find something to buy/sell
- Find somebody interested
- Think of a fair price
- Agree on a price
- Exchange goods

Internet era

- Find something to sell/buy
- Go to auction site
- Auctioneers determine price
- Goods travel by post

M A R K E T ! ! ! !

The more bidders, the better the price

Role of financial markets

Centralised markets

- Exchange goods
- Speed
- Good prices
- Low transaction costs

The more traders, the better the prices

Financial markets

Real goods:

- Stocks
- Commodities (oil, gold, . . .)
- Foreign exchange

Financial markets

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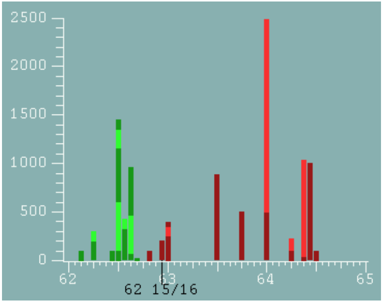
Risk-related goods:

- Bonds
- Futures
- Options
- Insurance
- Structured products
- . . .

Double auction

eBay[2]

- One item
- Several buyers: bids
- Several sellers: asks



1 year = 1 Tb of data

- Daily CERN-like experiment
- Much noise
- What to measure?
- How many computers?

Symmetries

- buy $\equiv$ sell
- buy,sell $\equiv$ sell/buy

Efficient markets

- $E[p(t+1)] = p(t)$
- $r_\tau(t) = \log p(t+\tau) - \log p(t)$: price return

$$E[r(t)r(t+T)] = 0$$

(necessary condition)

Why symmetry in markets?

Why symmetry in markets?

Because markets hate asymmetry

Any asymmetry is detected and corrected

Quality $\rightarrow$ reputation

Modelling

Simplest

- Bachelier: Gaussian uncorrelated random walk
- Mandelbrodt: Levy uncorrelated random walk
- Now: power-law random walks with long memory (stay awake)

Typical agent-based model

- N agents
- $S_i(t)$ state of agent $i \in \{-1, 0, 1\}$
- $S_i(t+1) = \mathrm{sign}[\sum_j J_{i,j} S_j(t) + h_i S_i(t) + F]$
- $\log p(t+1) = \log p(t) + \sum_i S_i(t)$

Buy/sell

Long position

- Time t, buy N shares
- Time t', sell N shares
- Capital gain:

$$N[p(t'+1)-p(t+1)]-T(N)$$

where $T(N)$ transaction costs

Buy/sell

Long position

- Time t, buy N shares
- Time t', sell N shares
- Capital gain:

$$N[p(t'+1)-p(t+1)]-T(N)$$

where $T(N)$ transaction costs

Short position

- Time t, sell N shares
- Time t', buy N shares
- Capital gain:

$$-N[p(t'+1)-p(t+1)]-T(N)$$

where $T(N)$ transaction costs

Broken symmetries: $T(N)$, maximum gain

Limit/market order

- Market order: BUY NOW!
- Limit order: buy at price p

Bouchaud et al:

- Cost limit order $=$ Cost market order   electronic markets
- Cost limit order $<$ Cost market order   markets w/ human

Microscopic asymmetry

- More buy limit orders than sell orders
- Limit order markets have a long memory:

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  - Buy 100, Sell 100 NOT $\equiv$ Sell 100, Buy 100

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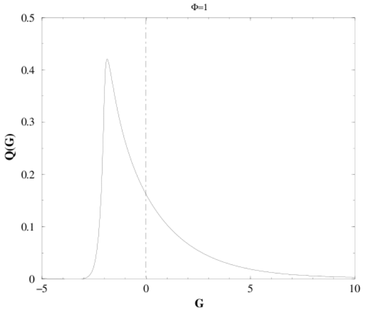
  - Buy 100, Sell 100 NOT $\equiv$ Sell 100, Buy 100
  - Buy 100, Buy 10 NOT $\equiv$ Buy 10, Buy 100

Winning ratios

Measures of success:

- % of successful trades
- $\text{won}/(\text{won} + \text{lost})$

Distribution of gains (trend followers) (Bouchaud Potters):



Heterogeneity

- information cost
- trading frequency
- wealth
- risk profile
- annual turnover

trading frequency $\leftrightarrow$ wealth, risk profile, turnover

Asymmetric information

information cost $\rightarrow$ heterogeneity

- processing power
- sources of information
- education
- honesty (insider information)

(In-)efficient markets

- $E[r(t)] = 0$ TRUE
- $E[r(t) - T|X] \neq 0$?
- What is X?
- How many of them?

Symmetry breaking, phase transition:

$$H = \sum_X E[r(t) - T|X]^2 > 0$$

H in real markets?

- $H > 0$ ALWAYS
- Risk?
- $\frac{H}{(\delta H)^2}$ significant?
- $H > 0$ stabilizing?

Agent-based models

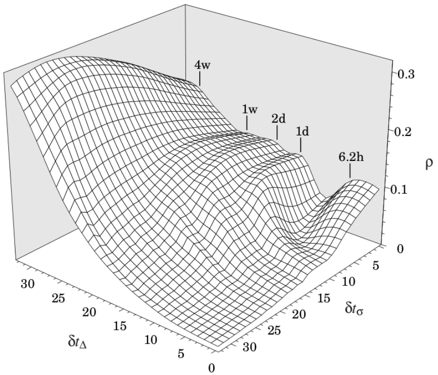
Usual aim: reproduce market behavior

Most important ingredient

- predictability H
- central to real life
- *how does it disappear?*

Trading frequency

- Who knows more about the market?
- Information spread?

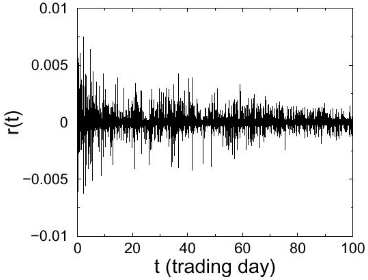


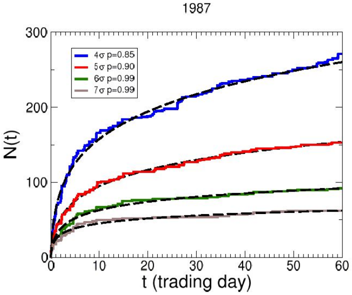
Time reversal asymmetry

$$X(p(t)) = X(p(-t))$$

- Humans not time reversal
- News not time reversal
- Markets?
  - who cares?

Measures of time reversal asymmetry



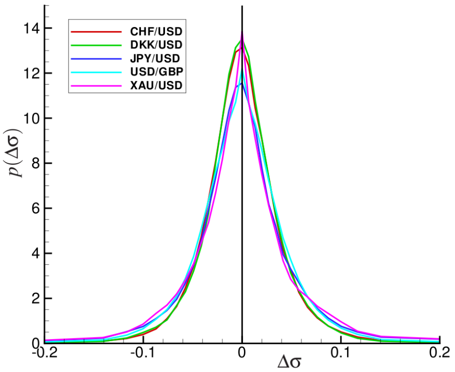


- Visually: Omori law
- autocorrelation NO
- asymmetry past/future.

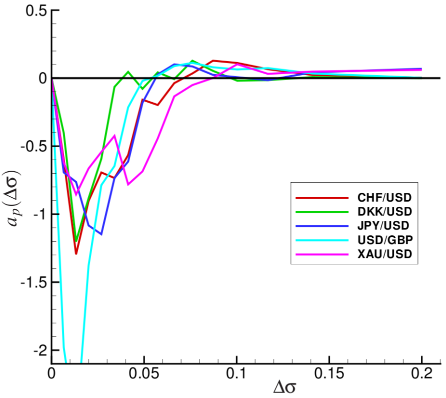
Zumbach volatility asymmetry

- price returns noisy
- most likely asymmetric
- Historical volatility over T units of time of τ seconds: $\sigma_h(T,\tau)$
- Realized volatilty over T units of time of τ seconds: $\sigma_r(T,\tau)$

$P(\delta\sigma)$



$P(\delta\sigma)$



The mathematics of time reversal asymmetry

- 100s of models
- Only models with heterogeneous time scales OK

$$\sigma = \sum_n 2^{-\alpha n} \sigma_h(T_0 2^n, \tau)$$

- Information flows between time scales
- Put by hand

New route: Baldovin and Stella

Unit of time $\tau = 1$

- $$r_{t,T} = \ln p(t+T) - \ln p(t)$$
- $t \neq t'$
  $$E(r_{t,1}, r_{t',1}) = 0$$
  (no linear arbitrage)
- price follows random walk
  $$E[r^2] \propto T^{2D}$$

$$P_T(r) = \frac{1}{T^D} g\left(\frac{r}{T^D}\right)$$

"Renormalisation": $T \to 2T$

- $$P_{2T}(r) = \frac{1}{2^D} P_T\left(\frac{r}{2^D}\right)$$
- $r = r_1 + r_2$

$$\begin{aligned} P_{2T}(r) &= \int \mathrm{d}r_1 \mathrm{d}r_2 \, p^{(2)}_{2T}(r_1, r_2) \delta(r - r_1 - r_2) \\ P_T(r_1) &= \int \mathrm{d}r_2 \, p^{(2)}_{2T}(r_1, r_2) \\ P_T(r_2) &= \int \mathrm{d}r_1 \, p^{(2)}_{2T}(r_1, r_2) \end{aligned}$$

Consequences

Scaling:

$$E[(r_1 + r_2)^2] = E[(r_1)^2] + E[(r_2)^2]$$
$$(2T)^{2D} = 2(T^{2D})$$

$$\rightarrow D = 1/2$$

Consequences 2

Characteristic function

$$\tilde{p}^{(2)}_{2T}(k_1, k_2) \leftrightarrow p^{(2)}_{2T}(r_1, r_2)$$

$$\begin{aligned}
P_{2T}(r) &= \int \mathrm{d}r_1 \mathrm{d}r_2 \, p^{(2)}_{2T}(r_1, r_2)\delta(r - r_1 - r_2) \\
\tilde{p}^{(2)}_{2T}(k, k) &= \tilde{g}(\sqrt{2T}k) \\
P_T(r_1) &= \int \mathrm{d}r_2 \, p^{(2)}_{2T}(r_1, r_2) \\
\tilde{p}^{(2)}_{2T}(k, 0) &= \tilde{g}(\sqrt{T}k)
\end{aligned}$$

Consequences 2

$$\tilde{p}^{(2)}_{2T}(k_1, k_2) = \tilde{g}\left(\sqrt{Tk_1^2 + Tk_2^2}\right)$$

- Kind of multiplication in characteristic function space
- Sqrt introduces a dependency between r_1 and r_2.
- Generalisation to n returns:

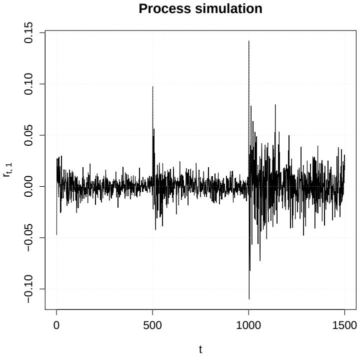
$$\tilde{p}^{(2)}_{2T}(k_1, \cdots, k_n) = \tilde{g}\left(\sqrt{Tk_1^2 + \cdots + Tk_n^2}\right)$$

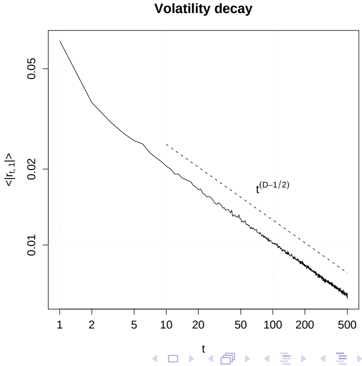
$\rightarrow$ Multivariate distribution from univariate distribution

Problems

- Constant volatility
- Time reversal invariant

Solution: dilute time so that $E[r^2(t)r(t+\tau)^2] \propto \tau^{-0.2}$

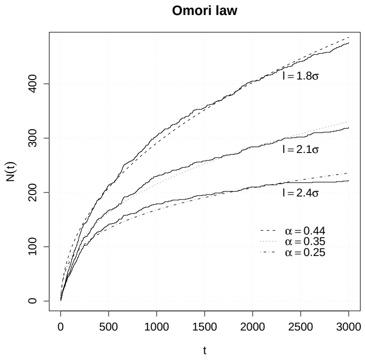




Asymmetry of time

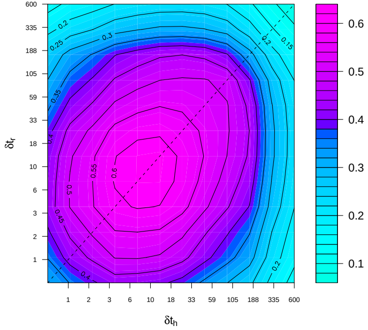
(with P.P. Peirano)

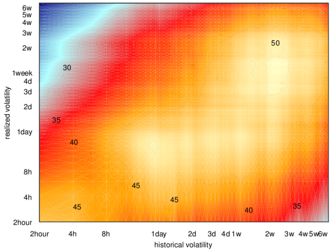
Omori's Law



Symmetric asymmetry of time

Zumbach-Lynch mugshot: B-S and USD/CHF (from ZL)





Visible time scale
No information cascade between time scales
External shocks only

Conclusions

- Markets are asymmetric
- Time reversal invariance breaking sorts out models
- Time scales: crucial ingredient
- Time scales and efficiency: open problem