# Common Welfare, Strong Currencies and the Globalization Process


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# OVERVIEW

- The von Neumann Equation & the Statistical Mixture of States
- The Replicator Dynamics & EGT
- Relationships between Quantum Mechanics & Game Theory
- Quantum Replicator Dynamics & the Quantization Relationships
- Games through Statistical Mechanics & QIT
- From Classical to Quantum
- On a Quantum Understanding of Classical Systems
- Some Crazy ideas & Conclusions

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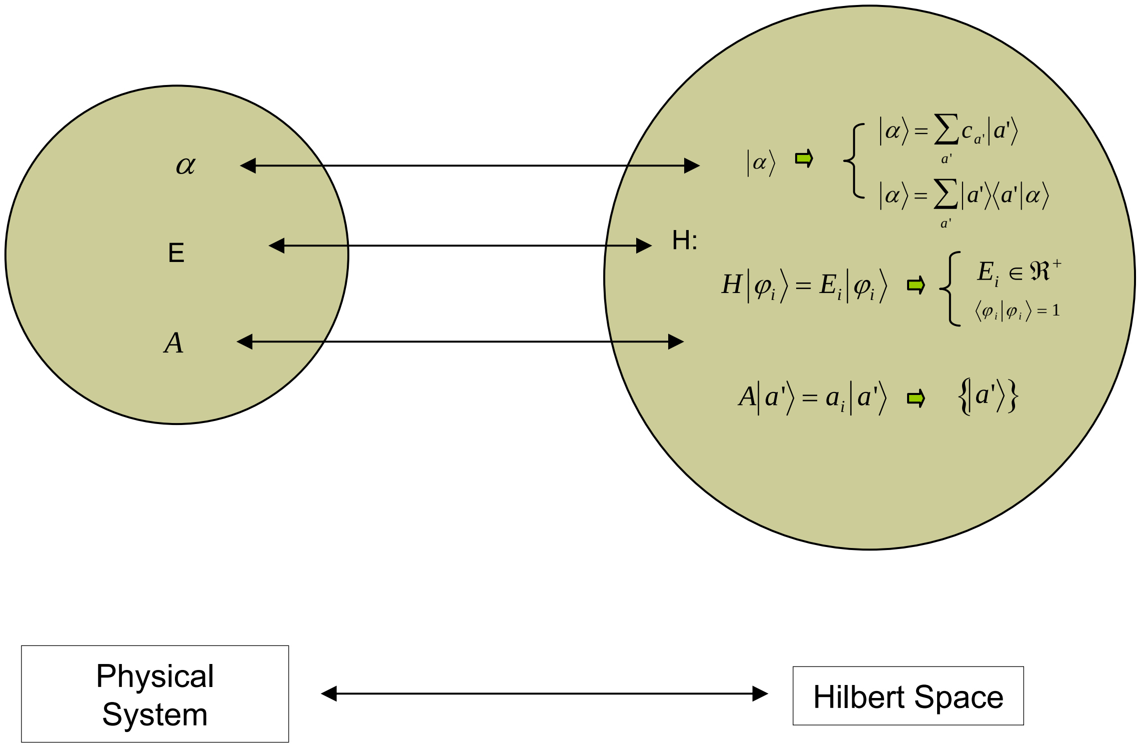
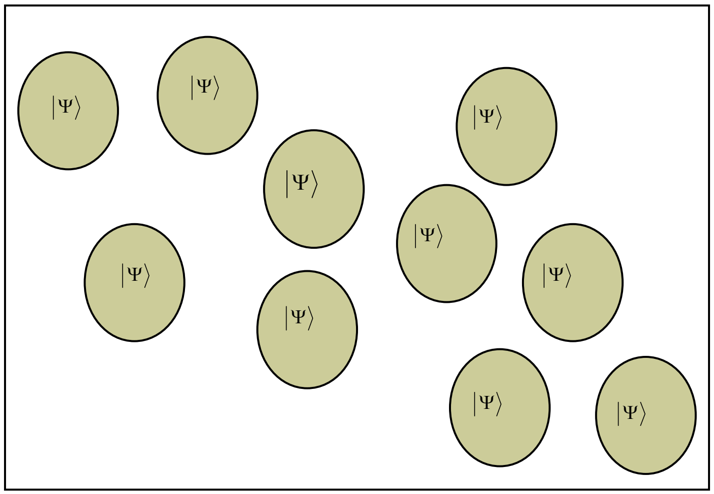
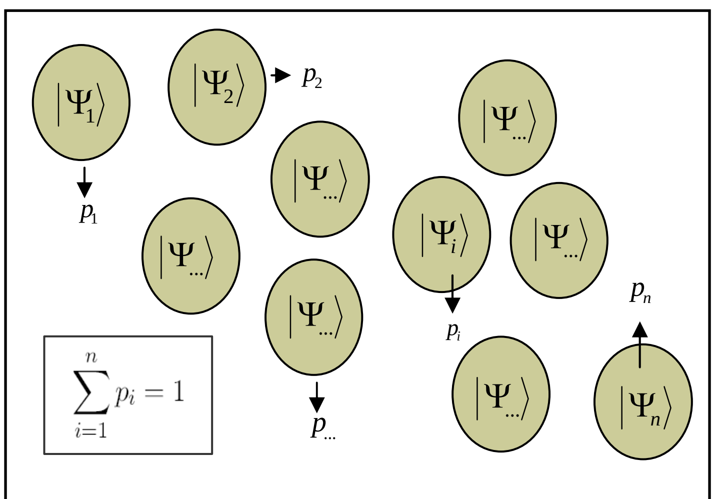
# Quantum Mechanics Foundations



# Density Operator and the Statistical Mixture of States



Pure Ensemble




Mixed Ensemble

$$\rho(t) = \sum_{i=1}^{n} p_i \left|\Psi_i(t)\right\rangle \left\langle\Psi_i(t)\right|$$

# Density Operator

$$\rho(t) = \sum_{i=1}^{n} p_i \left|\Psi_i(t)\right\rangle \left\langle\Psi_i(t)\right|$$

a) $\rho$ es Hermitiano.

b) $Tr\rho(t) = 1$

c) $\rho^2(t) \leqslant \rho(t)$

d) $Tr\rho^2(t) \leqslant 1$

Populations

$$\rho_{nn} = \sum_{i=1}^{n} p_i \left|c_n^{(i)}\right|^2$$

Coherences

$$\rho_{np} = \sum_{i=1}^{n} c_n^{(i)}(t) c_p^{(i)*}(t) p_i$$

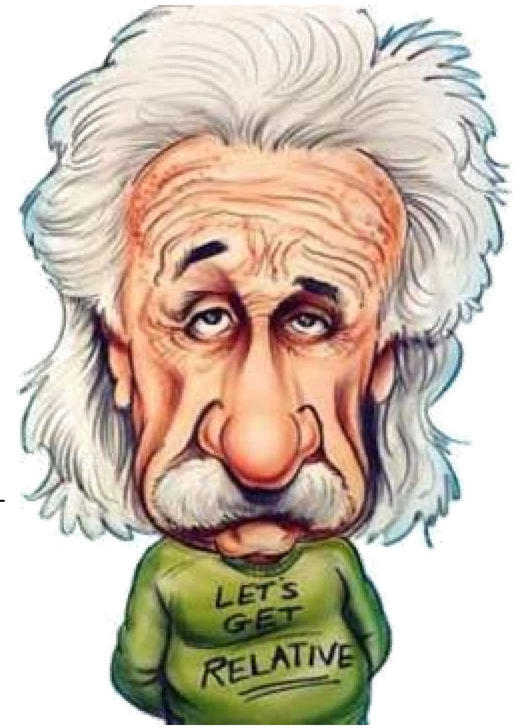
Von Neumann Equation:

$$i\hbar \frac{d\rho(t)}{dt} = [H(t), \rho(t)]$$

# Game Theory

$$E\left(s_i, s_j\right)$$

$\left(s_i\right)$ $\left(s_j\right)$




Simetric Game G = (S, E)

$$S_1 = \left(s_1, s_2, \ldots, s_i, \ldots, s_n\right)$$

$$S_2 = \left(s_1, s_2, \ldots, s_j, \ldots, s_n\right)$$

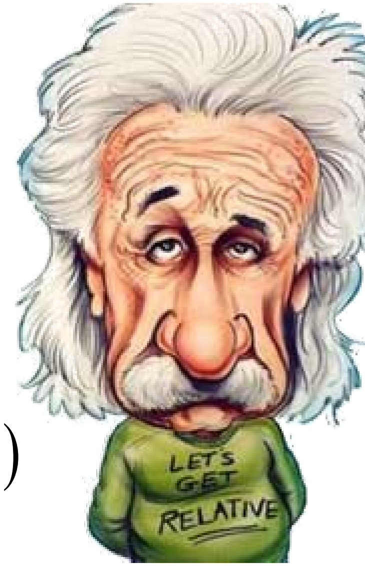
# Strategies & Equilibrium Points

*Mejor Respuesta* a $(q)$ es $(p)$ tq.
$E(p,q)$ sea máxima.

*Punto de Equilibrio* es $(p,q)$ si $(p)$ y $(q)$ son mejores respuestas mutuas.

*Estricta Mejor Respuesta* a $(q)$ es $(r)$ si es su única mejor respuesta.

*Punto de Equilibrio Estricto* es $(p,q)$ si $(p)$ y $(q)$ son estrictas mejores respuestas mutuas.




$(p)$

Mejor Respuesta a $(p)$ la cual es diferente de $(p)$ es llamada *Mejor Respuesta Alternativa*.

$(q)$

# Nash Equilibrium

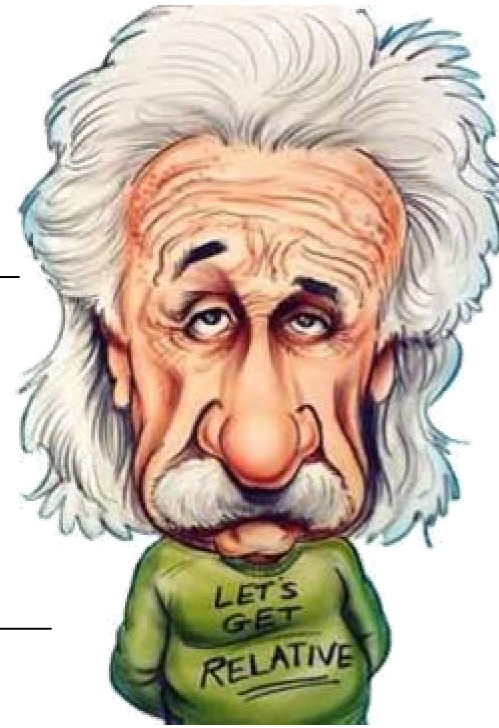
$S_1 = (s_1, s_2, \ldots, s_i, \ldots, s_n)$

$E(p, p)$

$(p)$ $(p)$

$E(r, p)$

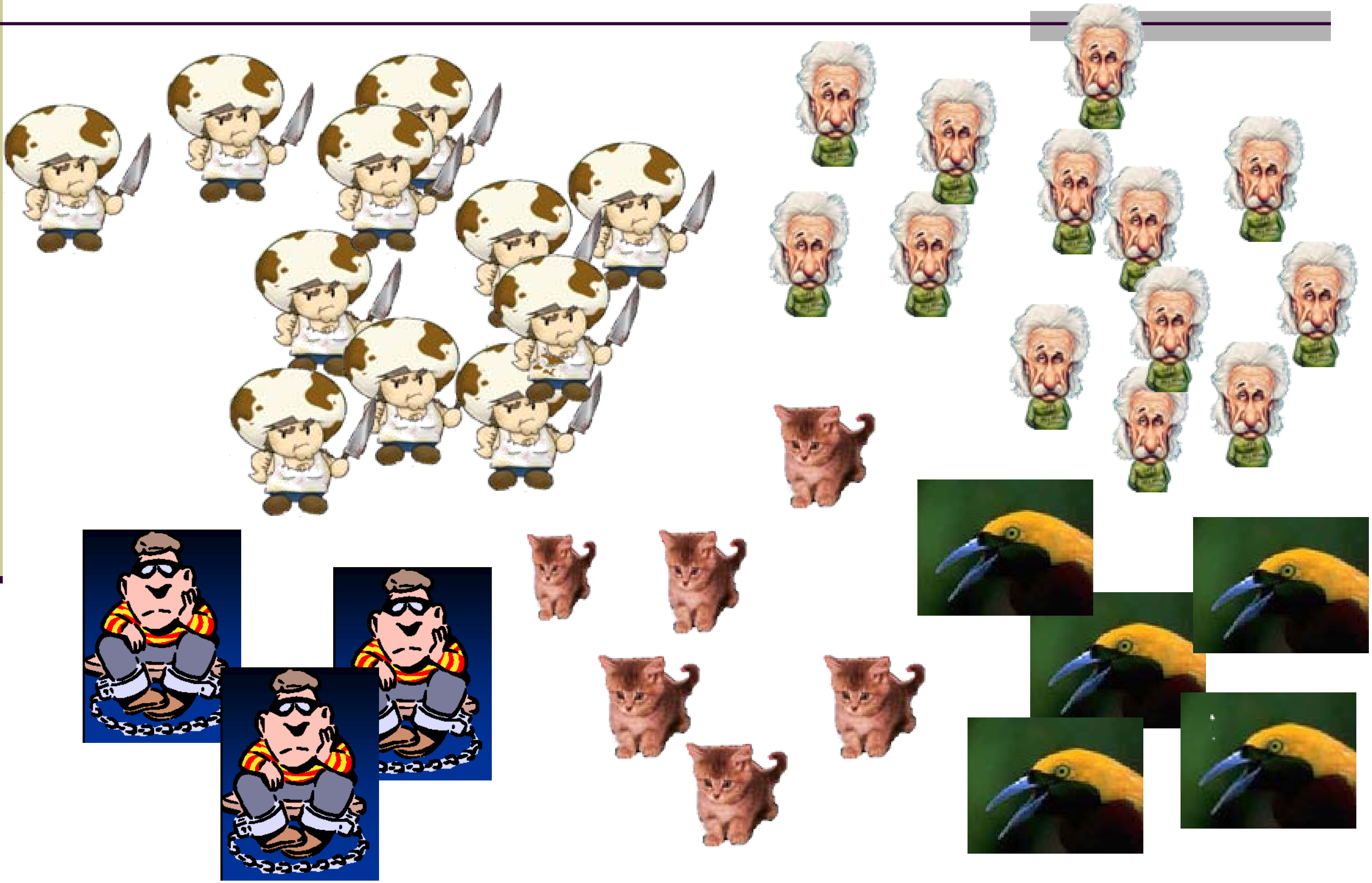
$(r)$ $(p)$

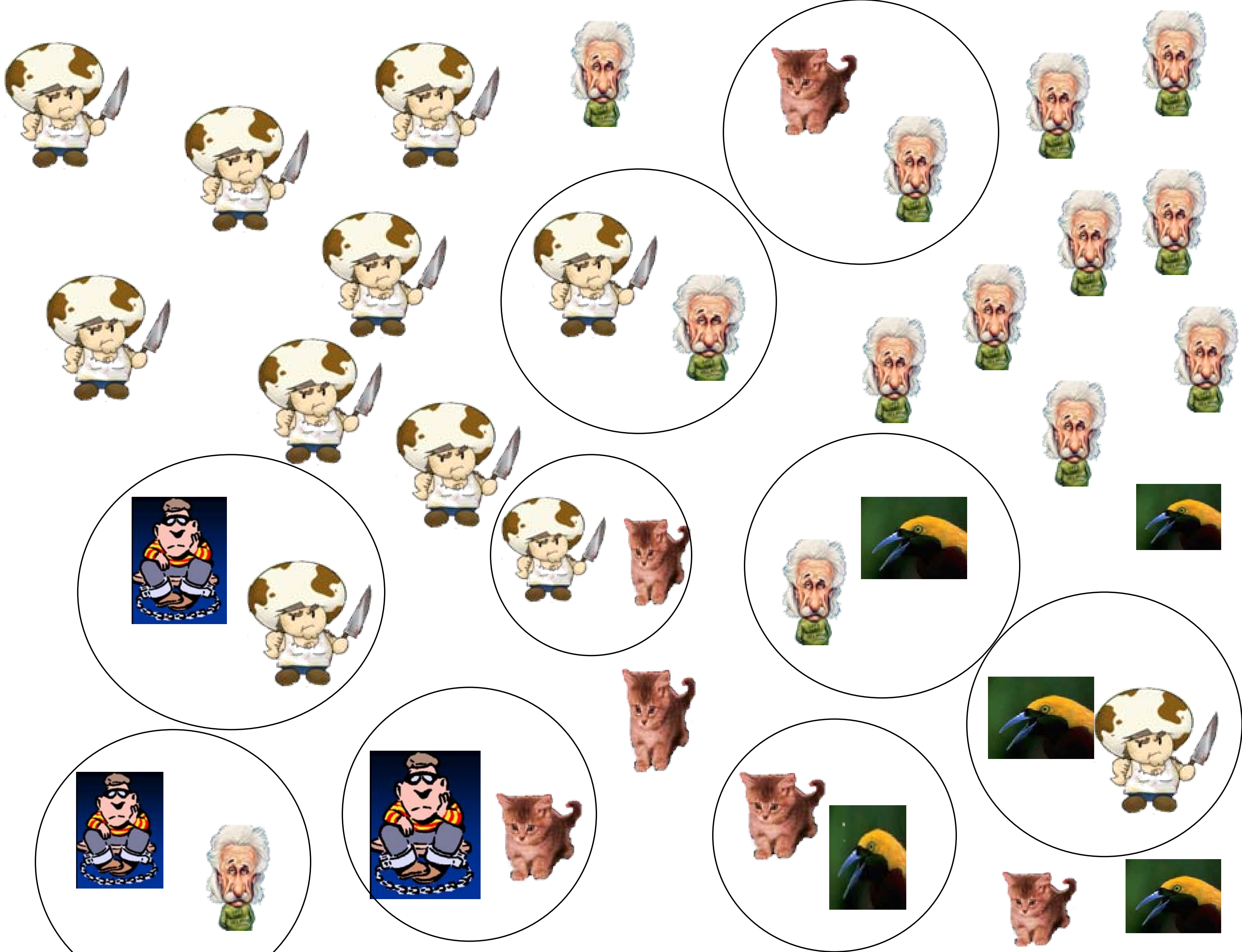


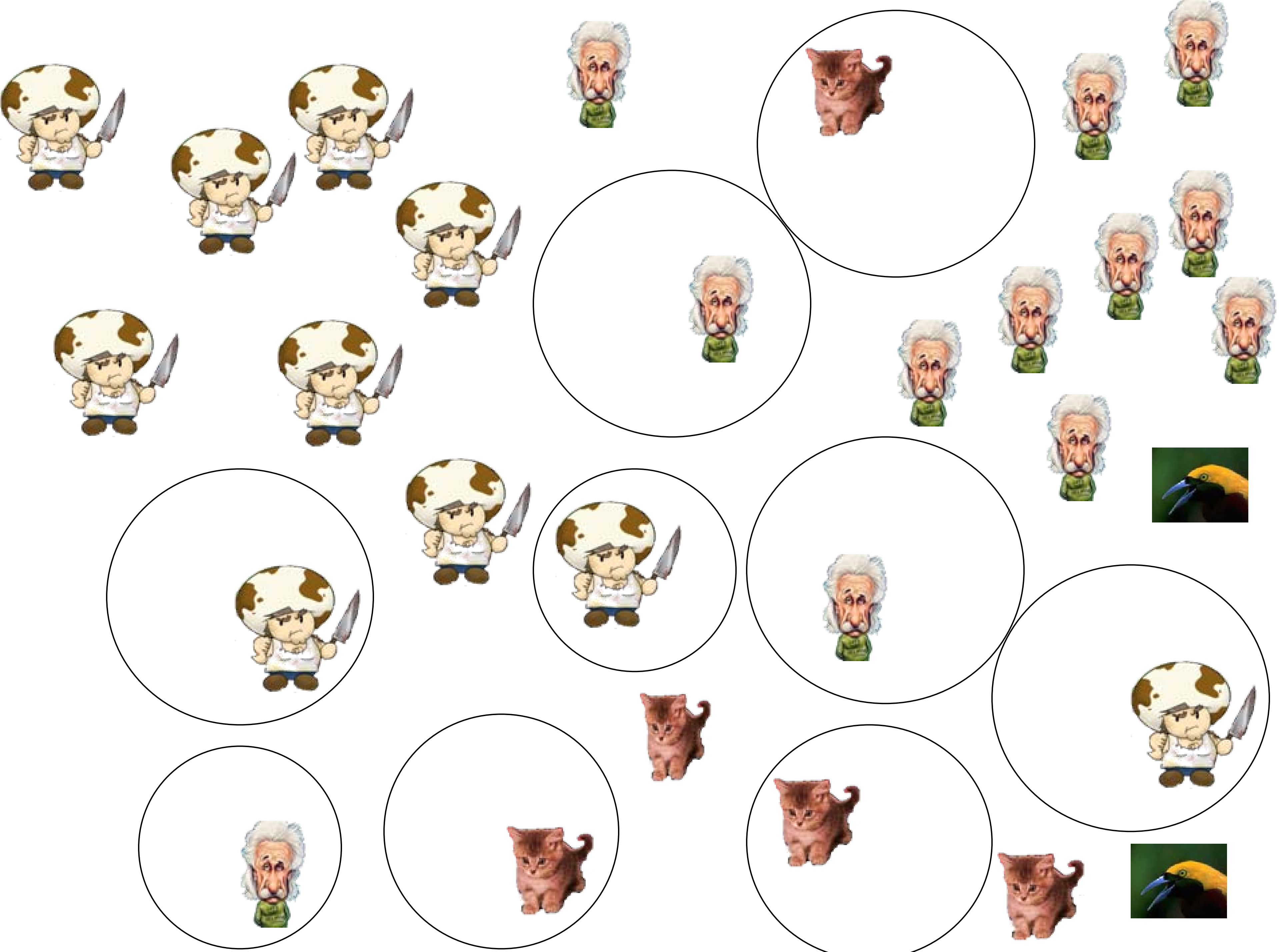

$S_2 = (s_1, s_2, \ldots, s_j, \ldots, s_n)$

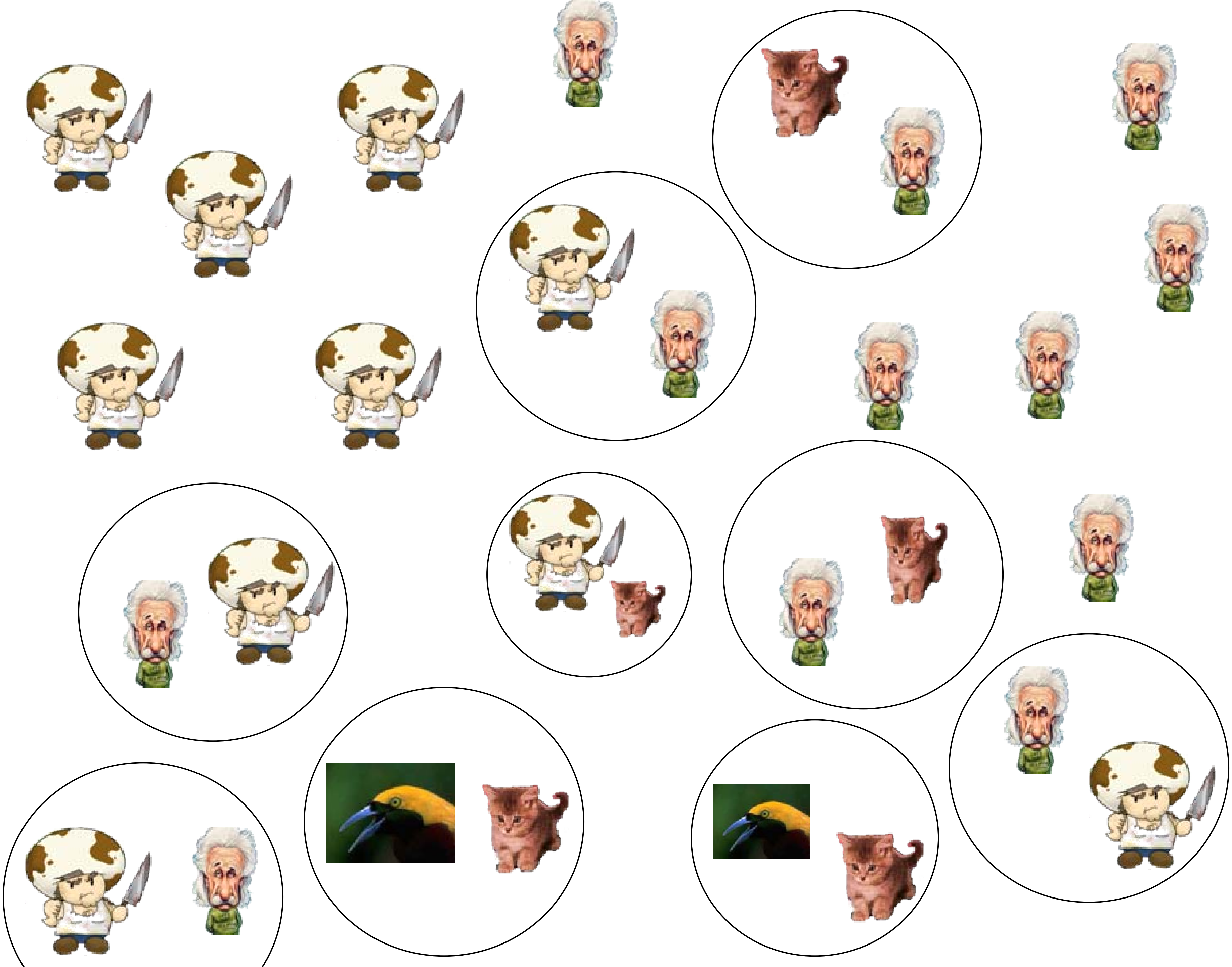
$$E(p,p) \geq E(r,p)$$

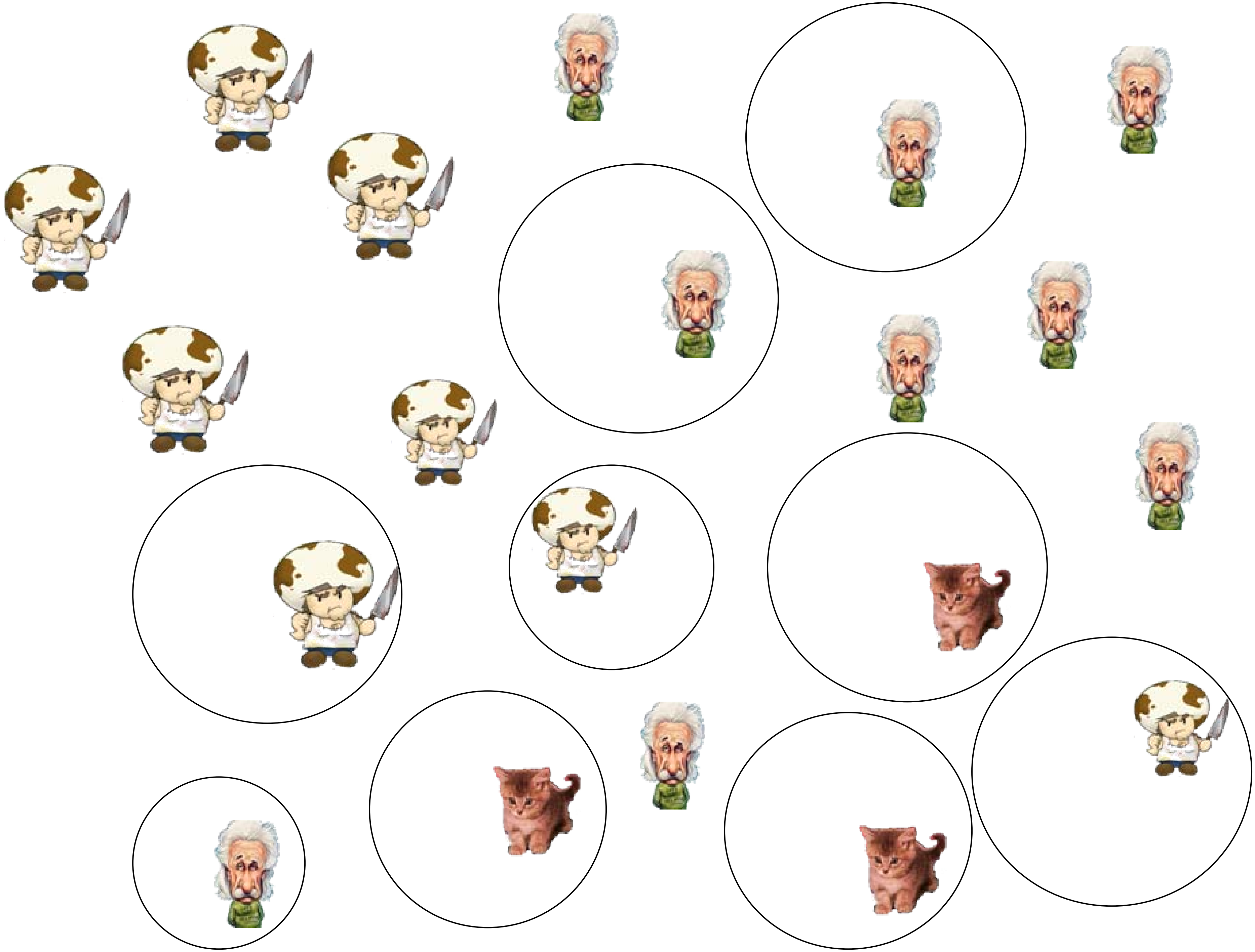
# The Replicator Dynamics & EGT

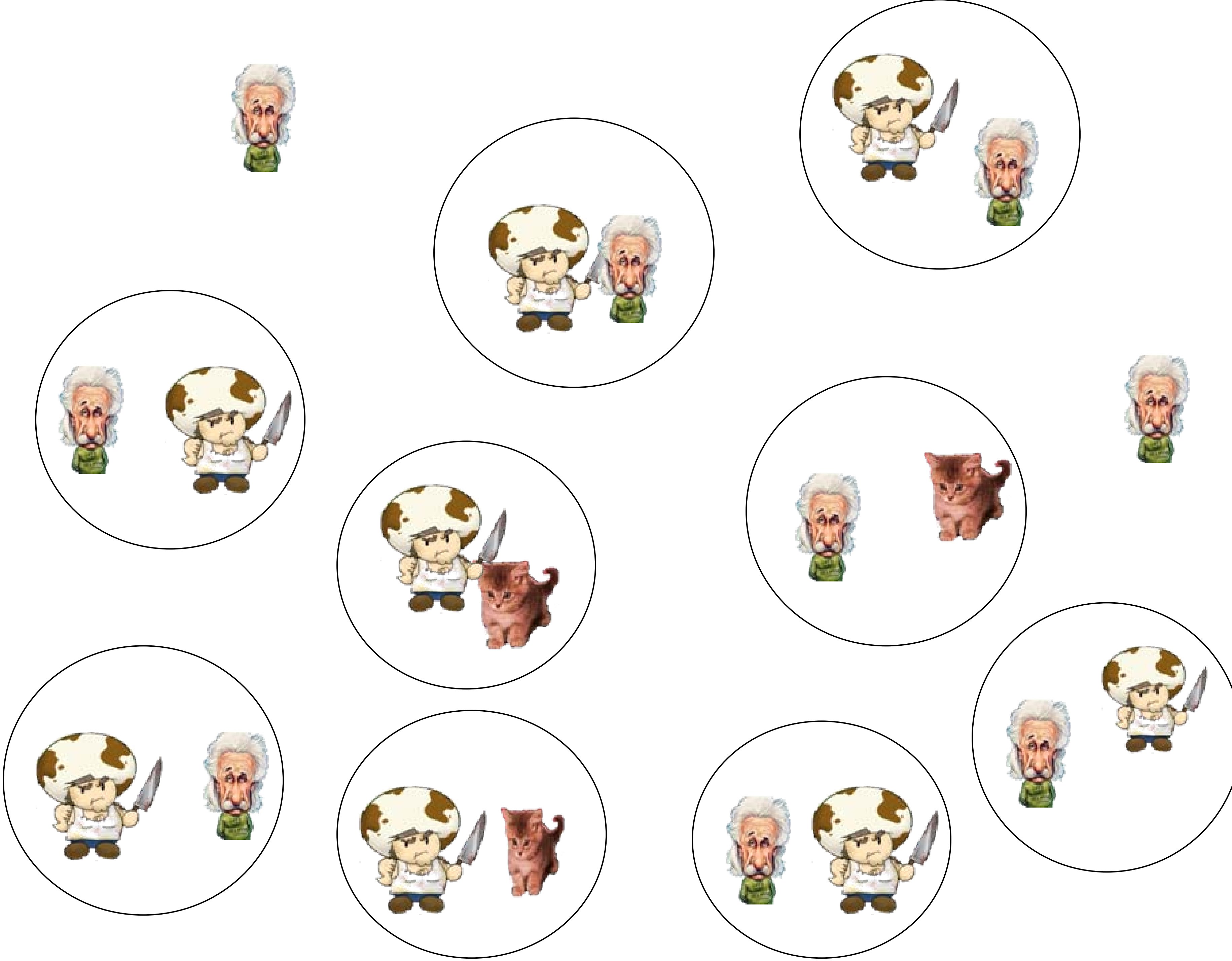


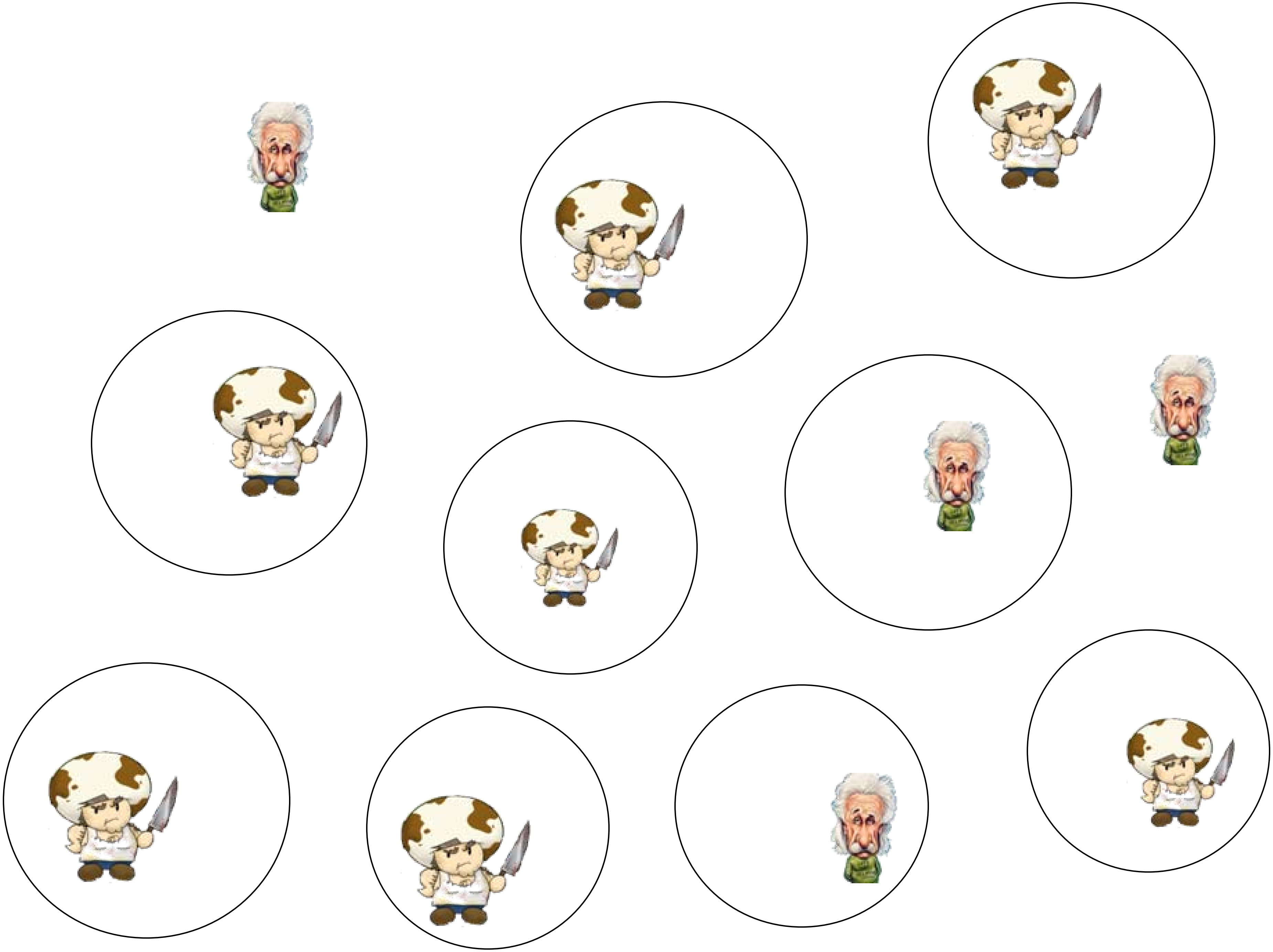


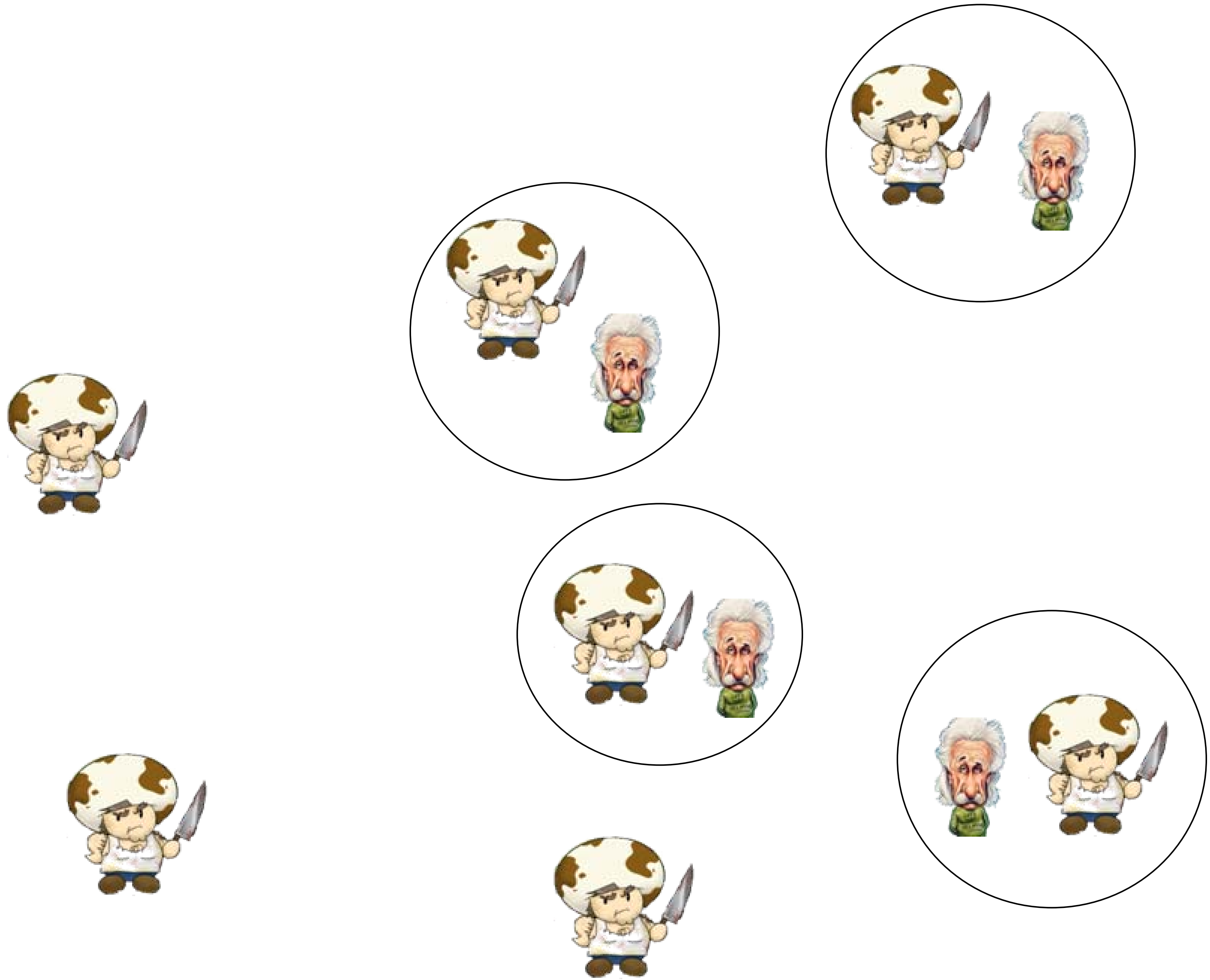


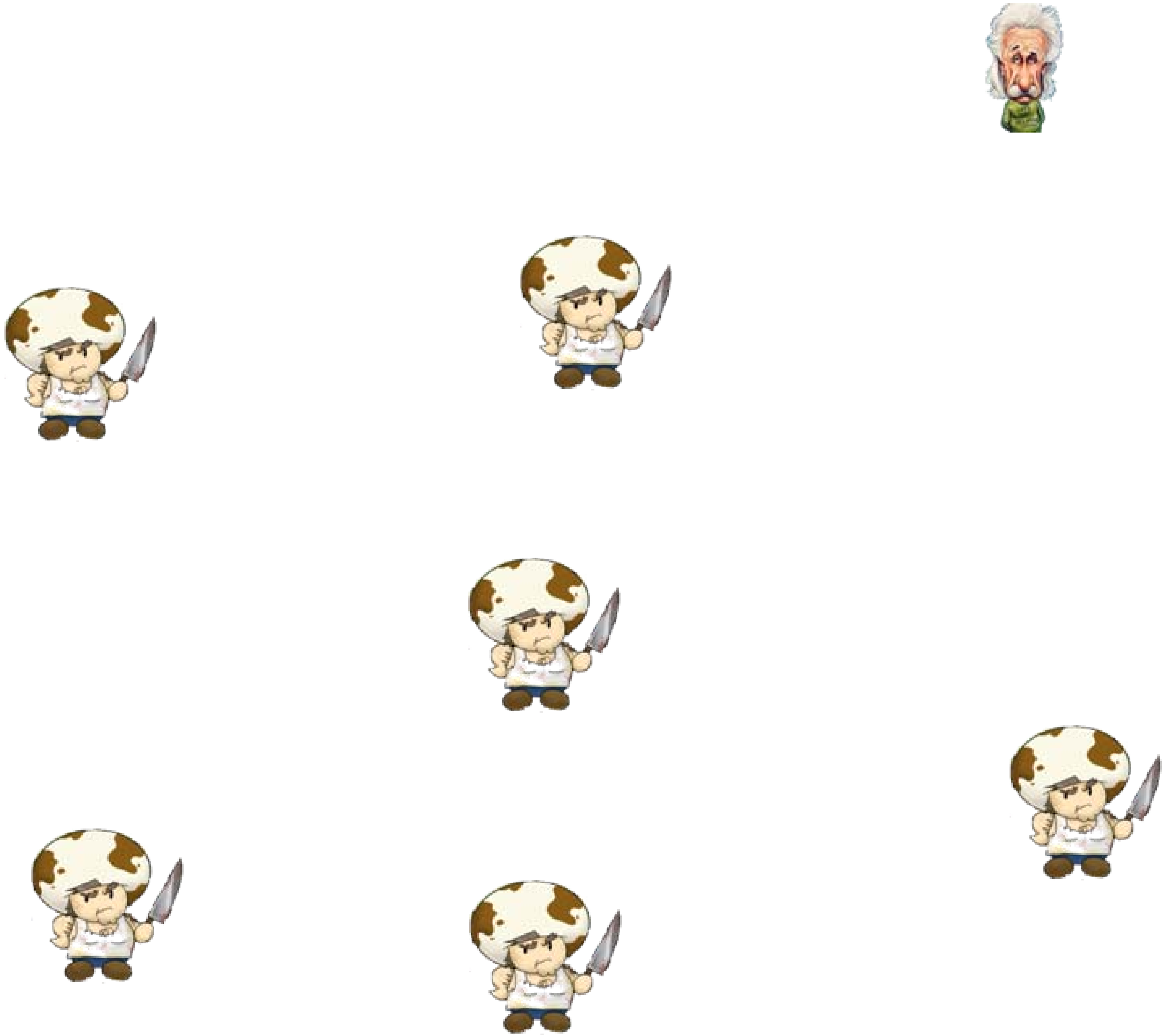
















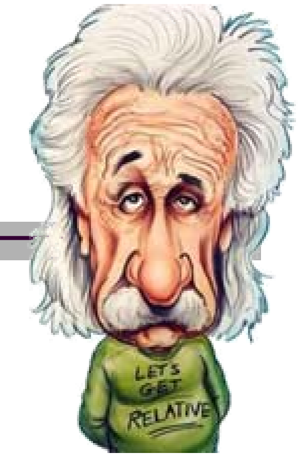







# Evolutionary Stable Strategies (ESS)

G = (S, E)




$(p)$ $(r)$

$$E(p,p) > E(r,p)$$

If $$E(p,p) = E(r,p)$$

then $$E(p,r) > E(r,r)$$

# The Replicator Dynamics

$$\frac{dx_i(t)}{dt} = \left[f_i(x) - \langle f(x) \rangle\right] x_i(t)$$

Relative frequencies vector

$x$

Payoff Function

$$f_i(x) = \sum_{j=1}^{n} a_{ij} x_j$$

Average Payoff Function

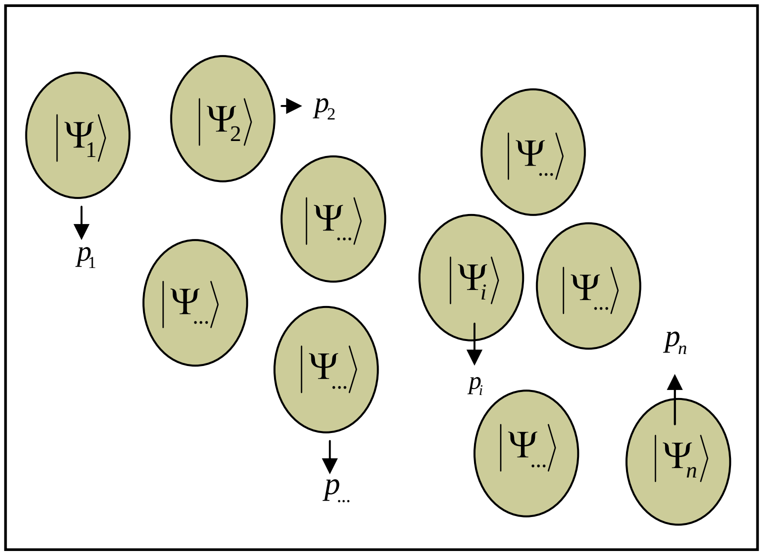
$$\langle f(x) \rangle = \sum_{k,l=1}^{n} a_{kl} x_k x_l$$

$$\frac{dx_i}{dt} = \left[\sum_{j=1}^{n} a_{ij} x_j - \sum_{k,l=1}^{n} a_{kl} x_k x_l\right] x_i$$

# Relationships between Quantum Mechanics & Game Theory

| Quantum Mechanics | Game Theory |
|---|---|
| n system members | n players |
| Quantum states | Strategies |
| States superposition | Strategies superposition |
| Density operator | Relative frequency vector |
| Von Neumann equation | Replicator Dynamics |
| System Equilibrium | Payoff |
| Maximum entropy | Maximum payoff |
| Altruism | Altruism or selfish |
| Collective Welfare principle | Minority Welfare principle |

# Relationships between Quantum Mechanics & Game Theory




$$i\hbar\frac{d\rho(t)}{dt} = [H(t), \rho(t)]$$

$$\frac{dx_i(t)}{dt} = [f_i(x) - \langle f(x)\rangle]\, x_i(t)$$

# Matrix Representation of the Replicator Dynamics

$$\frac{dx_i(t)}{dt} = \left[\sum_{j=1}^{n} a_{ij}x_j - \sum_{k,l=1}^{n} a_{kl}x_k x_l\right] x_i(t)$$

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^{n} a_{1j}x_j - \sum_{k,l=1}^{n} a_{kl}x_k x_l & 0 & \ldots & 0 \\ 0 & \sum_{j=1}^{n} a_{2j}x_j - \sum_{k,l=1}^{n} a_{kl}x_k x_l & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & \ldots & \sum_{j=1}^{n} a_{1j}x_j - \sum_{k,l=1}^{n} a_{kl}x_k x_l \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{pmatrix}$$

$$U = (Ax)_i - x^T Ax \longrightarrow \frac{dx(t)}{dt} = Ux(t)$$

$$\frac{dx(t)}{dt} = Ux(t)$$

X $(x_i)^{-1/2}$

$$v = U\hat{x} \quad \longrightarrow \quad \hat{x}_i = (x_i)^{1/2}$$

$$v_i = \frac{1}{(x_i)^{1/2}} \frac{dx_i}{dt}$$

$$\begin{pmatrix} \frac{1}{\sqrt{x_1}} \frac{dx_1}{dt} \\ \frac{1}{\sqrt{x_2}} \frac{dx_2}{dt} \\ \cdots \\ \frac{1}{\sqrt{x_n}} \frac{dx_n}{dt} \end{pmatrix} = U \begin{pmatrix} \sqrt{x_1} \\ \sqrt{x_2} \\ \cdots \\ \sqrt{x_n} \end{pmatrix}$$

$v = U\hat{x}$ $\longrightarrow$

$$\begin{pmatrix} \frac{1}{\sqrt{x_1}}\frac{dx_1}{dt} \\ \frac{1}{\sqrt{x_2}}\frac{dx_2}{dt} \\ ... \\ \frac{1}{\sqrt{x_n}}\frac{dx_n}{dt} \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^n a_{1j}x_j - \sum_{k,l=1}^n a_{kl}x_kx_l & 0 & ... & 0 \\ 0 & \sum_{j=1}^n a_{2j}x_j - \sum_{k,l=1}^n a_{kl}x_kx_l & ... & ... \\ ... & ... & ... & ... \\ 0 & ... & ... & \sum_{j=1}^n a_{1j}x_j - \sum_{k,l=1}^n a_{kl}x_kx_l \end{pmatrix} \begin{pmatrix} \sqrt{x_1} \\ \sqrt{x_2} \\ ... \\ \sqrt{x_n} \end{pmatrix}$$

$\boxed{v = U\hat{x}}$ X $\hat{x}^T$

$$\begin{pmatrix} \frac{1}{\sqrt{x_1}}\frac{dx_1}{dt} \\ \frac{1}{\sqrt{x_2}}\frac{dx_2}{dt} \\ ... \\ \frac{1}{\sqrt{x_n}}\frac{dx_n}{dt} \end{pmatrix} \begin{pmatrix} \sqrt{x_1} & \sqrt{x_2} & ... & \sqrt{x_n} \end{pmatrix} = U \begin{pmatrix} \sqrt{x_1} \\ \sqrt{x_2} \\ ... \\ \sqrt{x_n} \end{pmatrix} \begin{pmatrix} \sqrt{x_1} & \sqrt{x_2} & ... & \sqrt{x_n} \end{pmatrix}$$

$$\boxed{v = U\hat{x}} \quad \times \quad \hat{x}^T$$

$$G = \frac{1}{2} v\hat{x}^T$$

$$g_{ij} = \frac{1}{2}\frac{(x_j)^{1/2}}{(x_i)^{1/2}}\frac{dx_i}{dt}$$

$$G = \frac{1}{2} U\hat{x}\hat{x}^T$$

$$\boxed{(U\hat{x}\hat{x}^T)_{ij} = \left[\sum_{k=1}^{n} a_{ik}x_k - \sum_{k,l=1}^{n} a_{kl}x_k x_l\right](x_i x_j)^{1/2}}$$

$$G = \frac{1}{2} v \hat{x}^T$$

$$G^T = \frac{1}{2}(\hat{x} v^T)$$

$$g_{ij}^T = \frac{1}{2} \frac{(x_i)^{1/2}}{(x_j)^{1/2}} \frac{dx_j}{dt}$$

$$G = \frac{1}{2} U \hat{x} \hat{x}^T$$

$$G^T = \frac{1}{2}(\hat{x} \hat{x}^T U^T)$$

$$(\hat{x}\hat{x}^T U^T)_{ij} = (x_j x_i)^{1/2} \left[ \sum_{k=1}^n a_{jk} x_k - \sum_{k,l=1}^n a_{kl} x_k x_l \right]$$

$$\frac{dX}{dt} = G + G^T$$

$$x_{ij} = \frac{1}{2}\frac{(x_j)^{1/2}}{(x_i)^{1/2}}\frac{dx_i}{dt} + \frac{1}{2}\frac{(x_i)^{1/2}}{(x_j)^{1/2}}\frac{dx_j}{dt}$$

$$x_{ij} = (x_i x_j)^{1/2}$$

$$\begin{aligned}(G+G^T)_{ij} &= \frac{1}{2}\sum_{k=1}^{n} a_{ik}x_k(x_i x_j)^{1/2} \\ &+\frac{1}{2}(x_j x_i)^{1/2}\sum_{k=1}^{n} a_{jk}x_k \\ &-\sum_{k,l=1}^{n} a_{kl}x_k x_l (x_i x_j)^{1/2}\end{aligned}$$

# Replicator Dynamics Lax Form

$$\frac{dX}{dt} = G + G^T$$

$$x_{ij} = (x_i x_j)^{1/2}$$

$$\begin{aligned}(G+G^T)_{ij} \quad = \quad & \frac{1}{2}\sum_{k=1}^{n} a_{ik}x_k(x_i x_j)^{1/2} \longrightarrow G_1 \\ & +\frac{1}{2}(x_j x_i)^{1/2}\sum_{k=1}^{n} a_{jk}x_k \longrightarrow G_2 \\ & -\sum_{k,l=1}^{n} a_{kl}x_k x_l (x_i x_j)^{1/2} \longrightarrow G_3\end{aligned}$$

$$\boxed{\frac{dX}{dt} = QXX + XXQ - 2XQX}$$

$G_1 = QX$ $\quad G_2 = XQ$ $\quad G_3 = 2XQX$

$$\frac{dX}{dt} = [[Q, X], X]$$

$q_{ii} = \frac{1}{2}\sum_{k=1}^{n} a_{ik}x_k$

$x_{ij} = (x_i x_j)^{1/2}$

$$\boxed{\frac{dX(t)}{dt} = [\Lambda(t), X(t)]} \longrightarrow$$

a) $Tr(X) = 1$

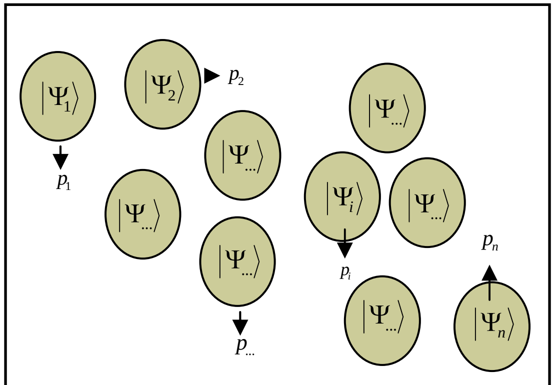
b) $X^2 = X$

c) $X^T = X$

$\downarrow$

$$(\Lambda)_{ij} = \frac{1}{2}\left[\sum_{k=1}^{n} a_{ik}x_k(x_i x_j)^{1/2} - (x_j x_i)^{1/2}\sum_{k=1}^{n} a_{jk}x_k\right]$$

# Relationships between Quantum Mechanics & Game Theory




$$\frac{dx_i(t)}{dt} = \left[f_i(x) - \langle f(x)\rangle\right] x_i(t)$$

$$i\hbar\frac{d\rho(t)}{dt} = [H(t), \rho(t)]$$

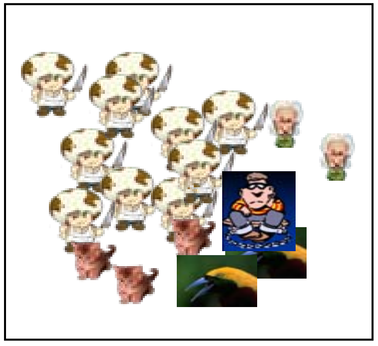
$$\frac{dX(t)}{dt} = [\Lambda(t), X(t)]$$

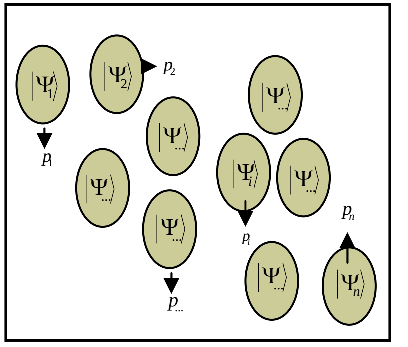
# Specific Resemblances

| Quantum Statistical Mechanics | Evolutionary Game Theory |
|---|---|
| n system members | n population members |
| Each member in the state $\left\|\Psi_k\right\rangle$ | Each member plays strategy $s_i$ |
| $\left\|\Psi_k\right\rangle$ with $p_k \rightarrow \rho_{ij}$ | $s_i \rightarrow x_i$ |
| $\rho, \ \sum_i \rho_{ii} = 1$ | $X, \ \sum_i x_i = 1$ |
| $i\hbar\frac{d\rho}{dt} = \left[\hat{H}, \rho\right]$ | $\frac{dX}{dt} = [\Lambda, X]$ |
| $S = -Tr\left\{\rho \ln \rho\right\}$ | $H = -\sum_i x_i \ln x_i$ |

| Density Operator | Relative freq. Matrix |
|---|---|
| $\rho$ is Hermitian | $X$ is Hermitian |
| $Tr\rho(t) = 1$ | $TrX = 1$ |
| $\rho^2(t) \leqslant \rho(t)$ | $X^2 = X$ |
| $Tr\rho^2(t) \leqslant 1$ | $TrX^2(t) = 1$ |

# Quantization Relationships





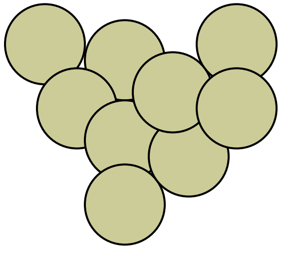

$$x_i \to \sum_{k=1}^{n} \langle i\,|\Psi_k\rangle\, p_k \,\langle \Psi_k\,|i\rangle = \rho_{ii},$$

$$(x_i x_j)^{1/2} \to \sum_{k=1}^{n} \langle i\,|\Psi_k\rangle\, p_k \,\langle \Psi_k\,|j\rangle = \rho_{ij}.$$

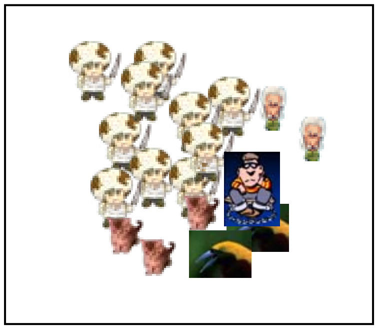
$(s_i) \longrightarrow |\Psi_k\rangle p_k$

$(x_i) \longrightarrow (\rho_{ii})$

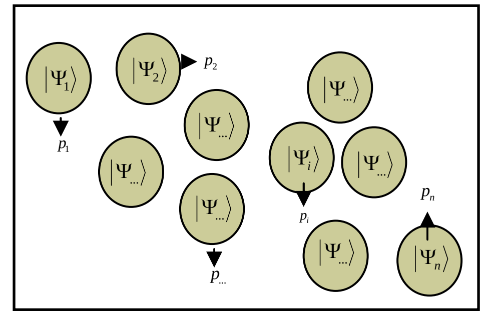
$(x_i x_j)^{1/2} \longrightarrow (\rho_{ij})$



# Quantum Replicator Dynamics



$\longrightarrow$




$$\frac{dX(t)}{dt} = [\Lambda(t), X(t)] \longrightarrow i\hbar\frac{d\rho(t)}{dt} = [H(t), \rho(t)]$$

$$X \longrightarrow \rho$$

$$\Lambda \longrightarrow -\frac{i}{\hbar}\hat{H}$$

# Games through Statistical Mechanics & QIT

$A \downarrow s_i^A \qquad B \downarrow s_j^B$

$$H(A,B) \equiv -\sum_{i,j} \breve{x}_{ij} \log_2 \breve{x}_{ij}$$

JOINT ENTROPY

$$H(A,B) \leq H(A) + H(B)$$

Measures our total uncertainty about the pair (A,B)

$A \downarrow s_i^A \qquad B \downarrow s_j^B$

$$H(A:B) \equiv -\sum_{i,j} x_{ij} \log_2 x_{i:j}$$

CORRELATION ENTROPY

$$H(A:B) \equiv H(A) + H(B) - H(A,B)$$

A measure of how much information A and B have in common and have an idea of how its strategies or states are correlated.

$A \downarrow \qquad B \downarrow s_j^B$

$$H(A \mid B) \equiv H(A,B) - \breve{H}(B)$$

CONDITIONAL ENTROPY

$$H(A \mid B) \qquad H(A:B) \leq H(A)$$

How uncertain we are about the value of A, given that we know the value of B

$$H(A \mid B,C) \leq H(A \mid B)$$

Our uncertainty about the decisions of player $A$ knowing how $B$ and $C$ plays is smaller or at least equal than our uncertainty about the decisions of $A$ knowing only how $B$ plays

CONDITIONAL ENTROPY

$$H(A) \geq H(A:B) \geq H(A:C)$$

MARKOV CHAIN

$$A \rightarrow B \rightarrow C$$

the information can only reduces in time. Also any information $C$ shares with $A$ must be information which $C$ also shares with $B$, $H(C:B) \geq H(C:A)$.

# Games through Statistical Mechanics & QIT

Shannon & von Neumann Entropies

$$H(A) \equiv H(p_1, ..., p_n) \equiv -\sum_{i=1}^{n} p_i \log_2 p_i. \qquad S(t) = -Tr\{\rho \ln \rho\}$$

**Entropy** can be regarded as a quantitative *measure of disorder*.

**MAXIMUM:** **lnN** Completely random ensemble
**MINIMUM:** **0** Pure ensemble

$\beta = \frac{1}{\tau}$

$$\rho_{ii} = \frac{e^{-\beta E_i}}{\sum_k e^{-\beta E_k}}$$

$\beta \to 0$ ("high – temperature limit") completely random ensemble

$\beta \to \infty$ ("low – temperature limit") pure ensemble

Z partition function
$S = \ln Z + \beta \langle E \rangle$

**Entropy** can be MAXIMIZED subject to different constrains. Generally, the result is a probability distribution function.

For example: If we maximize $S(t) = -Tr\{\rho \ln \rho\}$
subject to the constrains $\delta Tr(\rho) = 0$ and $\delta \langle E \rangle = 0$

The result is:

$$\rho_{ii} = \frac{e^{-\beta E_i}}{\sum_k e^{-\beta E_k}}$$

which is the CONDITION that the density operator must satisfy to our system tends to maximize its entropy.

$$\begin{aligned} \frac{dS(t)}{dt} &= \frac{11}{6}\sum_i \frac{d\rho_{ii}}{dt} \\ &\quad -6\sum_{i,j} \rho_{ij}\frac{d\rho_{ji}}{dt} \\ &\quad +\frac{9}{2}\sum_{i,j,k} \rho_{ij}\rho_{jk}\frac{d\rho_{ki}}{dt} \\ &\quad -\frac{4}{3}\sum_{i,j,k} \rho_{ij}\rho_{jk}\rho_{kl}\frac{d\rho_{li}}{dt} + \zeta. \end{aligned}$$

# On the Quantum Understanding of Classical Systems

If our systems are analogous and thus exactly equivalents,
*our physical equilibrium (maximum entropy)* should be also
*exactly equivalent* to our *socieconomical equilibrium (NE or ESS).*

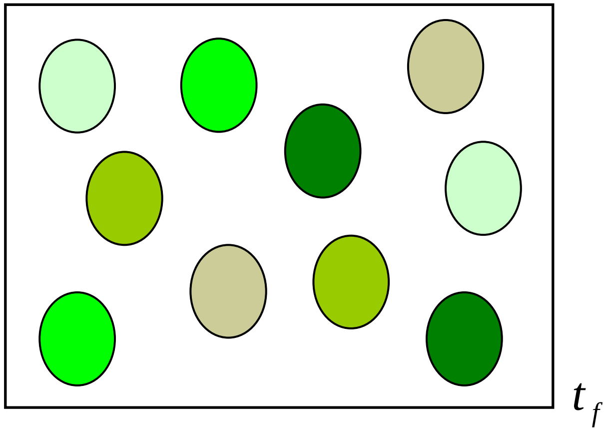
Also suggested From an absolutely different point of view: by:

(Dragulescu & Yakovenko 2000). In a closed economic system, money is conserved. Thus, by analogy with energy, the equilibrium probability distribution of money must follow the exponential Boltzmann-Gibbs law characterized by an effective temperature equal to the average amount of money per economic agent.

•(Darooneh 2006). The maximum entropy principle is used for pricing the insurance. Darooneh obtained the price density based on this principle, applied it to multi agents model of insurance market and derived the utility function. The main assumption in his work is the correspondence between the concept of the equilibrium in physics and economics.

•(TopsØe 1979;1993) also has suggested that thermo dynamical equilibrium equals game theoretical equilibrium.

# *Physical equilibrium (maximum entropy)*




$$S_f > S_2 > S_1 > S_0$$
$$S_f \max = \ln N$$

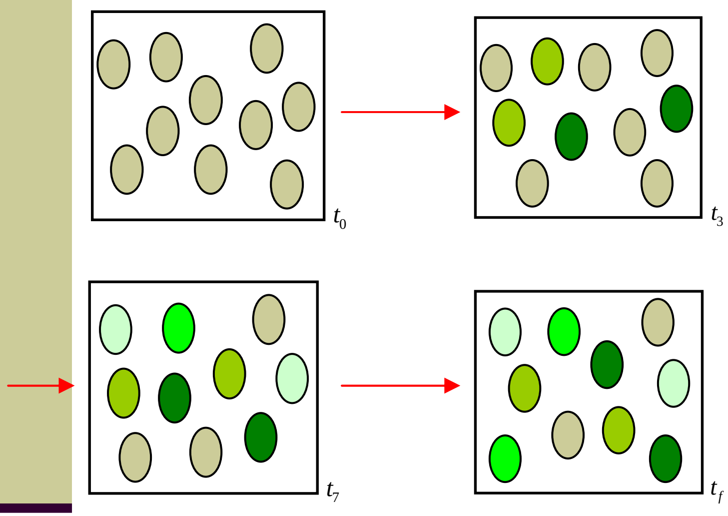
$$p_1 = \ldots = p_f = \frac{1}{N}$$

$\Omega$max

The purpose and *maximum payoff* of a physical system is its maximum entropy state i.e. its equilibrium.

The system and its members will vary and rearrange themselves to reach the best possible state for each of them which is also the best possible state for the whole system.

# On the Quantum Understanding of Classical Systems

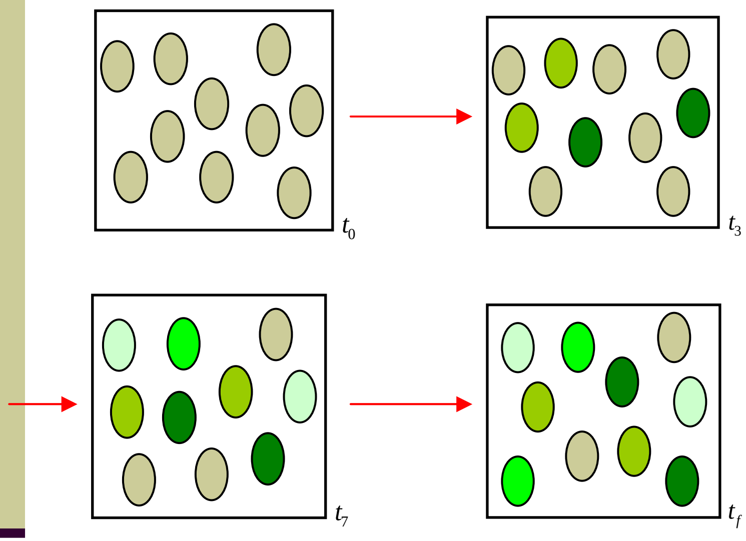



Microscopical cooperation between quantum objects to improve their states with the purpose of reaching or maintaining the equilibrium of the system.

All the members of our quantum system will play a game in which its maximum payoff is the EQUILIBRIUM of the system.

They act as a whole besides individuals like they obey a rule in where they prefer the welfare of the collective over the welfare of the individual.

# On the Quantum Understanding of Classical Systems




This equilibrium is represented in the maximum system entropy in where the system "resources" are fairly distributed over its members.

A system is stable only if it maximizes the welfare of the collective above the welfare of the individual.

A system where its members are in NE (or ESS) is exactly equivalent to a system in a maximum entropy state.

# Some Crazy Ideas & Conclusions

1. The **quantum analogue** of the replicator dynamics is the von Neumann equation.

2. A population is represented by a quantum system in which each subpopulation playing strategy …

3. Quantum mechanics could be used to **explain** more correctly biological and economical processes and even <u>encloses</u> theories like games and evolutionary dynamics.

4. Although both systems analyzed are described through two apparently different theories (quantum mechanics and game theory) <u>**both are analogous**</u> and thus exactly equivalents.

5. So, we can take some concepts and definitions from quantum mechanics and physics for the best understanding of the behavior of **economics and biology.**

# Some Crazy Ideas & Conclusions

6. We could maybe understand **nature like a game** in where its players compete for a common welfare and the equilibrium of the system that they are members

7. All the members of our system will play a game in which its **maximum payoff is the equilibrium of the system.** They act as a whole besides individuals like they obey a rule in where they prefer to work for the welfare of the collective besides the individual welfare.

8. A system is stable only it maximizes the welfare of the collective above the welfare of the individual. If it is maximized the welfare of the individual above **the welfare of the collective** the system gets unstable an eventually collapses.

9. A system where its members are in **NE (or ESS) is exactly equivalent to a system in a maximum entropy state** (its stability should be given by the welfare of the collective).

# Some Crazy Ideas & Conclusions

10. The GLOBALIZATION process has a behavior exactly equivalent to a system that is tending to an MAXIMUM ENTROPY state.

Big common markets (EU, NA, SA,ASIA,O),
And strong common currencies ($, €, £, *,…),

With the time The number of markets and currencies ($, €, £, *,…), will decrease until the system reaches its equilibrium i.e. only one currency in the world.