# Dynamic of Networks at the Edge of Chaos

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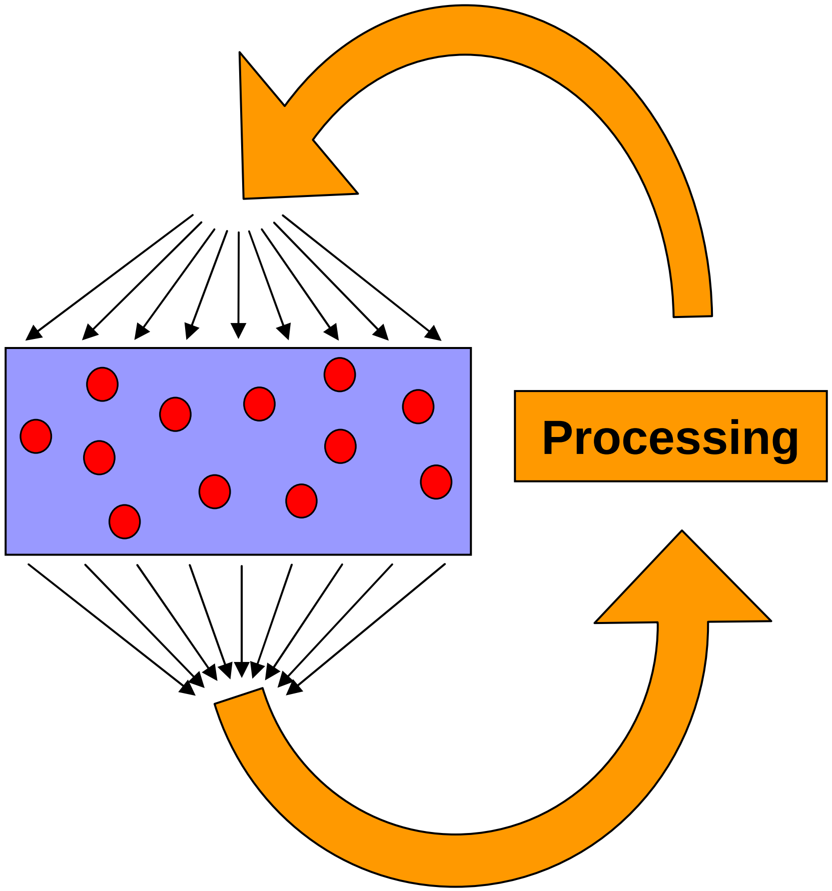
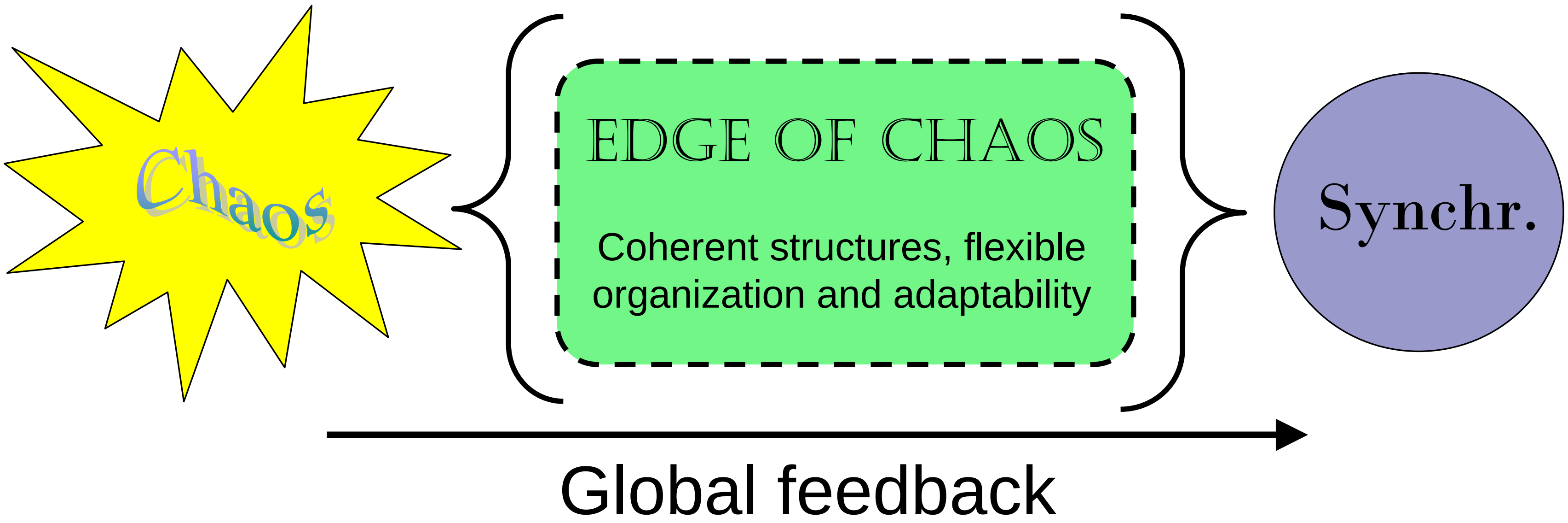
# Control by global feedback



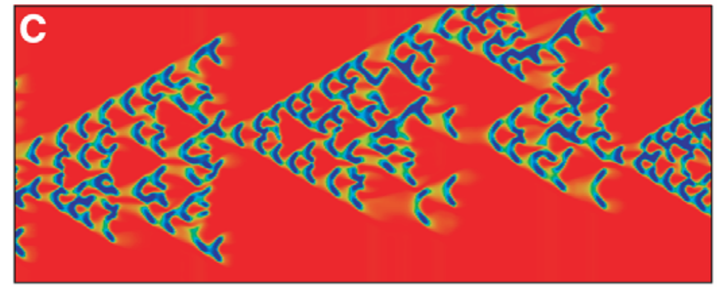

- A system of many elements produces dynamical signals.
- These signals are processed, and a control signal is generated.
- The control signal is then applied back to the system as extern al forcing.
- When the control signal is applied uniformly to all elements, we call this a **global feedback**

# A simple way of controlling chaos:



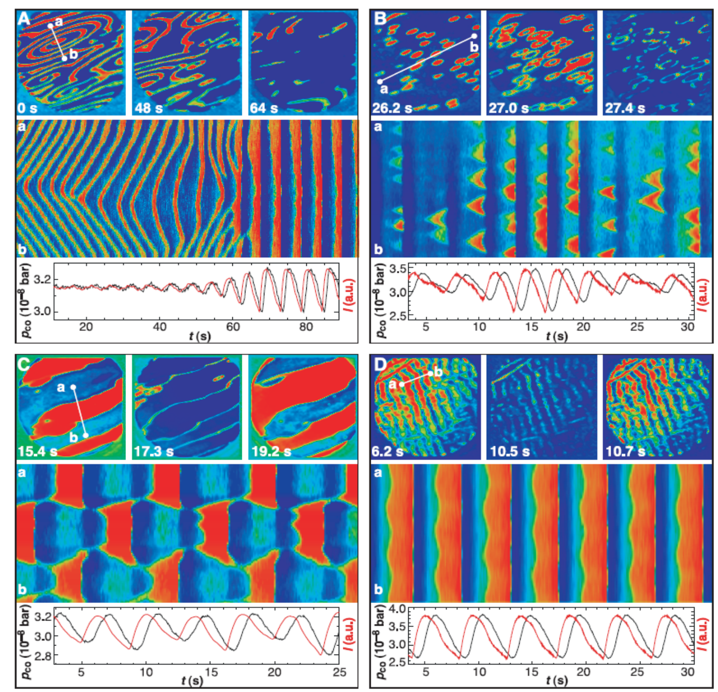

Global feedbacks allow us to bring a system to the edge of chaos, and to keep it there.

# Introduction: Control by global feedback in oscillatory media




- Complex Ginzburg-Landau equation

  Global feedback suppresses chaotic behaviour and induces uniform oscillations. Different coherent structures emerge at the edge of chaos




- Oscillatory surface chemical reactions

  Phase clusters, standing waves, intermittent turbulence, etc.

M. Kim et al. Science (2001)

Mikhailov & Showalter. Phys. Rep.(2006)

# Control of network dynamics...

We want to study transition to chaos under global feedback for dynamical networks, in contrast to spatially extended systems.

What could the transition scenario look like?
What will be the corresponding "coherent structures"?

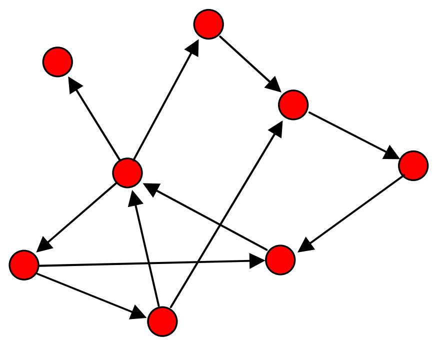
To answer these questions, we design a model:

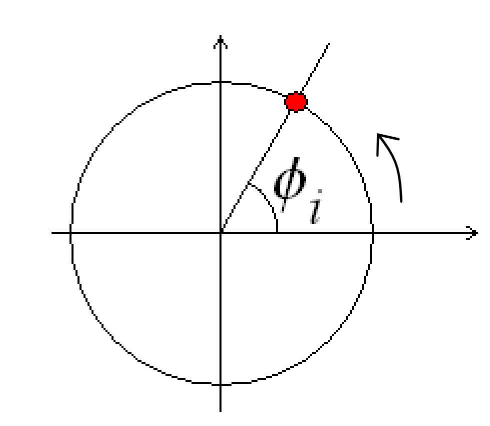
- The interaction pattern is given through a network of complex topology
- Very simple dynamical objects: phase oscillators
- Time delays are effectively accounted for through phase shifts in the interactions

# The model

$$\dot{\phi}_i = \omega + \sum_{j=1}^{N} T_{ij} \sin\left(\phi_j - \phi_i + A_{ij}\right)$$

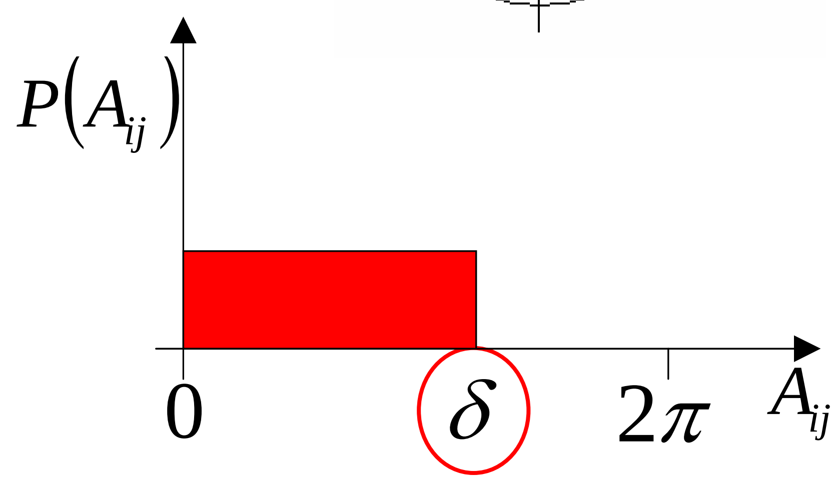
**network** **phase shifts**






The elements of matrices $T$ and $A$ are chosen randomly

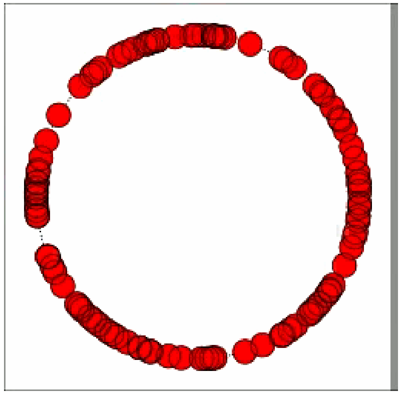
$$T_{ij} = \begin{cases} 1 & prob. \quad \rho \\ 0 & prob. \quad 1-\rho \end{cases}$$

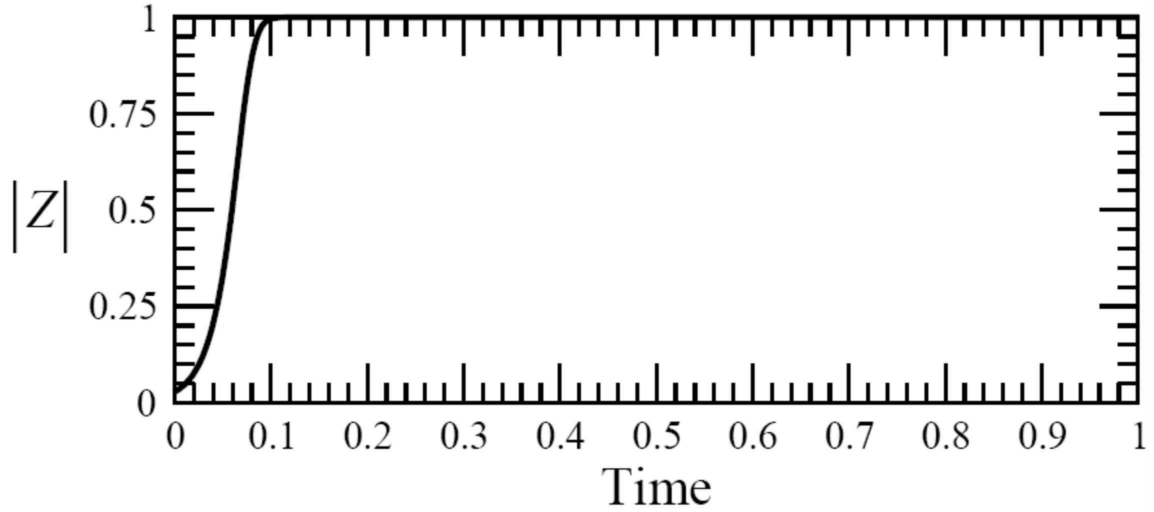


$$\dot{\phi}_i = \sum_{j=1}^{N} T_{ij} \sin\left(\phi_j - \phi_i + A_{ij}\right)$$

Global signal as an order parameter: $Z = \frac{1}{N}\sum_{i=1}^{N} \exp\left(\mathbf{i}\phi_i\right) = r\, e^{\mathbf{i}\Psi}$

$\rho = 0.1 \quad N = 100$



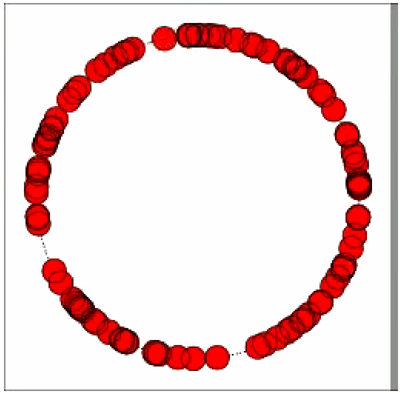


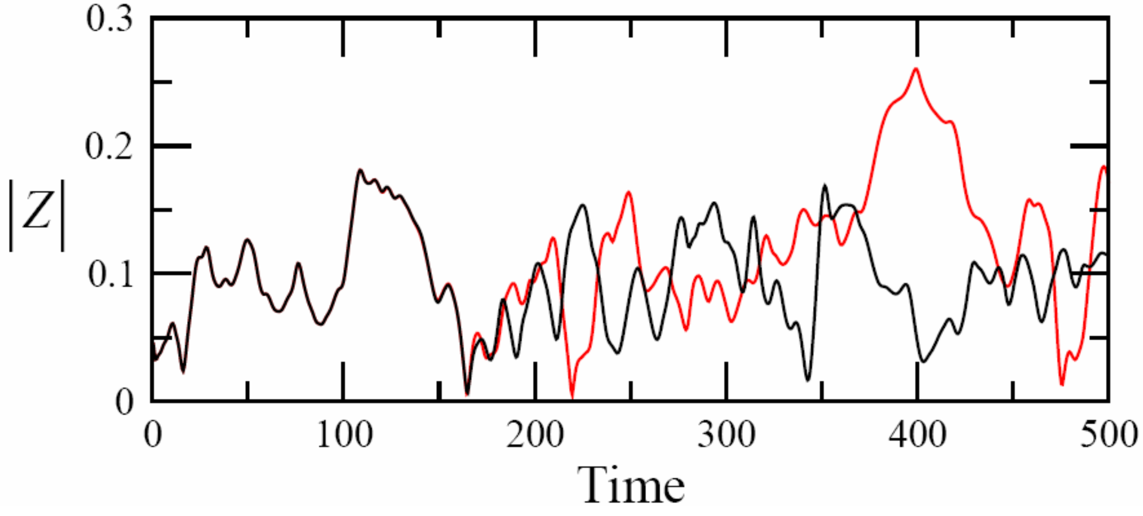

$\delta = 0$

$$\dot{\phi}_i = \sum_{j=1}^{N} T_{ij} \sin\left(\phi_j - \phi_i + A_{ij}\right)$$

Global signal as an order parameter: $Z = \frac{1}{N}\sum_{i=1}^{N} \exp\left(\mathbf{i}\phi_i\right) \quad = r\, e^{\mathbf{i}\Psi}$

$\rho = 0.1 \quad N = 100$






$\delta = 2\pi$

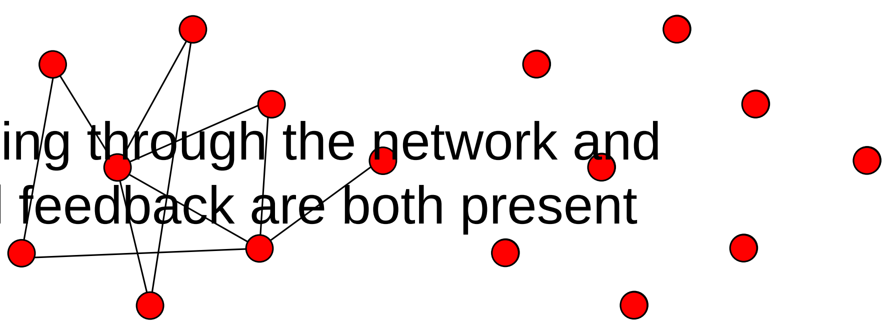
# Control through global feedback

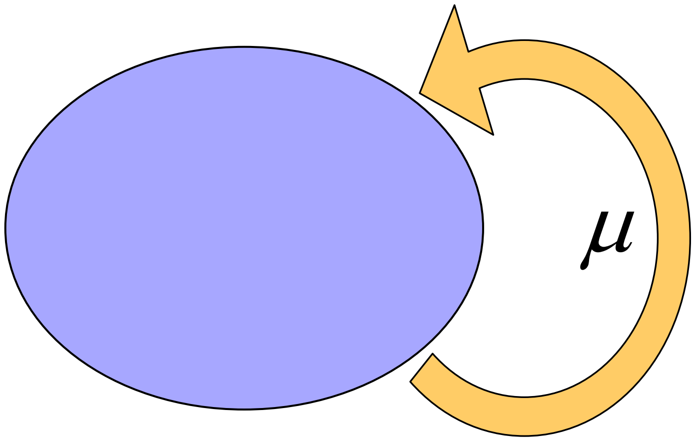
$$\dot{\phi}_i = \sum_{j=1}^{N} T_{ij} \sin\left(\phi_j - \phi_i\right) + \mu r \sin\left(\Psi - \phi_i\right)$$

$$Z = \frac{1}{N}\sum_{i=1}^{N} \exp\left(\mathbf{i}\phi_i\right) = r\, e^{\mathbf{i}\Psi}$$

Interactions

Global feedback






Coupling through the network and global feedback are both present

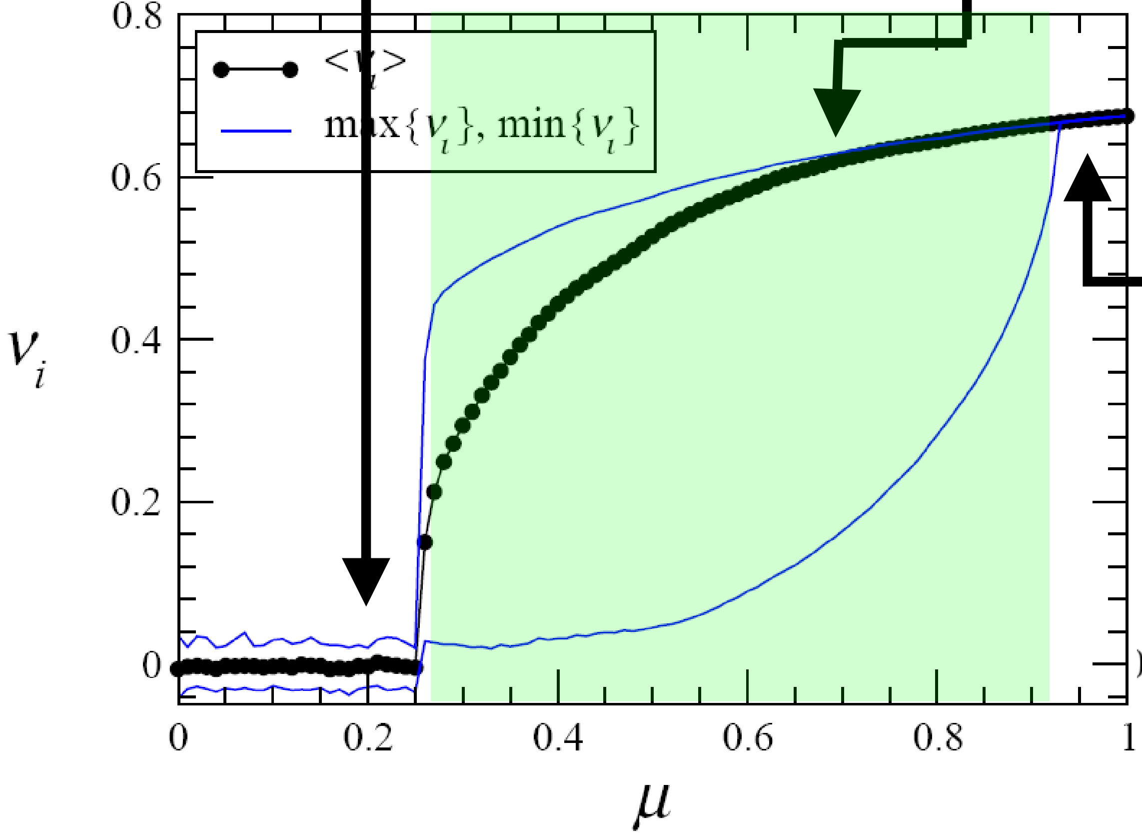
For numerical simulations of the system

$$\dot{\phi}_i = \sum_{j=1}^{N} T_{ij} \sin\left(\phi_j - \phi_i + A_{ij}\right) + \mu\, r \sin\left(\Psi - \phi_i\right)$$

$$Z = \frac{1}{N}\sum_{i=1}^{N} \exp\left(\mathbf{i}\phi_i\right) = r\, e^{\mathbf{i}\Psi}$$

we keep the network and the phase shifts fixed (in the chaotic regime), and control the system by changing the intensity of the global feedback.

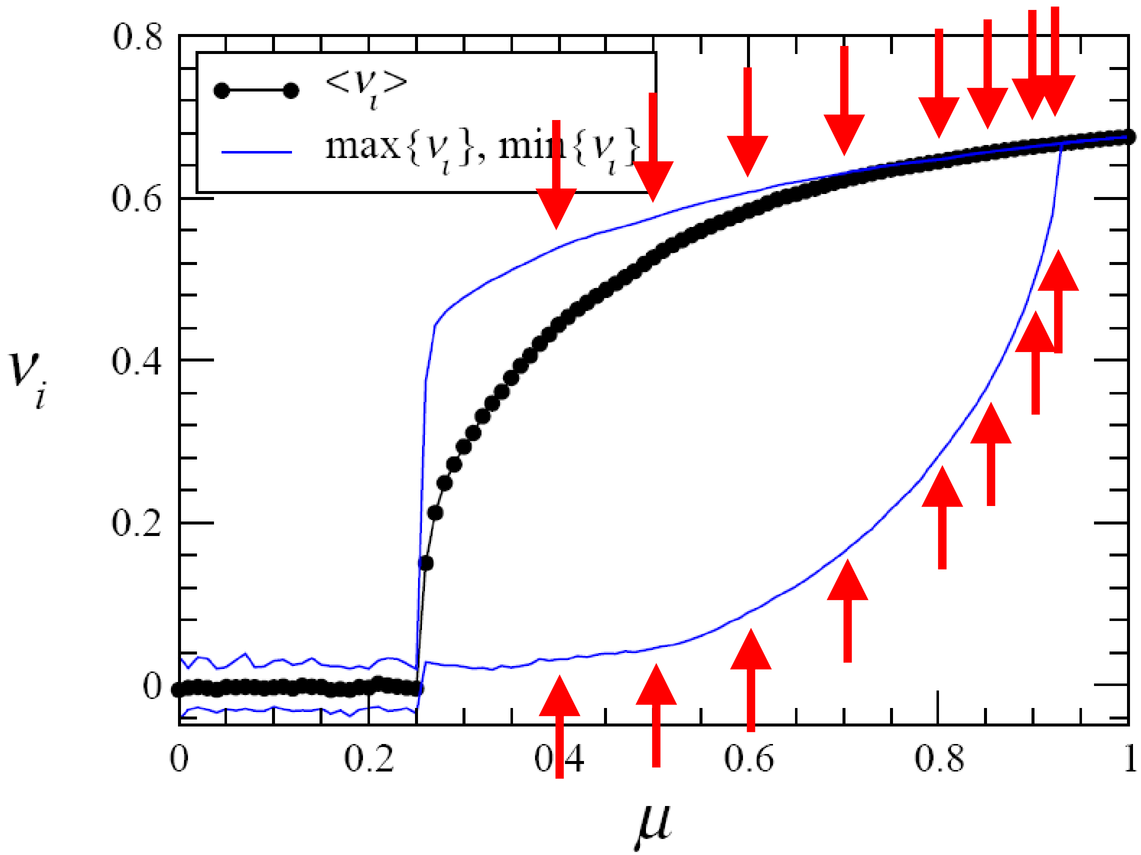
- We integrate these equations for a system of 1000 oscillators.
- We fix ❒■🗀✍🗀🗀⌛ and ♎■🗀✍🖰 ■ which shows chaos in the original system
- We look at the time evolution of the phases for different values of

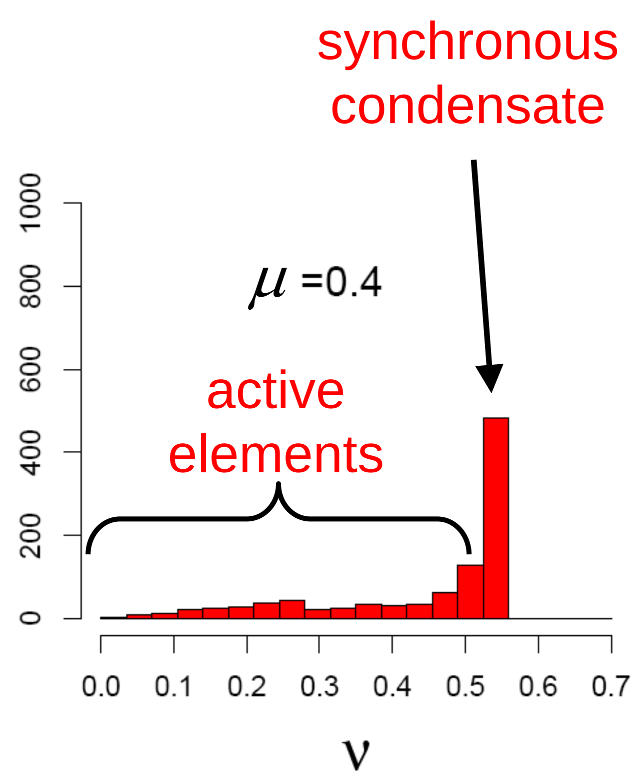


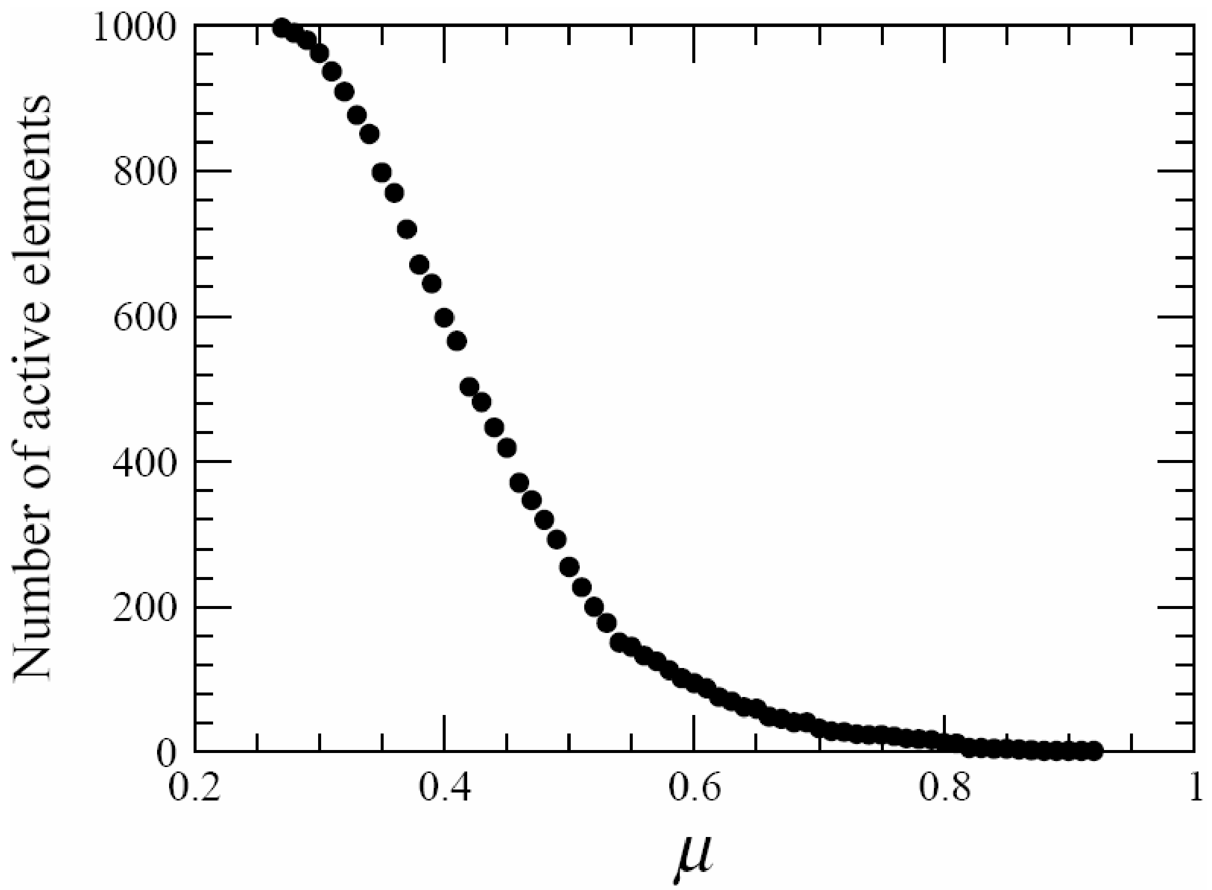

$$v_i = \frac{\phi_i(T_2) - \phi_i(T_1)}{T_2 - T_1}$$

We can watch the system in a co-rotating frame with the velocity of the synchronous condensate.

Elements with velocities different from that of the condensate are **active elements**






$N = 1000$

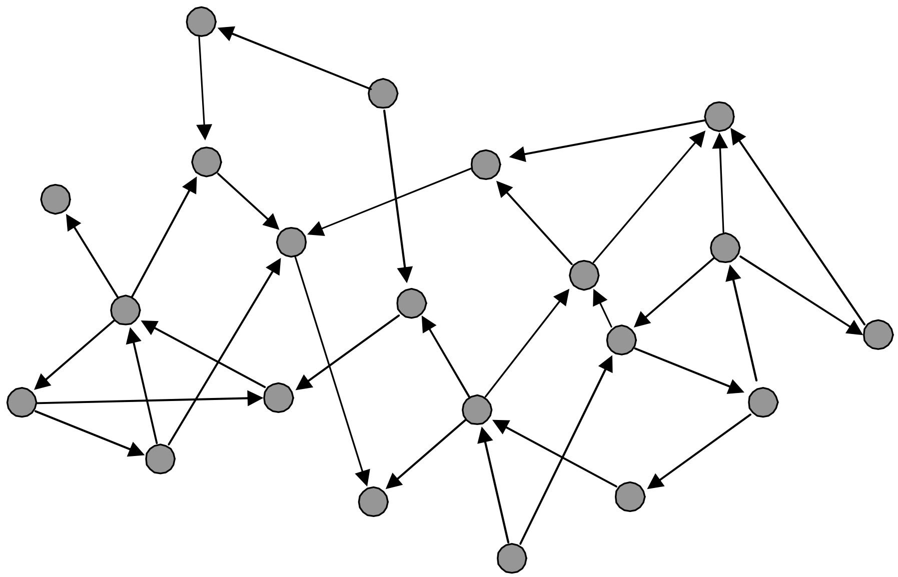
Number of active elements as a function of the global feedback intensity

# Subnetworks of active elements

For a given value of ❍ a certain number of oscillators will become active.

All other elements will belong to the synchronous condensate

If we disregard these passive elements and their connections, we can see how active elements interact

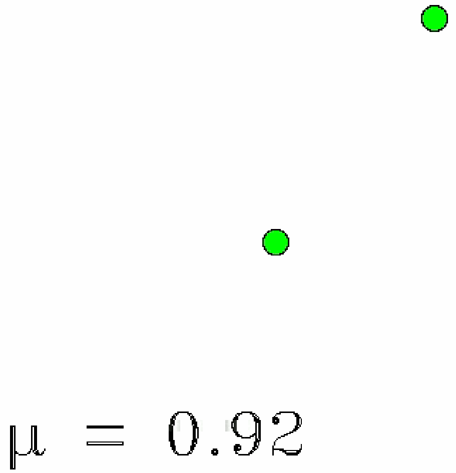


By changing ❍ we can control the emergence of subnetworks of active elements.

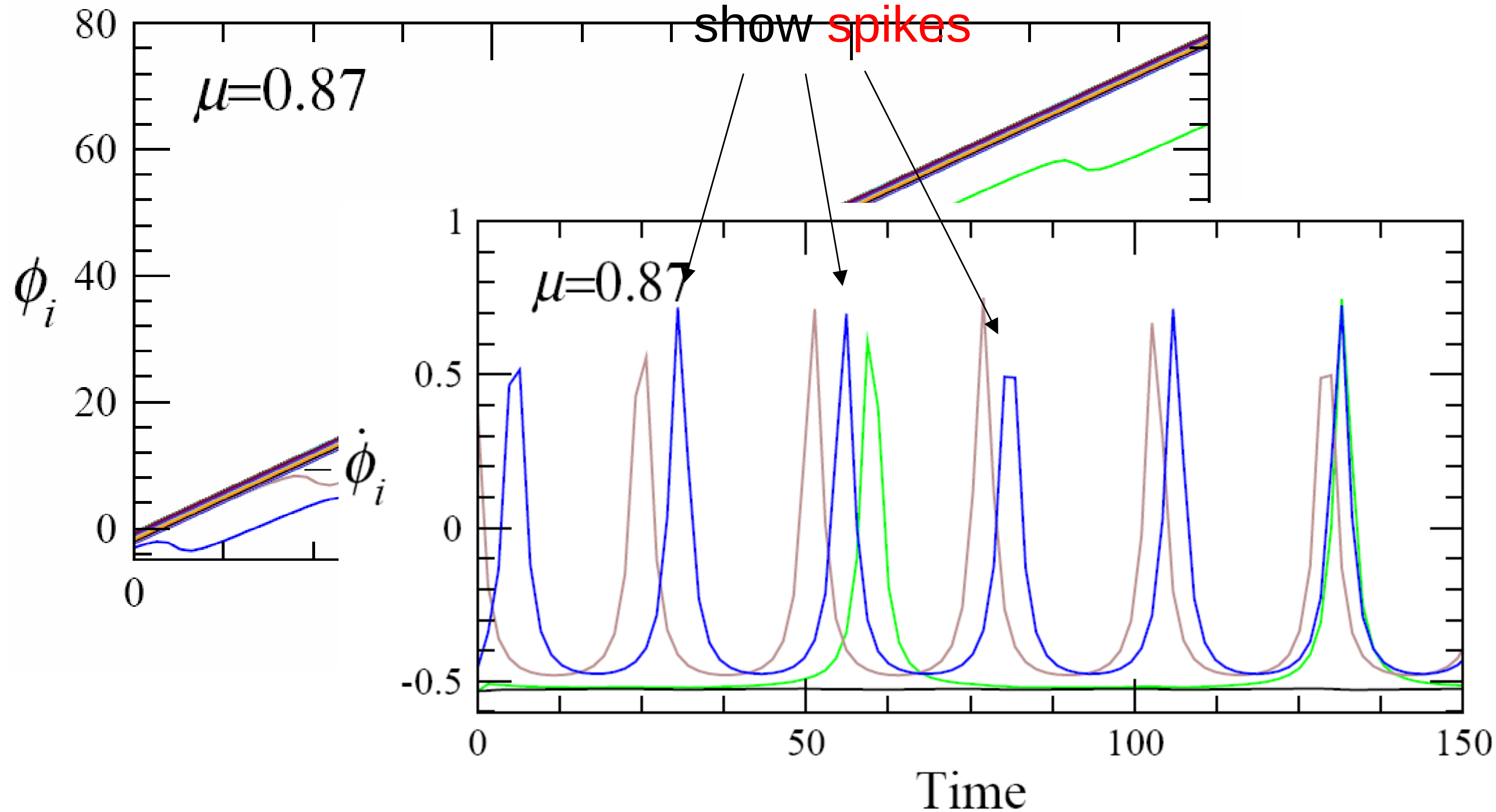
# Emergence of active subnetworks

For each value of ❍ we will have a certain subnetwork of active elements

- When decreasing ❍, more elements become active
- Isolated active elements aggregate into connected active components
- Connected components emerge, grow and merge with each other, forming larger structures



Velocities of active elements show spikes

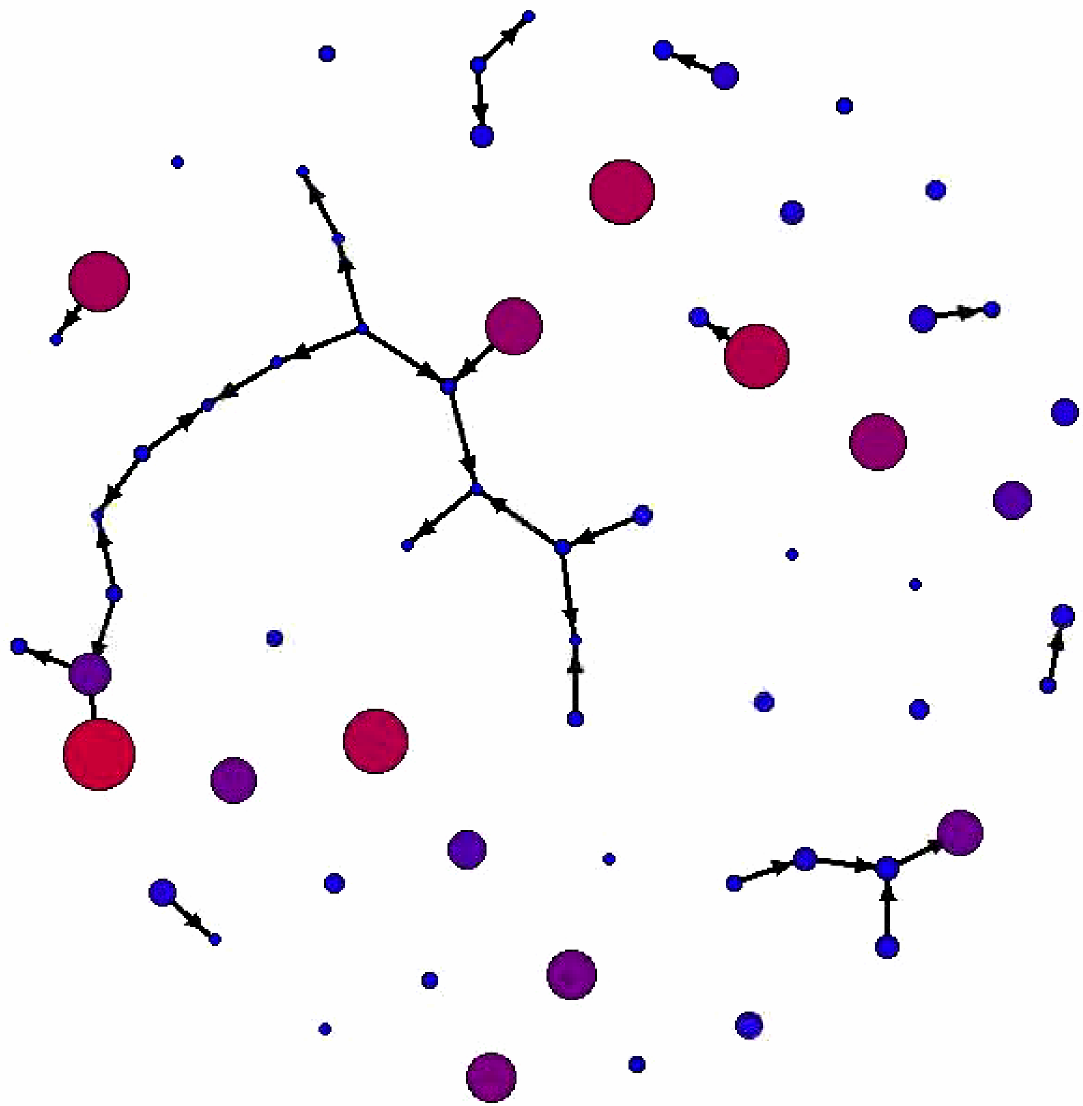



Dynamics of active elements is characterized by repeated phase slips. These oscillators perform excursions from the condensate, and their phases change by steps of 2□

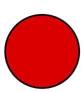
# Dynamics of active subnetworks

Time evolution of the velocities of active elements for fixed ❍.

$$\mu = 0.65$$

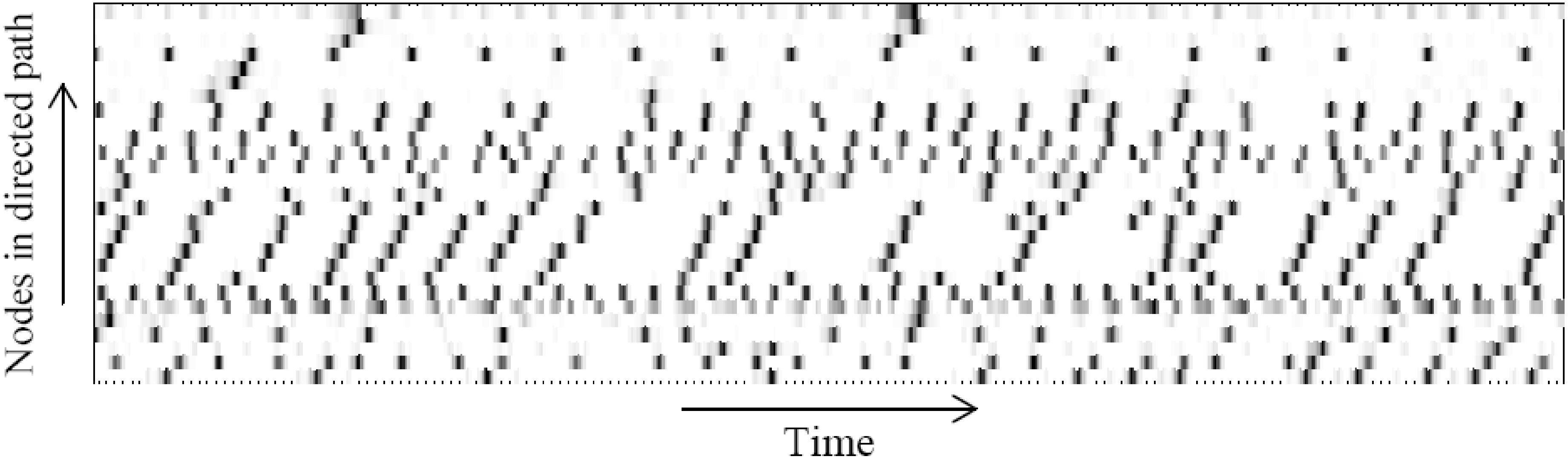


- Connected components have different dynamics
- Excitation can propagate along paths of active subnetworks

 = “spiking” elements

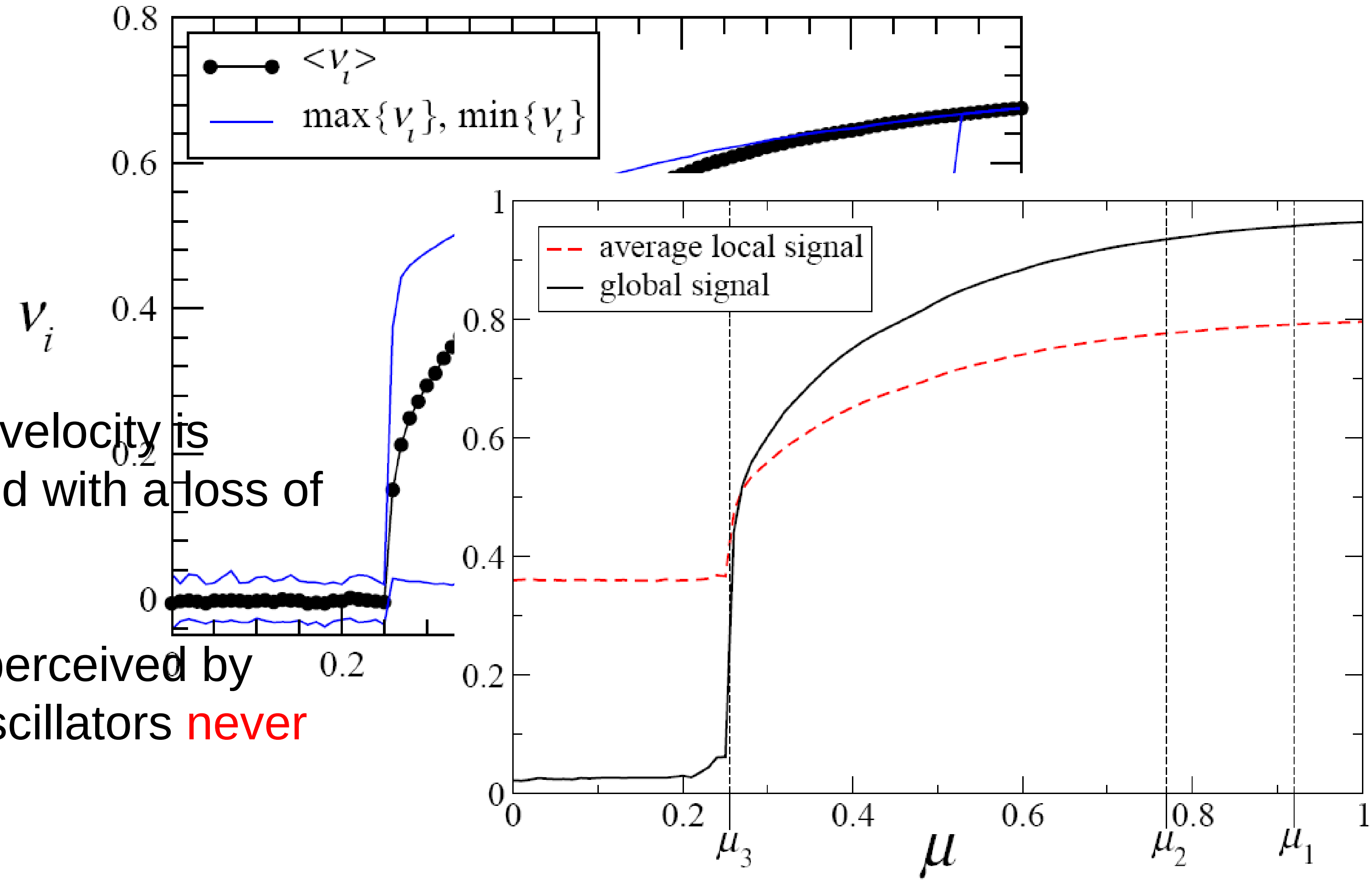
# Signal propagation

Raster plot for the velocities of elements along a given directed path.




Persistent trains of “velocity spikes” are present. Their dynamics are irregular

# Loss of synchronization



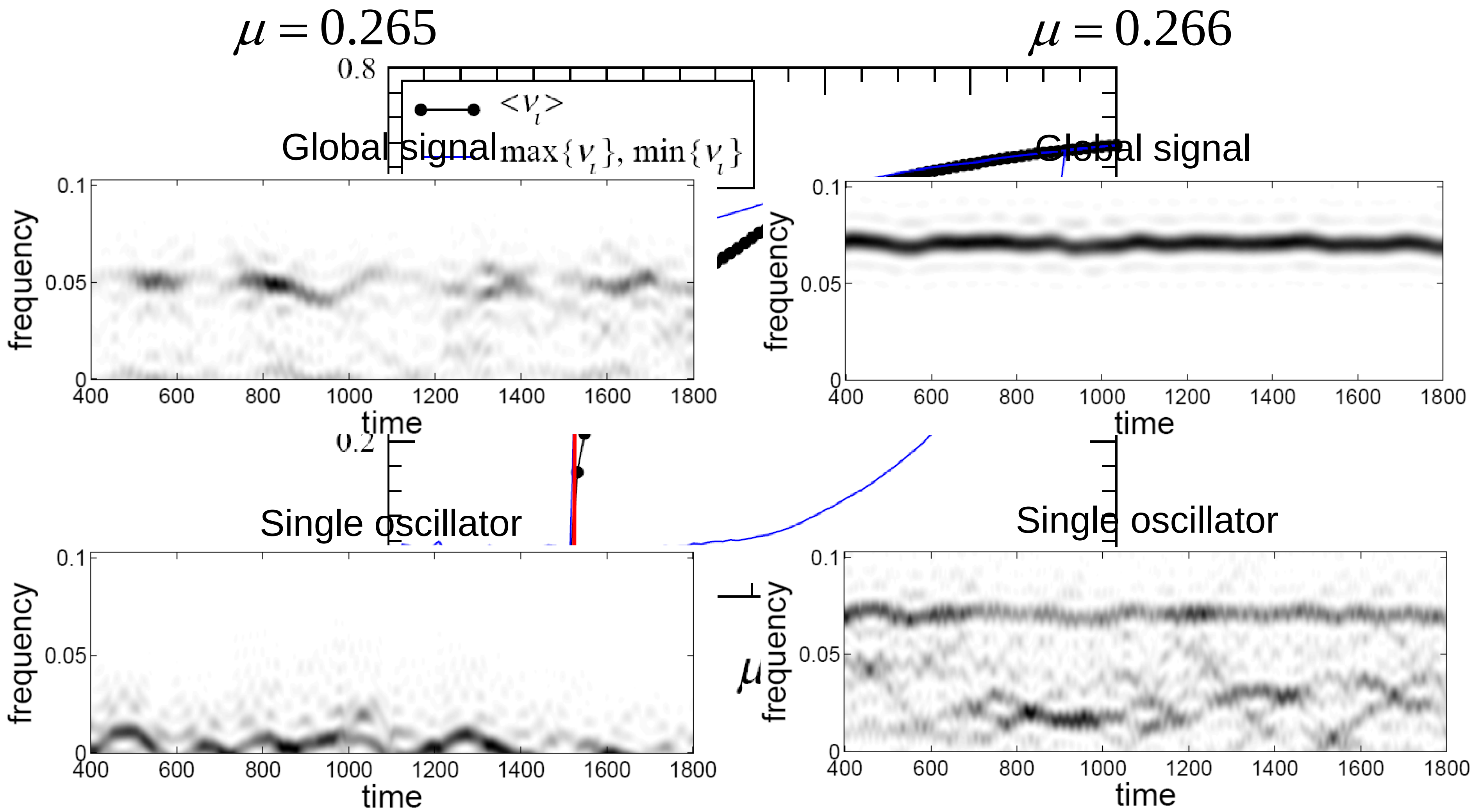

The drop in velocity is accompanied with a loss of coherence

The signal perceived by individual oscillators never vanishes

# Time-dependent spectra

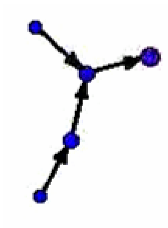
- We calculate the Fourier transform of the global signal in a window of time.
- Shifting the window in time, we get a power spectrum for each time moment.
- We compile this spectra in a time-dependent spectrum plot.
- We compare this plots before and after the transition.

# Time-dependent spectra

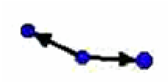


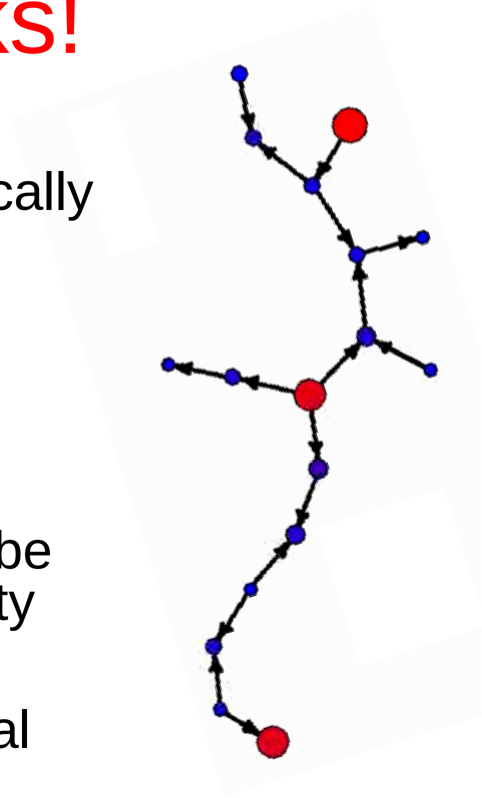
What could the transition scenario look like?
What will be the corresponding "coherent structures"?

# Emerging active networks!



- Active network structures are generated dynamically by a self-organization process.
- Emerging networks have different dynamics and signals can propagate along them
- Their sizes and patterns of dynamic activity can be controlled by varying the global feedback intensity
- Intermittency can lead to transient states of global coherence





VolkswagenStiftung

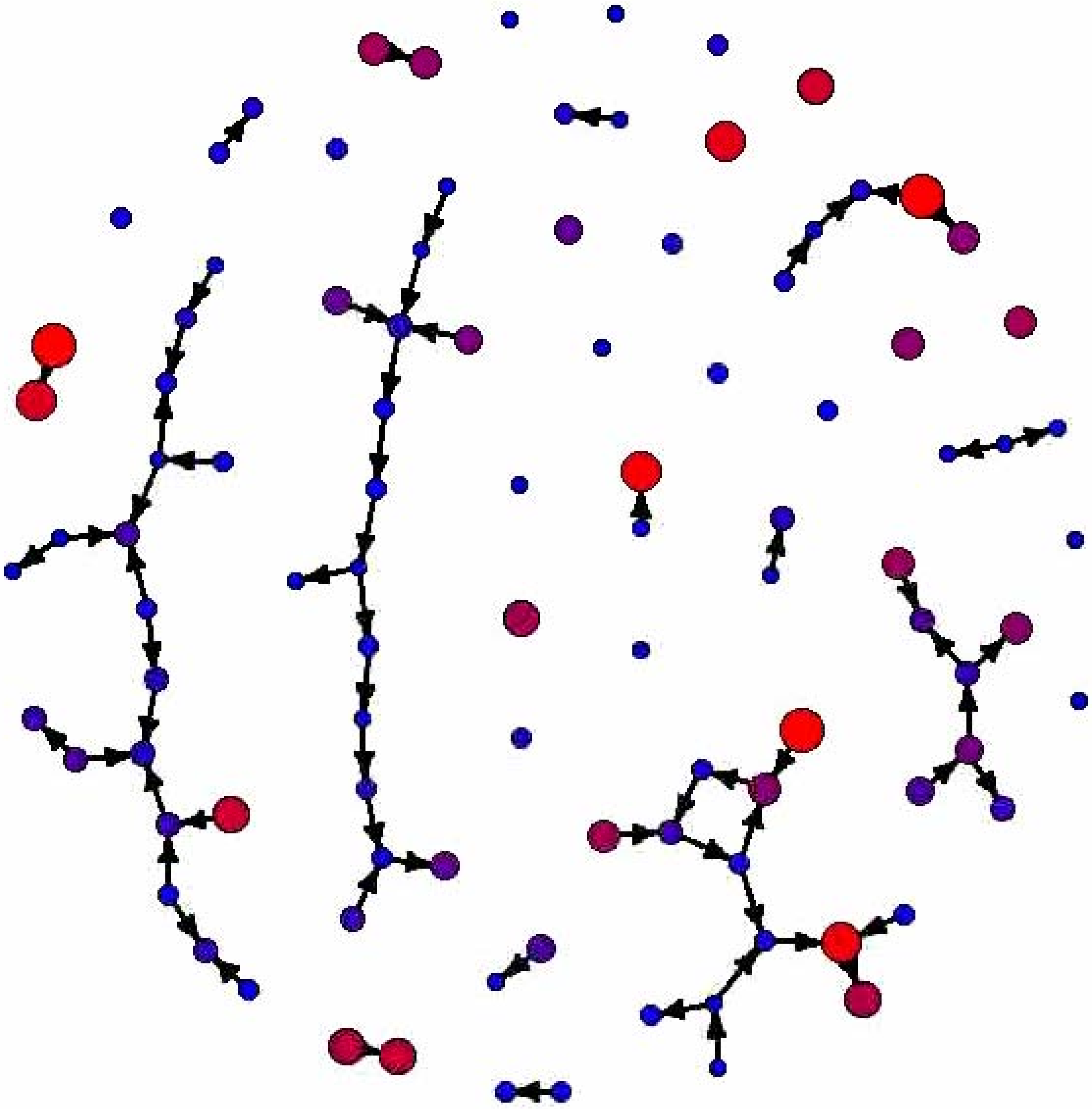
**Research project:**

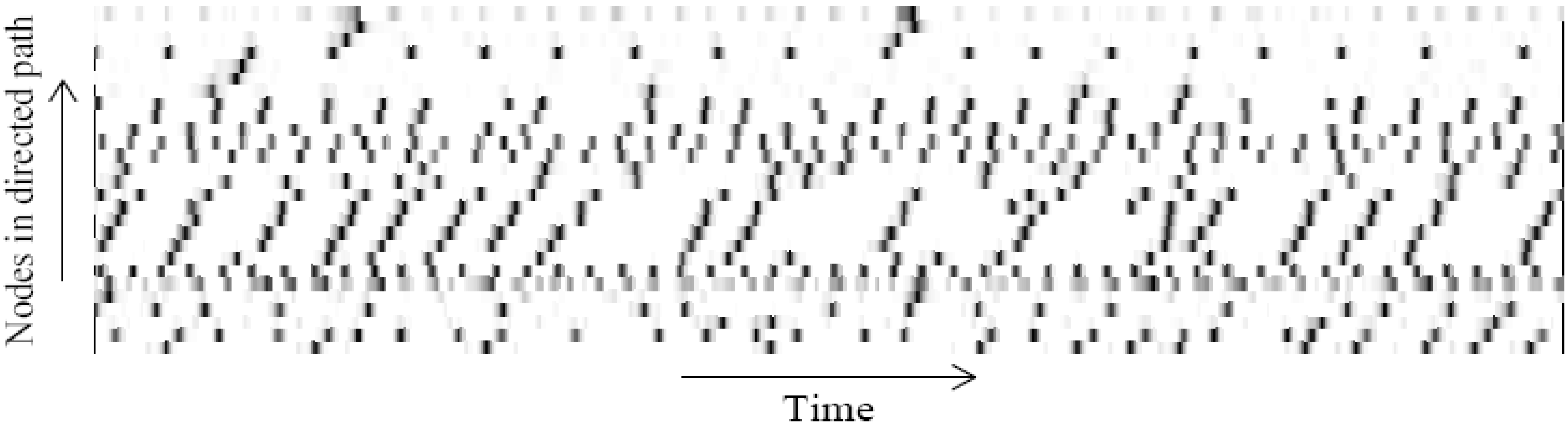
# Complex Self-Organizing Networks of Interacting Machines

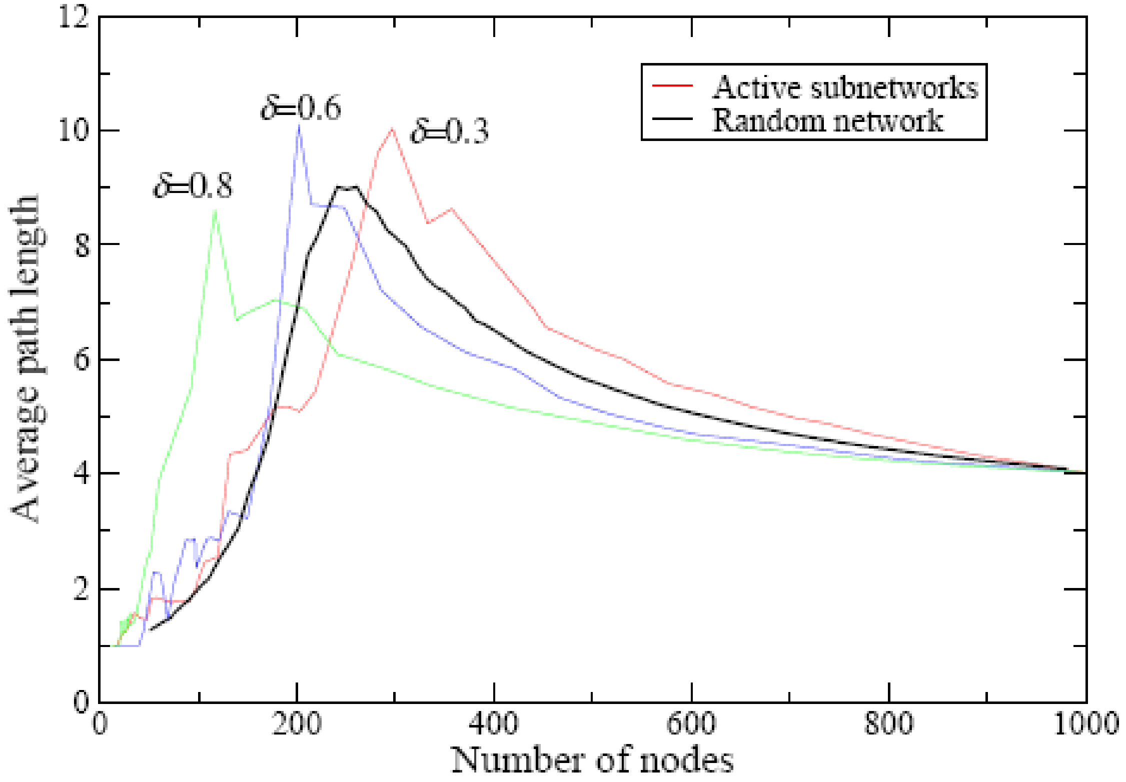
Principles of Design, Control and Functional Optimization

Four Teams:

| | |
|---|---|
| Prof. D. Helbing | (TU Dresden / ETH Zurich) |
| Prof. A. Mikhailov | (FHI Berlin) |
| Prof. M. Vingron | (MPI for Molecular Genetics, Berlin) |
| Prof. D. Ambruster | (Arizona State University) |



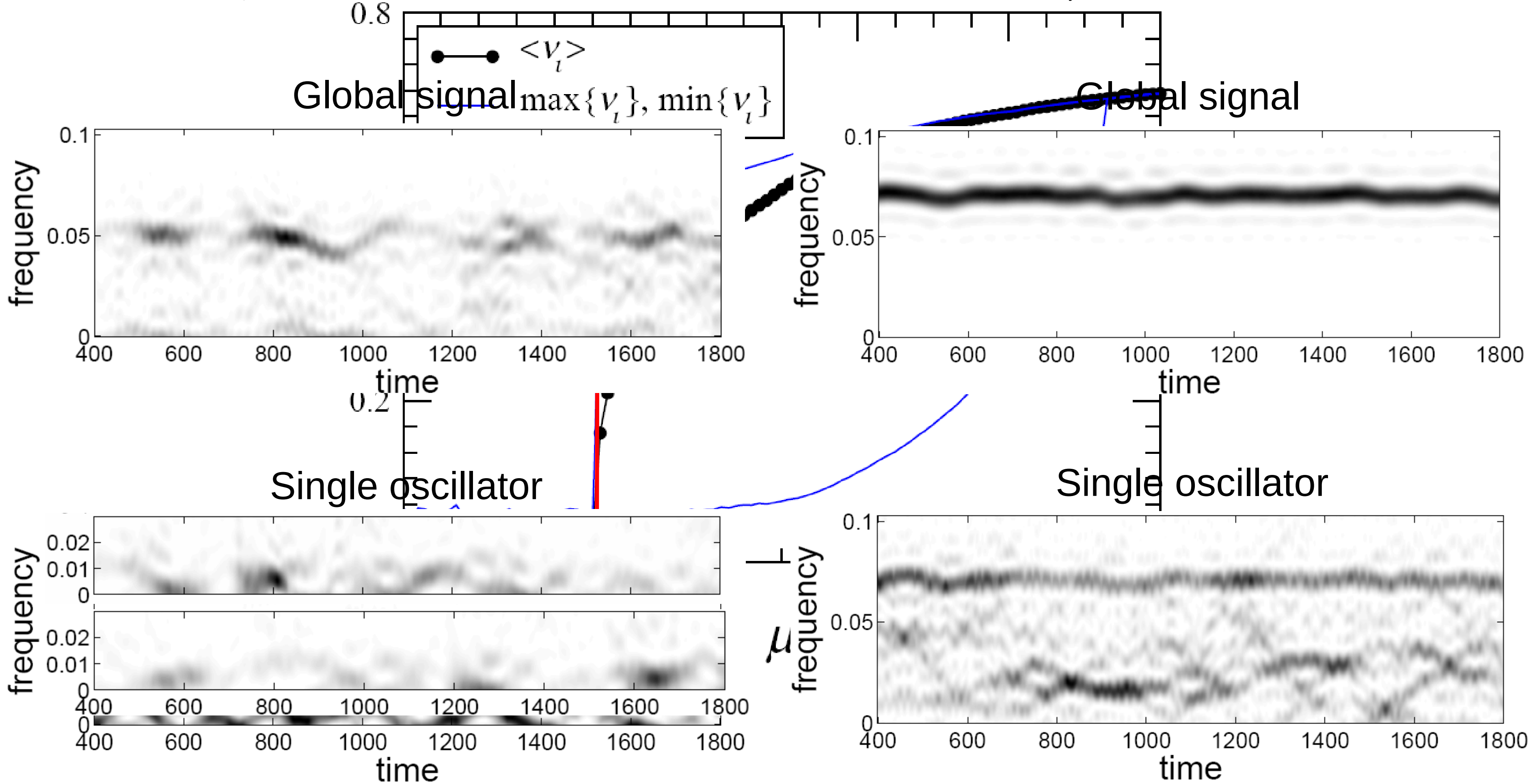

Nodes in directed path
Time

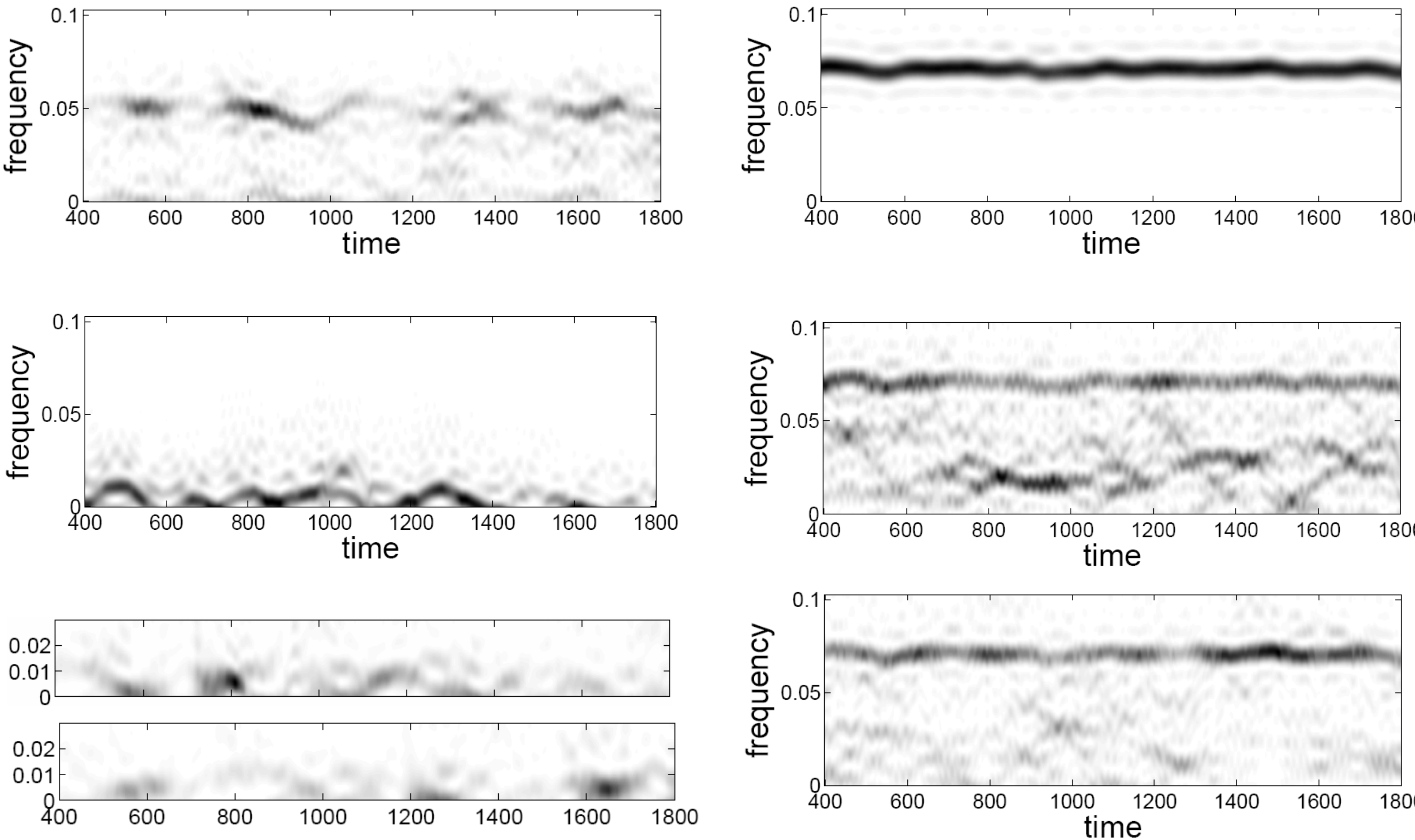


Active subnetworks
Random network
δ=0.8
δ=0.6
δ=0.3
Average path length
Number of nodes
0
2
4
6
8
10
12
200
400
600
800
1000

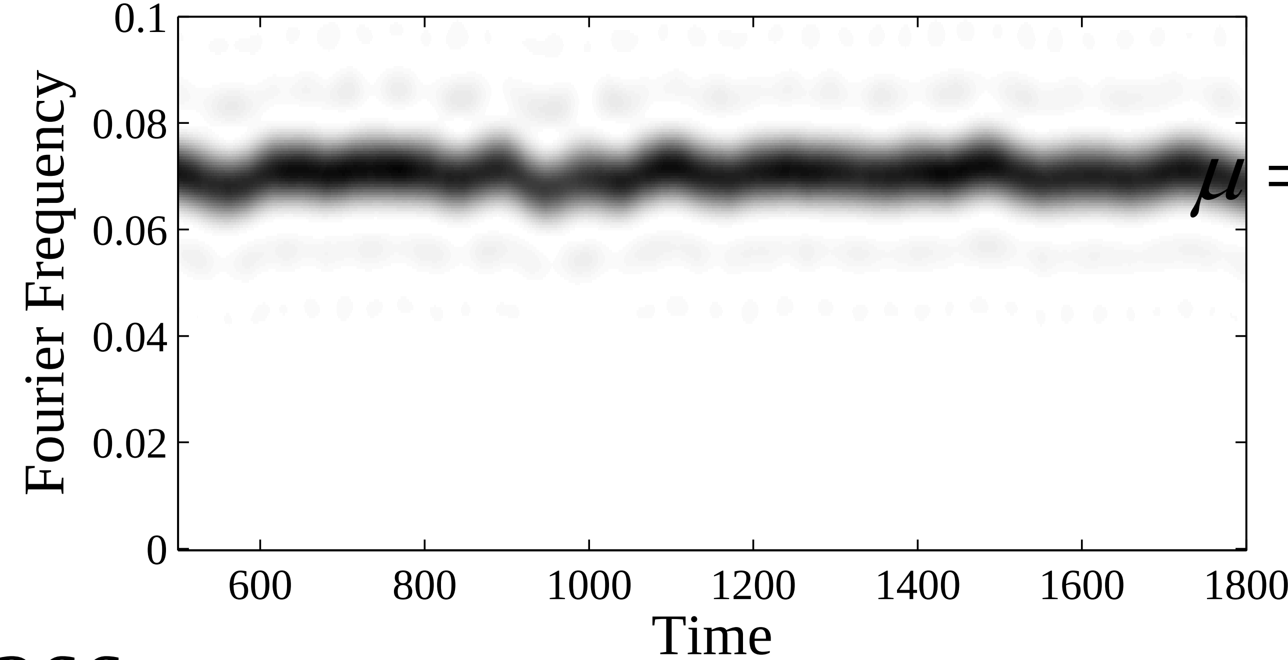
# Time-dependent spectra

$\mu = 0.265$ $\mu = 0.266$



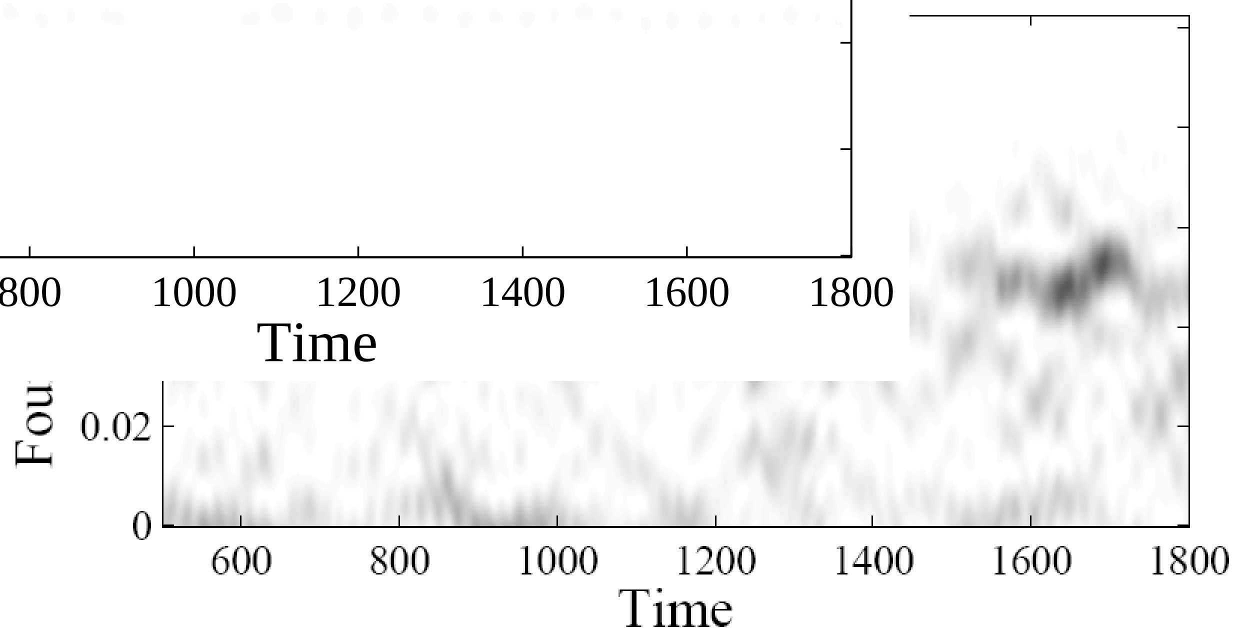

frequency
time
0.1
0.05
0
400
600
800
1000
1200
1400
1600
1800
0.02
0.01

# Time-dependent spectrum of global signal

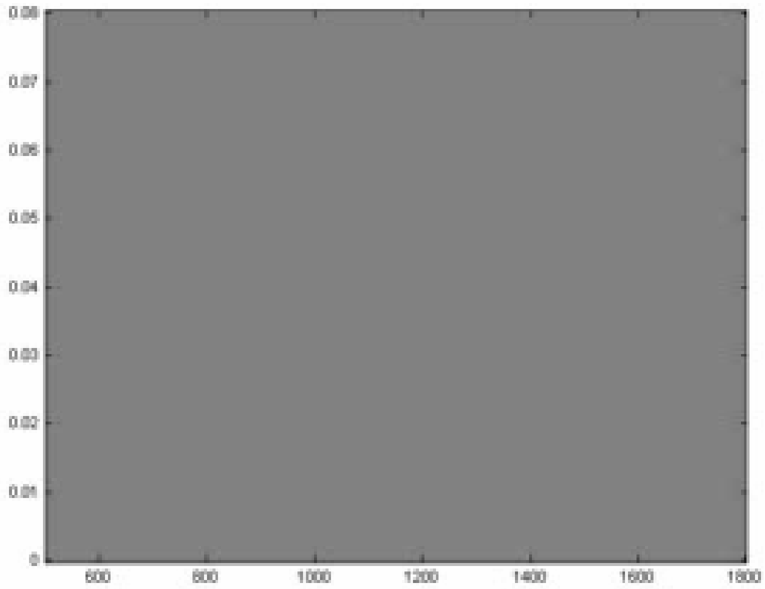


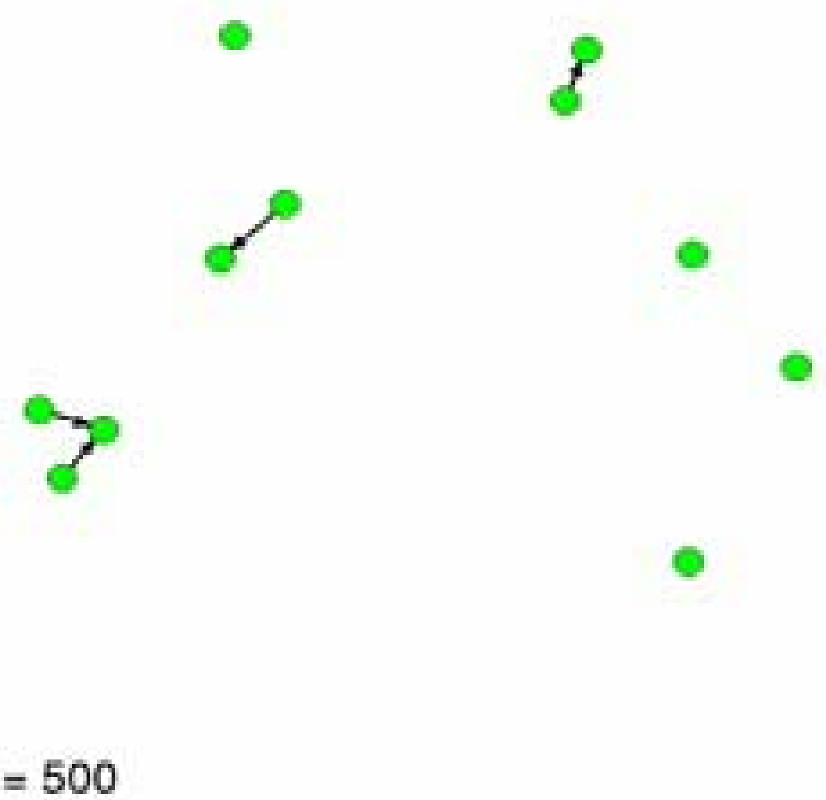

$$\mu = 0.265$$

$$\mu = 0.266$$



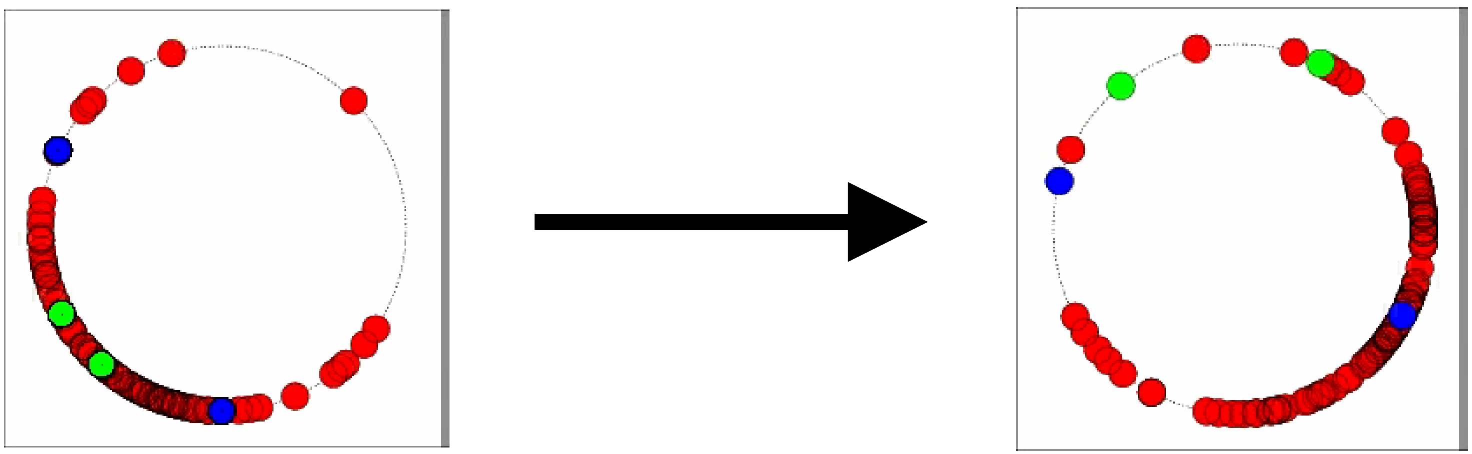
# Individual contributions to Fourier Spectrum



# Control through global feedback.

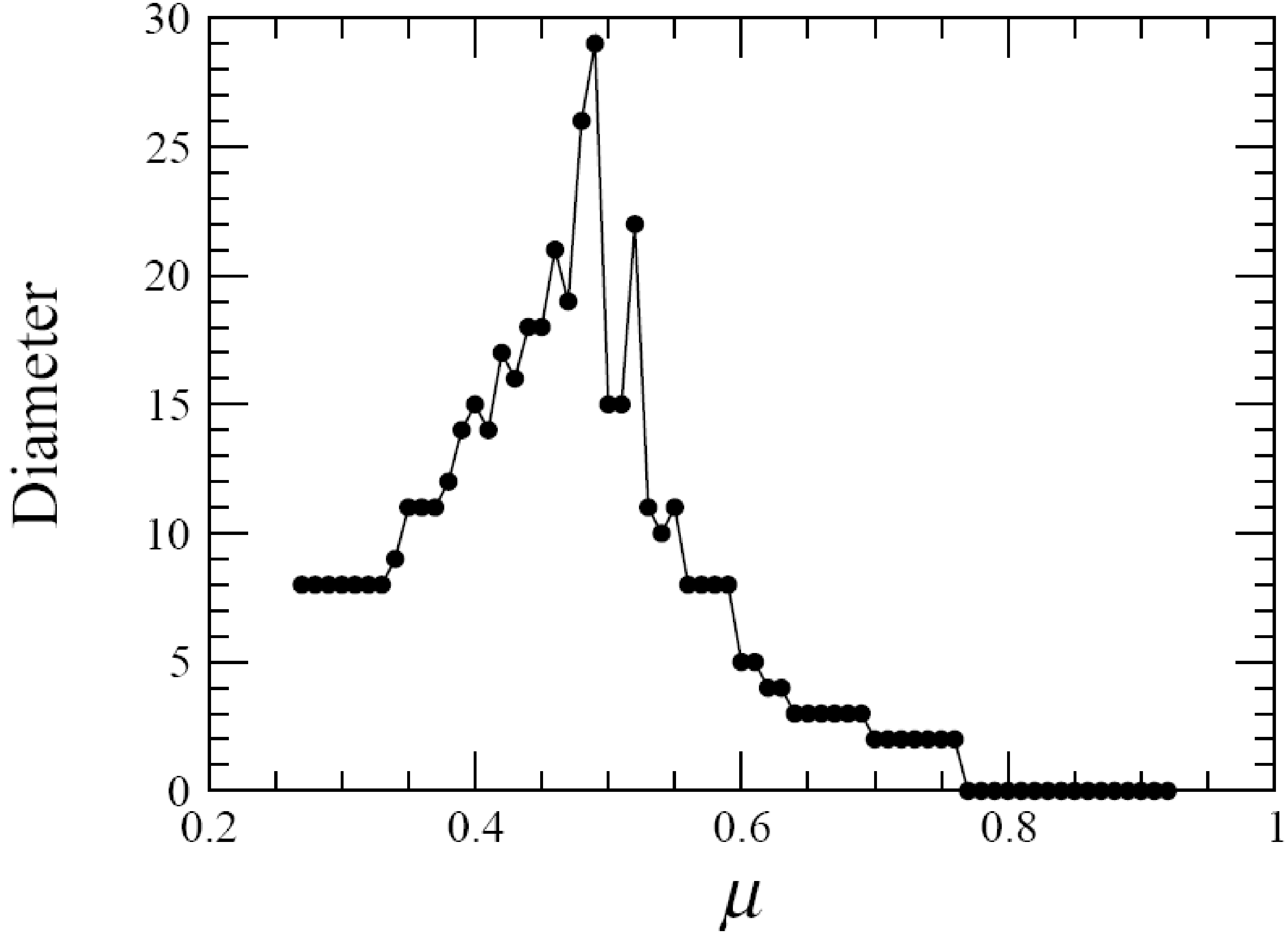
We can watch the system in a co-rotating frame.

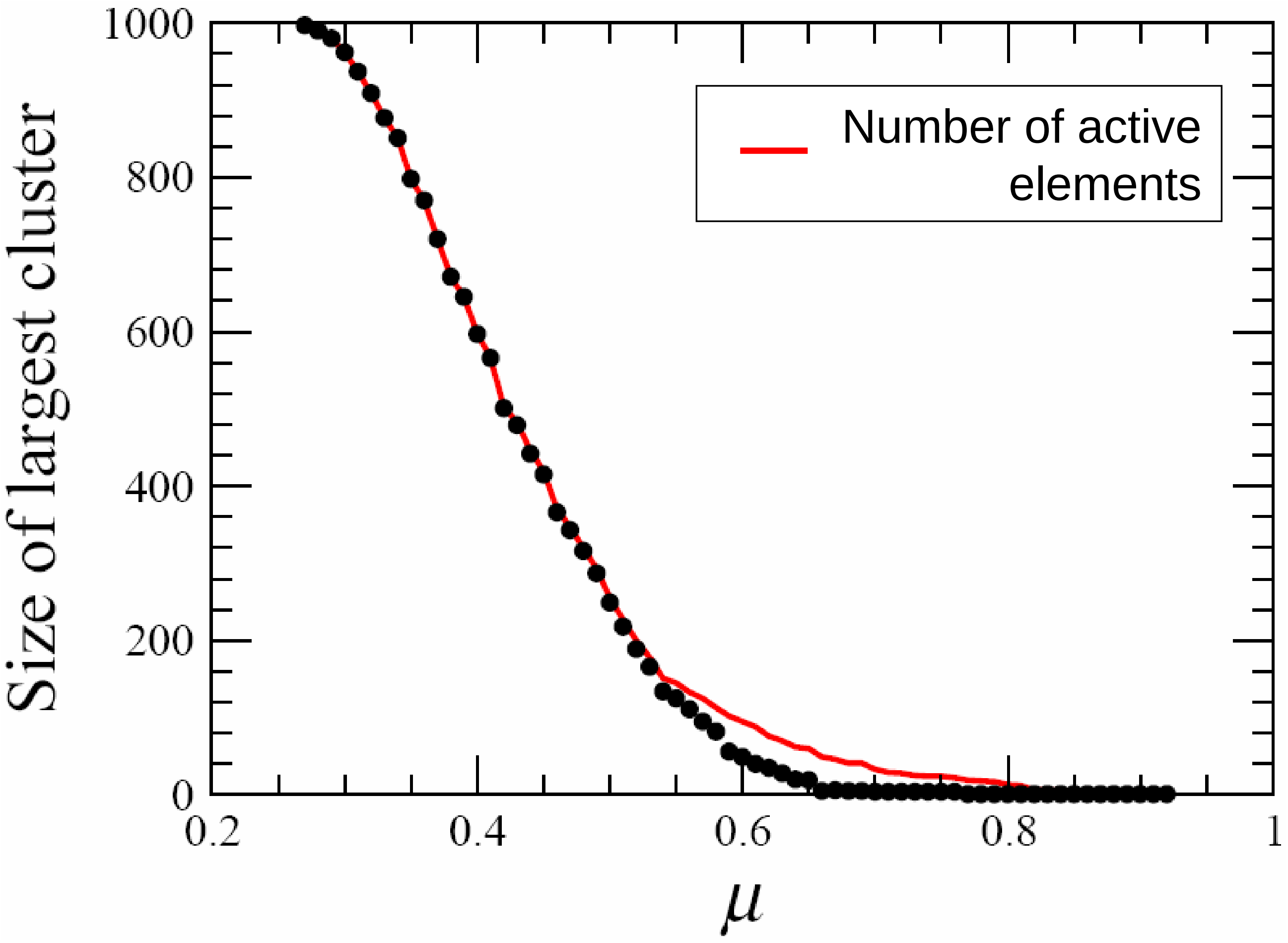


Nodes which dynamically deviate from the bulk are
active nodes

# Control through global feedback

Diameter (largest path amongst the existing shortest paths between pairs of nodes) of the subnetwork of active elements.






$N = 1000$

Size of the largest connected component in the subnetworks of active nodes as a function of the global feedback intensity