Levy-stable processes in economics

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Topics

- Levy-stable distributions and their properties
- A bit of history: the revenge of Mandelbrot
- Applications in economics
  - Business cycle theory
  - Diversification theory
  - Demand dynamics in experience-good markets
- Theory: economic-based generative processes
  - Matching
  - Information transmission
  - Choice-theoretic based GLV

General properties

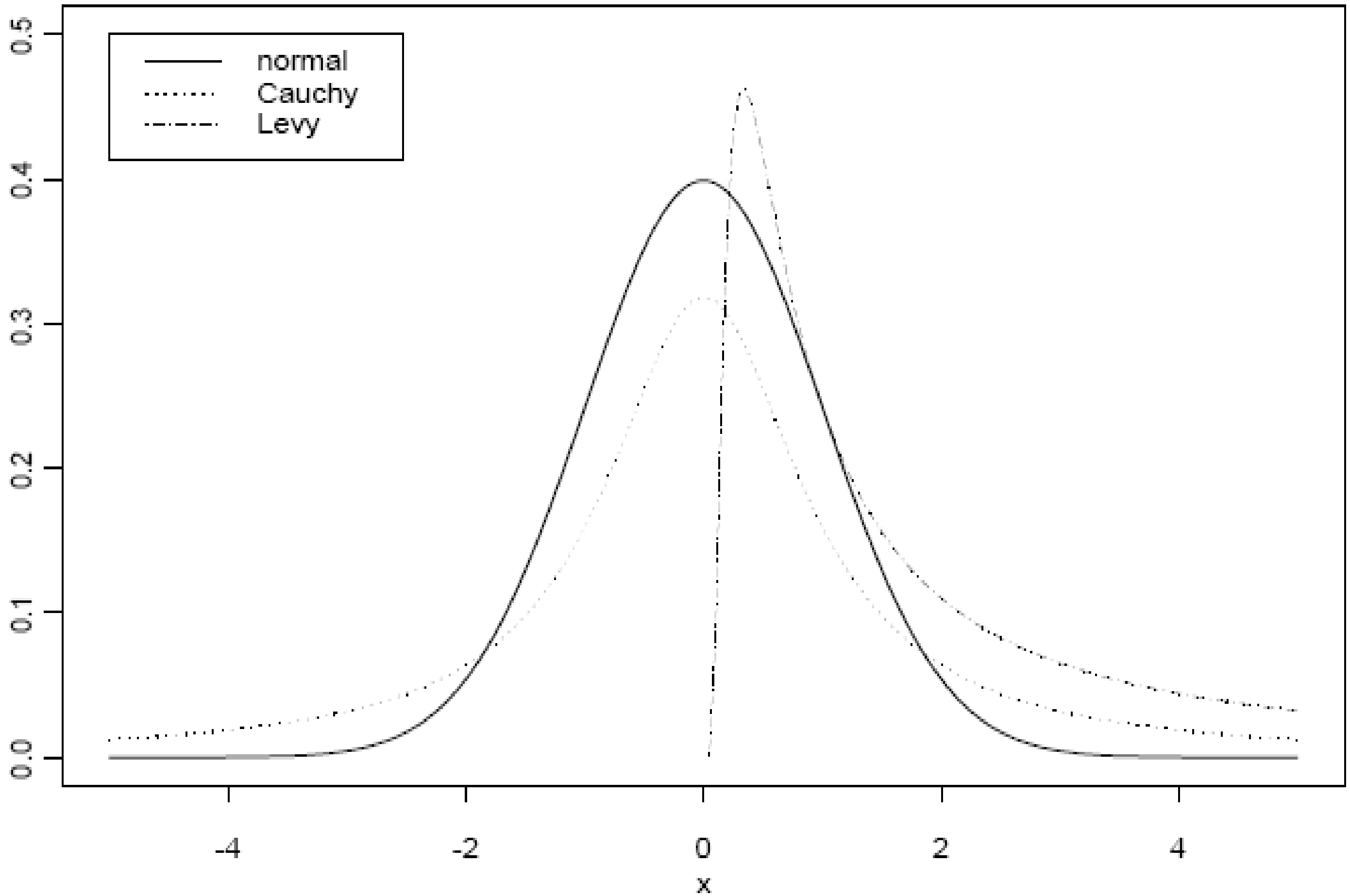
- A four-parameter family of distribution: S(α, β, δ, γ)
  where $\alpha \in (0, 2]$ = index of stability/characteristic exponents
  $\beta \in [-1, 1]$ = skewness parameter
  $\gamma > 0$ = scale parameter
  $\delta \in R$ = location parameter

- A stable distributed random variable X has characteristic function

$$E\exp(iuX) = \begin{cases} \exp\left(-\gamma^{\alpha}|u|^{\alpha}\left[1+i\beta(\tan\frac{\pi\alpha}{2})(\operatorname{sign} u)(|\gamma u|^{1-\alpha}-1)\right]+i\delta u\right) & \alpha \neq 1 \\ \exp\left(-\gamma|u|\left[1+i\beta\frac{2}{\pi}(\operatorname{sign} u)\log(\gamma|u|)\right]+i\delta u\right) & \alpha = 1 \end{cases}$$

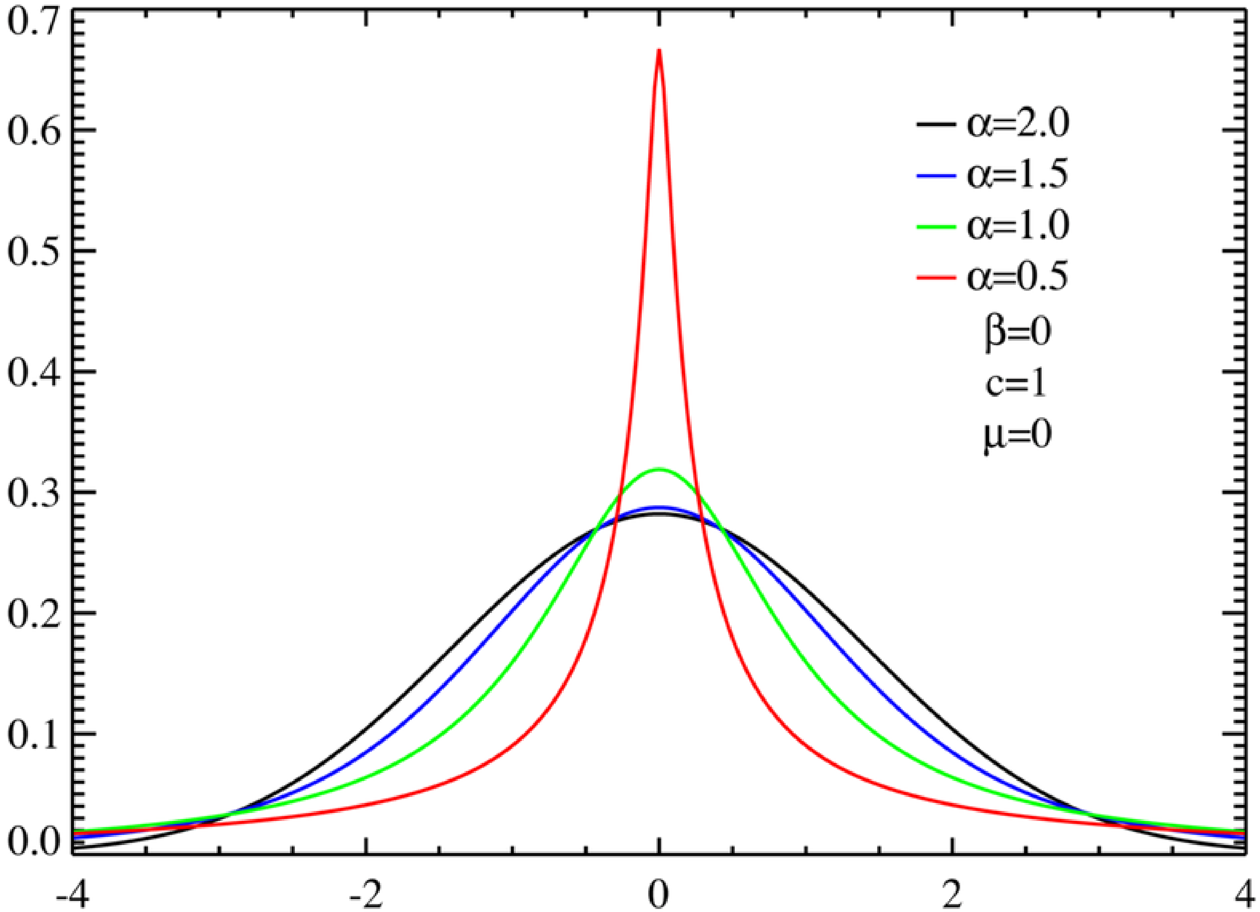
PDFs in closed forms only in three cases

- $\alpha = 2$ ⇨ Gaussian
- $\alpha = 1; \beta = 0$ ⇨ Cauchy
- $\alpha = 0.5; |\beta| = 1$ ⇨ Levy



Implications

- Tails are power-law distributed: $P(|X| > x) \sim x^{-\alpha}$
- Moments are infinite for $p > \alpha$



Levy-stable processes have a long history in economics

- Mandelbrot (1960`)
- The ARCH-GARCH counter-revolution (1970` - 1980`)
- The econophysics movement (1990`)

1. Business cycles

- Gabaix (2005), *The granular origin of aggregate fluctuations*

In real economies the FSD is a power law

→ shocks to few BIG firms cause a *slow-down* of the LLN

… an economy composed of N firms will display aggregate fluctuations with size proportional to $1/\ln(N)$, rather than $1/N^{1/2}$.

Shocks are assumed to possess finite variance.

1. Business cycles

What if shocks have infinite variance?

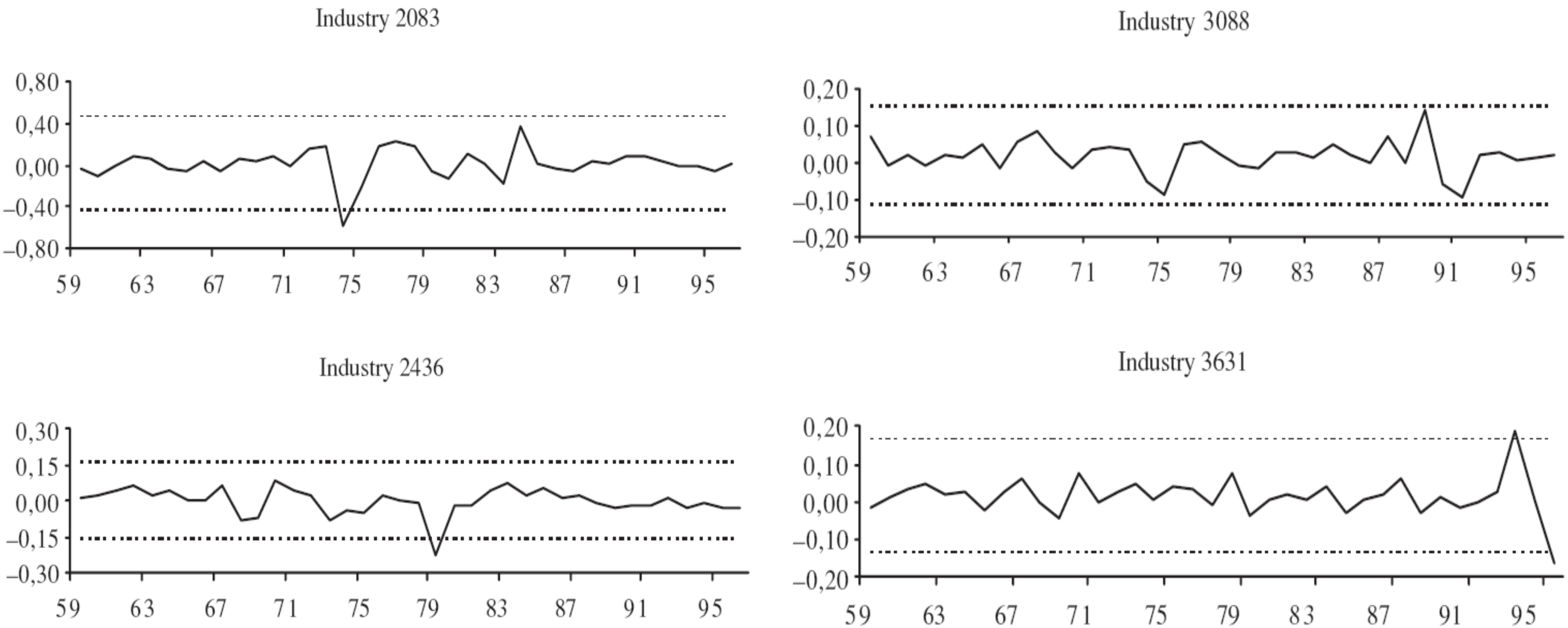
- Gaffeo (2008), *Levy-stable productivity shocks*

Main idea: take TFP growth rates for more than 400 sectors, and try to make some distribution fitting exercises.

Data are available for the USA.

Applications in economics

1. Business cycles



Are they just outliers?

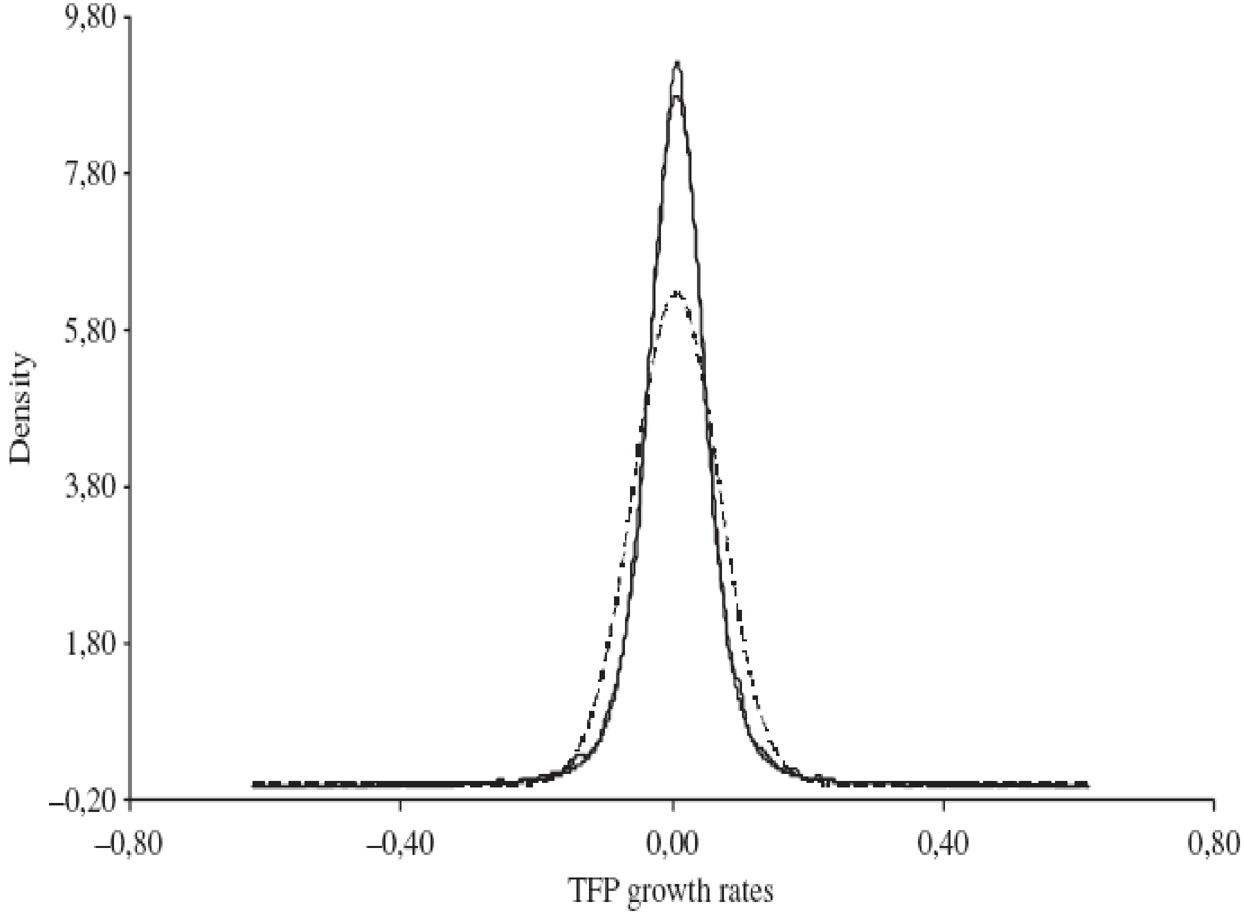
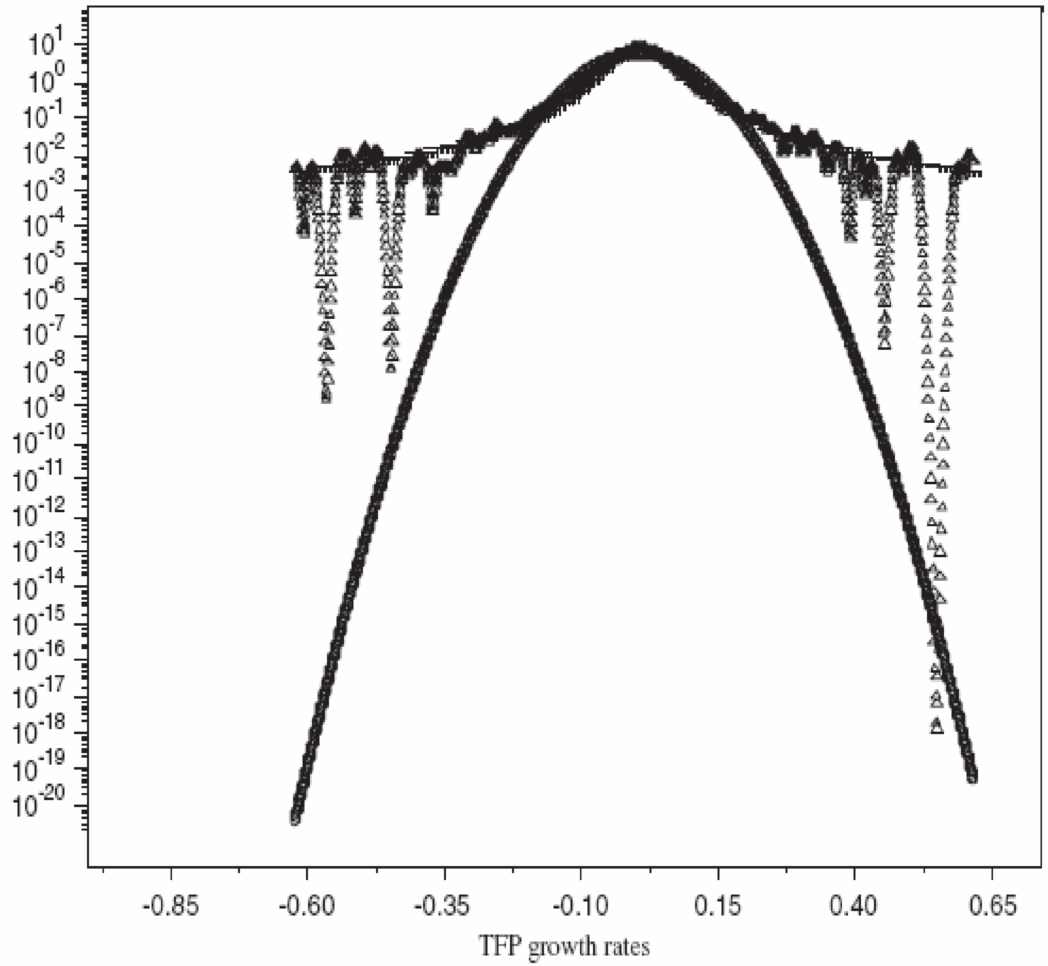


TABLE 2. TFP growth rate distribution parameter estimates, full sample; for maximum likelihood estimates, the 95% confidence bounds are reported

Method	α	β	γ	δ
Quantile	1.5609	−0.0095	0.0316	0.0069
ECF	1.7035	−0.0509	0.0330	0.0069
ML	1.6324 ± 0.0232	-0.0282 ± 0.0563	0.0323 ± 0.0005	0.0068 ± 0.0008



10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
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10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-18}
10^{-19}
10^{-20}
-0.85
-0.60
-0.35
-0.10
0.15
0.40
0.65
TFP growth rates

Method	α	β	γ	δ
Industry 20				
Quantile	1.3689	−0.1168	0.0329	0.0069
ECF	1.5219	−0.0284	0.0347	0.0085
ML	1.4208 ± 0.0713	−0.0869 ± 0.1226	0.0333 ± 0.0017	0.0057 ± 0.0026
Industry 21				
Quantile	1.4555	−0.0707	0.0283	−0.0021
ECF	1.6655	−0.3739	0.0307	−0.0026
ML	1.5517 ± 0.2520	−0.0353 ± 0.5235	0.0238 ± 0.0050	−0.0034 ± 0.0083
Industry 22				
Quantile	1.5656	−0.1540	0.0293	0.0113
ECF	1.8095	−0.4415	0.0313	0.0108
ML	1.7160 ± 0.0987	−0.2338 ± 0.2888	0.0307 ± 0.0019	0.0113 ± 0.0036
Industry 23				
Quantile	1.5448	−0.0151	0.0323	0.0049
ECF	1.7676	−0.2273	0.0340	0.0038
ML	1.6548 ± 0.0885	−0.0226 ± 0.2266	0.0329 ± 0.0018	0.0058 ± 0.0033
Industry 24				
Quantile	1.5675	0.0761	0.0315	0.0027
ECF	1.7693	0.1878	0.0330	00022
ML	1.6650 ± 0.1190	0.0412 ± 0.3109	0.0322 ± 0.0024	0.0038 ± 0.0044

Method	α	β	γ	δ
Industry 25				
Quantile	1.6696	−0.2020	0.0272	0.0032
ECF	1.8371	−0.5107	0.0279	0.0022
ML	1.8547 ± 0.1101	−0.0647 ± 0.6593	0.0281 ± 0.0021	000227 ± 0.0042
Industry 26				
Quantile	1.6535	−0.0619	0.0278	0.0044
ECF	1.6925	−0.3499	0.0275	0.0042
ML	1.6217 ± 0.1203	−0.1594 ± 0.2785	0.0267 ± 0.0021	0.0039 ± 0.0036
Industry 27				
Quantile	1.5939	−0.0036	0.0241	0.0014
ECF	1.8406	0.2039	0.0262	0.0025
ML	1.7702 ± 0.1214	0.0914 ± 0.4422	0.0259 ± 0.0020	0.0005 ± 0.0038
Industry 28				
Quantile	1.5922	−0.1312	0.0372	0.0133
ECF	1.7621	−0.1305	0.0390	0.0122
ML	1.6615 ± 0.0909	−0.1297 ± 0.2323	0.0380 ± 0.0022	0.0144 ± 0.0039
Industry 29				
Quantile	1.6149	−0.3042	0.0326	0.0094
ECF	1.7804	−1.0000	0.0345	0.0089
ML	1.6627 ± 0.2161	−0.3928 ± 0.5316	0.0334 ± 0.0046	0.0078 ± 0.0084
Industry 30				
Quantile	1.4814	−0.5087	0.0206	0.0205
ECF	1.8428	−1.0000	0.0233	0.0228
ML	1.7248 ± 0.1158	−0.6518 ± 0.2974	0.0228 ± 0.0017	0.0167 ± 0.0033

Method	α	β	γ	δ
Industry 31				
Quantile	1.4534	−0.0703	0.0348	0.0002
ECF	1.7092	−0.4116	0.0383	−0.0017
ML	1.5601 ± 0.1510	−0.1710 ± 0.1510	0.0366 ± 0.0036	0.0016 ± 0.0062
Industry 32				
Quantile	1.5830	−0.0627	0.0326	0.0084
ECF	1.7531	−0.0217	0.0342	0.0079
ML	1.7003 ± 0.0943	−0.1819 ± 0.2612	0.039 ± 0.0020	0.0096 ± 0.0037
Industry 33				
Quantile	1.6534	−0.1389	0.0369	0.0060
ECF	1.8060	−0.1600	0.0385	0.0058
ML	1.7279 ± 0.0922	−0.1673 ± 0.2850	0.0378 ± 0.0022	0.0066 ± 0.0041
Industry 34				
Quantile	1.6532	0.1550	0.0304	0.0025
ECF	1.7119	0.1742	0.0308	0.0018
ML	1.6793 ± 0.0792	0.0397 ± 0.2136	0.0305 ± 0.0015	0.0027 ± 0.0028
Industry 35				
Quantile	1.4936	0.2254	0.0315	0.0030
ECF	1.6454	0.2217	0.0332	0.0035
ML	1.6108 ± 0.0692	0.3144 ± 0.1519	0.0330 ± 0.0015	0.0021 ± 0.0026
Industry 36				
Quantile	1.6905	−0.1613	0.0337	0.0162
ECF	1.7283	−0.1256	0.0337	0.0174
ML	1.6667 ± 0.0807	−0.0217 ± 0.2122	0.0331 ± 0.0017	0.0144 ± 0.0030
Industry 37				
Quantile	1.5310	−0.0319	0.0309	0.0065
ECF	1.6906	0.0562	0.0330	0.0066
ML	1.6574 ± 0.1157	−0.0893 ± 0.2950	0.0325 ± 0.0024	0.0073 ± 0.0043
Industry 38				
Quantile	1.8398	0.4933	0.0341	0.0018
ECF	1.8561	0.1073	0.0348	−0.0011
ML	1.7858 ± 0.1087	0.0916 ± 0.4174	0.0343 ± 0.0024	0.0048 ± 0.0046
Industry 39				
Quantile	1.4864	−0.0555	0.0297	0.0106
ECF	1.6477	−0.1247	0.0309	0.0114
ML	1.5690 ± 0.1184	−0.0830 ± 0.2508	0.0301 ± 0.0023	0.0099 ± 0.0040

What about common shocks?

TABLE 4. Characteristic exponents of the errors distribution from cross-section linear regressions with stable disturbances

	α (s.e.)		α (s.e.)
1959	1.4967 (0.0715)	1978	1.5308 (0.0747)
1960	1.4935 (0.0717)	1979	1.5136 (0.0748)
1961	1.5686 (0.0739)	1980	1.4947 (0.0738)
1962	1.6281 (0.0800)	1981	1.5538 (0.0700)
1963	1.4500 (0.0704)	1982	1.5679 (0.0693)
1964	1.5908 (0.0735)	1983	1.4483 (0.0719)
1965	1.7071 (0.0827)	1984	1.5629 (0.0770)
1966	1.6389 (0.0757)	1985	1.5278 (0.0711)
1967	1.8781 (0.0666)	1986	1.5027 (0.0709)
1968	1.9031 (0.0594)	1987	1.5022 (0.0727)
1969	1.6302 (0.0742)	1988	1.6202 (0.0750)
1970	1.7838 (0.0701)	1989	1.5373 (0.0700)
1971	1.7503 (0.0713)	1990	1.6273 (0.0729)
1972	1.4675 (0.0768)	1991	1.5569 (0.0716)
1973	1.5068 (0.0795)	1992	1.6260 (0.0724)
1974	1.5160 (0.0746)	1993	1.5823 (0.0742)
1975	1.5342 (0.0772)	1994	1.5327 (0.0691)
1976	1.4383 (0.0717)	1995	1.5707 (0.0795)
1977	1.5102 (0.0748)	1996	1.5799 (0.0743)

Implications for business cycles

Let us start from Hulten (1978): the rate of increase of GDP caused by *iid* shocks to TFP τ to N sectors is

$$g_{\text{GDP}} = \sum_{i=1}^{N} \frac{S_i}{Y} \tau_i$$

If shocks have identical finite variance σ_τ^2, and each sector is $1/N$ of the total, then

$$\sigma_{\text{GDP}} = \frac{\sigma_\tau}{\sqrt{N}}$$

As the number of sectors gets large, the aggregate standard deviation becomes negligible.

Ex. If $\sigma = 6\%$ for 450 sectors, then aggregate volatility is 0.15%.

Implications for business cycles

If shocks are *iid* ~ S(α, 0, δ, 0), by the property of invariance under convolution we have

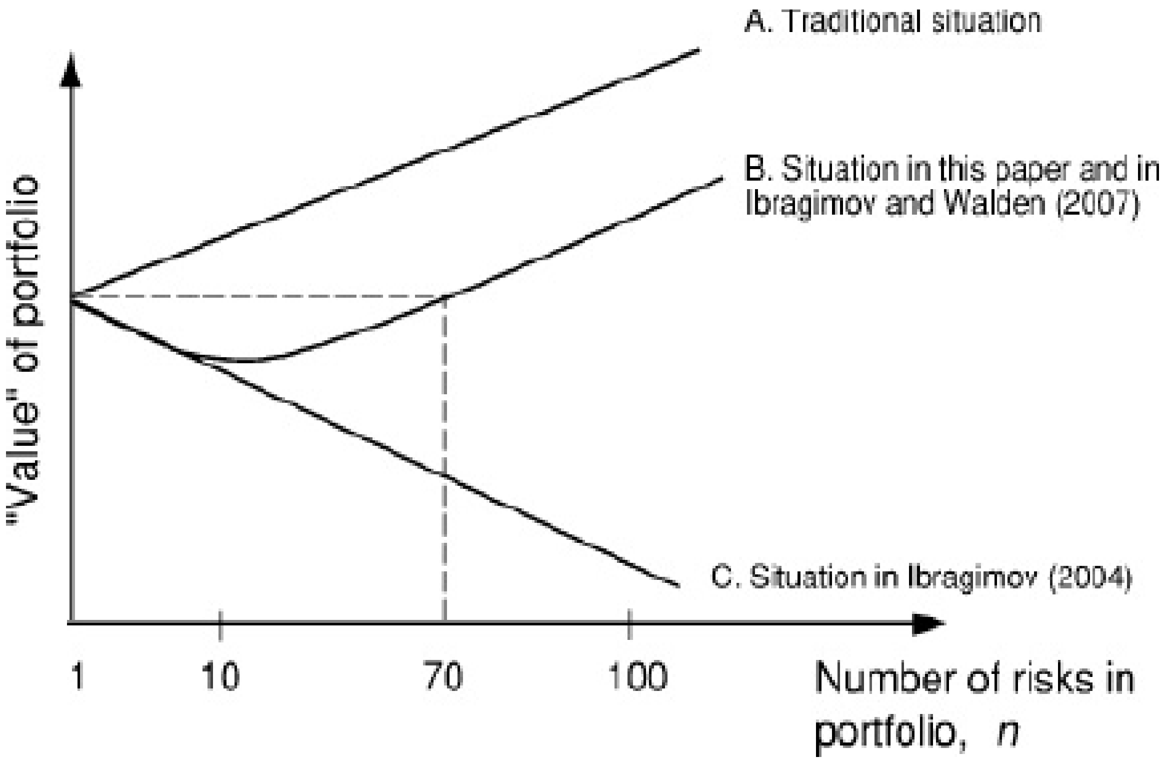
$$\tilde{\sigma}_{\text{GDP}} = \frac{\tilde{T}^{\frac{1}{2}}}{N^{\left(\frac{\alpha-1}{\alpha}\right)}}$$

where T is a stable-distributed random variable.

Hence, aggregate fluctuations decays with N at the rate $\frac{\alpha-1}{\alpha}$, that is much more slowly $N^{-\frac{1}{2}}$ than as implied by Gaussian shocks.

2. Diversification theory

• Ibragimov, Jaffe and Walden (2008), *Nondiversification traps in catastrophe insurance markets*



How much risky is our economic well-being?

$$Wealth_t = PDV_t(\Pi) + PDV_t(W)$$

Diversifiable in financial markets

...sent discounted ...ue of annual ...terest and dividends from financial assets

Present c... ...e of wag... busin... assets...

→ Hu... hous...

Is this fraction diversifiable?

≈ 10%

≈ 90%

How to manage the largest economic risks?

Example: Human capital

- Construct labor income indices pricing uncertainty on future labor income;
- Design a market for labor income risk-sharing;

Problems in creating a market for labor income risk-sharing

1) Moral hazard;

2) Psychological barriers in buying insurance;

3) Microstructure of the market:
   - Role of intermediares
   - Contract settlement
   - Liquidity

Ref.: Shiller (1993); Shiller and Schneider (1998).

A simple implementation

1) *FIs* offer insurance contracts incorporated into deposit account contracts;

2) Short position on an index related to the income from his occupation, long position on a portfolio of indices for other occupations;

3) Max overdraft facility used as a margin for labor insurance contract settlements.

Could it work?

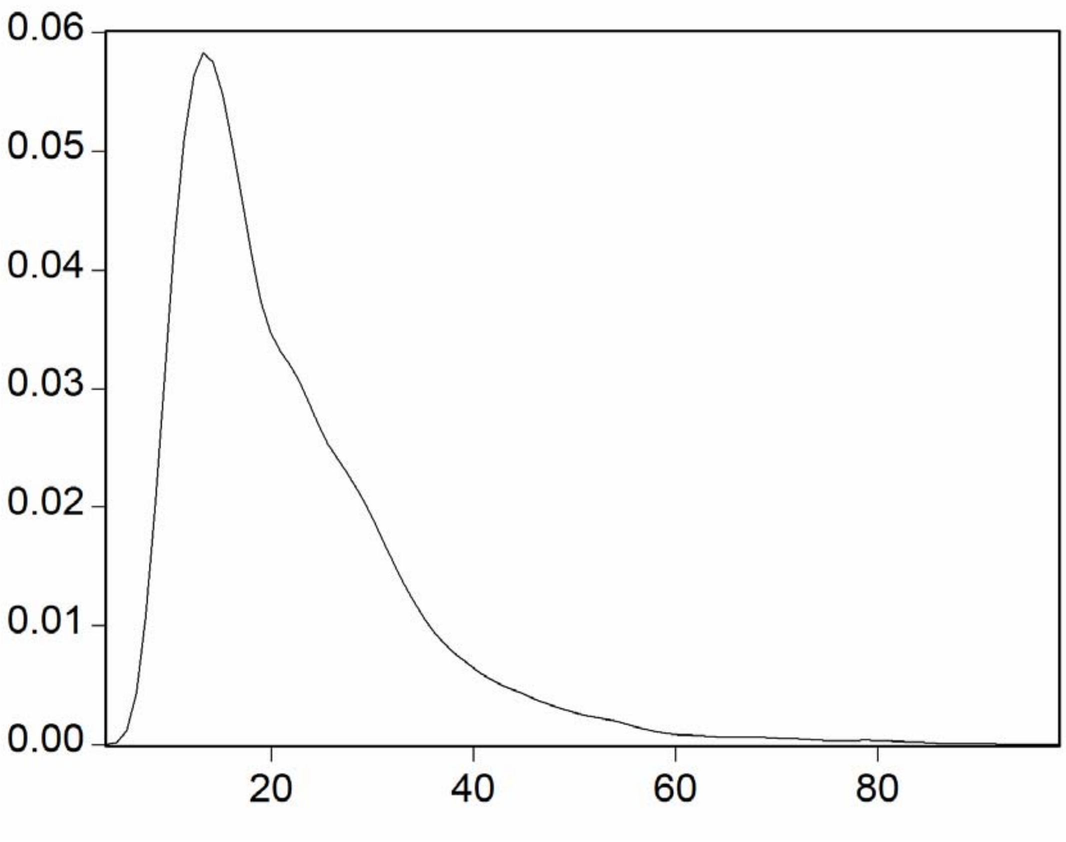
1) Sizeable diversifiable labor income risk;

2) Careful assessment of risk distributions

- Index and option pricing
- Optimal portfolio selection
- Intermediaries' risk management

A picture of the labor market in the U.S.

Average hourly wages for occupations at a 4-digit level, 2006



Number of data-points: 43607

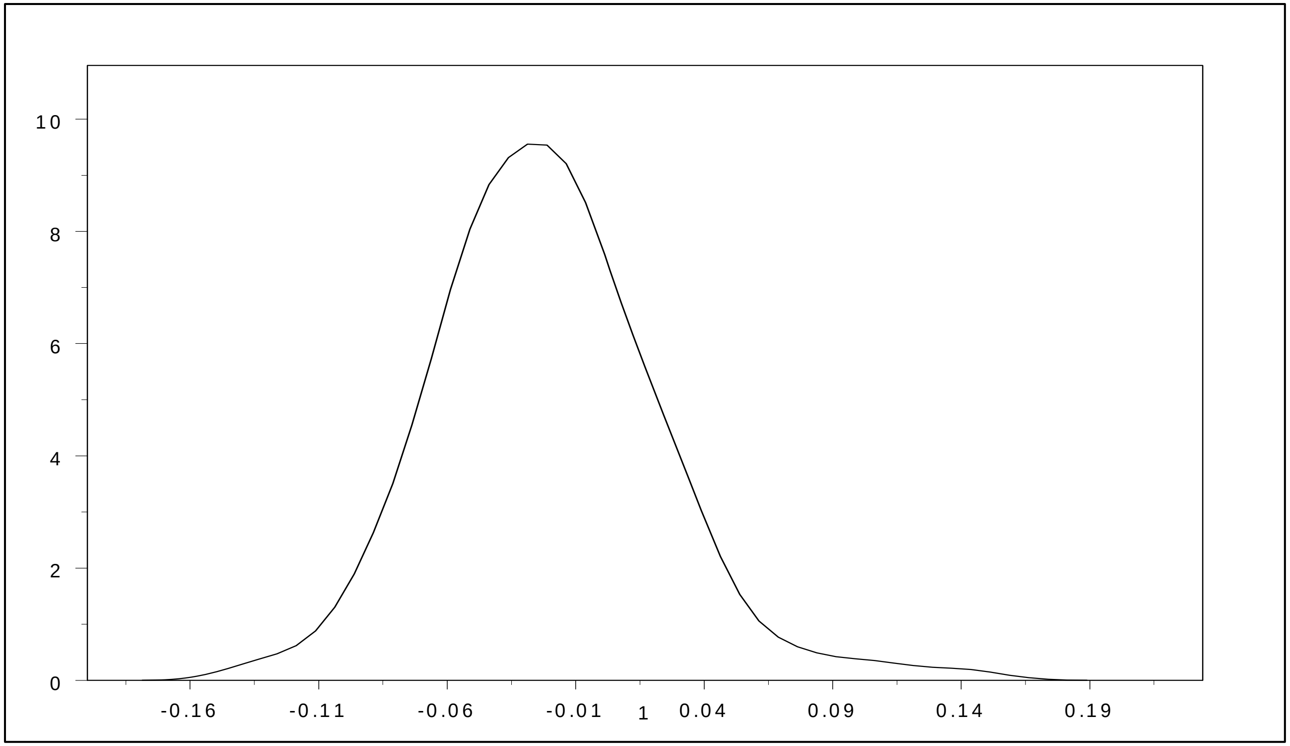
Descriptive statistics

Average hourly wages for occupations in 4-digit sectors, 2006

	Mean	Max	Min.	Std. Dev.	Skewness	Kurtosis	Obs.
[0, 20)	14.04	19.99	6.08	3.1217	0.0005	2.1073	24089
[20, 40)	27.36	39.98	20.00	5.2036	0.5501	2.3223	1643
[40, 60)	47.01	59.91	40.00	5.1928	0.5889	2.3216	2576
[60, 80)	68.24	79.97	60.01	5.6702	0.3901	2.0573	440
[80, 100)	84.34	95.46	80.00	3.6344	0.9430	3.1085	72
All	21.67	95.46	6.08	11.3236	1.6994	7.0849	43607

Is there enough variability?

cumulative growth rates of real hourly wages over a 5-year horizon - 293 industries

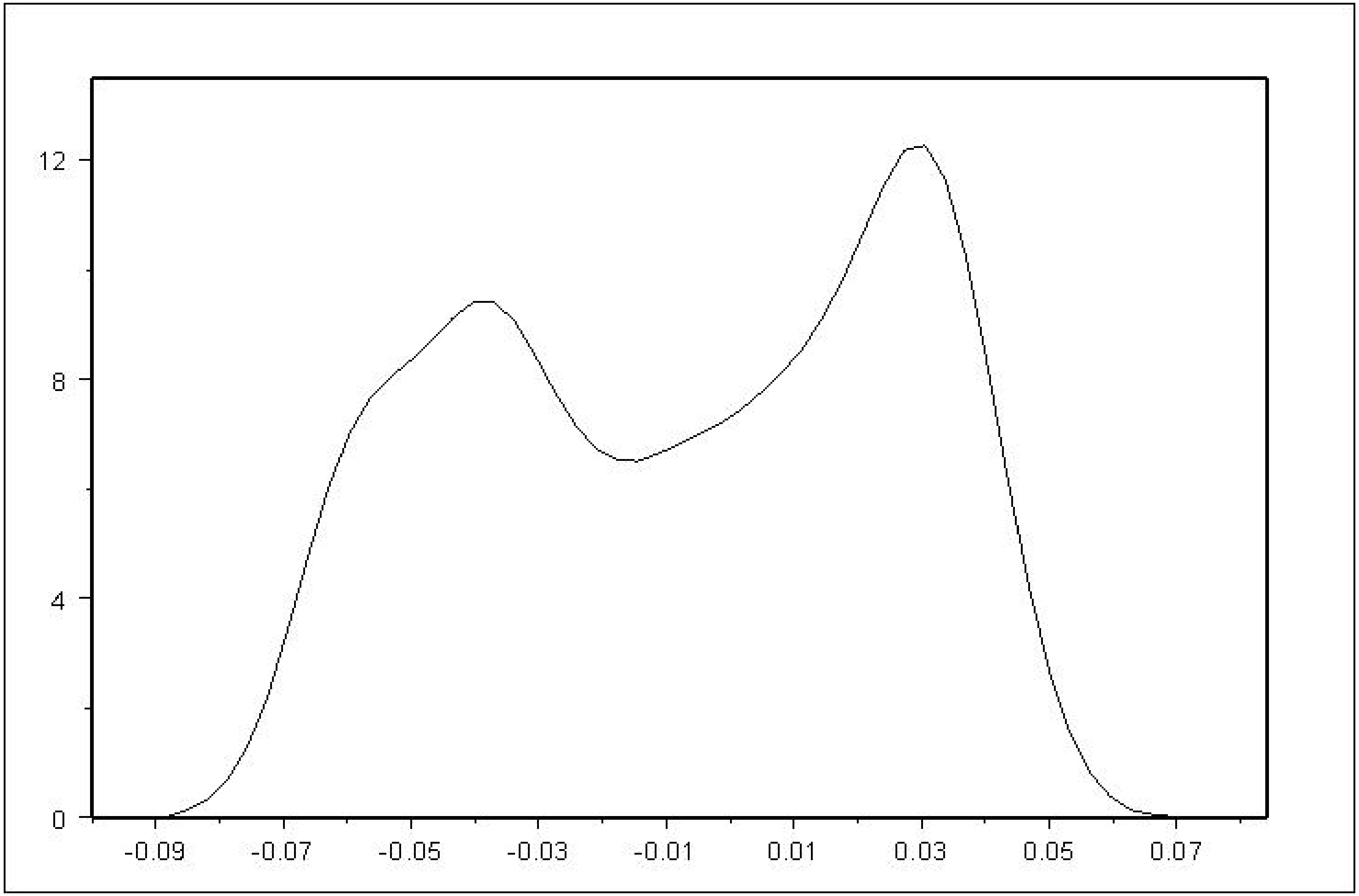


Occupational majors

1) Management
2) Business and financial operations
3) Computation and mathematical science
4) Architecture and engineering
5) Life, physical and social science
6) Community and social services
7) Education, training and library
8) Art, design, entertainment, sports and media
9) Healthcare practitioner and technical occupations
10) Healthcare support
11) Protective service
12) Food preparation and serving
13) Building and grounds cleaning and maintenance
14) Personal care and service
15) Sales and related occupations
16) Office and administrative support
17) Farming, fishing and forestry
18) Construction and extraction
19) Installation, maintenance and repair
20) Production
21) Transportation

winners and loosers

cumulative growth rates of real hourly wages over a 5-year horizon - 21 occupational major averages



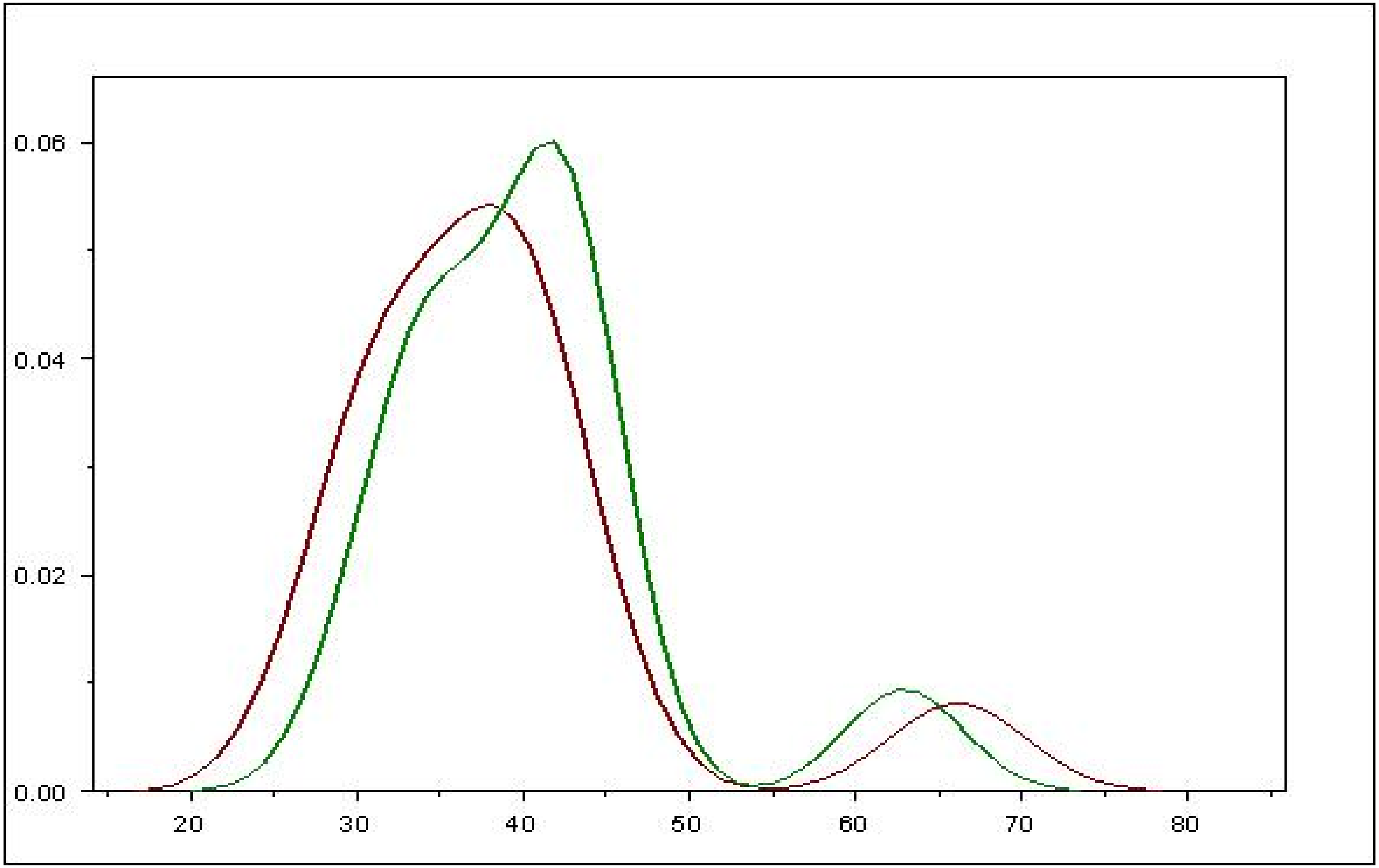
Major: Management

1) Advertisement and promotion mgs
2) Sales mgs
3) Administrative services mgs
4) Marketing mgs
5) Computer and information systems mgs
6) Financial mgs
7) Industrial production mgs
8) Purchasing mgs
9) Transport, storage and distribution mgs
10) Engineering mgs
11) Chief executives mgs
12) General and operations mgs

Wage dispersion

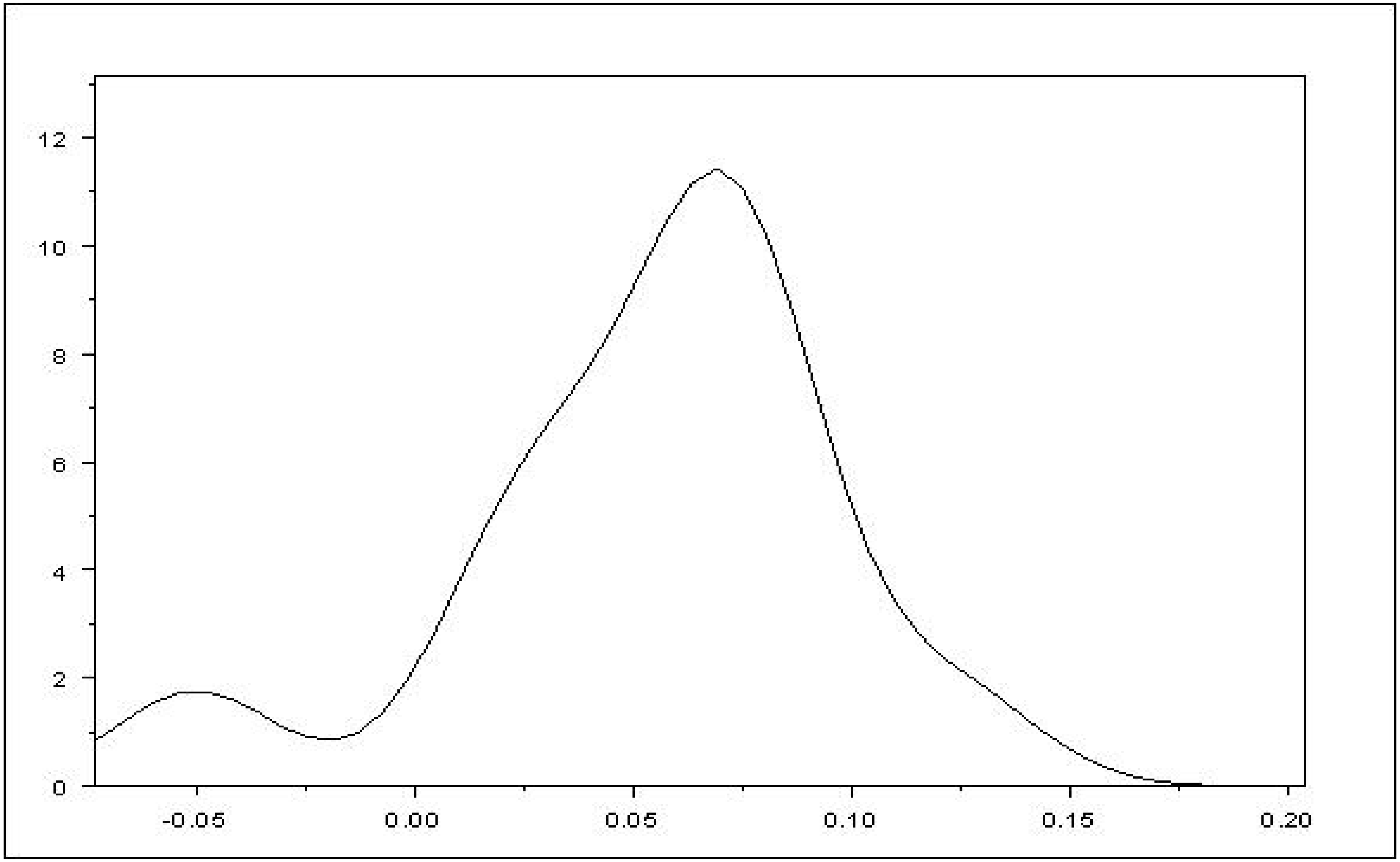
12 different management occupations.

—— 2002 —— 2006



winners and loosers

cumulative growth rates of real hourly wages over a 5-year horizon
- 12 managerial occupations



Could it work?

1) For sure, huge scope for risk-sharing;

2) Be careful in assessing the distributional features of occupational hedgeable risk

Estimation of occupation-specific growth uncertainty

$$g_{i,t,t+s} - \bar{g}_{t,t+s} = \mu_s'\left(z_{i,t} - \bar{z}_t\right) + u_{i,t,t+s}$$

Diversifiable risk

g = growth rate of average o… income over an s time horiz…

z = predictable variable(s…

Ref.: Athanasoulis and van Wincoop (2001).

Diversifiable labor income risk (OLS)

Major occupations	c	μ	σ
Management	-0.0002 (0.0051)	-0.2039 (0.0243)	0.087
Business and financial operations	-0.0004 (0.0043)	-0.2688 (0.0275)	0.072
Computer and mathematical science	-0.0031 (0.0063)	-0.2158 (0.0415)	0.104
Architecture and engineering	-0.0034 (0.0077)	-0.4248 (0.0519)	0.113
Life, physical and social science	-0.0065 (0.0102)	-0.5013 (0.0498)	0.156
Community and social services	-0.0029 (0.0184)	-0.3755 (0.0791)	0.144
Education, training and library	-0.0078 (0.0193)	-0.2993 (0.0679)	0.184
Art, design, entertainment, sport, media	-0.0032 (0.0085)	-0.1824 (0.0339)	0.131
Healthcare practitioner and technical	-0.0068 (0.0113)	-0.2958 (0.0555)	0.149
Healthcare support	-0.0031 (0.0129)	-0.1715 (0.0747)	0.105
Protective service	-0.0110 (0.0115)	-0.3389 (0.0502)	0.175

Diversifiable labor income risk (OLS)

Major occupations	c	μ	σ
Food preparation and serving	-0.0038 (0.0102)	-0.3743 (0.0613)	0.117
Building and grounds cleaning	-0.0009 (0.0047)	-0.2921 (0.0319)	0.078
Personal care and service	-0.0025 (0.0145)	-0.5808 (0.0576)	0.157
Sales and related	-0.0048 (0.0091)	-0.0785 (0.0232)	0.152
Office and administrative support	0.0003 (0.0022)	-0.0987 (0.0149)	0.077
Farming, fishing and forestry	-0.0037 (0.0144)	-0.2499 (0.0757)	0.127
Construction and extraction	-0.0050 (0.0087)	-0.3842 (0.0509)	0.129
Installation, maintenance and repair	-0.0003 (0.0036)	-0.1174 (0.0203)	0.061
Production	-0.0007 (0.0058)	-0.1626 (0.0242)	0.096
Transport	0.0086 (0.0058)	-0.1043 (0.0236)	0.098

Diversifiable labor income risk (Levy errors)

Major occupations	c	μ	α
Management	0.0031 (0.0041)	-0.1829 (0.0205)	1.5427 (0.1064)
Business and financial operations	0.0004 (0.0036)	-0.2404 (0.0250)	1.6728 (0.0987)
Computer and mathematical science	0.0095 (0.0046)	-0.1091 (0.0343)	1.4995 (0.0937)
Architecture and engineering	-0.0017 (0.0058)	-0.2582 (0.0528)	1.5702 (0.1246)
Life, physical and social science	0.0065 (0.0074)	-0.3137 (0.0452)	1.5366 (0.1103)
Community and social services	0.0144 (0.0123)	-0.1963 (0.0883)	1.2843 (0.2176)
Education, training and library	-0.0063 (0.0137)	-0.1483 (0.0471)	1.4801 (0.1511)
Art, design, entertainment, sport, media	0.0004 (0.0076)	-0.1277 (0.0340)	1.7380 (0.0968)
Healthcare practitioner and technical	-0.0031 (0.0080)	-0.2317 (0.0505)	1.4719 (0.1317)
Healthcare support	-0.0052 (0.0130)	-0.1769 (0.0719)	1.9218 (0.1583)
Protective service	-0.0204 (0.0083)	-0.2685 (0.0383)	1.4517 (0.1041)

Diversifiable labor income risk (Levy errors)

Major occupations	c	μ	σ
Food preparation and serving	0.0150 (0.0065)	-0.0767 (0.0660)	1.1077 (0.1271)
Building and grounds cleaning	-0.0084 (0.0038)	-0.3067 (0.0254)	1.6663 (0.1031)
Personal care and service	0.0518 (0.0104)	-0.0792 (0.0622)	1.1344 (0.1328)
Sales and related	-0.0064 (0.0058)	-0.0233 (0.0145)	1.4996 (0.0923)
Office and administrative support	0.0017 (0.0020)	-0.0830 (0.0144)	1.7559 (0.0908)
Farming, fishing and forestry	-0.0037 (0.0144)	-0.2499 (0.0747)	2.0000 (0.0000)
Construction and extraction	-0.0055 (0.0064)	-0.1908 (0.0413)	1.4109 (0.1154)
Installation, maintenance and repair	0.0038 (0.0024)	-0.0974 (0.0145)	1.4470 (0.0974)
Production	0.0003 (0.0036)	-0.0985 (0.0172)	1.3875 (0.0938)
Transport	0.0034 (0.0032)	-0.0946 (0.0159)	1.3548 (0.0837)

Management occupations (G-Levy errors)

Management	c	μ	α	β	γ
Advertisement and promotion	0.0083	-0.3034	1.6868	-0.2032	0.0975
Sales	0.0107	-0.3571	1.6497	-0.4275	0.0629
Administration services	0.0034	-0.3812	1.8260	-0.3059	0.0764
Marketing	-0.0077	-0.3627	1.6031	0.0976	0.0676
Computer and information systems	0.0080	-0.4285	1.8236	-0.7142	0.0609
Finance	0.0008	-0.3574	1.8516	-0.1161	0.0575
Industrial production	0.0031	-0.3917	1.4196	-0.1042	0.0388
Purchasing	-0.0252	-0.3383	1.6346	0.7926	0.0626
Transport, storage and distribution	-0.0223	-0.4532	1.6678	0.5164	0.0688
Engineering	0.0031	-0.3380	1.8644	-0.2879	0.0499
Chief executives	-0.0028	-0.3729	1.9223	0.7847	0.0545
General and operations	-0.0016	-0.2419	1.8590	0.4740	0.0434

Applications in economics

3. Demand dynamics in creative good markets

- De Vany (2005), *Hollywood economics*

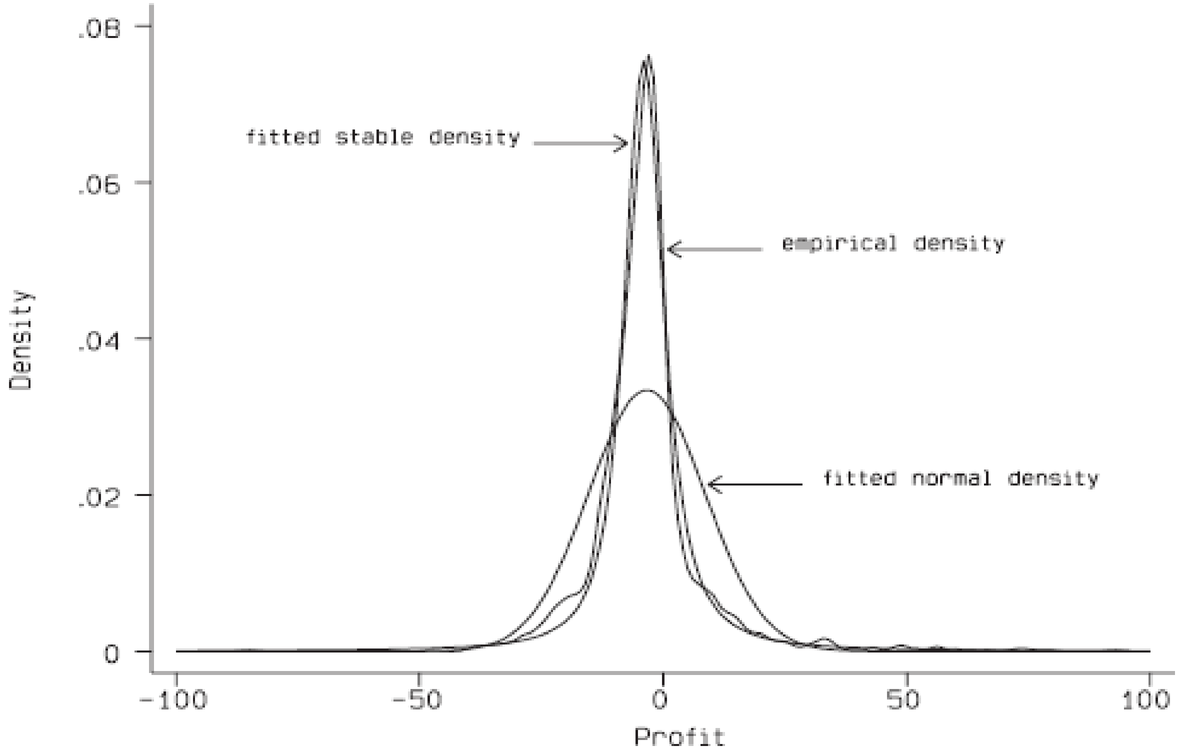


Fig. 1. Empirical and fitted density functions of absolute profit.

3. Demand dynamics in creative good markets

Creative markets display:

- *Nobody knows* principle
- The sample average profit is not stationary, as extreme events dominate the average
- Conditional expectations do not converge *Success breeds success*

3. Demand dynamics in creative good markets

This is good also for books

Gaffeo, Scorcu, Vici (2008), *Demand distribution dynamics in creative industries: the market for books in Italy*

Table 1
Estimates of the scaling exponent α for all three markets

Sample	Italian novels			Foreign novels			Non-fiction		
	a	b	c	a	b	c	a	b	c
94.1	1.39	1.12	1.31	1.38	1.34	1.36	1.32	1.25	1.27
94.2	1.33	1.14	1.18	1.34	1.23	1.27	1.33	1.32	1.33
94.3	1.21	1.05	1.09	1.51	1.46	1.46	1.37	1.29	1.32
94.5	1.04	1.15	1.12	1.33	1.42	1.41	1.13	1.26	1.23
94.6	0.95	0.99	0.98	1.11	1.09	1.09	1.01	1.07	1.09
95.1	1.07	1.06	1.06	1.2	1.2	1.21	1.35	1.31	1.34
95.2	1.17	1.18	1.16	1.19	1.16	1.18	1.29	1.26	1.27
95.3	1.18	1.05	1.12	1.28	1.25	1.26	1.39	1.1	1.16
95.5	1.06	1.03	1.06	1.07	1.07	1.06	1.16	1.14	1.16
95.6	1.01	0.93	0.95	0.91	1.02	1.04	1.11	1.06	1.08
96.1	1.13	1.13	1.12	1.12	1.03	1.06	1.4	1.26	1.31
96.2	1.2	1.19	1.18	1.15	1.03	1.05	1.44	1.28	1.33
96.3	1.26	1.14	1.18	1.19	1.14	1.17	1.45	1.53	1.5
96.5	1.15	1.12	1.12	1.2	1.09	1.11	1.3	1.25	1.26
96.6	0.98	0.89	0.91	1.1	1	1.07	1.09	0.97	1.01

a: White's robust OLS estimates; b: robust regression estimates (Hamilton); c: median regression estimates. All parameters statistically significant at the 5% level. The goodness of fit R^2 is higher than 0.94 in each case.

Theory: economic-based generative processes

1. Matching

Gabaix and Landier (2008), *Why has CEO pay increased so much?*

Consider the market for managers, each one endowed with a given amount of talent.

In the upper tail of any well-behaved distribution for talent $T(x)$, $T'(x)$ [marginal talent] is approximately a power function x^{α}.

It is possible to show that competitive matching generates a PL relation between CEO pay and firm size, and a PL of the pay distribution.

2. Information transmission

Gaffeo, Scorcu, Vici (2008)

Generalized to *M* possible choices the *Information Contagion* model by Arthur and Lane (1993).

Each consumer is endowed with a constant absolute risk aversion utility function defined on the internal representations associated to the quality of the *M* issued books:

$$u(\mu_m) = \begin{cases} -\exp(-2\lambda\mu_m) & \text{if } \lambda > 0 \\ \mu_m & \text{if } \lambda = 0 \end{cases} \tag{6}$$

so that the objective function of the *i*th agent is to maximize a linear function of the mean and the variance of the posterior probability associated to the quality of the book *m*[1]:

$$u_m = \frac{1}{n_m + \alpha_m}(n_m\mu_m^* + \alpha_m n_m - \lambda\sigma_{ob}^2) \tag{7}$$

where the constant λ measures the degree of risk aversion: the larger λ, the more risk averse the agent is. Upon computing u_m for each book in $(1,M)$, consumers choose the book with the highest expected utility.

2. Information transmission

Gaffeo, Scorcu, Vici (2008)

We end up with an infinite Polya urn function.

as we let the probability of a new ball being placed in an existing urn (in our case, a new customer purchases an incumbent book) be proportional to s_m^γ, with the parameter $\gamma \in \mathbf{R}$, Theorems 3.1, 4.1 and 4.2 in Chung et al. (2003) state that

(i) if $\gamma > 1$, one bin dominates;
(ii) if $\gamma = 1$, the limit probability distribution function associated to the random vector $(s_1, \ldots, s_M)$ satisfies

$$P[S_m = s_m] \propto c s_m^{-(1+\alpha)} \qquad (10)$$

that is a power law distribution with $\alpha = \frac{1}{1-p}$, and c is a constant;
(iii) if $-\infty < \gamma < 1$, the distribution of bin sizes decreases exponentially under rather mild conditions.

Theory: economic-based generative processes

3. GLV

Delli Gatti, Gaffeo, Gallegati (2008), *A look at the relationship between industrial dynamics and aggregate fluctuations*

Three basic ideas

1. The firms` financial position matters

2. Agents are heterogeneous as regards how they perceive risk associated to economic decisions

3. Firms interact through the labour and equity markets

Main assumptions:

I firms operate in an homogeneous good market to maximize expected profits.

$$\max_{y} E(\pi_{it} - C_{it}) = y_{it} - R\left(\frac{w_t y_{it}}{\phi} - a_{it}\right) - \frac{c}{2(1-z_{it})}\left[\left(\frac{Rw_t}{\phi} - z_{it}\right)y_{it}^2 - Ra_{it}y_{it}\right]$$

The expected relative price is a random variable with a common mean equal to 1, and variance $V(u_{it}) = \frac{(1-z_{it})^2}{3}$, where z_i is a random variable.

As the bankruptcy cost c grows large, the reaction function of firm i becomes

$$y_{it} \cong \frac{R}{2\left(\frac{Rw}{\phi} - z_{it}\right)} a_{it} = h_{it} a_{it}$$

Finally, the wage rate is determined on an aggregate labor market according to the linear rule $w_t = bn_t$.

The evolution of the equity base at the individual level is given by:

$$a_{it+1} = u_{it} y_{it} - R(w_t n_{it} - a_{it}) + \gamma_i \bar{a}_t$$

where $\bar{a}_t$ is the average capitalization of firms at time t (*hot market* effect).

Solving the model

Assuming rational expectations for any *i* and *t*, as we take the cross-sectional average we obtain:

$$\bar{a}_{t+1} = \left(\bar{h}_t + R\right)\bar{a}_t - R\left(\frac{Ib\bar{h}_t^{\,2}\bar{a}_t^{\,2}}{\phi^2}\right) + \bar{\gamma}\bar{a}_t$$

A suitable change of variable allows us to express the per-capita dynamics as:

$$x_{t+1} = \Gamma x_t \left(1 - x_t\right)$$

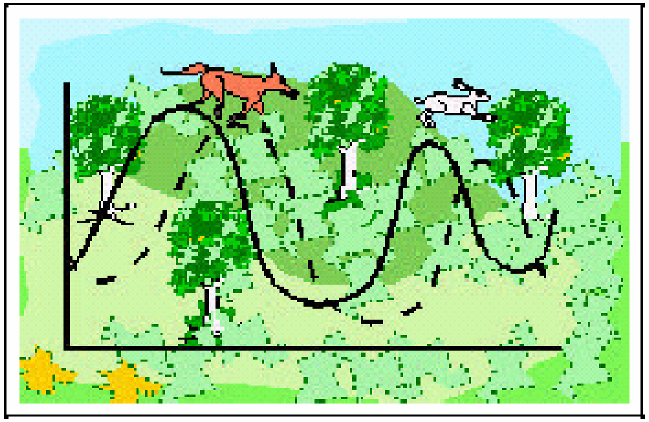
where $x_t = R\dfrac{Ib\bar{h}_t^{\,2}}{\phi^2}\bar{a}_t$, and $\Gamma_t = \bar{h}_t + R + \bar{\gamma}$.

LOGISTIC MAP

deterministic cycles if $3 < \Gamma_t < 3.57$

chaotic behavior if $3.57 < \Gamma_t < 4$.

The rational



Aggregate behavior based on the Lotka-Volterra dynamics

During an upswing, the increase of output induces higher profits and more equity funds. Higher production means also rising employment and higher wages, however. The increased wage bill calls for more bank loans which, when repaid, will depress profits and the production and the equity level as well. The labour requirement thus decreases, along with the real wage, while profits raise. This restores profitability and the cycle can start again.

The firms' size distribution

The model can be expressed, at an individual level, as a

Generalized Lotka-Volterra system (Solomon and Levy, 1996)

The dynamics is based on

i) a stochastic autocatalytic term representing production and how it impacts on equity;

ii) a drift term representing the influence played – via a *hot market* effect – by aggregate capitalization on the financial position of each firm

iii) a time dependent saturation term capturing the competitive pressure exerted by the labour market

The firms' size distribution

Let $\varphi_i(t) = \frac{a_i(t)}{\bar{a}(t)}$ be the relative equity of firm i.

It can be shown that under rather general conditions

$$P(\varphi) \sim \varphi^{-1-\alpha} \exp\left[\frac{-2\gamma}{\sigma^2 \varphi}\right]$$

with $\alpha = 1 + \frac{2\gamma}{\sigma^2}$.

The distribution $P(\varphi)$ is unimodal, as it peaks at $\varphi_0 = \frac{1}{1 + \frac{\sigma^2}{\gamma}}$.

Above φ_0 it behaves like a power law with scaling exponent α
below φ_0 it vanishes very fast.

Implications

1) α depends on:

i) how much rationed firms are in issuing new risk capital
$\Rightarrow$ how much capital markets are affected by adverse selection and moral hazard phenomena;

ii) how much heterogeneous individuals are as regards the perceived riskyness associated to their final demand.

2) Our model suggests that the degree of industrial concentration should be country-specific.

3) γ, that is a proxy for agency costs in capital markets, tunes at the same time the qualitative dynamic features of aggregate fluctuations and the longitudinal characteristics of microeconomic units.

Thank you all!

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