September 16, 2008

The case of the Altruist meme:

To share or not to share: that is the question!

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"Cast thy bread upon the waters: for thou shalt find it after many days. Give a portion to seven, and also to eight; for thou knowest not what evil shall be upon the earth.",Solomon 11,1-2

Altruism:

What are we talking about??

Selfless concern for the welfare of others...

1) it is directed towards helping another,

2) it involves a high risk or sacrifice to the actor,

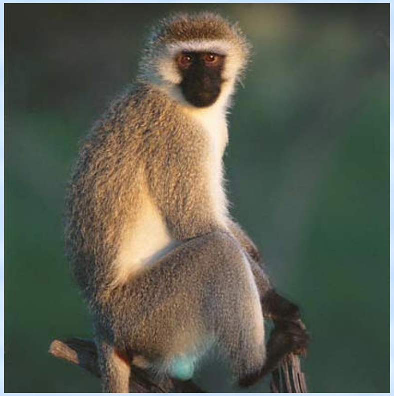
3) it is accomplished by no external reward,

4) it is voluntary.

Some examples of animal altruism

Dogs often adopt orphaned cats, squirrels, ducks and even tigers.

Wolves and wild dogs bring meat back to members of the pack not present at the kill.



Vervet Monkeys give alarm calls to warn fellow monkeys of the presence of predators, even though in doing so they attract attention to themselves, increasing their personal chance of being attacked.

Dolphins support sick or injured animals, swimming under them for hours at a time and pushing them to the surface so they can breathe.

Some examples of animal altruism

According to the research of Gerald Wilkinson, vampire bats have a "buddy system" in which a bat who has had a successful night of feeding will regurgitate blood for its less fortunate companion.

In numerous bird species, a breeding pair receives support in raising its young from other "helper" birds, including help with the feeding of its fledglings. Some will even go as far as protecting an unrelated bird's young from predators

Altruism:

How come it is evolutionary stable strategy?

A long debate started with Darwin and his group selection and continued with:

- Behavioural manipulation (for example, by certain parasites that can alter the behaviour of the host)
- Bounded rationality (for example, Herbert Simon)
- Kin selection (including eusociality)
- Reciprocal altruism, mutual aid.

Altruism:

How come it is evolutionary stable strategy?

Our solution:

Multiplicative dynamics does it for "free" -

What might be seen like an altruistic behaviour (phenotype)

is really a selfish concern for the survival of one's genes/memes

Multiplicative Vs. Additive Random dynamics:

i = 1..N

Multiplicative

$$X_i(t+1) = \begin{cases} a \cdot X_i(t) & \text{with probability } p \\ b \cdot X_i(t) & \text{with probability } q = 1 - p \end{cases}$$

Additive

$$X_i(t+1) = \begin{cases} X_i(t) + C & \text{with probability } p \\ X_i(t) + D & \text{with probability } q = 1 - p \end{cases}$$

A Caricature of the principal:

an agent is going through the following dynamics in each time step: $i = 1..N$

$$X_i(t+1) = \begin{cases} 0 & \text{with probability } 1/2 \\ 3 \cdot X_i(t) & \text{with probability } 1/2 \end{cases}$$

On average, the agents are suppose to grow like $(3/2)^T$

However, each agent (with probability 1) will get to 0 with average life time of 2 steps...

Overcoming the paradox:

If the agents decide to *share* their wealth and redistribute it after each iteration:

- In a synchronous updating mechanism, the average life time of each of the agents in a group of N sharing individuals will be multiplied by 2^{N-1}

- In an A-synchronous updating mechanism even a group of two sharing individuals will *NEVER* reach 0!

(half of the common wealth is in a "*safe*" place)

Both updating mechanisms exist in nature.

Let us consider the more general stochastic process:

$$i = 1..N$$

$$X_i(t+1) = \begin{cases} a \cdot X_i(t) & \text{with probability } p \\ b \cdot X_i(t) & \text{with probability } q = 1 - p \end{cases}$$

naïve calculation:

$$<X_i(t+1)>_i = (p\cdot a+(1-p)\cdot b)\cdot <X_i(t)>_i$$

F≡Arithmetic mean

Which leads to:

$$<X_i(T)>_i = (F^T)\cdot <X_i(0)>_i$$

naïve calculation:

$$\langle X_i(t+1)\rangle_i=(p\cdot a+(1-p)\cdot b)\cdot\langle X_i(t)\rangle_i$$

F≡Arithmetic mean

Which leads to:

$$\langle X_i(T)\rangle_i=(F^T)\cdot\langle X_i(0)\rangle_i$$

IS THAT SO ???

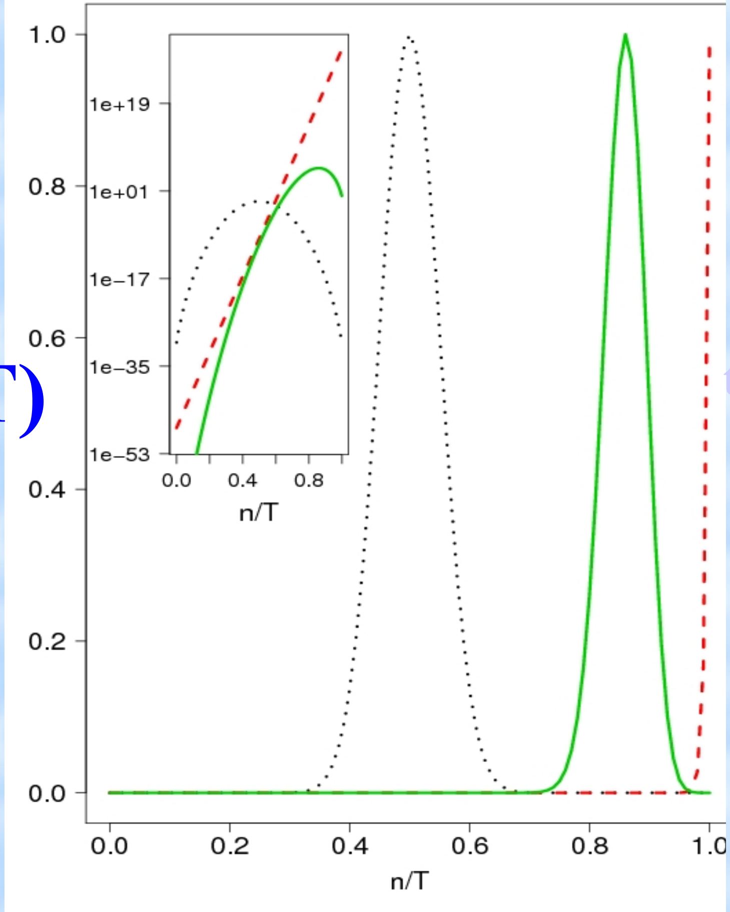
...Only for *exponentially large* number of agents:

i.e. : $M \sim e^{AT}$ where A is defined by the parameters of the game (a,b and p)

For example: $p=q=0.5$, $a=2$, $b=1/3$, $T=100$

$$<X_i(T)>_i=\sum P(T)_i X_i(T)$$

time evolution



What is going on??

As some of you may know:

for long times: the *typical* value dominants

(the expectation value for the log of the wealth)

$$w \equiv W/T$$

$$l \equiv L/T$$

$$X_i(T) = a^W \cdot b^{L=T-W} \cdot X_i(0) =$$

$$X_i(0) \cdot e^{W \cdot \log(a) + L \cdot \log(b)} =$$

$$X_i(0) \cdot e^{\{w \cdot \log(a) + l \cdot \log(b)\} \cdot T} =$$

$$\rightarrow X_i(0) \cdot e^{\{p \cdot \log(a) + q \cdot \log(b)\} \cdot T} \equiv X_i(0) \cdot (a^p \cdot b^q)^T$$

$T \rightarrow \infty$

r≡Geometric mean

In the two previous examples:

for $a=3$, $b=0$ ===> $F=1.5 > 1$ while $r=0 < 1$

and

for $a=2$, $b=1/3$ ===> $F=7/6 > 1$ while $r=(2/3)^{0.5} < 1$

Partial summary:

If one takes the limit $N \to \infty$ then the arithmetic mean determine the mean behaviour of the system.

If the limit $T \to \infty$ is taken first, then the geometric mean sets the mean dynamics of the system.

What we may remember from hi-school is that

arithmetic mean ≥ geometric mean

In this work we focused on the case where:

arithmetic mean >1> geometric mean

which brings the problem to a matter of life and death !!

How could the arithmetic mean be restored without having exponential number of realizations??? (could it?)

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The answer is ....YES – by sharing:

The average (over time) of the growth rate of N sharing individuals could be calculated to be:

$$r_N = \sum_{k=0}^{N} \binom{N}{k} q^k p^{N-k} \cdot \ln(a \cdot \frac{k}{N} + b \cdot \frac{N-k}{N})$$

where it can be shown to behave like $r_N - \log(F) = o(1/N)$

Intuition:

Without sharing:

A A B B A B A A A B A B B A B B B $= A^8 B^9$ +

B B B B B A B A A A A B A B B A B $= A^7 B^{10}$ +

A A B B A B A B B A B A A A B B A $= A^9 B^8$ +

B B B B B B B B B B B B B B B B A $= A^1 B^{16}$ +

A A A A A A A A A A A A A A A A A $= A^{16} B^1$ +

B B A B A A A A A B A A B B A B A $= A^{10} B^7$

Intuition:

With sharing:

A A B B A B A A A B A B B A B B B
\+ +
B B B B B A B A A A A B A B B A B
\+ +
A A B B A B A B B A B A A A B B A
\+ +
B B B B B B B B B B B B B B B A
\+ +
A A A A A A A A A A A A A A A A
\+ +
B B A B A A A A B A A B B A B A

*

Partial summary:

In the cases discussed before

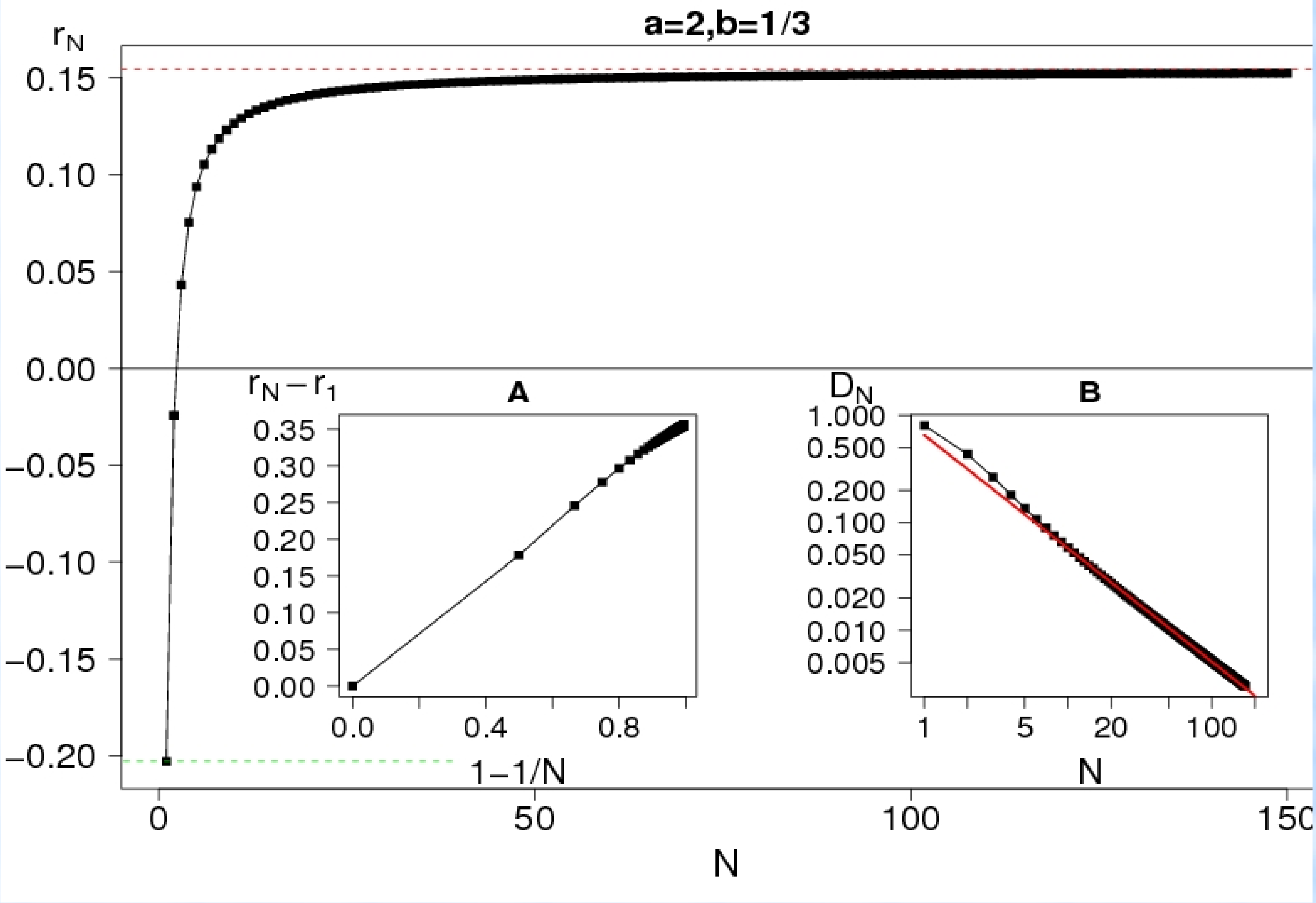
(arithmetic mean >1> geometric mean)

There exist N_{crit}

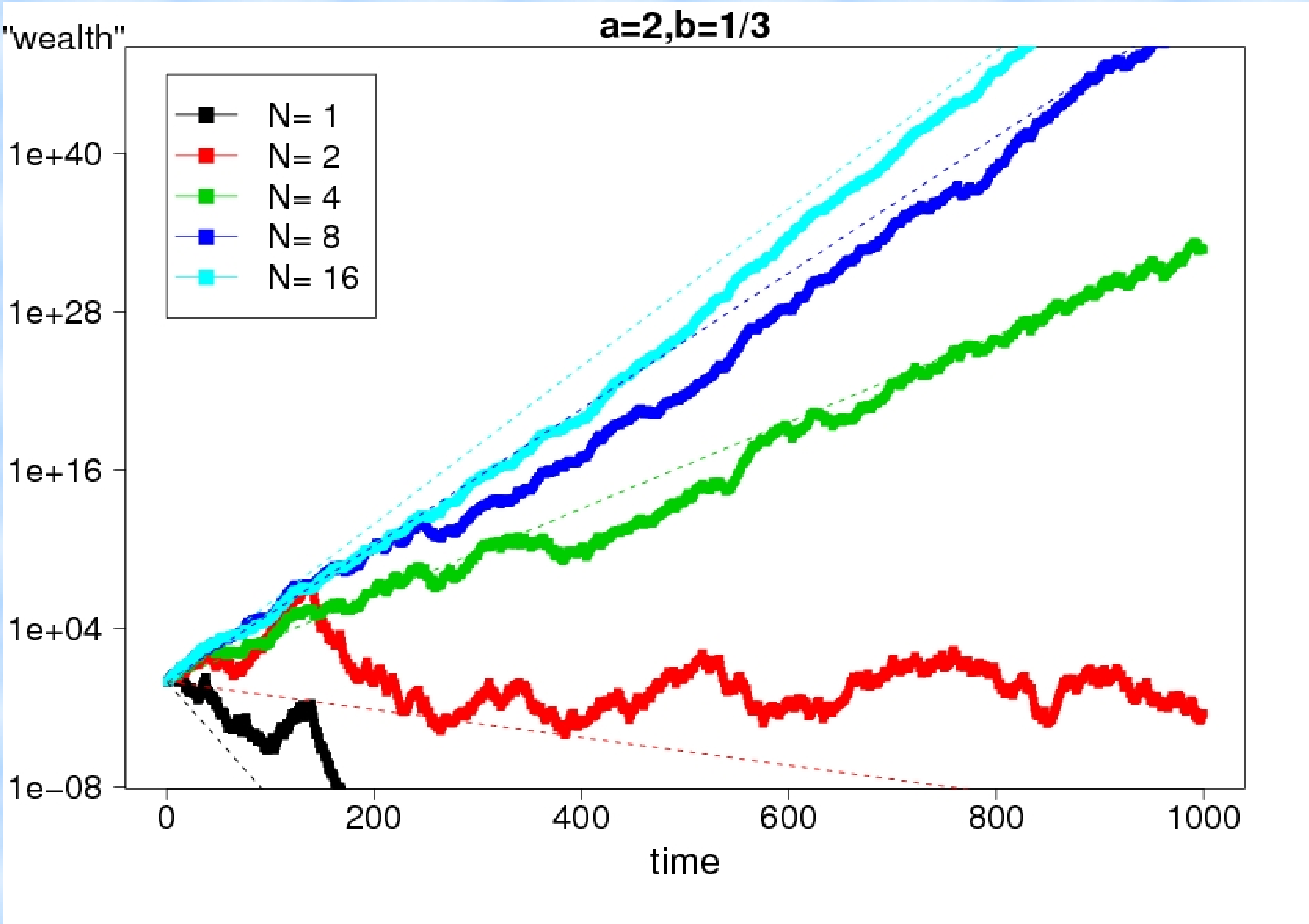
Which distinguishes between LIFE and DEATH

and makes this altruistic meme a stable evolutionary strategy!!

Asymptotic growth rate - calculation



Asymptotic growth rate – comparison with simulations



What would have J.kelly say about all that?

The Gambler problem: invest 1$ and in case of winning one gets (d+1)$. In case of loosing- null.

$$X_i(t+1)= \begin{cases} (1+f\cdot d)\cdot X_i(t) & \text{with probability } p \\ (1-f)\cdot X_i(t) & \text{with probability } q=1-p \end{cases}$$

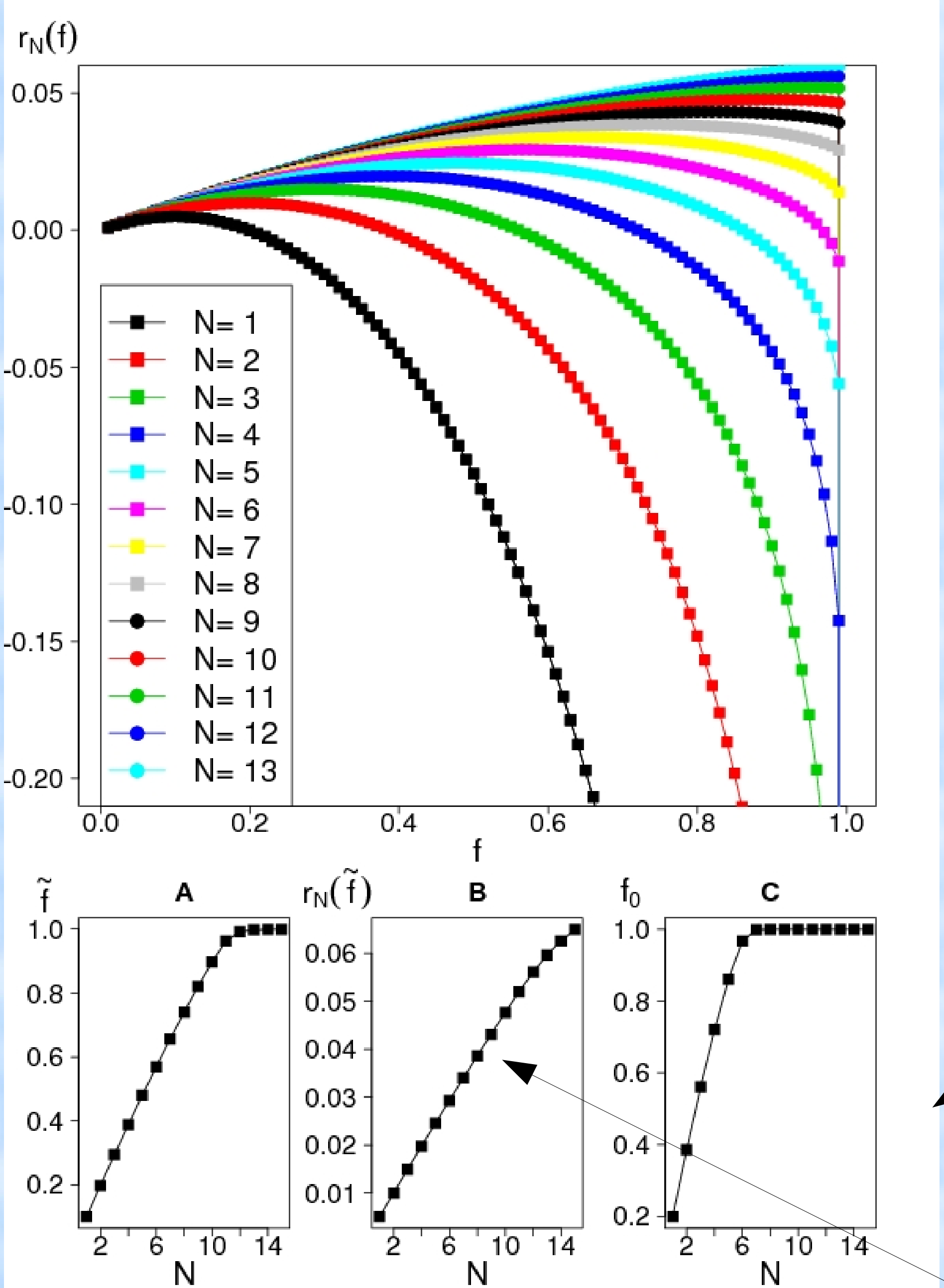
When $f=1$, the geometrical mean is 0

Kelly introduced myopic (static) strategy:

Do not risk all you have....($f\neq 1$)

P=0.55,

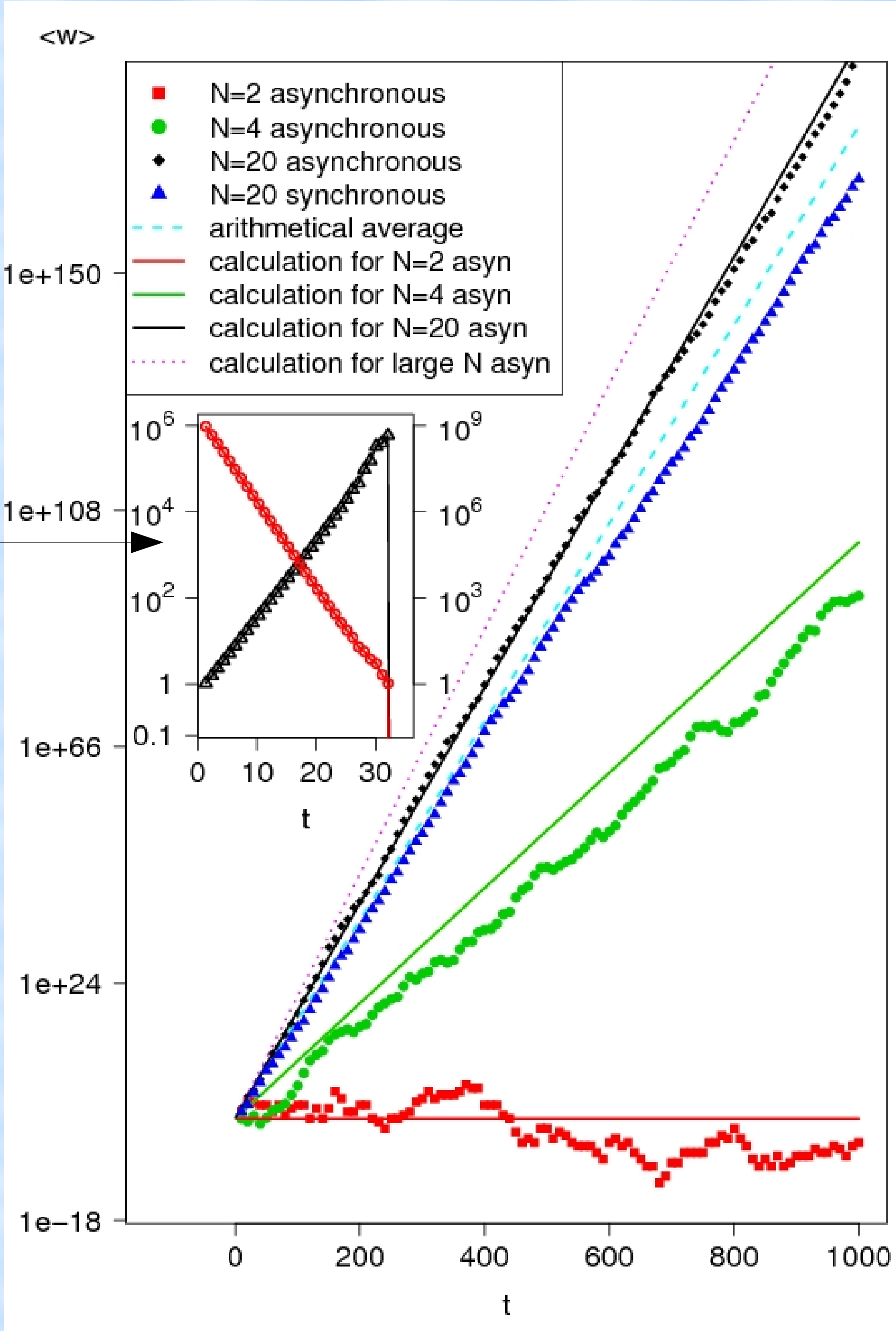
d=1



by sharing -risk is reduced!!

and the growth is higher..

averaged over 10^6 realizations of nonsharing individuals!



a=0

b=3

Asynchronous

Limited Generosity:

If one decides to share only a fraction D out of the difference between it's wealth and the average wealth, what happens?

In formula:

After each “reaction step” we make diffusion step:

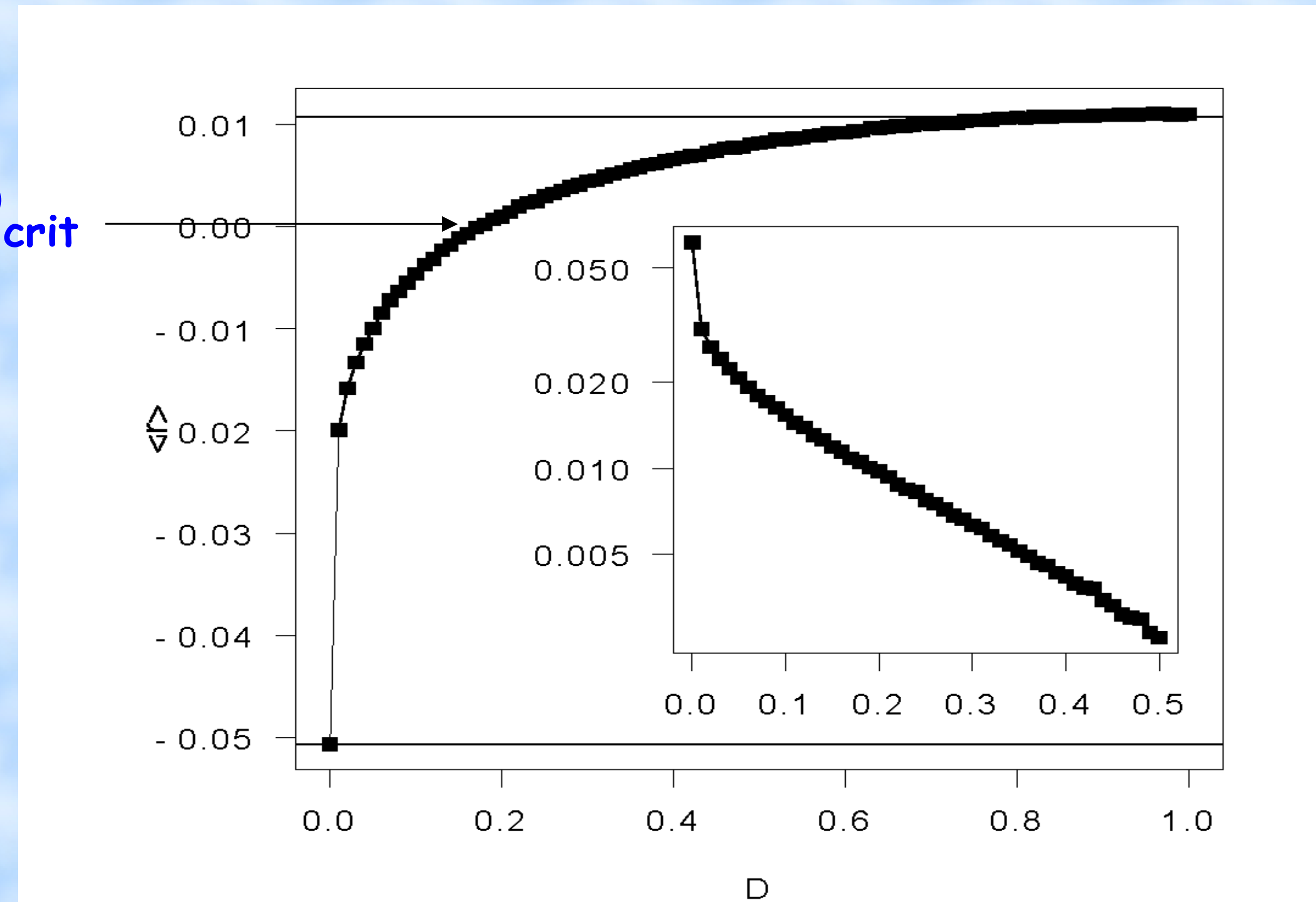
$$X_i(t+1)-X_i(t)=D\cdot\{<X_i(t)>_i-X_i(t)\}$$

Limited Generosity:

If one decides to share only a fraction D out of the difference between it's wealth and the average wealth,

for $N > N_{crit}$ there exist D_{crit} !

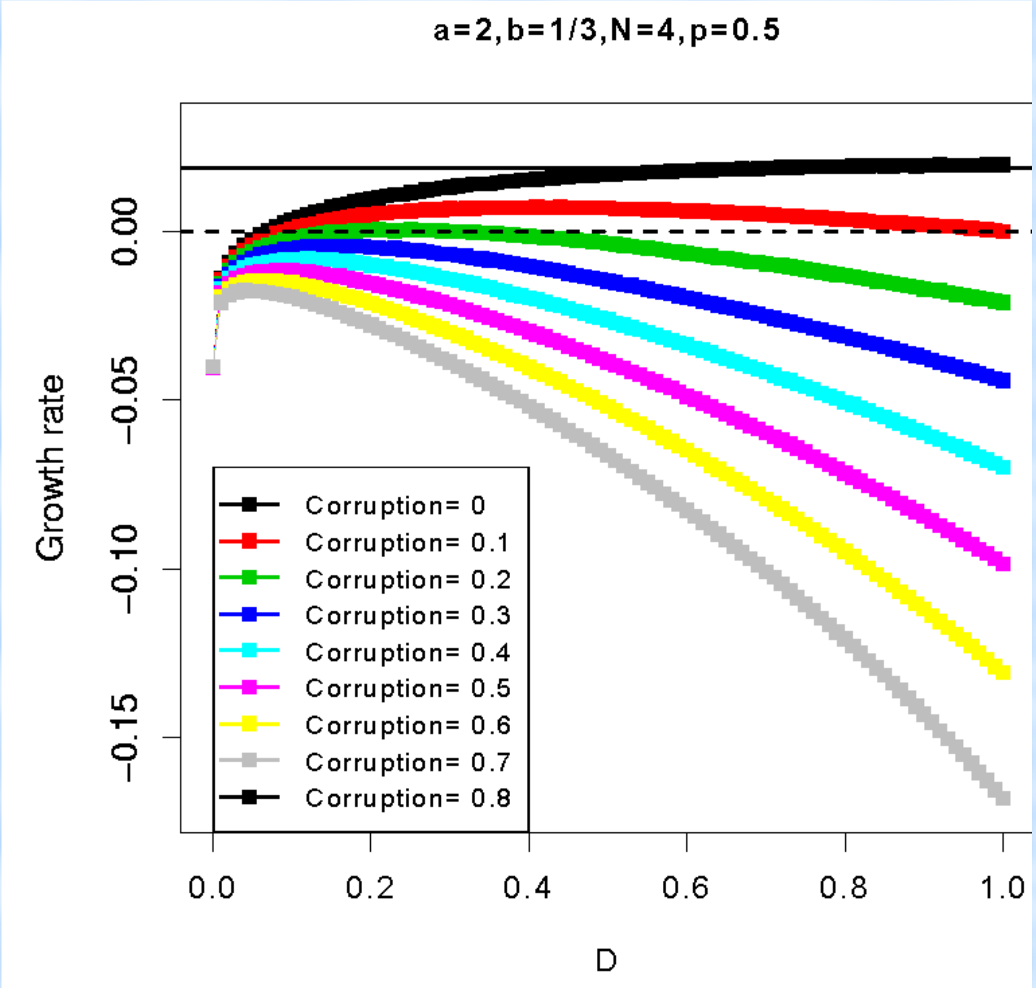
D_{crit}



p=q=0.5,
a=2,
b=1/3,
N=4

Limited Generosity:

What happens when you have politicians that are taking a fraction of whatever one intended to give to others ??



An optimal level of generosity appears (≠1):

Tell me the level of corruption in society and I'll tell you the level of generosity you have to have..

Does this theoretical analysis pass natural selection??

YES!

If one design a proper simulation set-up to take into account *competition for limited resources, multiplication and mutation*, it turns out that the dominant strategy has D≠0:

Choose agent *i* randomly

“Reaction”

$a \cdot X_i(t)$ or $b \cdot X_i(t)$

“Diffusion”/donation

if $X_i(t) > \langle X_i(t) \rangle_i$

then donate:

$D \cdot (\langle X_i(t) \rangle_i - X_i(t))$

Selection”

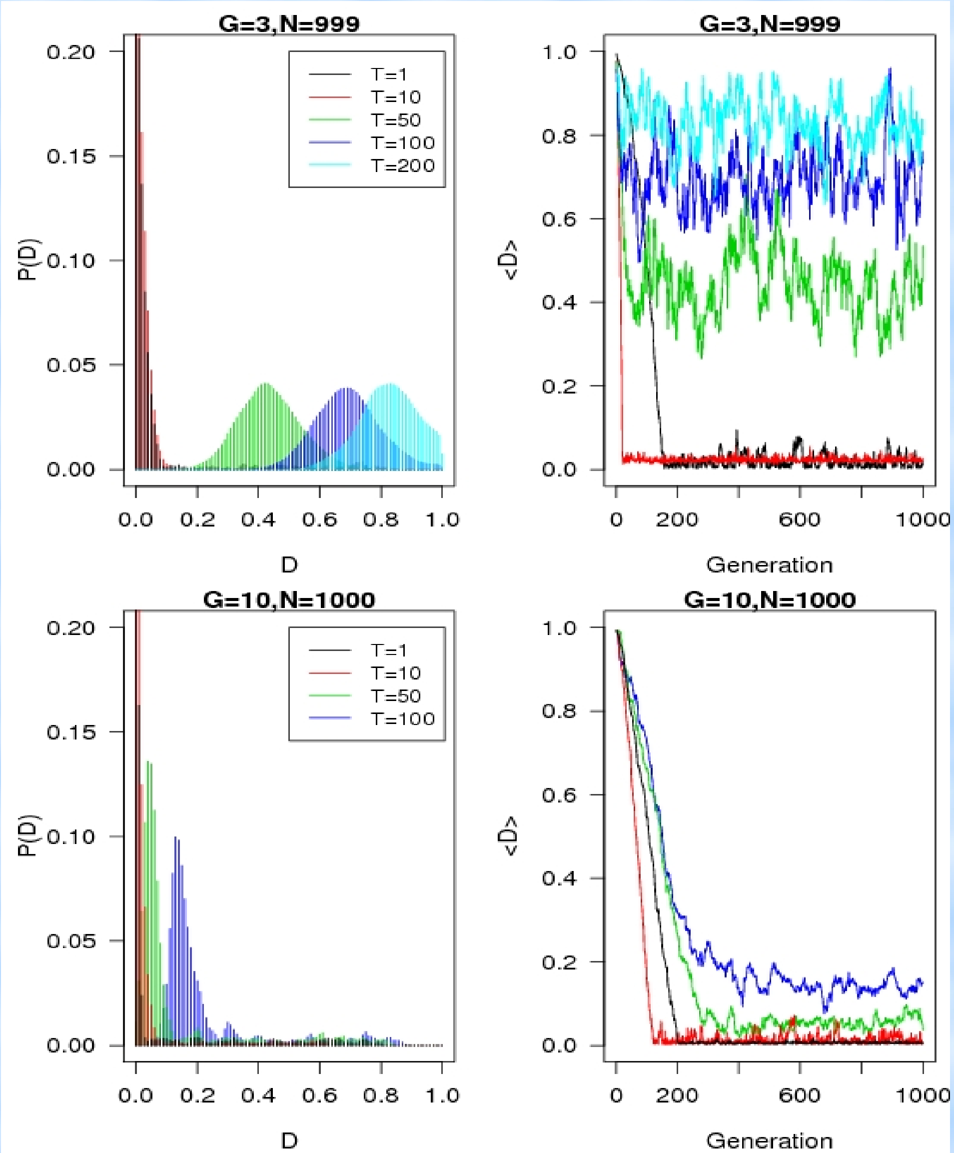
Choose agent *j* randomly

(with probability 1/T), and:

if $X_i(t) > X_j(t)$

then $D_j(t)=D_i(t)$ and $X_j(t)=X_i(t)$

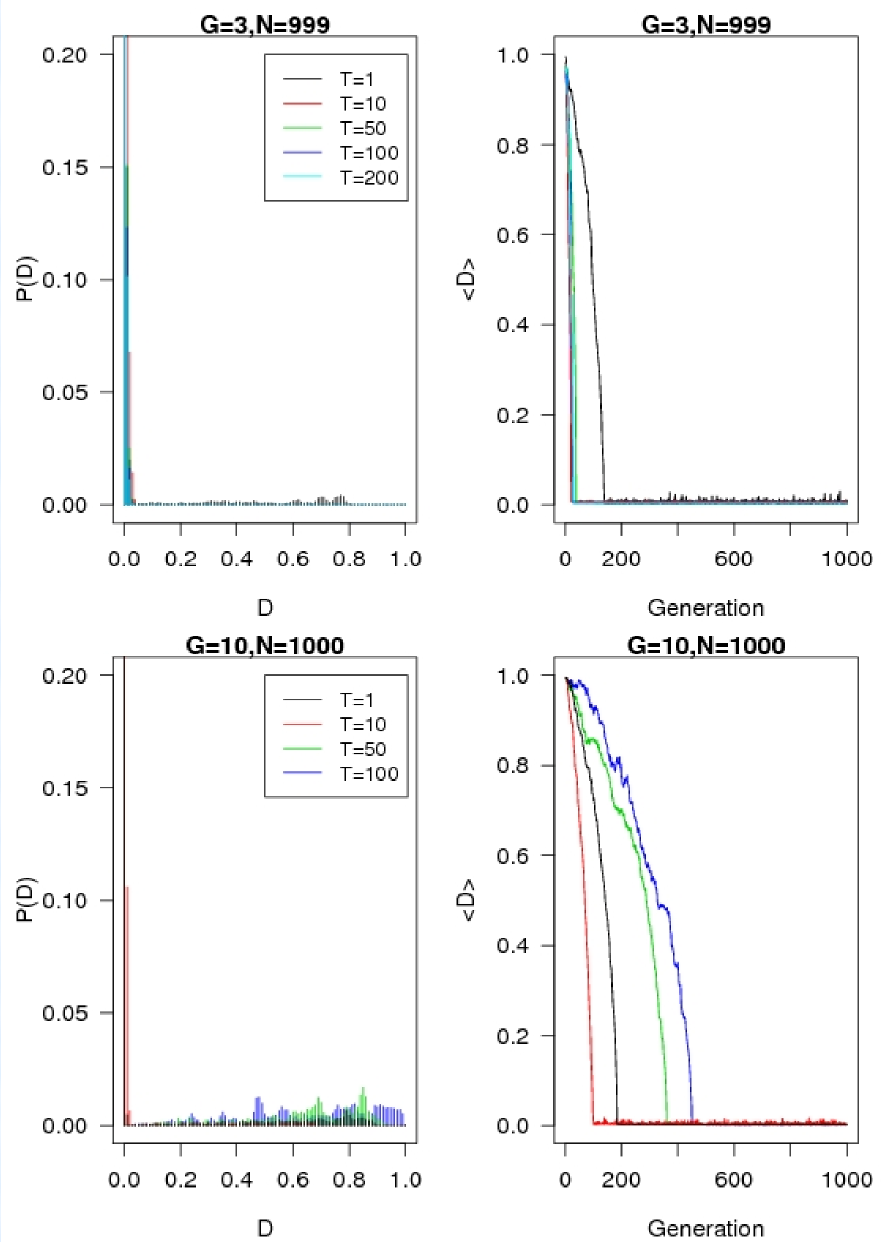
The evolution of the average "altruism level" as a function of generation length - T



"Reaction"

$a \cdot X_i(t)$ or $b \cdot X_i(t)$

The same thing for additive dynamics:



"Reaction"

$X_i(t)+a$ or $X_i(t)+b$

Conclusions:

We offer a solution to the altruism stability paradox by noticing that in RANDOM MULTIPLICATIVE INVIRONMENT it is in the SELFISH interest of oneself to donate to his/her peers and by this to ensure that he will not be alone in this hostile environment called life.

This "Selfish Altruism" is not distinguishable behaviourally from "pure" altruism and as such could be evolutionary stable !

Blagodarya

Multumesc

Dziekuje

Grazie mille

NAGYON KÖSZÖNÖM

Gracias

Thank You very much

Tika hoki

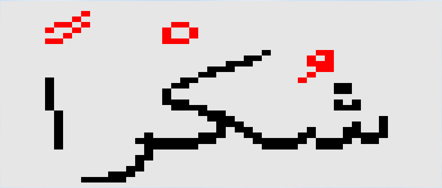
Spasibo

dunke schoen

תודה רבה

Shukriya

Arigato



merci beaucoup

A-synchronous: how should we approach it?

keeping a fraction $(N-1)/N$ in a safe place:

i.e. In Kelly's terminology:

$$f \rightarrow f/N$$

$$t \rightarrow t \cdot N$$

Calculating the growth rate for $N >> 1$ A-synchronous sharing individuals gives us result that is actually *BETTER* than the arithmetical mean!!