Complexity in Jerusalem
Sept. 15, 2008

Minimal Agent Based Model for Economics: Stylized facts and their Self-organization

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***Physics, Complexity, Economics*:**

***Physics*: try to discover the laws of nature**

***Economics*: are there laws to be discovered?**
evolutive elements, adaptivity,
the whole society is involved

***Complexity*: new vision and possible point of contact**

MODELS AND BASIC PROBLEMS

*Ising * (1911)*
Scaling, Criticality (64 - 70)
and RG Group (>72)
Percolation (‘70-’80)*
Glasses Spin Glasses* etc.(>74)
Deterministic Chaos* (78)
Fractal Geometry (‘80-’90)
Polymers and Soft Matter
***Dynamical Systems** and Turbulence*
Fractal Growth Physical Models:
DLA/DBM (82-84)*
Selforganized Criticality
Sandpile* (87)
Granular Systems (‘90)
Minority Game (‘97)
Rare Events
Complex Networks (>2000)

INTERDISCIPLINARY APPLICATIONS

Condensed Matter problems
Phase Transitions
Magnetic Systems
Bio-inspired Problems
Astrophysics
Geophysics
Information Theory
Optimization
Economics and Finance
Social Sciences

(Random Walk, Bachelier 1900)

Classic theory of economics:

(New Scientist editorial, 2008)

- ***Situation of equilibrium with agents (quasi) rational and informed***
- ***Important price changes correspond to new information which arrives on the market***
- ***This information modifies the ratio between offer and demand and then also the price***
- ***Relation cause - effect***

Probelms with the classic theory:

- *Great cathastrofic events like the ‘87 crash, the Inernet bubble of 2000 and the recent case of the Subprimes do not seem to have any relation with specific events or new information*
- *Also the Stylized Facts at smaller scales cannot be really explained within the standard model*
- *Breaking of the cause-effect relation: then what is the real origin of large price changes?*

New perspective:

- ***The market seems to evolve spontaneously towards states with intrinsic instability which then collapse or explode triggered by minor or irrelevant perturbations***

- ***Importance of herding effects especially in situations of uncertainity with respect to the fundamentals of economics (fear, panic, euphoria)***

- ***Breaking of the cause-effect relation and of the traditional economic principles:***
  New type of agent models are necessary to capture these new phenomena

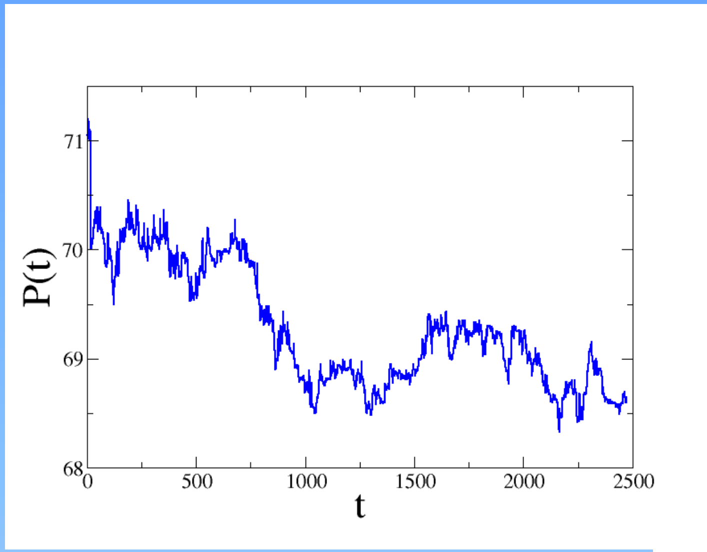
Basic Stylized Facts (Universal?):

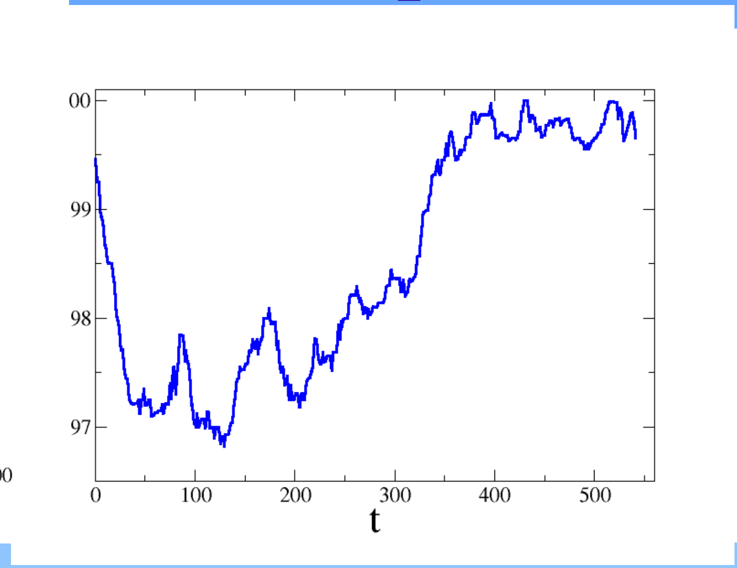
- ***Arbitrage -- Random Walk (B&S)***
- ***Fat tails, Volatility Clustering etc.***

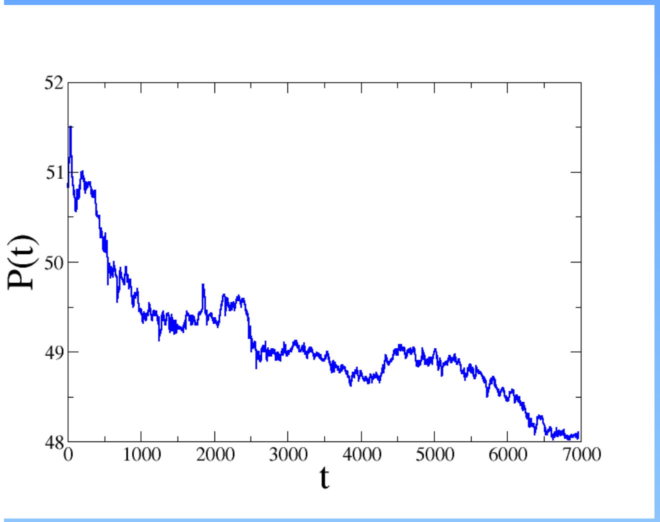
AND ALSO

- ***Non stationarity***
- ***Self-organization***

NYSE stock-price data







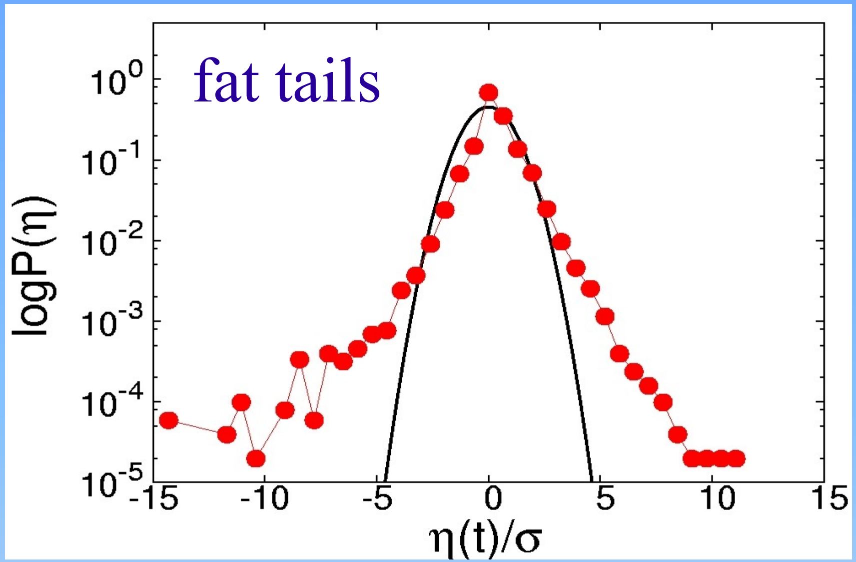
Arbitrage condition: no correlations between price returns

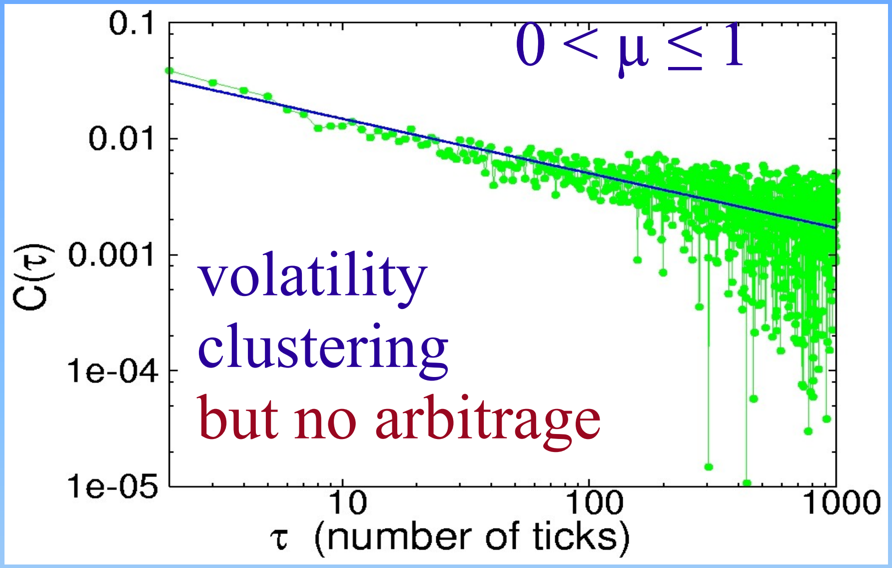
Simplest model: Random Walk

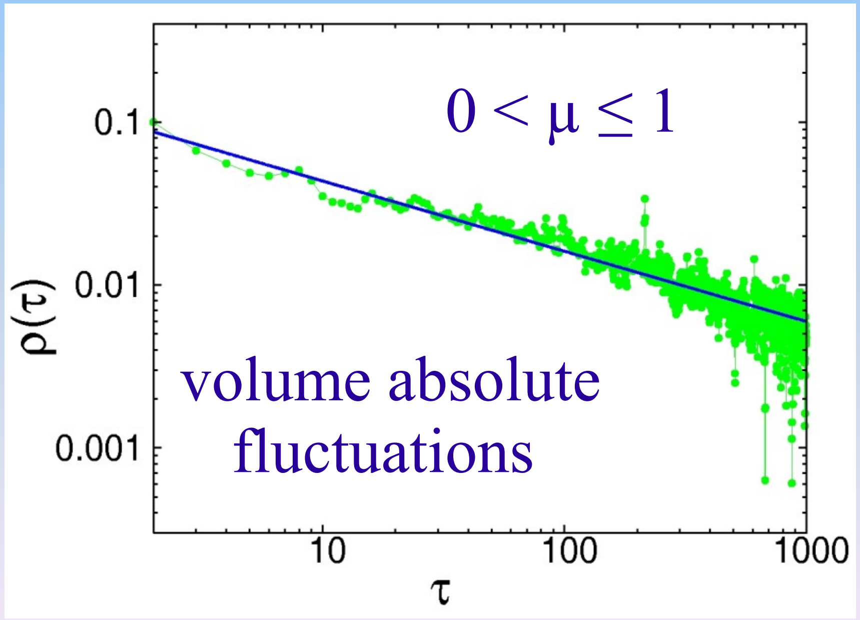
Persistent deviations from RW: Stylized Facts

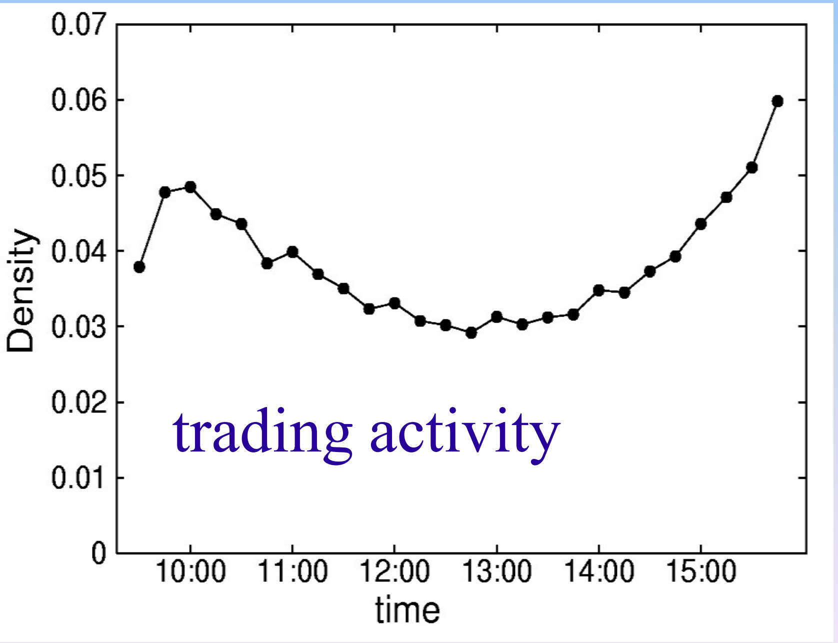
Origin of Stylized Facts: Agent Based Models

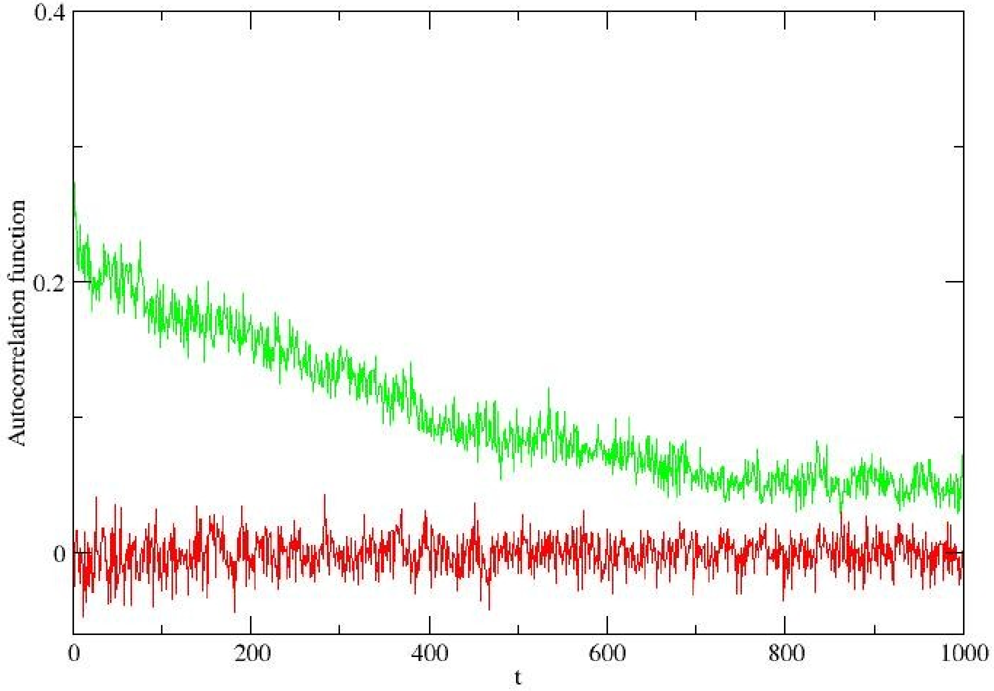
Stylized Facts of Financial Data





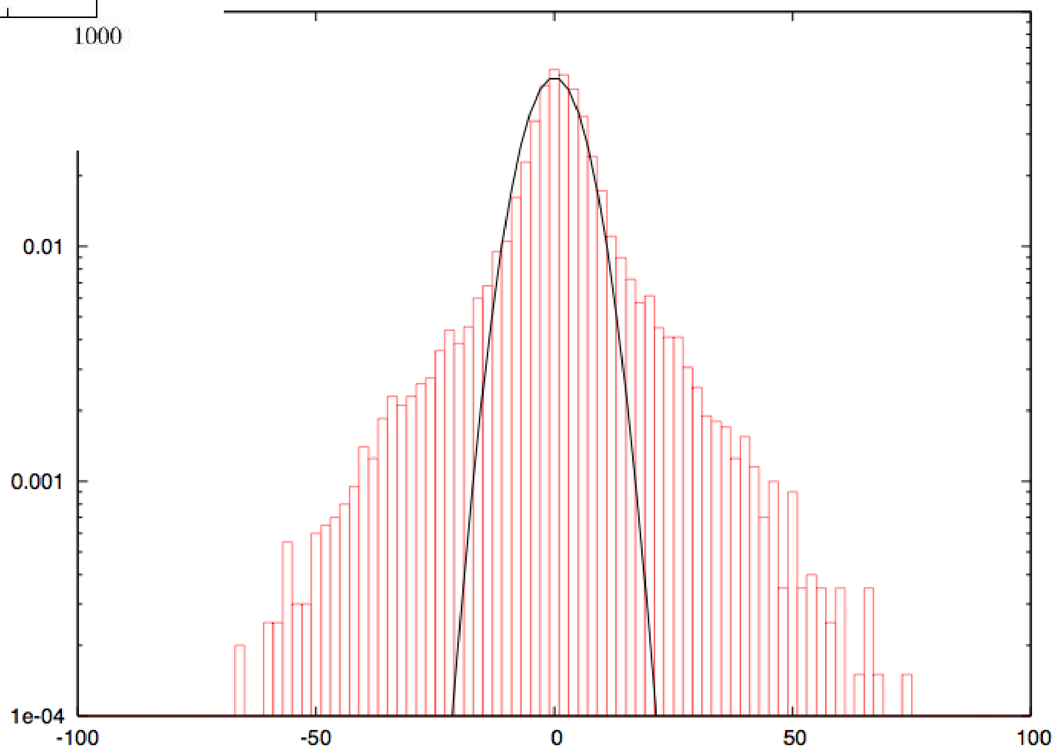






Autocorrelation functions of returns and square returns

General behavior, but how universal ? (R. Cont, >2000)



Probability density function of price-returns

ABM models to reproduce Stylized Facts
Lux & Marchesi Model

Four basic elements:

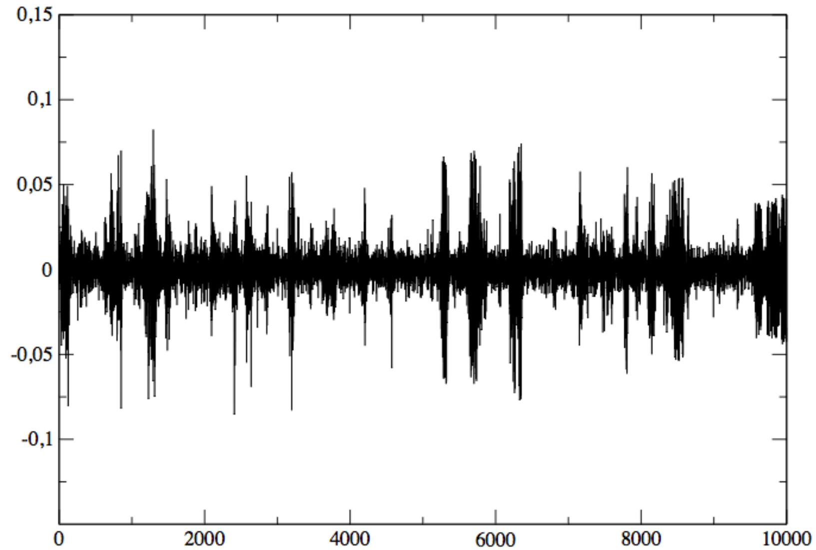
- Chartists: follows the market trend, evaluate historical series (INSTABILITY)
- Fundamentalists: believe that a fundamental price (Pf) exists and try to drive the price towards Pf (STABILITY)
- Herding: Agents tend to follow the others
- Price behavior: F and C agents look at signals from the price

T. Lux, M. Marchesi, Nature (2000) and
Int. J. of Theo. and Appl. Finance, **3**, 675-702, 2000

Puzzle of the N-dependence (Egenter, Lux, Stauffer, ‘99)

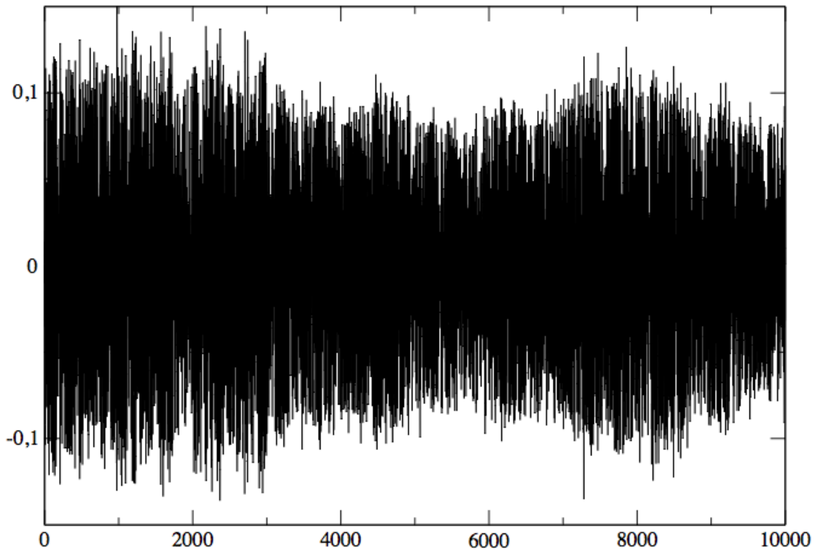
Price Returns

Intermittent behavior: OK

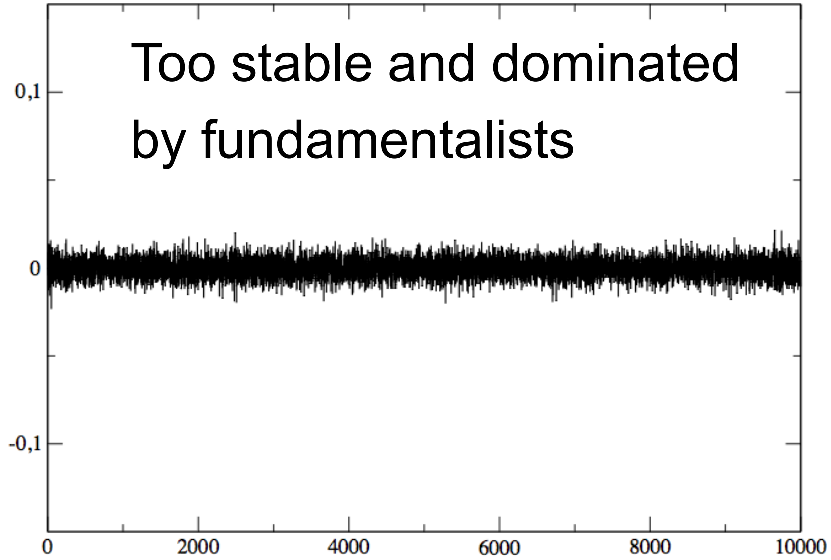


N=500

Changes of opinion are too fast



N =50



N 5000

Other problems of the LM model

Stability with respect to other parameters

(Parameters are changed one by one)

- Herding parameters ν, β, γ and tc
- ν from 2-3 to 20-30 or 0.2-0.3: unstable
- β from 6 to 0.6 or 60: stable
- γ from 0.01 to 0.1 or 0.001: unstable
- tc from 0.02 to 0.2 and 0.002: unstable

Nature of the fluctuations in bubbles

- Bubbles are characterized by unrealistic, very fast fluctuations between optimists and pessimists

In summary: not much progress in the past 10 years:

LM model seems to be too complicated

Minimal ABM model

(V. Alfi, M. Cristelli, L.P., A. Zaccaria 2008)

N players: N_F fundamentalists N_C chartists

- Basic elements similar to LM but much simpler, workable model. Also analytical approach is possible.
- Price formation and excess demand proportional to trader's signals (effective N*). Simple dynamical system.
- Specific origin of Stylized Facts (finite size effects)
- Nonstationarity (effective N* fluctuations)
- Self-organization to the quasi-critical (intermittent) state

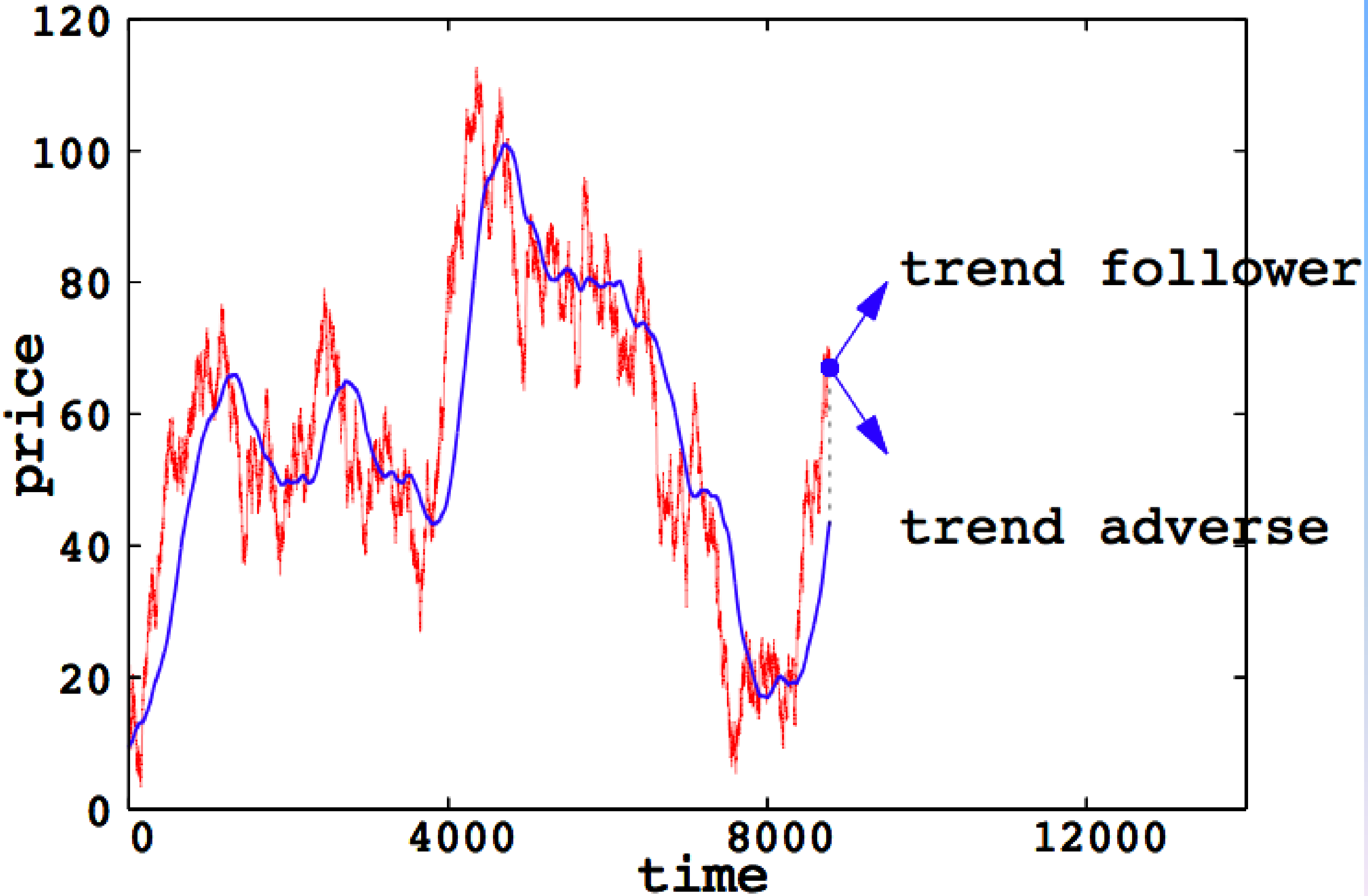
A new description of Chartists: The Effective Potential Model

- Directional elements are strongly connected with agents’ strategies.
- This model is based on a random walk with an active potential defined on the walk dynamic
- The centre of the potential is the moving average of the trajectory of the walker
- No optimists and pessimists, only destabilizing chartists (spurious oscillation between o. and p. in LM bubbles)
- Important simplification: Only 2 types of agents (instead of 3)
  Rate probabilities reduce from 6 to 2
  Analytical results and systematic simulations

M.Takayasu, T.Mizuno and H.Takayasu, preprint 2005, [physics/0509020].

V. Alfi, F. Coccetti, M. Marotta, L. Pietronero, M.Takayasu, Physica A, **370**, 30-37, 2006

Moving Average



The basic ansatz is that price dynamics P(t) can be described in terms of a stochastic equation of the type:

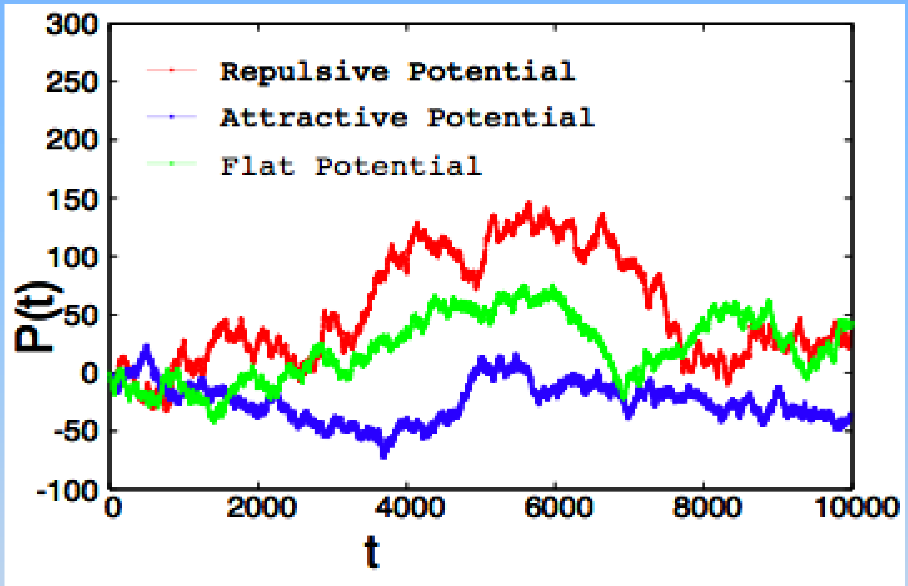
$$P(t+1) - P(t) = b(t)F[P(t) - P_M(t)] + \sigma(t)\omega(t)$$

- $P(t+1) - P(t)$: Next increment
- $b(t)$: The pre-factor b(t) gives the sign and the strength of the potential
- F: F is the Force
- $\omega(t)$: Random noise with unitary variance and zero mean

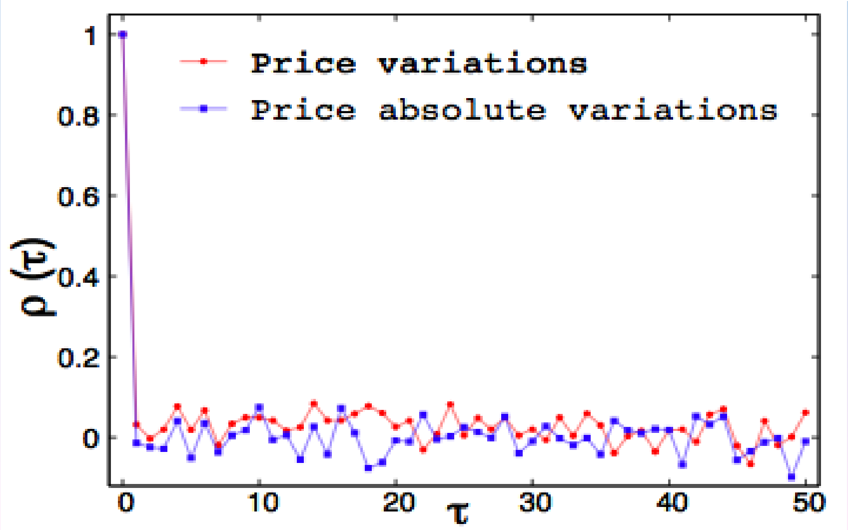
$$P_M(t) = \frac{1}{M}\sum_{k=1}^{M} P(t-k)$$

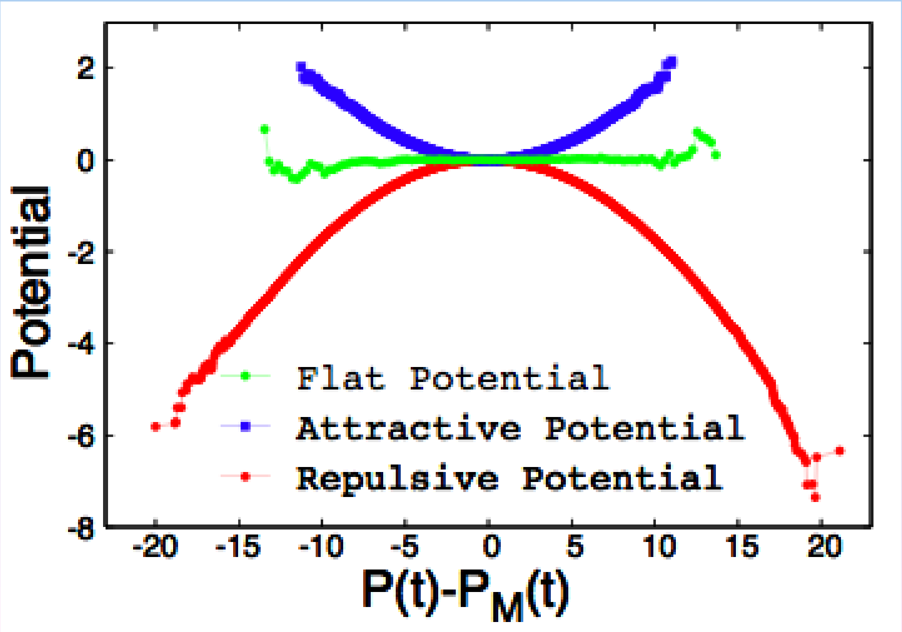
The Moving Average

The interesting point is that one can identify a non trivial situation in terms of effective potential but in absence of simple correlations.



RW+quadratic potential model





ABM model with moving average-based strategies (V. Alfi, L.P., A. Zaccaria 2008)

(Linear dynamics to start: more stable and easy to treat)

N players: N_F fundamentalists N_C chartists

each time step, each agent can change its strategy with probabilities

$$P_{CF} \sim (K + \frac{N_F}{N}) \exp(\gamma |p - p_F|) \quad P_{FC} \sim (K + \frac{N_C}{N}) \exp(\frac{b|p - p_M|}{M-1})$$

Price formation

$$p(t+1) = p(t) + \sigma\xi + b\frac{p(t) - p_M(t)}{M-1} N_C + \gamma(p_f - p) N_F$$

Stochastic Multiplicative Process

Origin of the Finite size effects

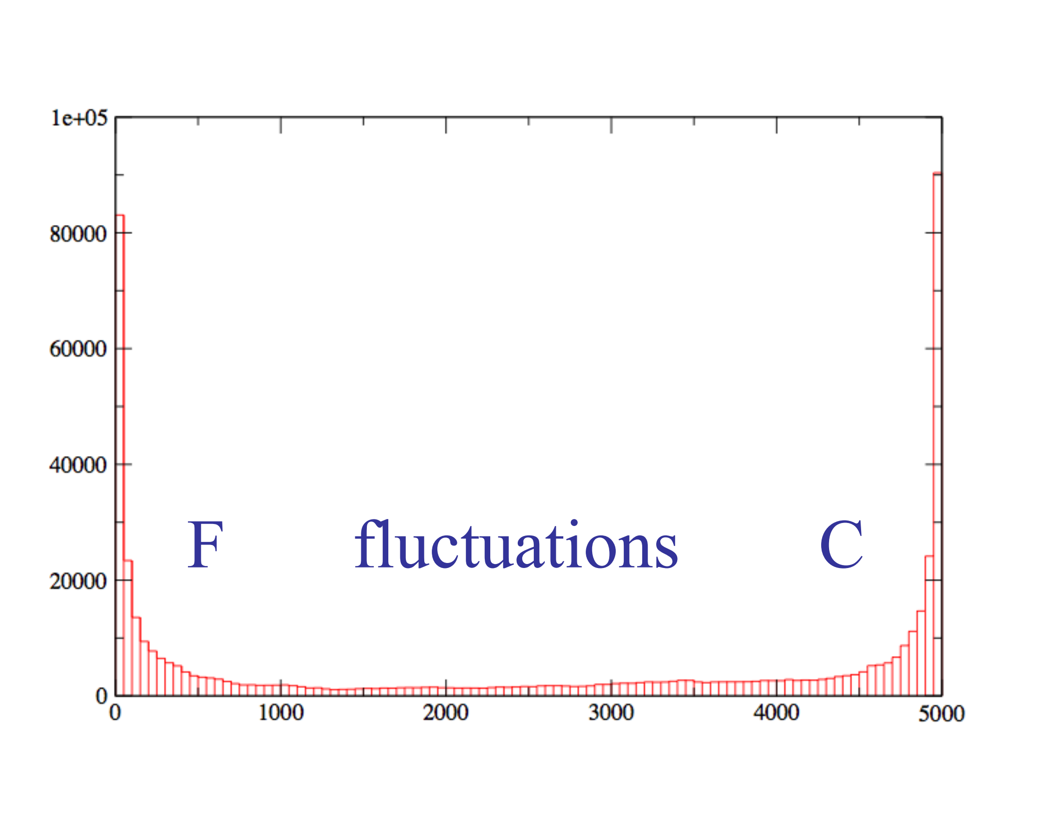
$$N = N_F + N_C$$

$$P_{C \to F} = \beta(K + \frac{N_F}{N})$$

$$P_{F \to C} = \beta(K + \frac{N_C}{N})$$

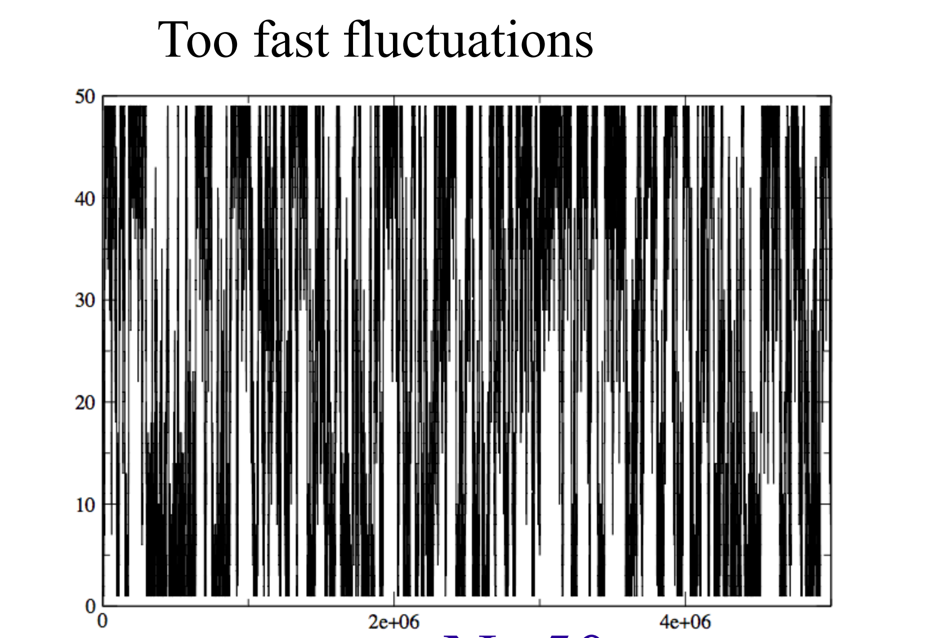
$$x = \frac{N_F - N_C}{N}$$

$$P_e(x) = \frac{1}{L} \frac{1}{1 - x^2}$$

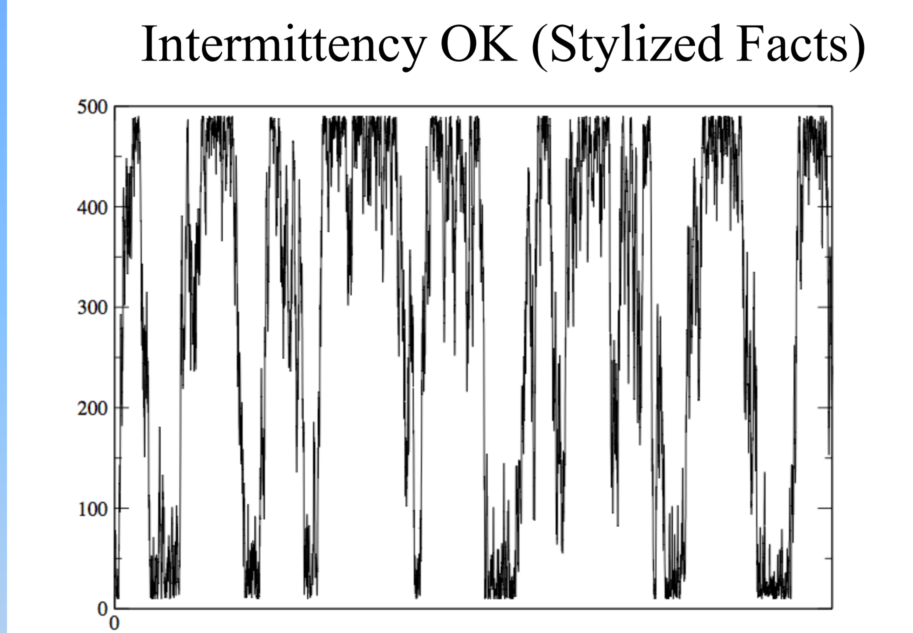


Alfarano&Lux 2006

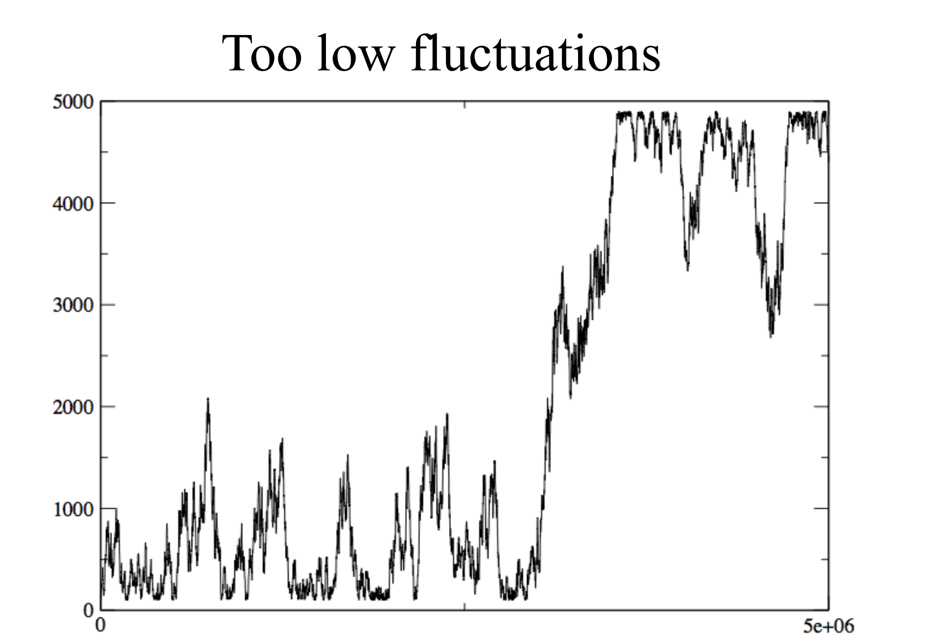
Nc $\beta = 0.005$



N=50

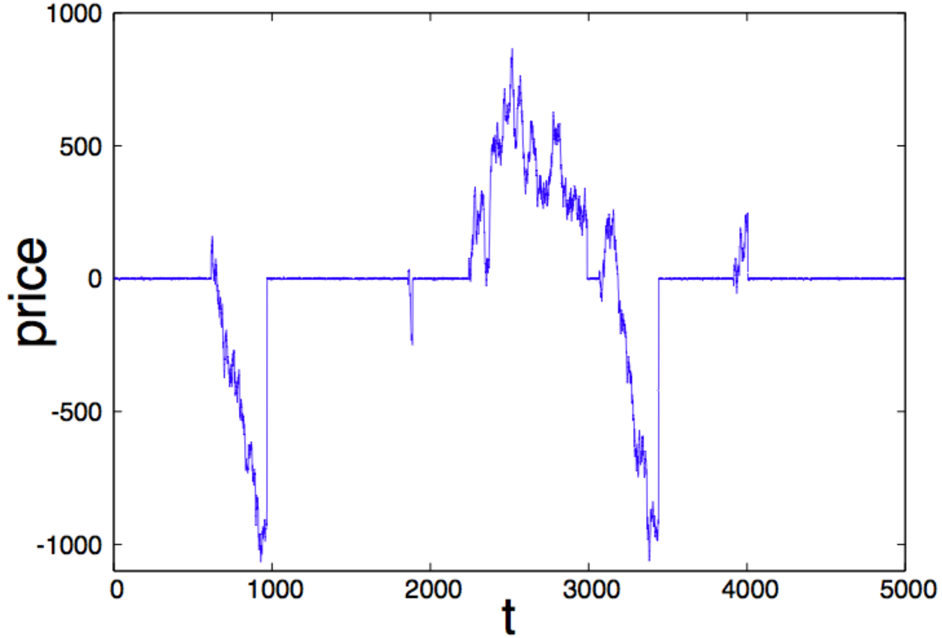


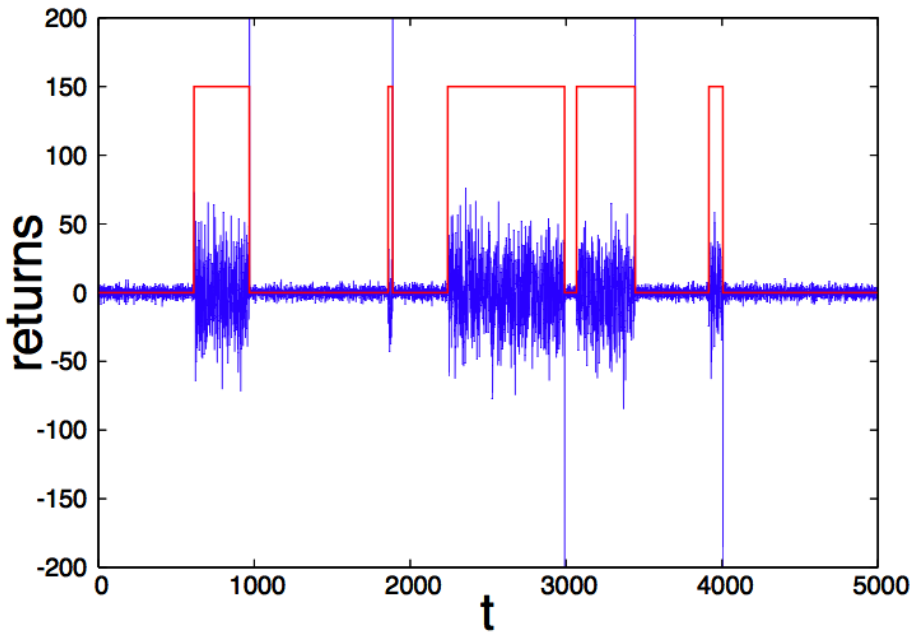
N=500



N=5000

NB: For N diverging fluctuations are suppressed. Therefore Stylized Facts correspond to finite size effects





N=1

M=10

b=5·10-4

K=0.05

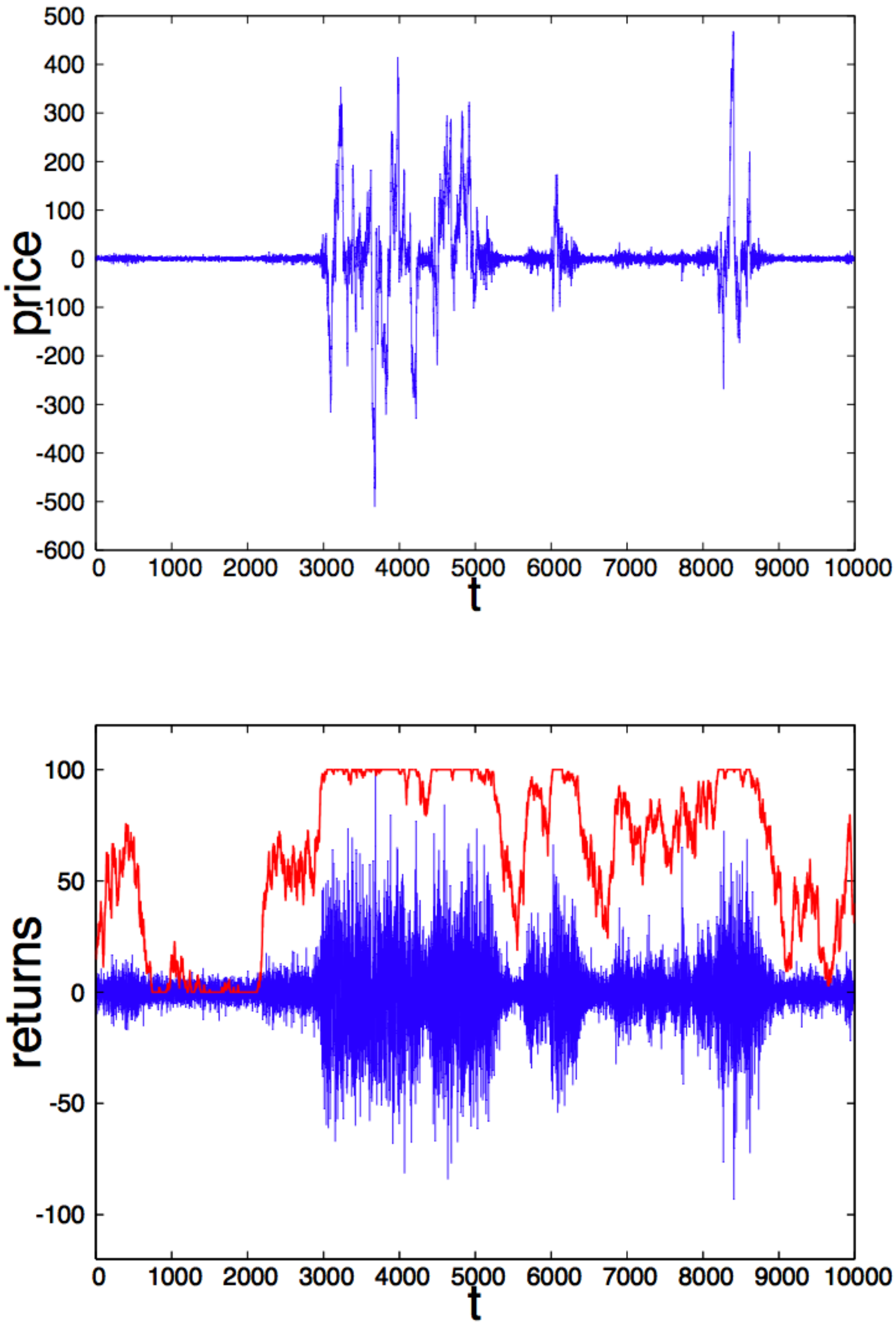
B=1

g=0.1

s=1

NB: even a single agent can show some intermittency

(Pf = 0)



N=100

N=100

M=10

b=1·10-3

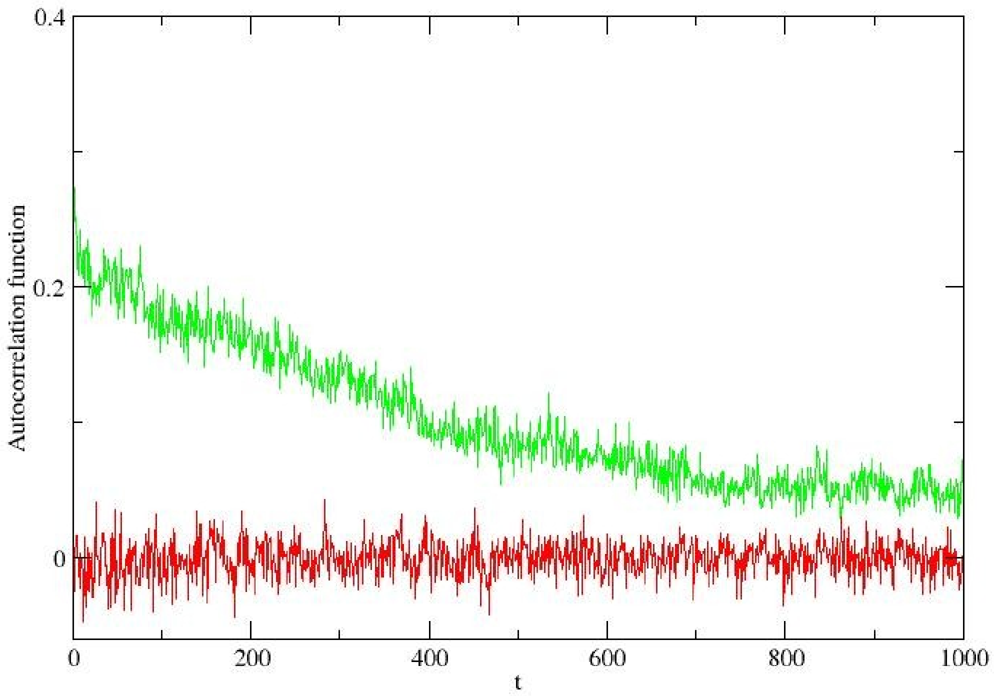
K=0.002

B=1

g=0.1

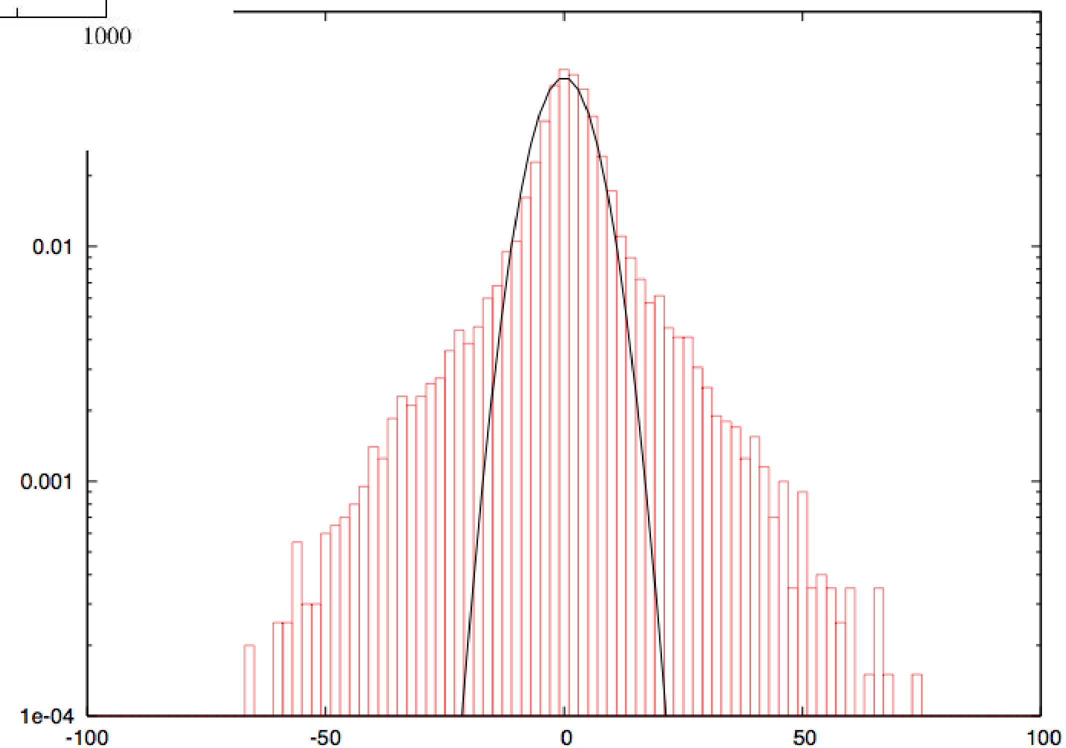
s=1

NB: All the parametrs are now in full control



Autocorrelation functions of returns and square returns

NB: SF arise from Finite Size Effects



Probability density function of price-returns

More Realistic Case

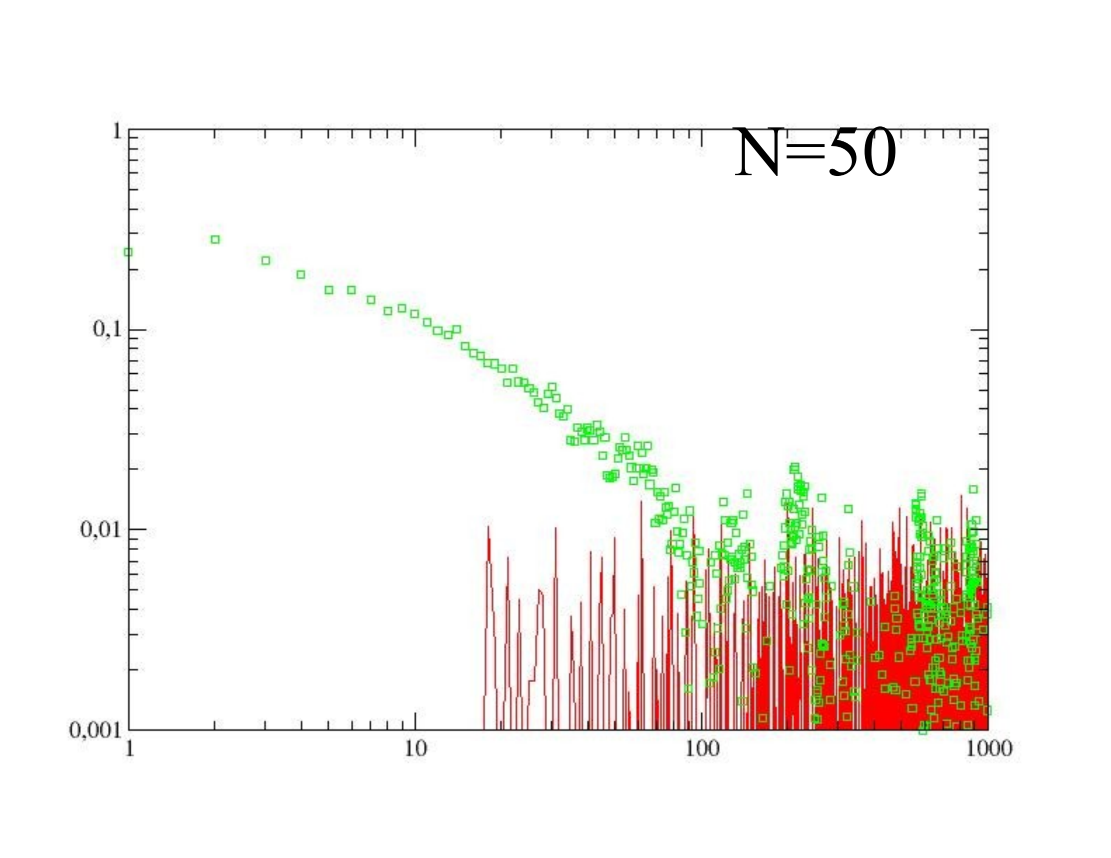
<u>Really Heterogeneous:</u>

<u>Agents adopt different timescales for their analysis</u>

<u>Possible origin of the “apparent” power law behavior</u>

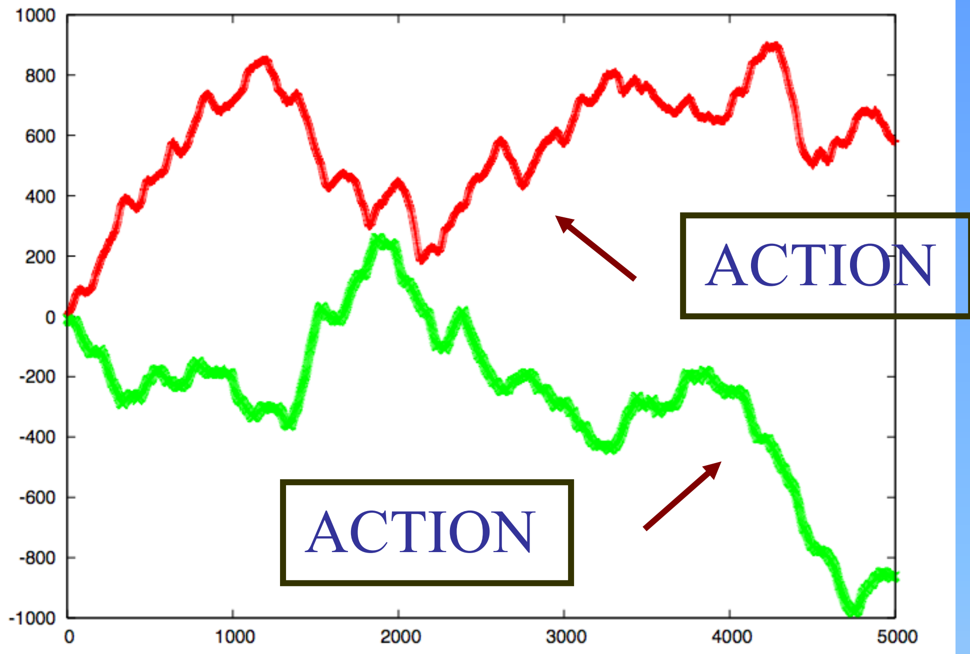
Transition probability with exponential

Moving average drawn from a uniform distribution between 10-50



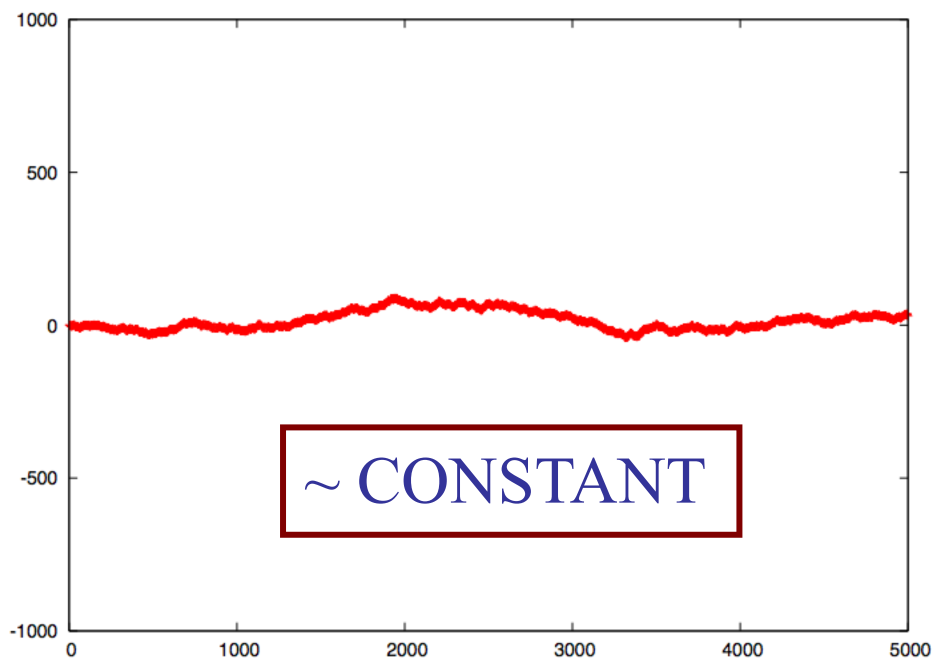
Basic question: what is really N or N*?

- In general the number of agent N is fixed in the Agent Models This idea originates probably from Stat Phys but it is rather unrealistic for trading (Nonstationarity)
- Even with N fixed traders may decide NOT to play or to play variable amounts of shares (volume)
- We can consider a sort of effective action N* which is strongly fluctuating in various ways but also N can actually vary (route to SOC)



N_C and N_F detect interesting signals and are stimulated to take an action

In this case N* increases



In this case little action is stimulated

N* drops

ACTION $\longrightarrow$ INCREASE OF N*

This resembles the GARCH phenomenology but
At a microscopic level

$$\sigma(t+1) = \mathrm{f}(\sigma(t); \Delta p(t))$$

Following our concept:

$$\text{action } \Delta p(t) \rightarrow \text{increase of } N^* \rightarrow$$

$$\text{increase of } \sigma \rightarrow \text{increase of } |\Delta p(t+1)|$$

Therefore there is a multiplicative nature of correlations which leads to a persistence in the value of σ (high or low).

↓

CONCEPTUAL FRAMEWORK FOR FAT TAILS ANDVOLATILITY CLUSTERING (NONSTATIONARITY)

MICROSCOPIC AGENT-LIKE INTERPRETATION OF GARCH PHENOMENOLOGY

Why no arbitrage ?

Any action ⟶ $\sigma(N^*)$ increases

but price trend is much more complex

$$\Delta p(t+1) = \mathrm{f}(N^*; N_c; N_F; p_M(t); p_F)$$

Therefore: much more information is crucial for the sign of the price return

Towards Self-organization

Asymmetric case: Basically Fundamentalists with bubbles due to Chartists

If the transition probabilities are symmetric the equilibrium distribution is bimodal or unimodal depending on the parameters

With asymmetric transition probabilities the scenario is richer

$$P_{CF} = (a_1 + b_1 \frac{N_F}{N})$$

$$P_{FC} = (a_2 + b_2 \frac{N_C}{N})$$

TENDENCY TO FUNDAMENTALISM

$$P_{CF} = B(1+\delta)(K + \frac{N_F}{N})$$

$$K = \frac{1}{3N}$$

bimodal region

$$P_{FC} = B(1-\delta)(K + \frac{N_C}{N})$$

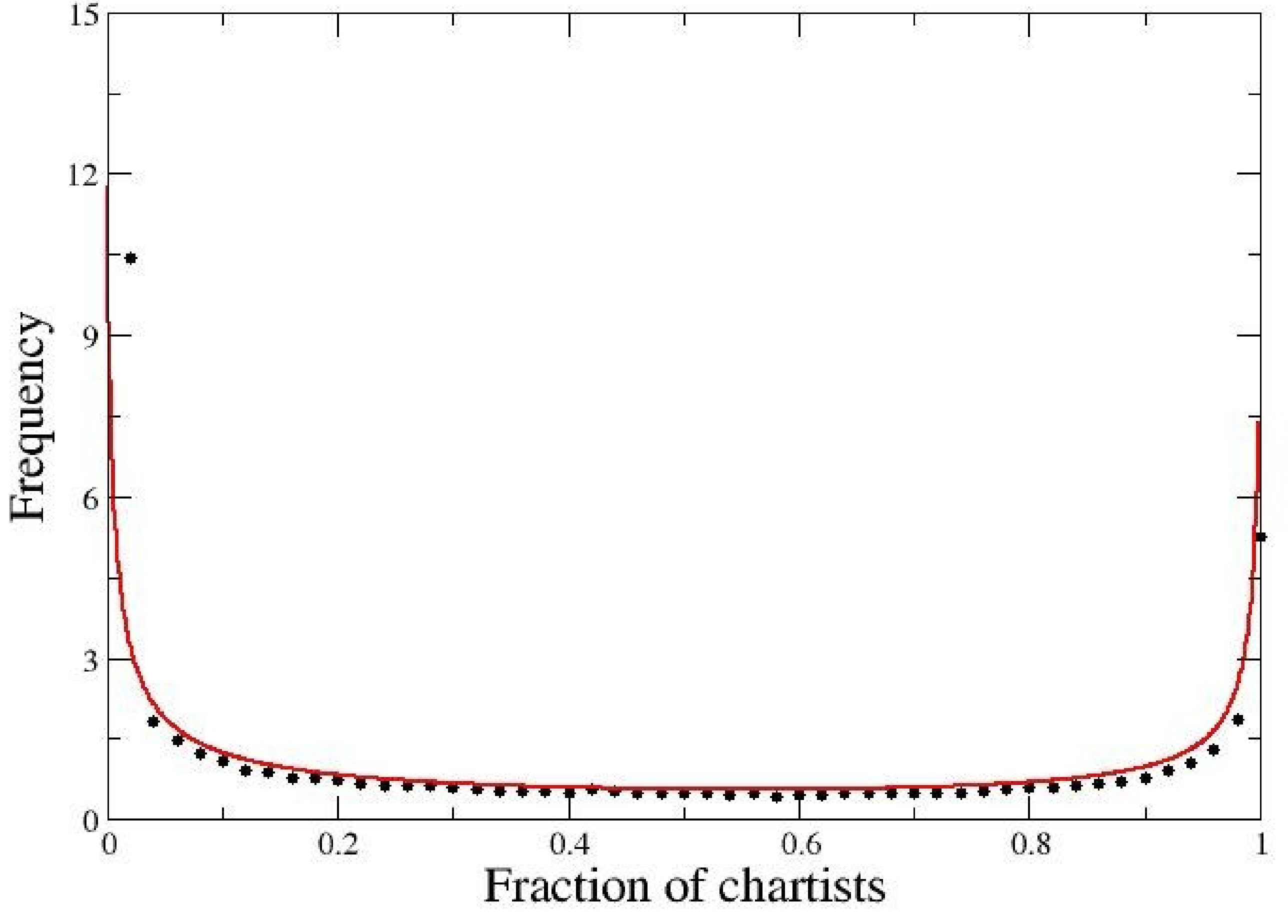
$$P_{eq} \sim \nu^{r(1-\delta)-1}(1-\nu)^{r(1+\delta)-1} \exp\left(-\delta \nu N\right)$$

$$\nu = \frac{N_C}{N}$$

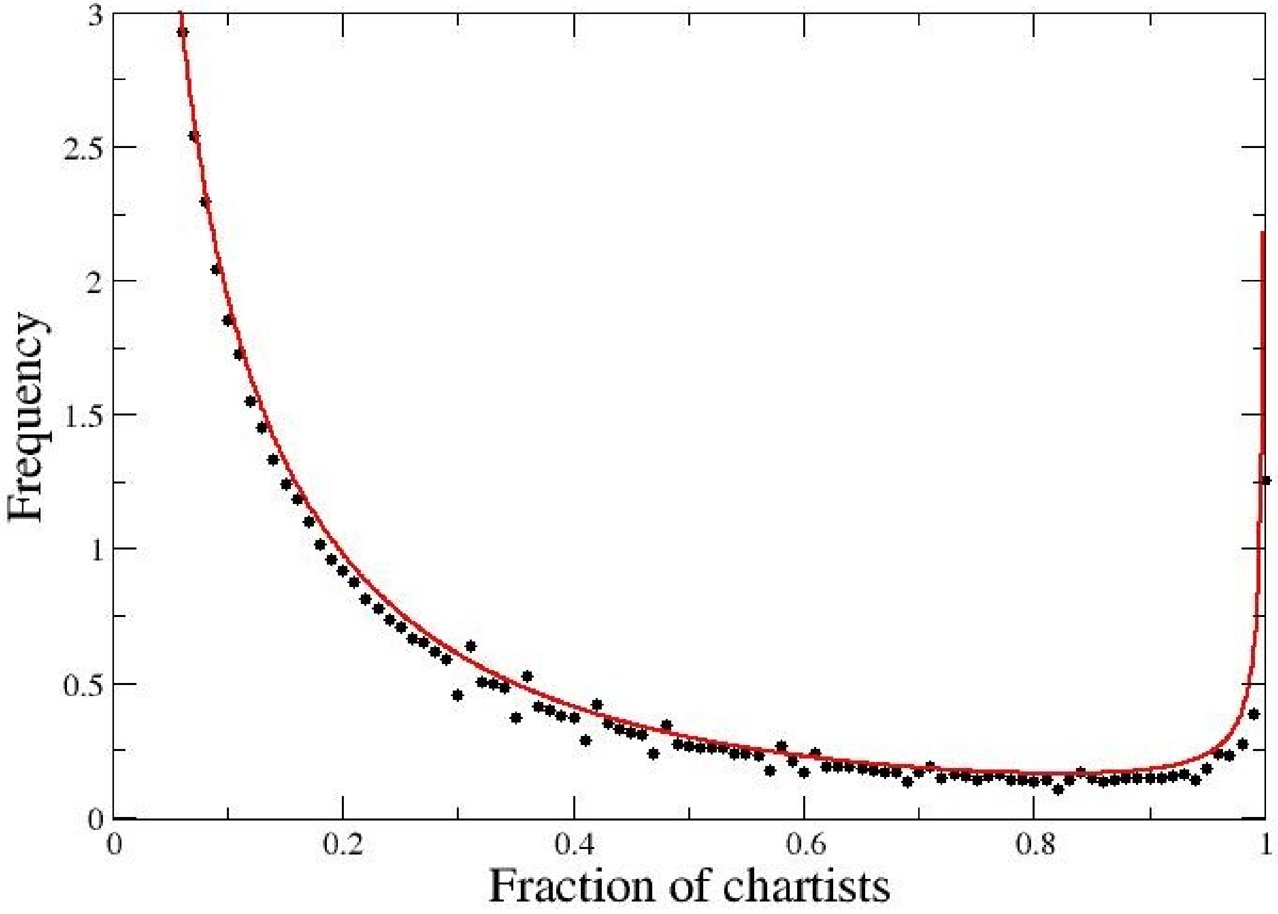
relative number of chartists

For large value of N chartists are suppressed

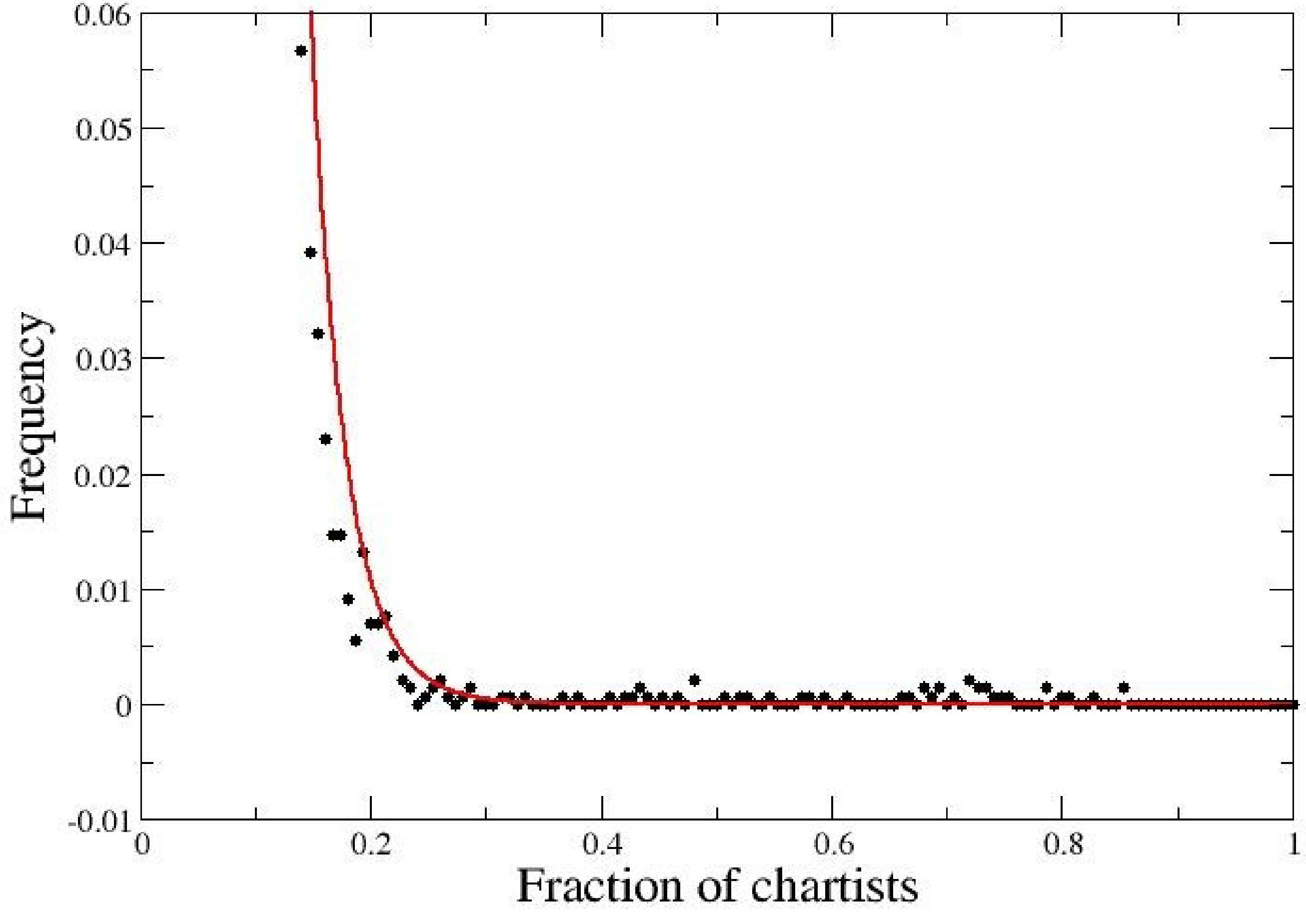
N=50



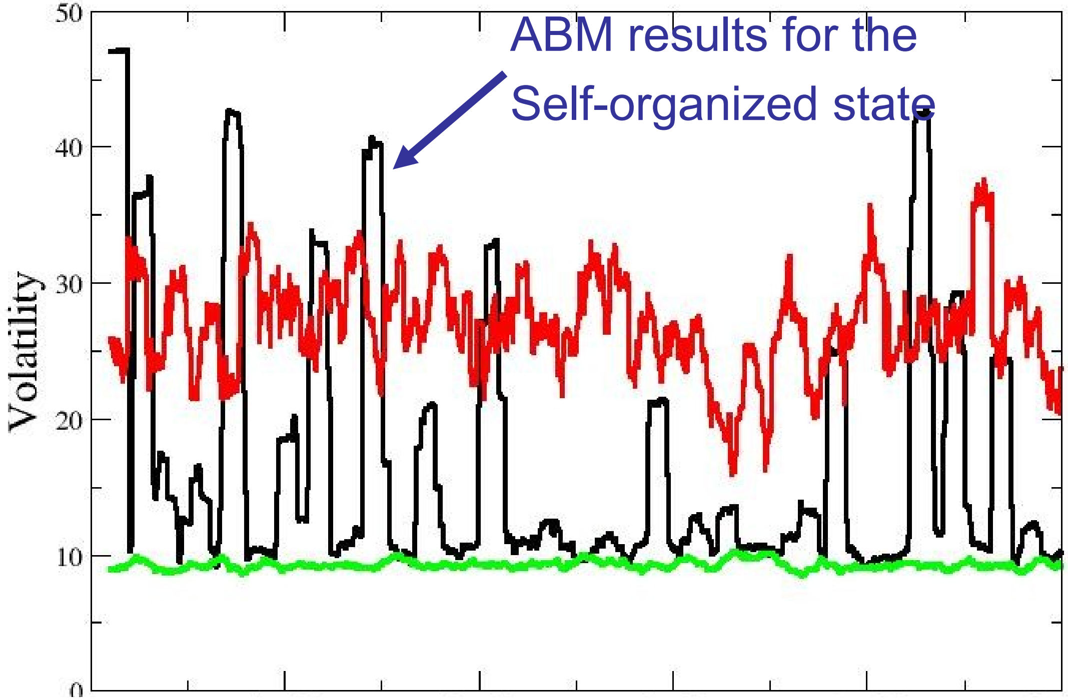
N=500

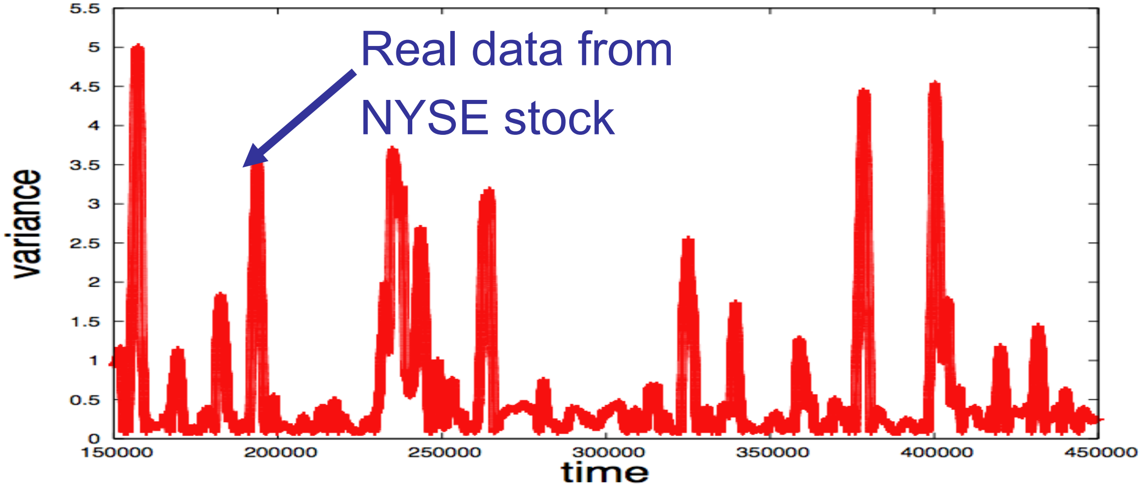


N=5000

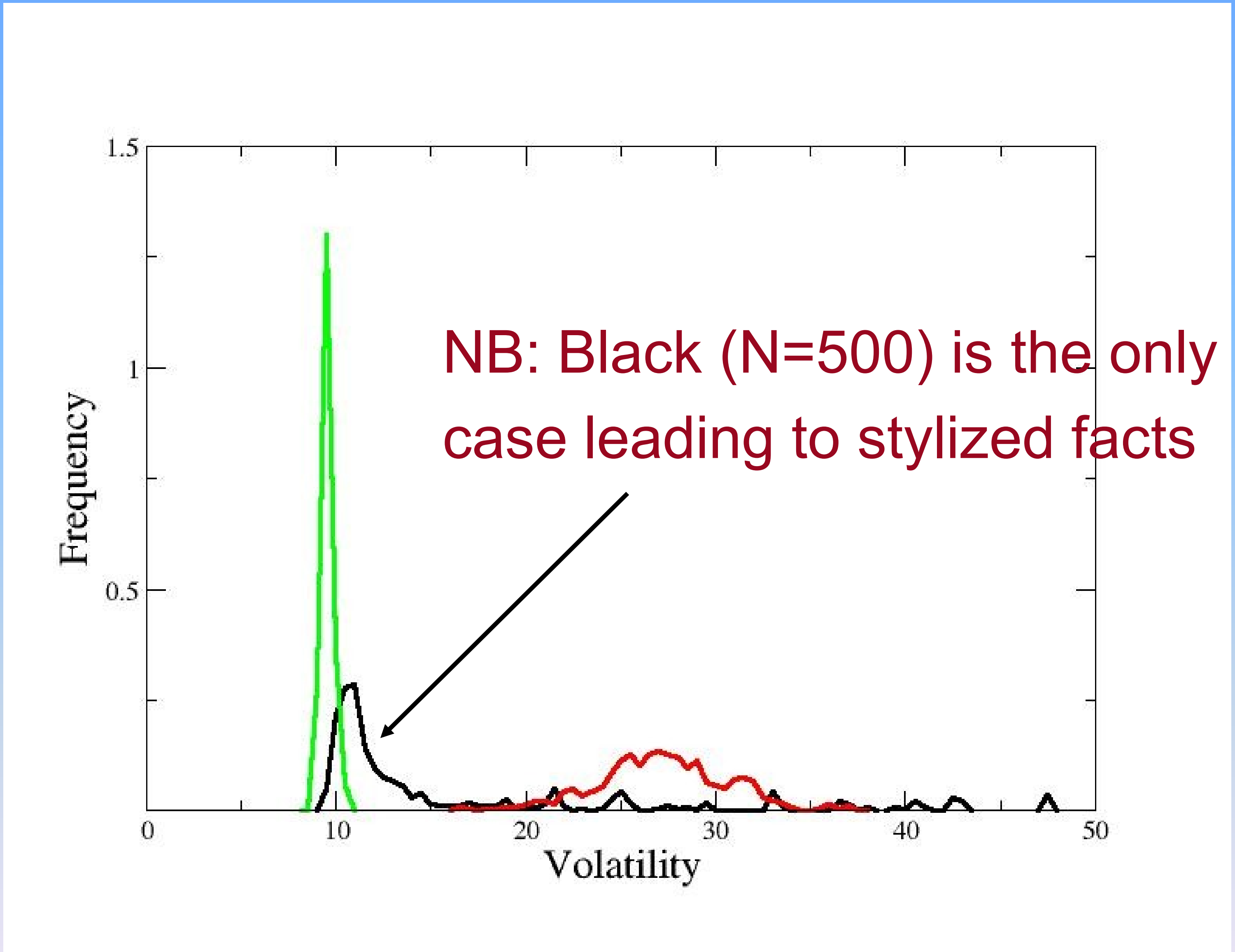

Frequency
Fraction of chartists
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
0
0.2
0.4
0.6
0.8
1

(red N=50; black N=500; green N=5000)





(red N=50; black N=500; green N=5000)



Basic criterion for Self-Organization:

- Agents decide whether trading (or not) depending on the price movements they observe

- Stable prices: Less trading
- Large Action (price movements): More trading

Caution: some agents may prefer a stable market and be scared by fluctuations. This would require an analysis of different time scales and, in any case, these agents certainly do not produce the Stylized Facts

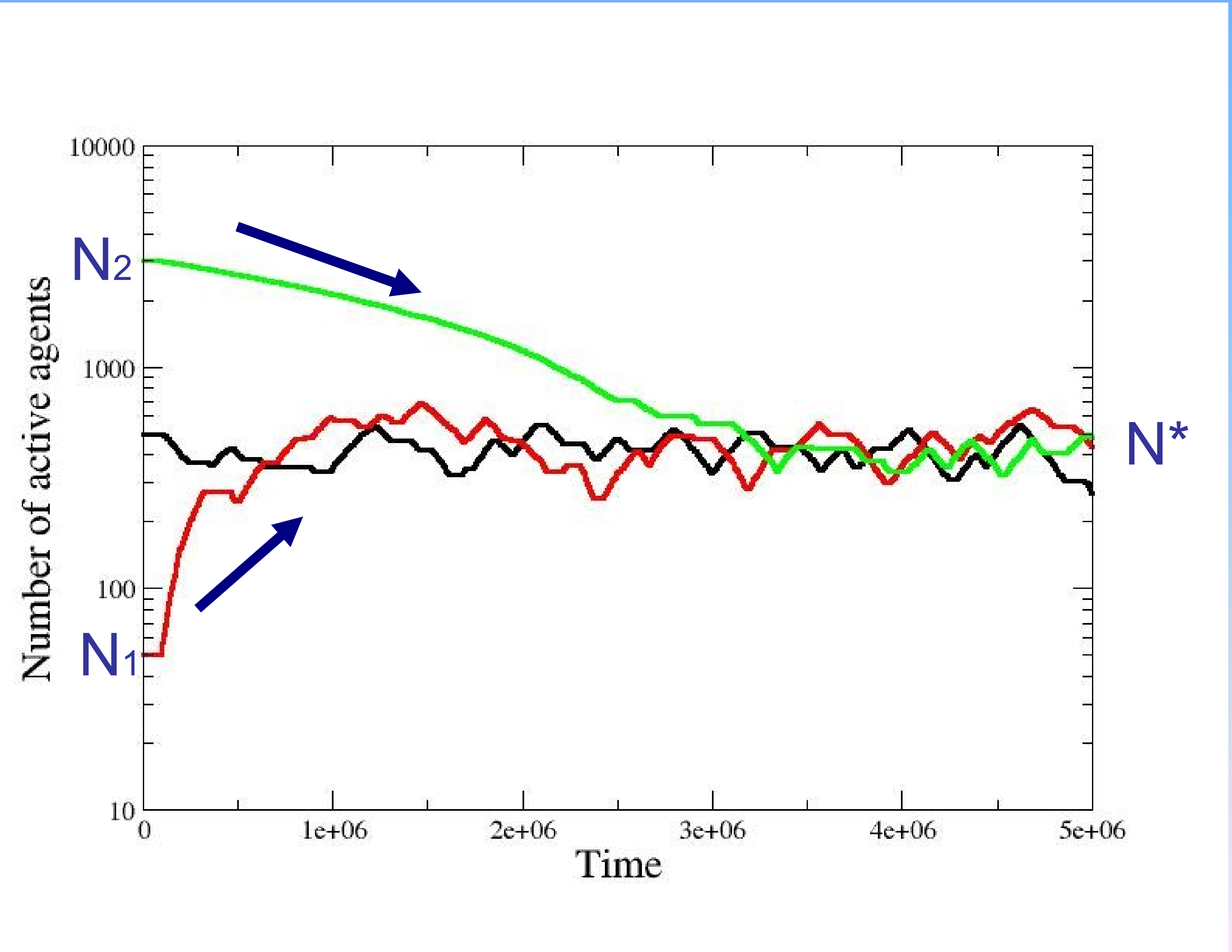
Each agent calculate the price-volatility on the previous T steps

$$\sigma(t,T) = \frac{1}{T-1}\sum_{i=t}^{t-T}(p_i - \bar{p})^2$$

On the basis of the calculated volatility each agent has a probability to enter/leave the market if the volatility is above/under a certain threshold

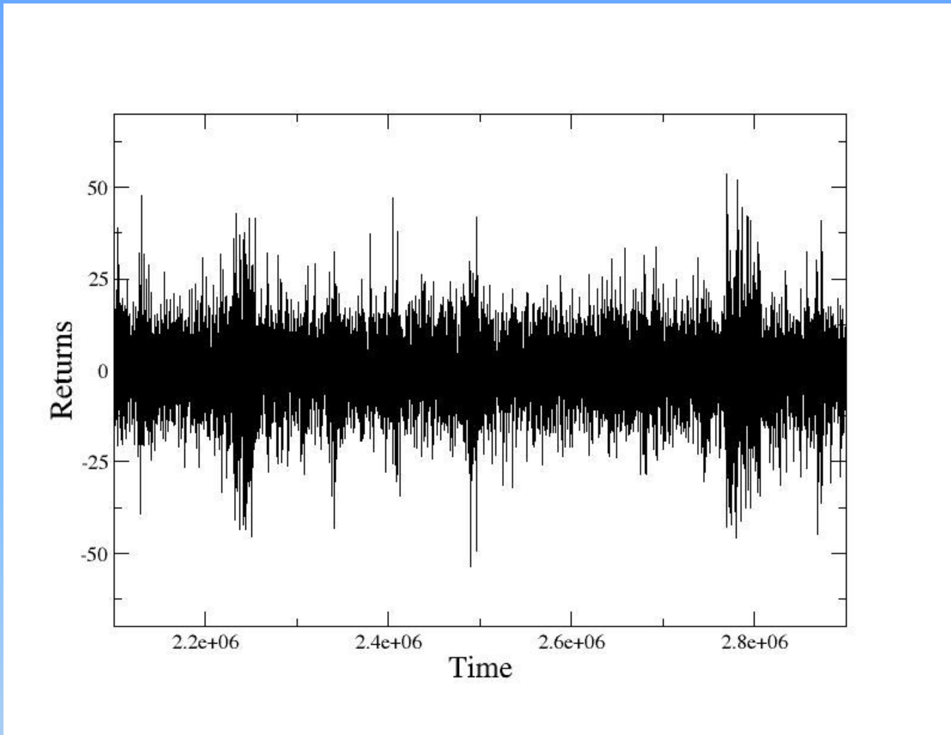
$$\begin{aligned} \sigma(t,T) &> \Theta_{in} \\ \sigma(t,T) &< \Theta_{out} \end{aligned}$$

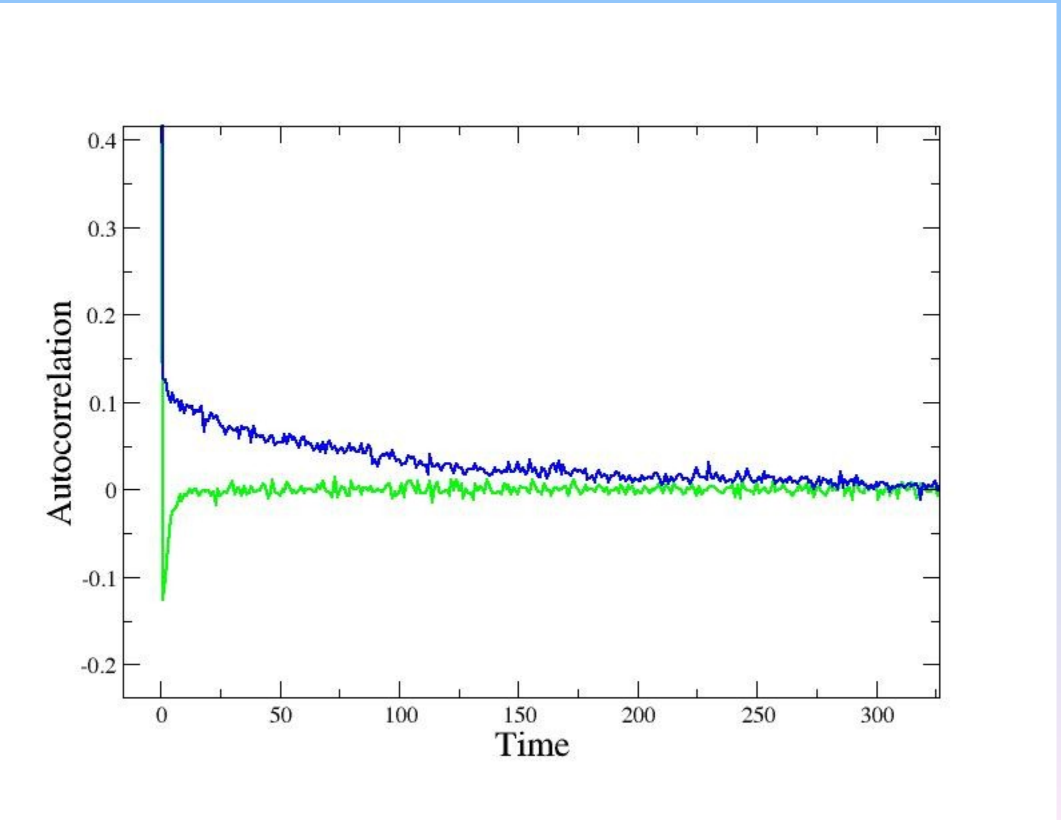
Self-Organization in action: Different starting N (50, 500, 3000) evolve and finally converge to the Quasi-critical state (N=500) which corresponds to the Stylized Facts



Stylized Facts at Convergence

returns

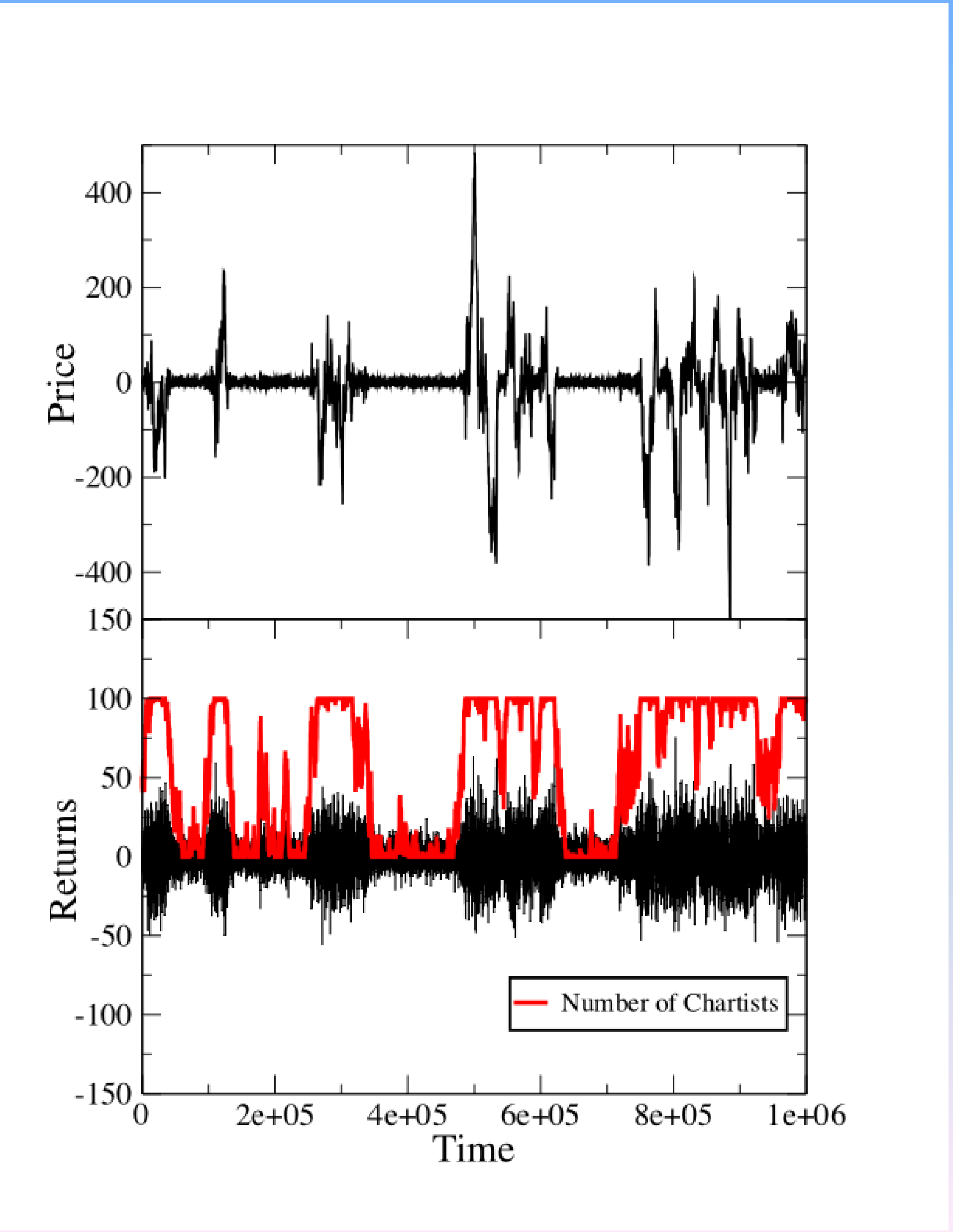




Autocorrelations of returns and square-returns

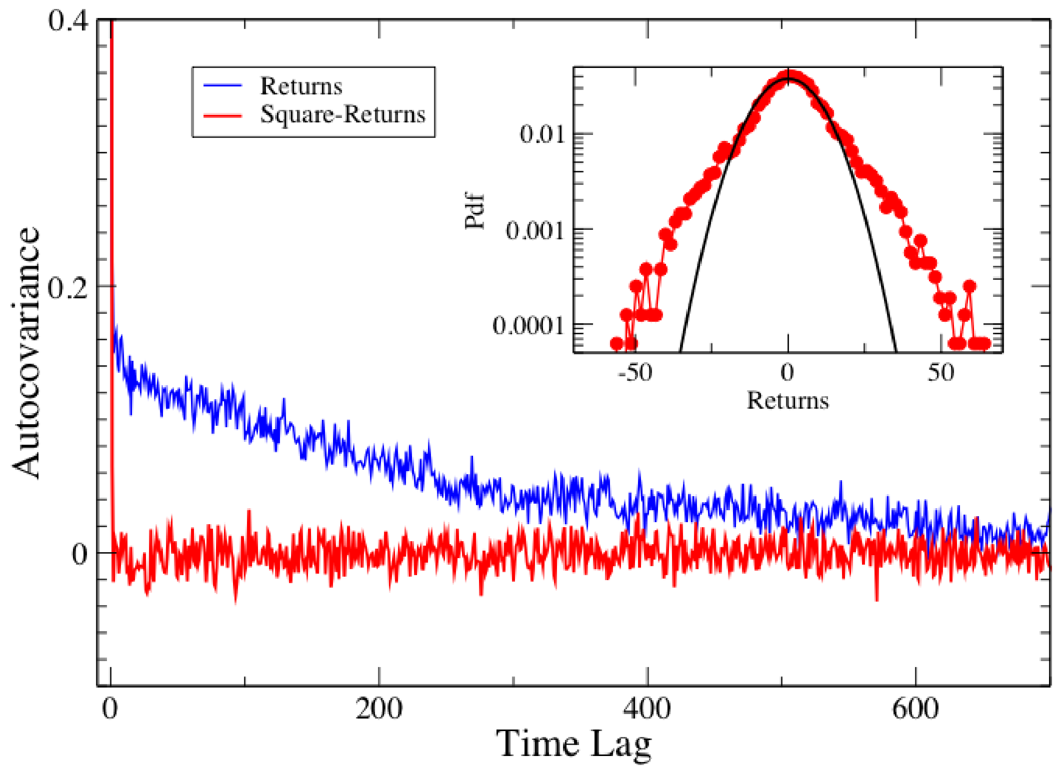
Into the details about what gives what:

Case N = 100 and no price effect



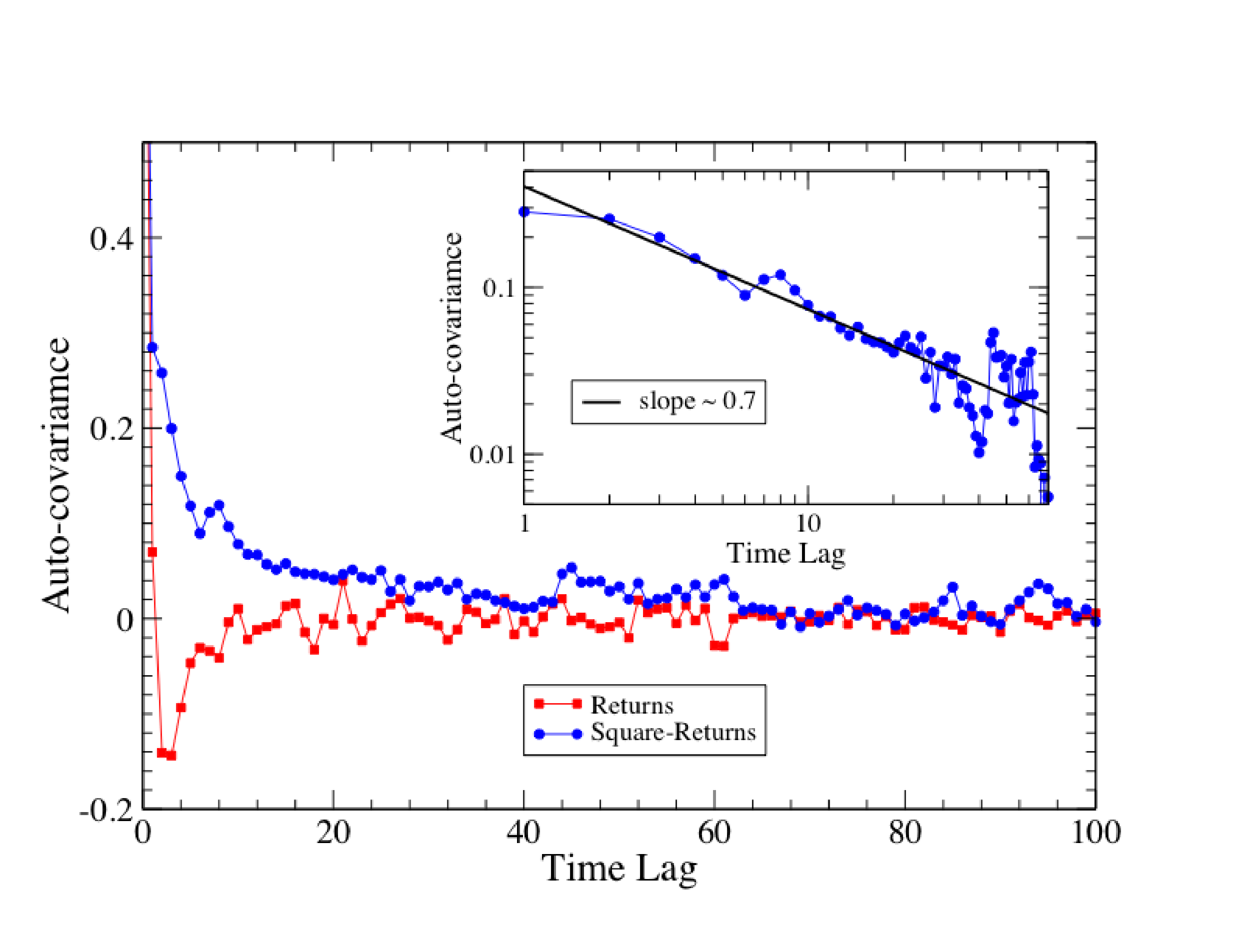
Paper1 fig.6 a

Linear dynamics; N = 100; no price effect



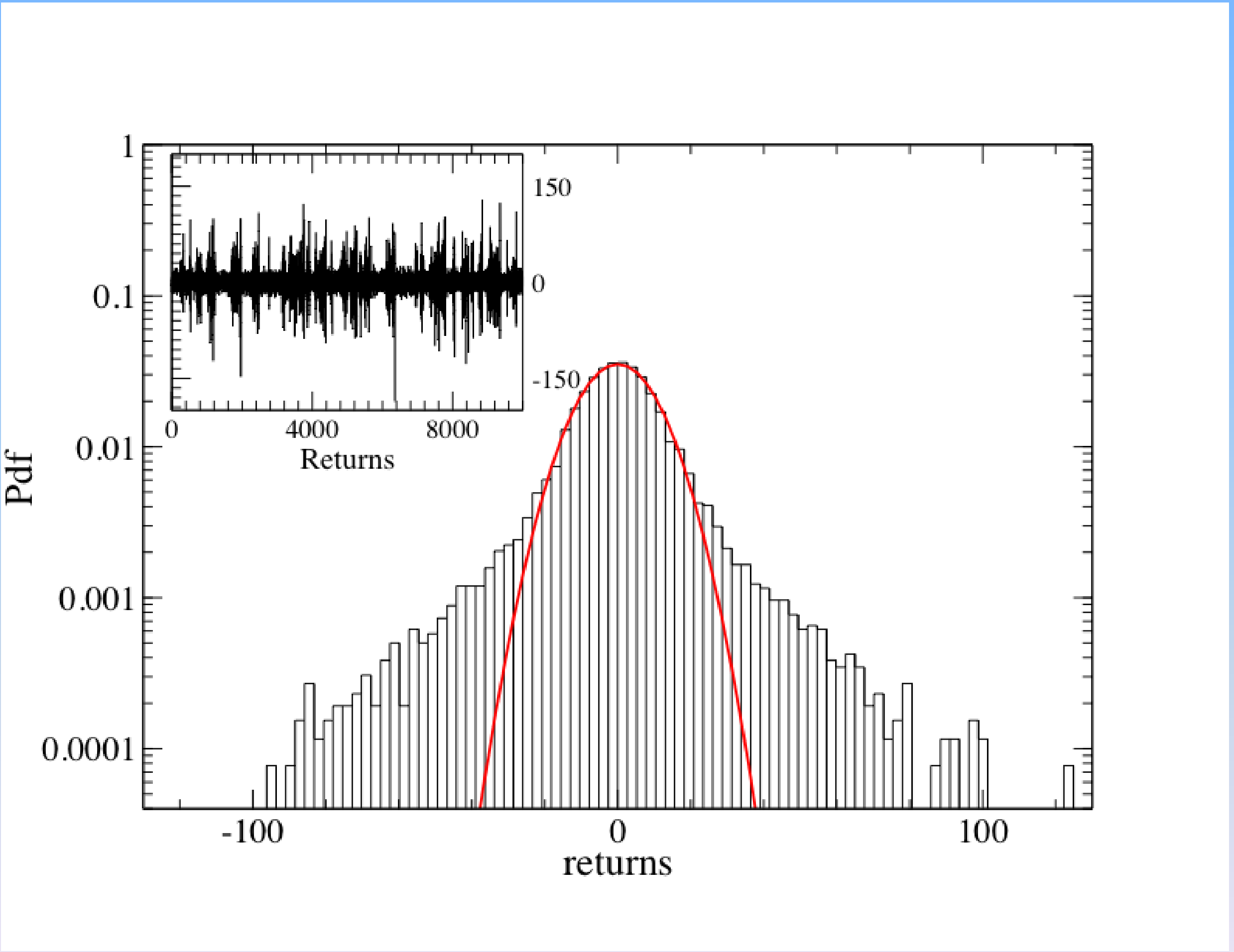
Paper1 fig.6 b

Linear dynamics; N = 500; Price effect included; Hetherogeneity with respect to thir time horizon Volatility clustering is decreased because the behavior is less coherent



Paper1 fig. 7 a

…. but Fat Tails are much stronger



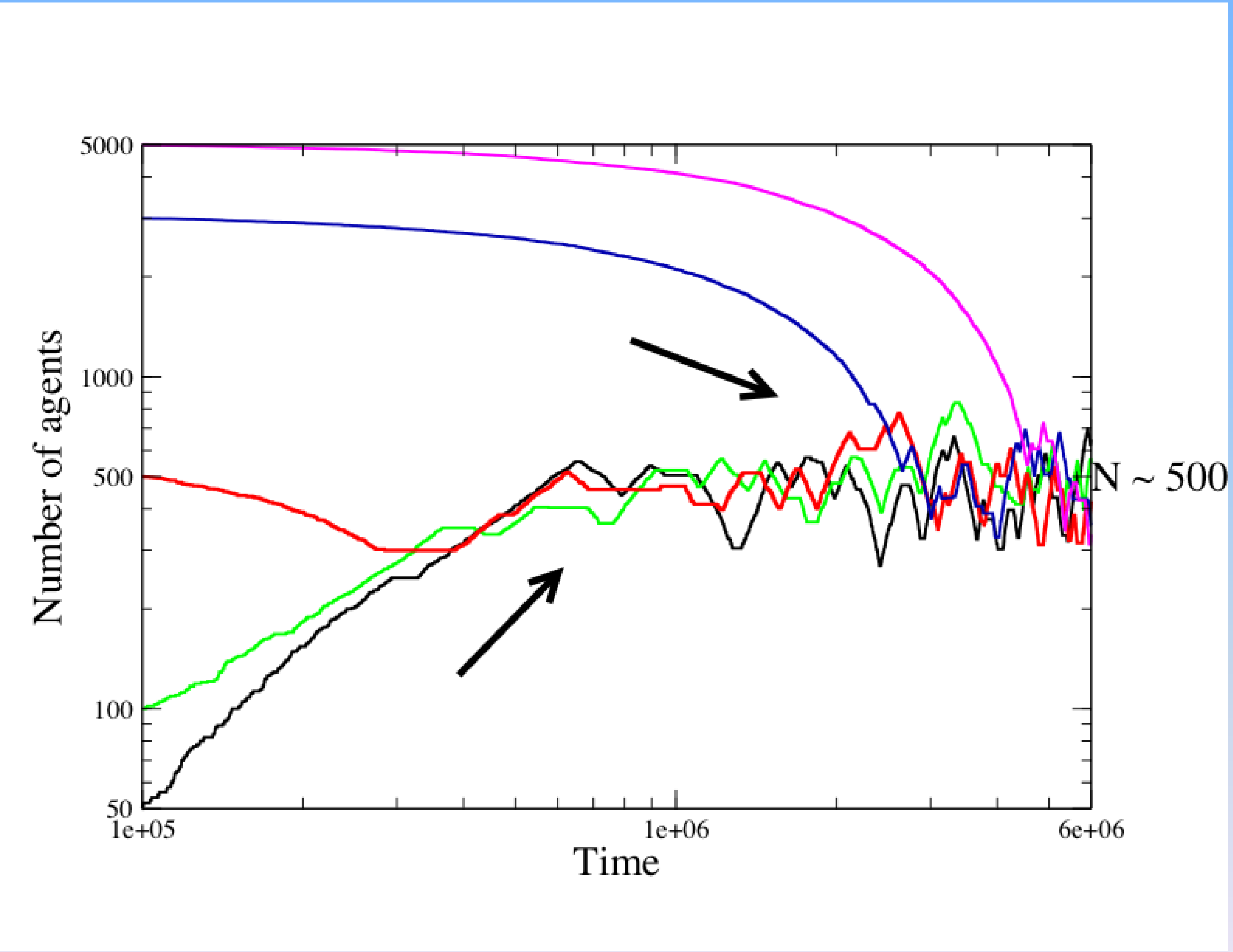
Paper1 fig.7 b

Self-Organized-Intermittency (SOI)

Convergency is faster from small N

Not really critical in the standard sense

But there may be more timescales

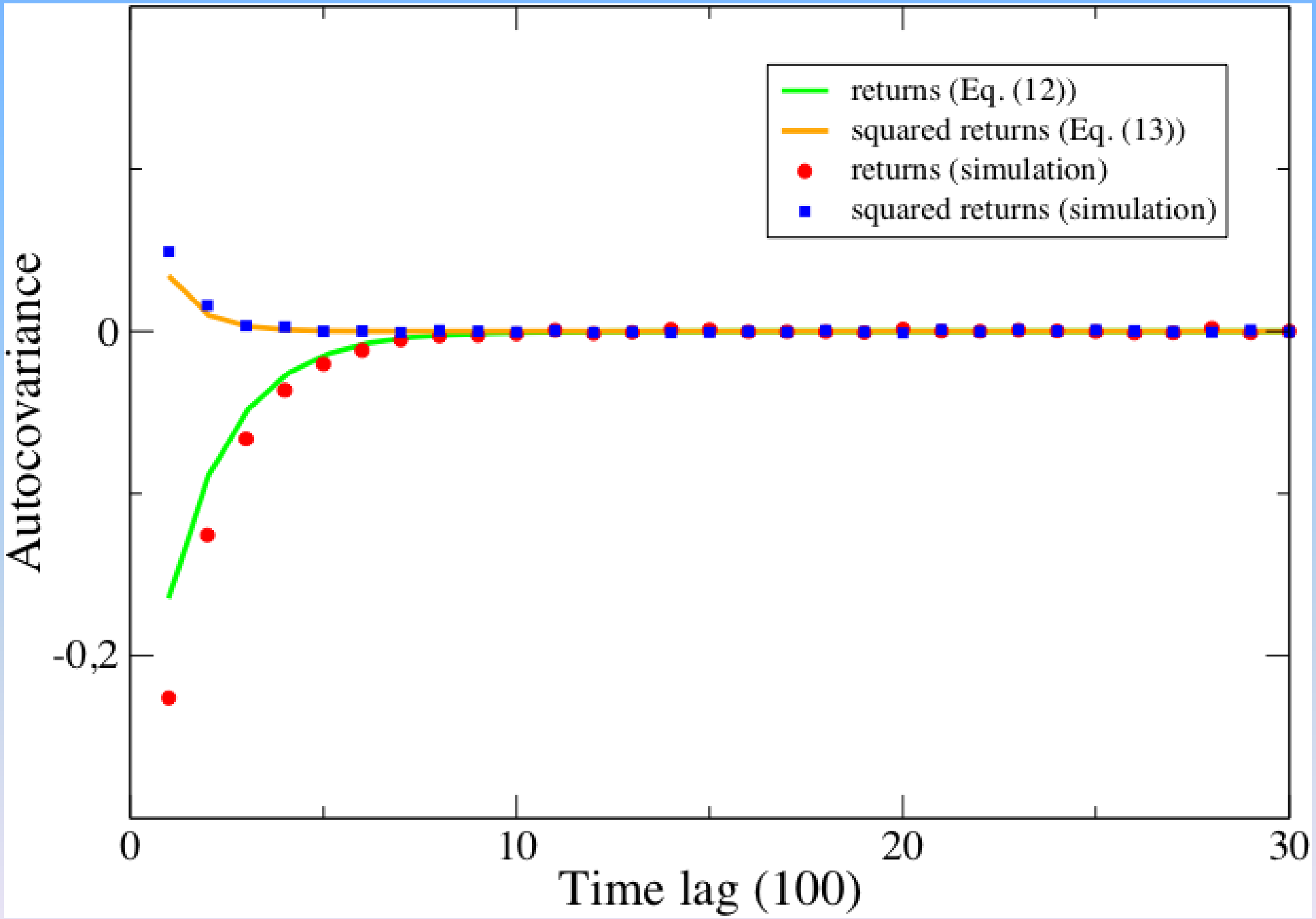


Paper1 fig. 8

Limiting cases:

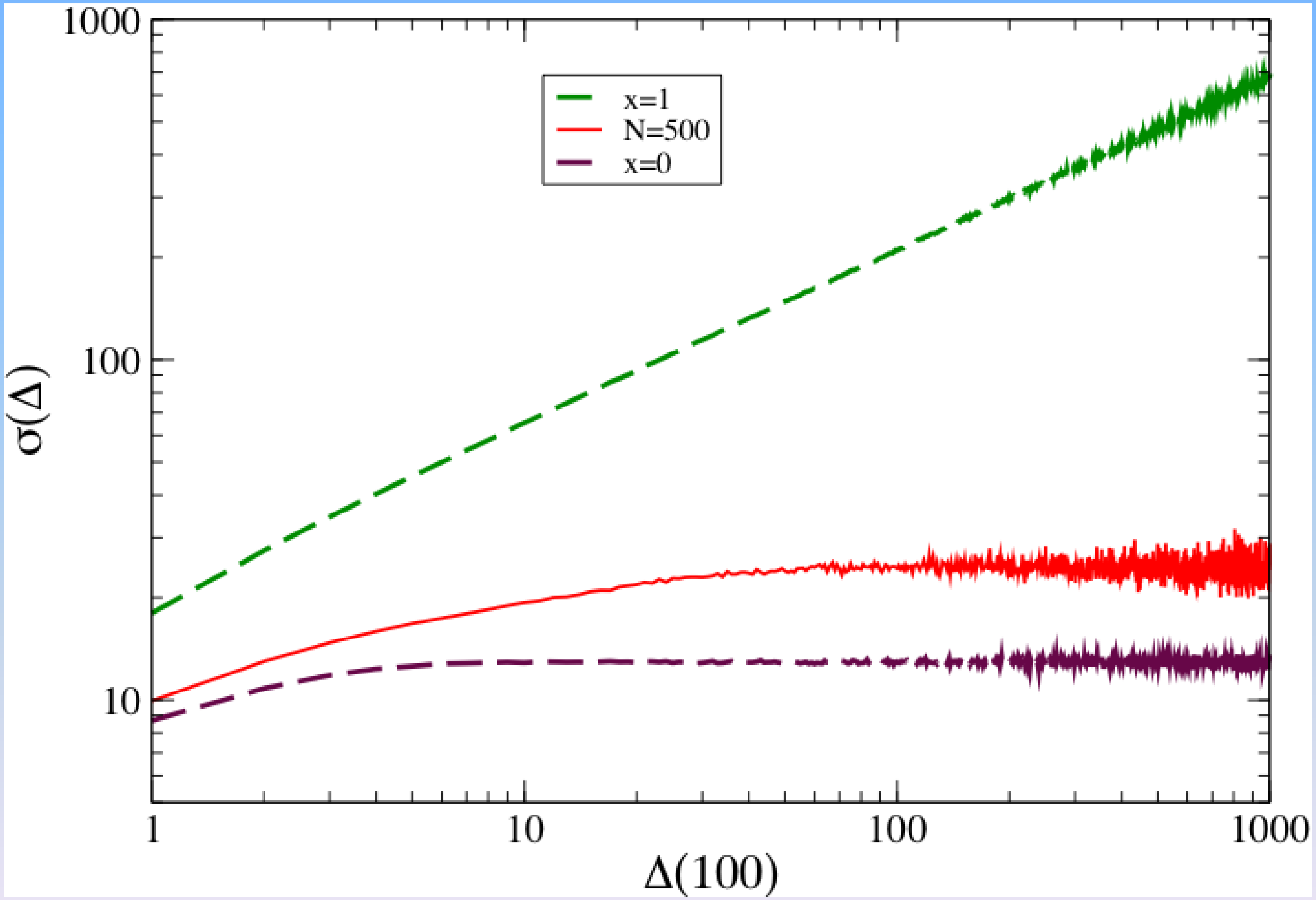
Only chartists and only Fundamentalists

Volatility Clustering disappears for both limits



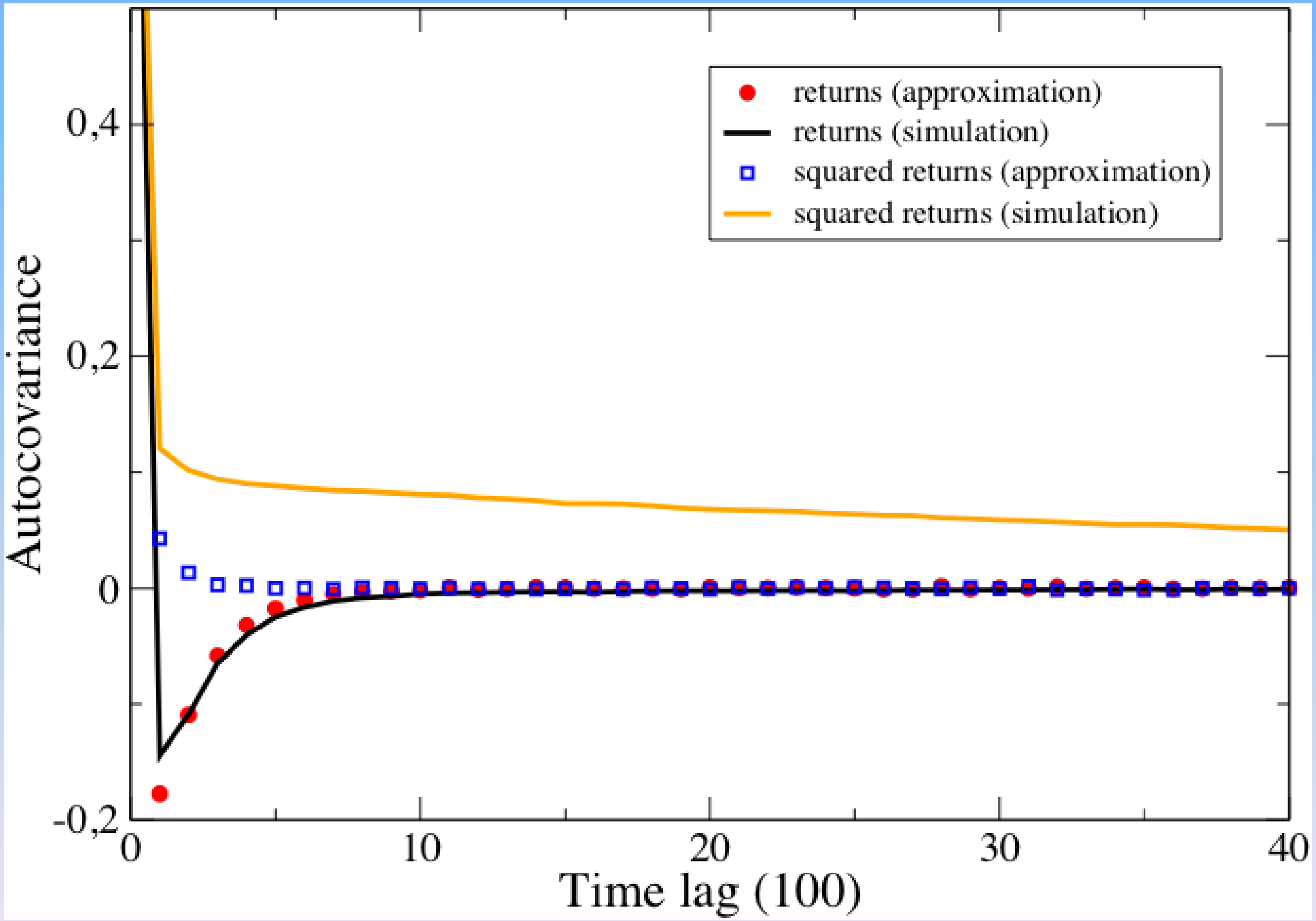
Paper2 fig. 1

Diffusion properties for the two limiting cases and for the mixed one (red)



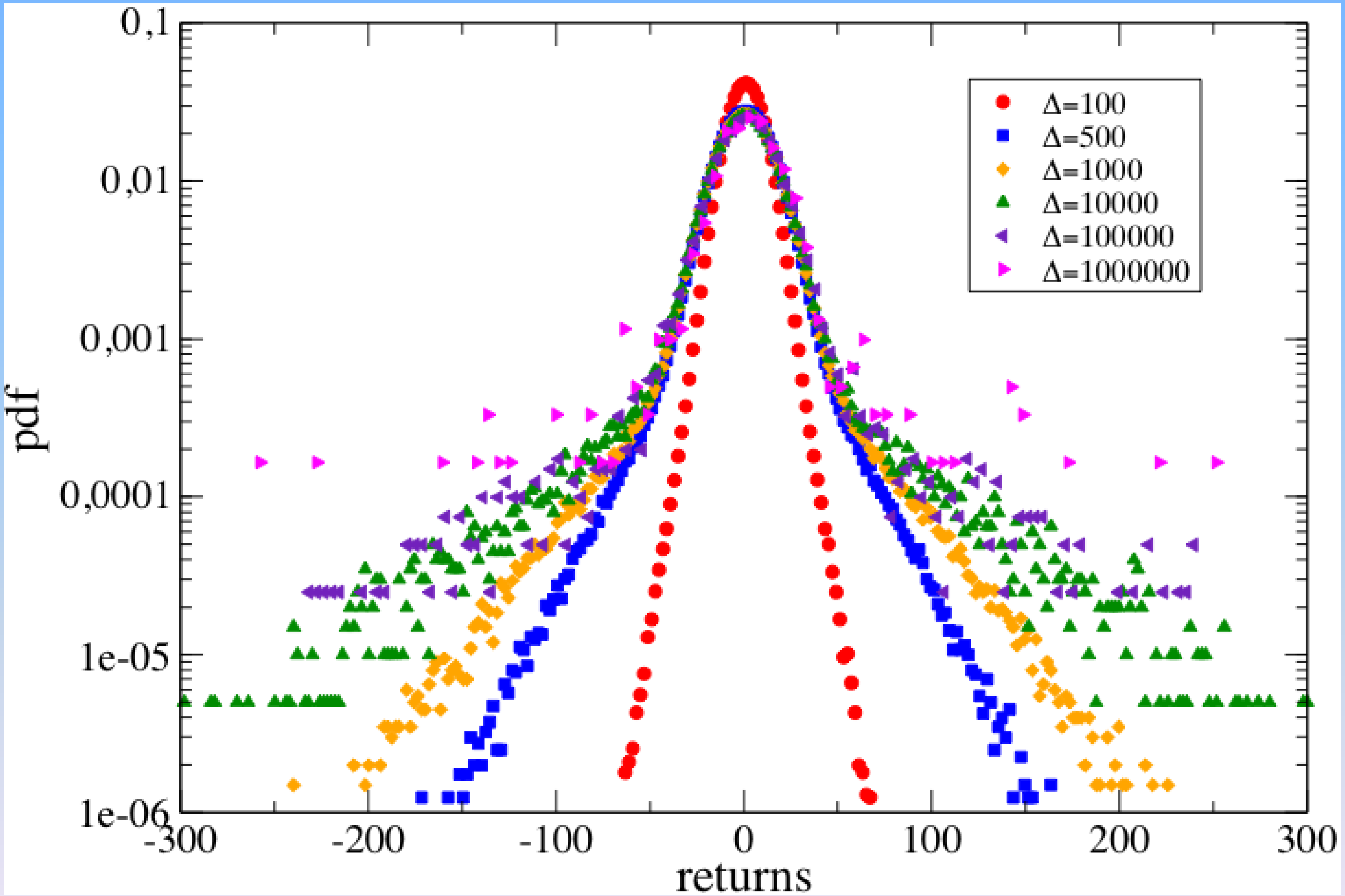
Paper2 fig. 6

Simple approximation: superposition of only C and only F limiting cases. Volatility Clustering is not reproduced. Population dynamics is important



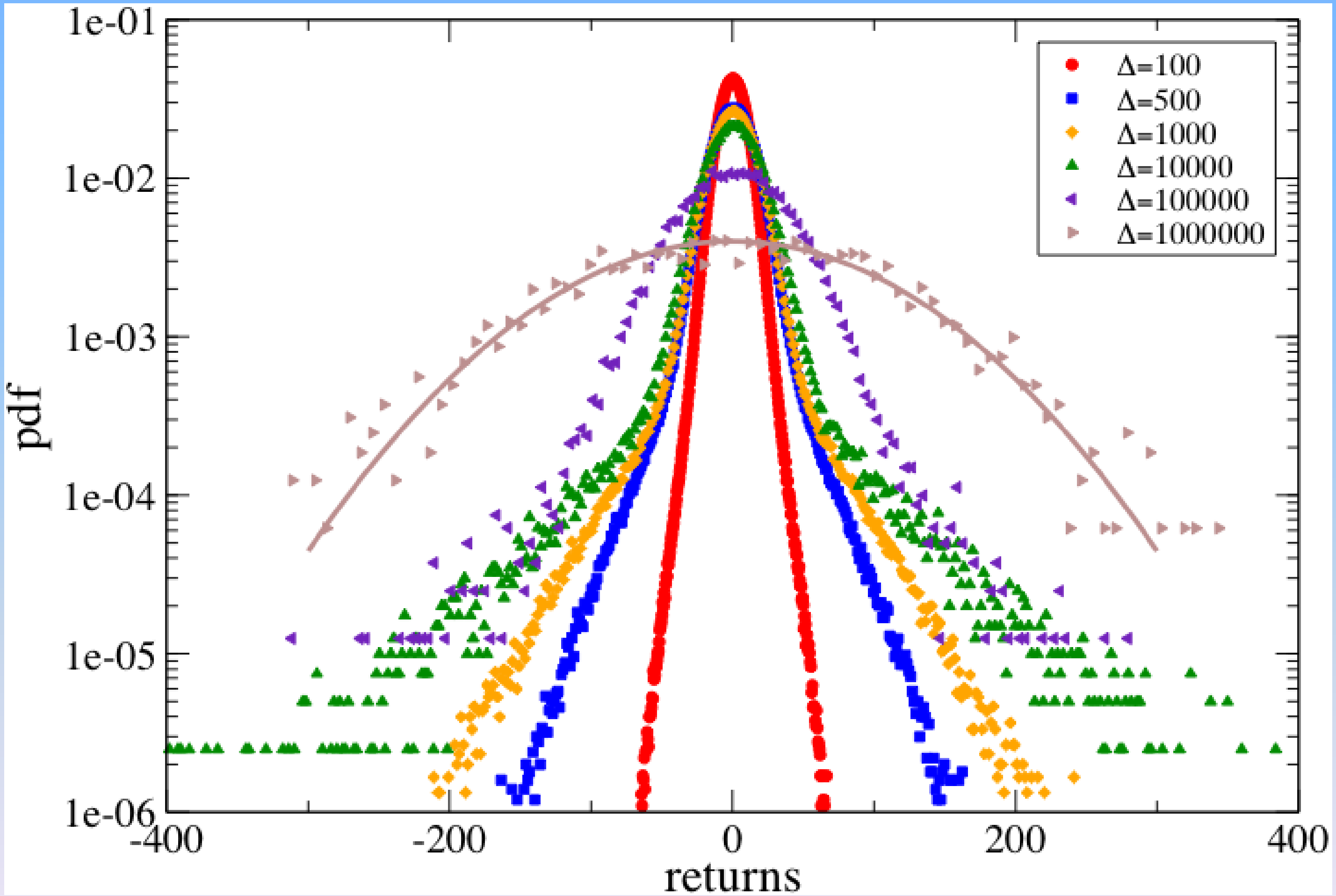
Paper2 fig. 11

Prob. density funct. for different time intervals: no transition to gaussianity. BUT in this model Pf = const



Paper2 fig.7

Adding a Random Walk for Pf(t)
At large times transition to gaussainity

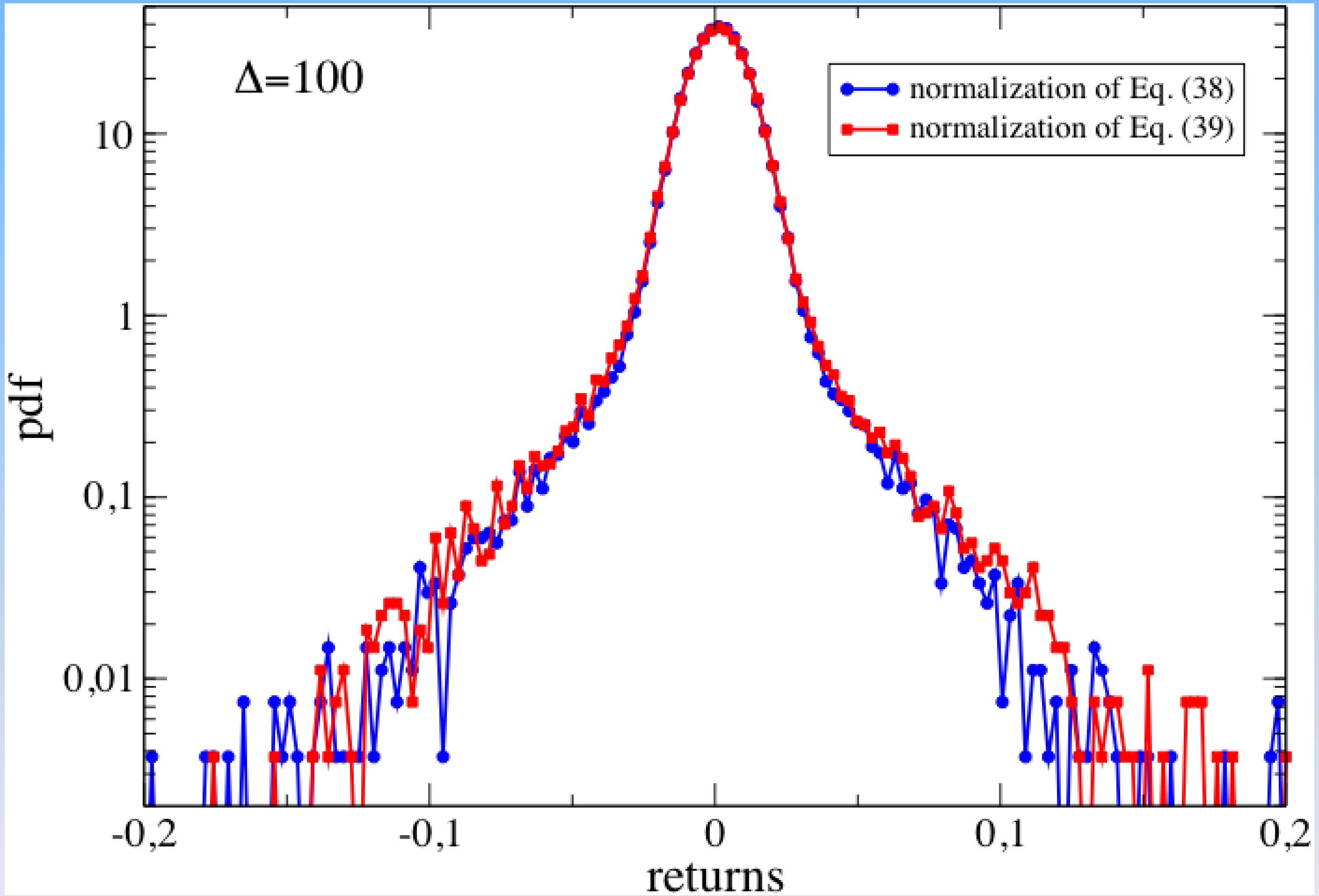


Paper2 fig.13

Multipicative or log dynamics

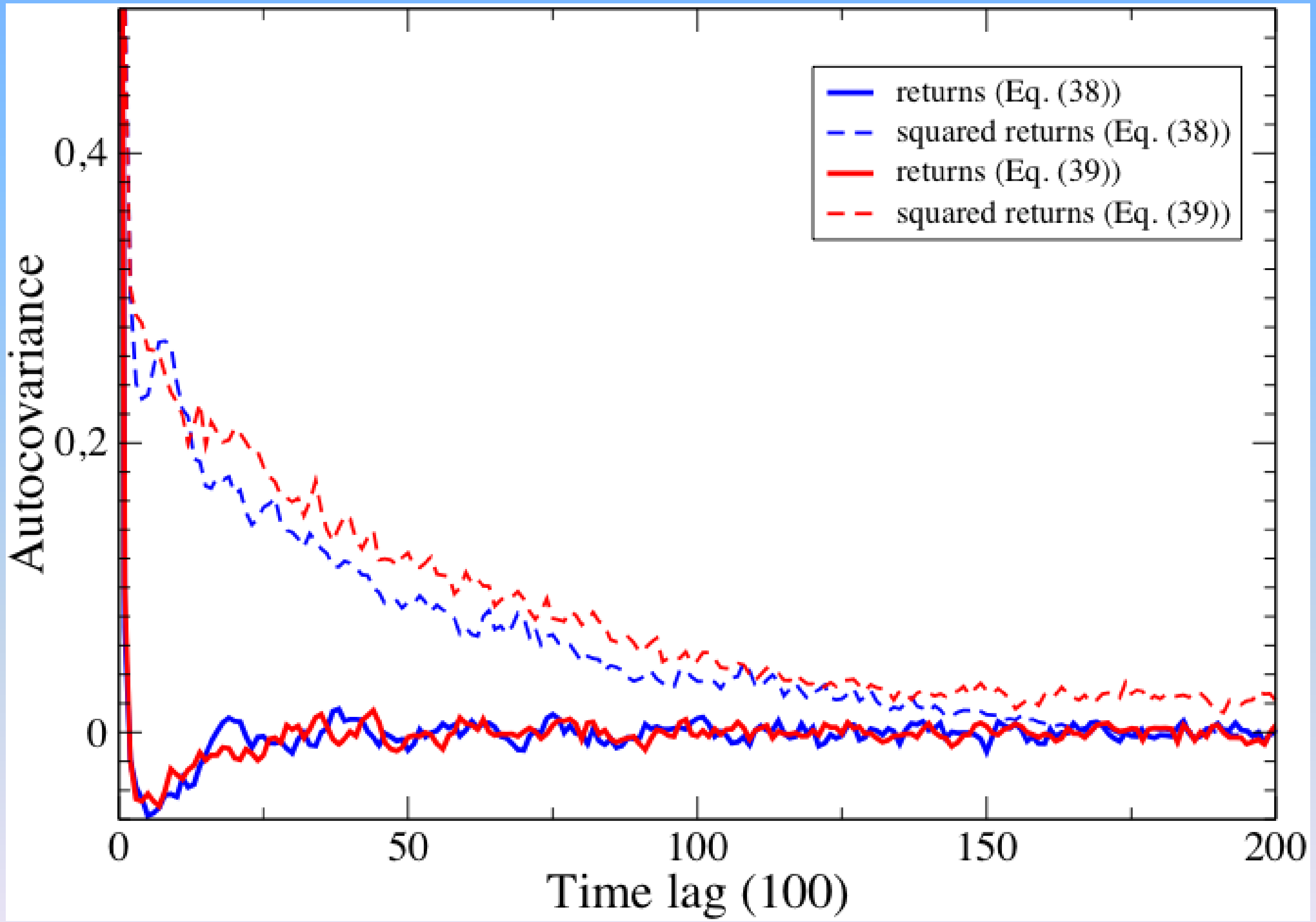
- Less stable for simulations
- Analytical results become much harder
- Additional subtle differencies

Fat tails for the multiplicative dynamics



Paper2 fig. 15 a

Multiplicative dynamics: autocorelations



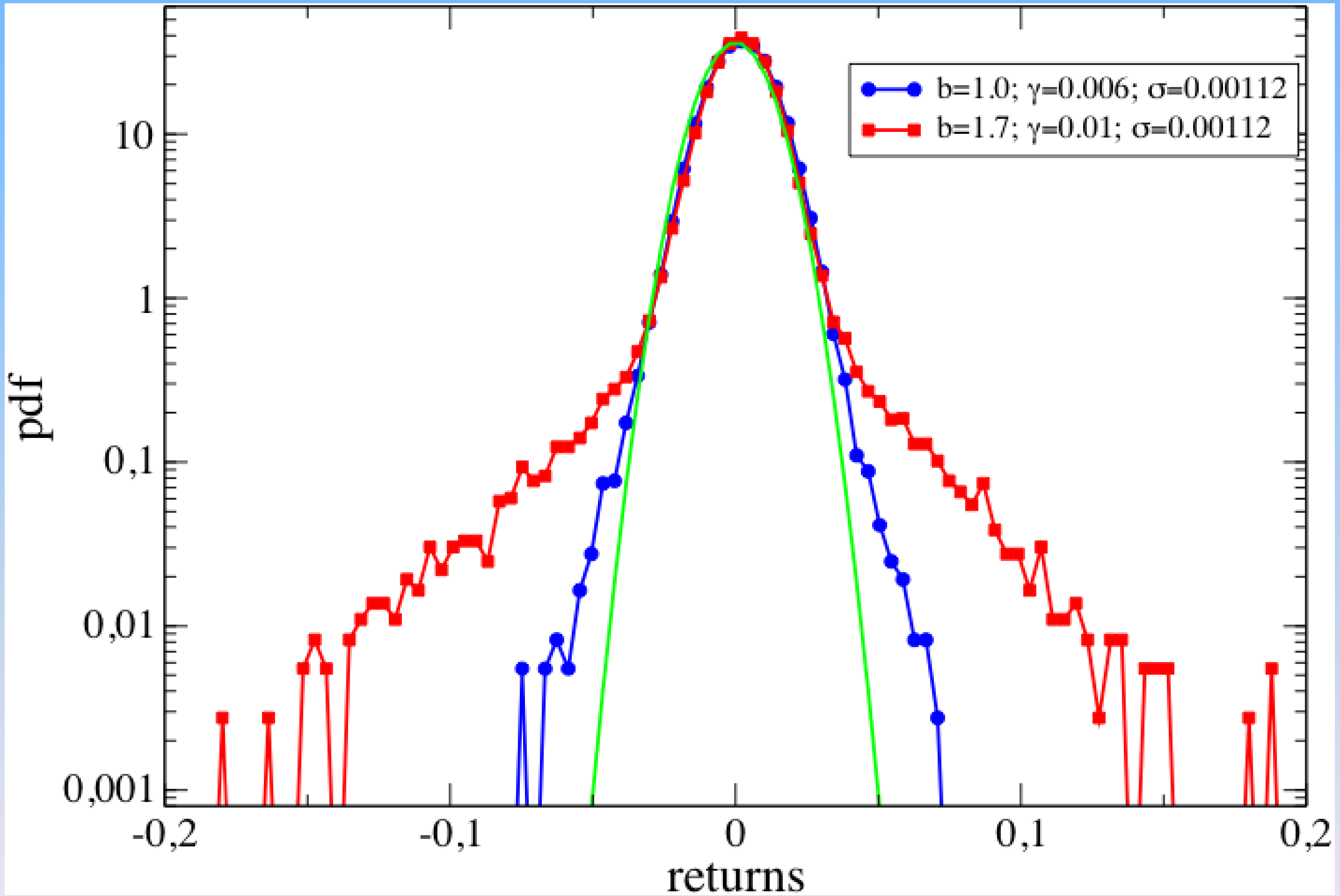
Paper2 fig. 15 b

Multiplicative dynamics:

Extreme sensitivity to parameter region

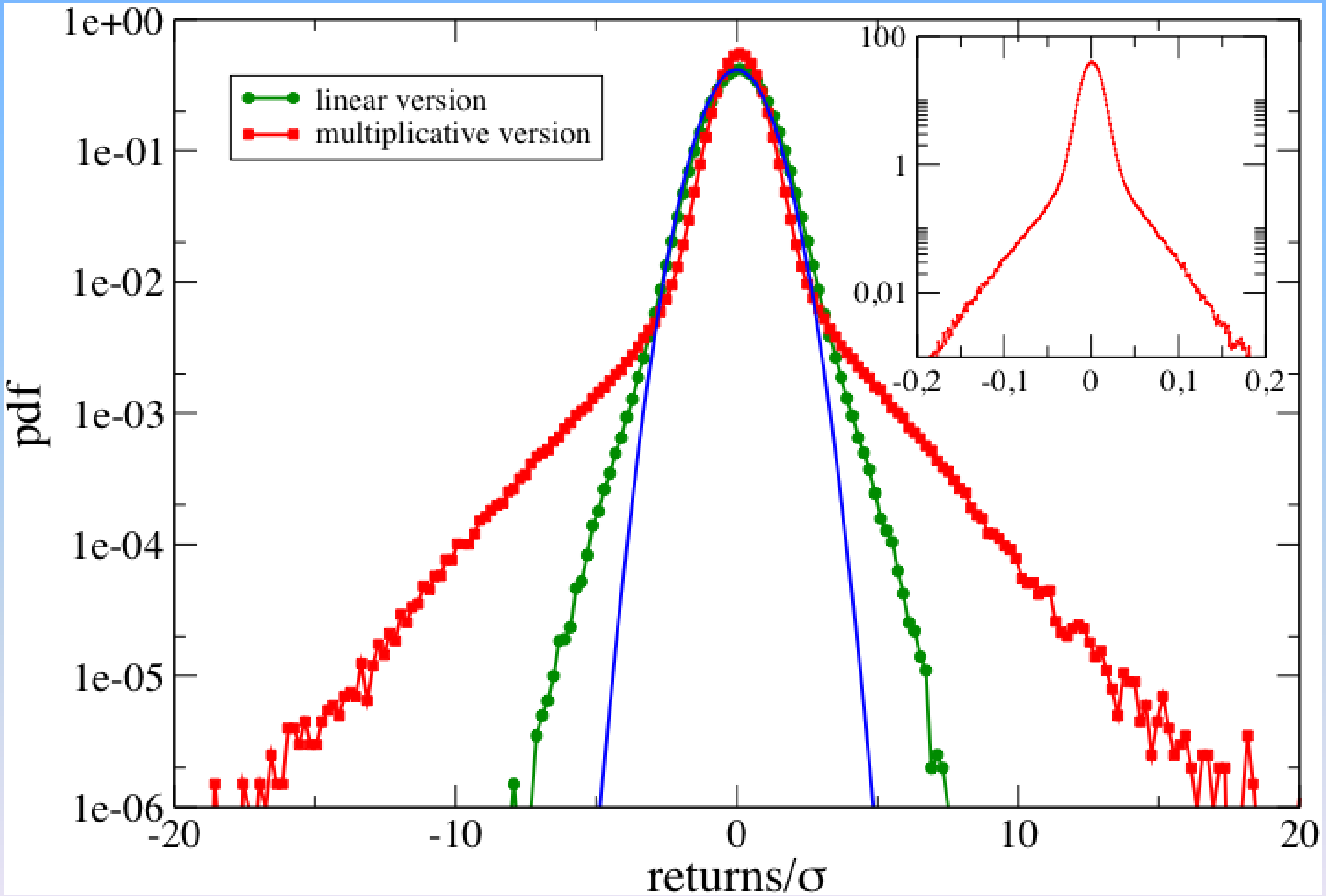
Slightly different papramenters lead to

very different Fat Tails



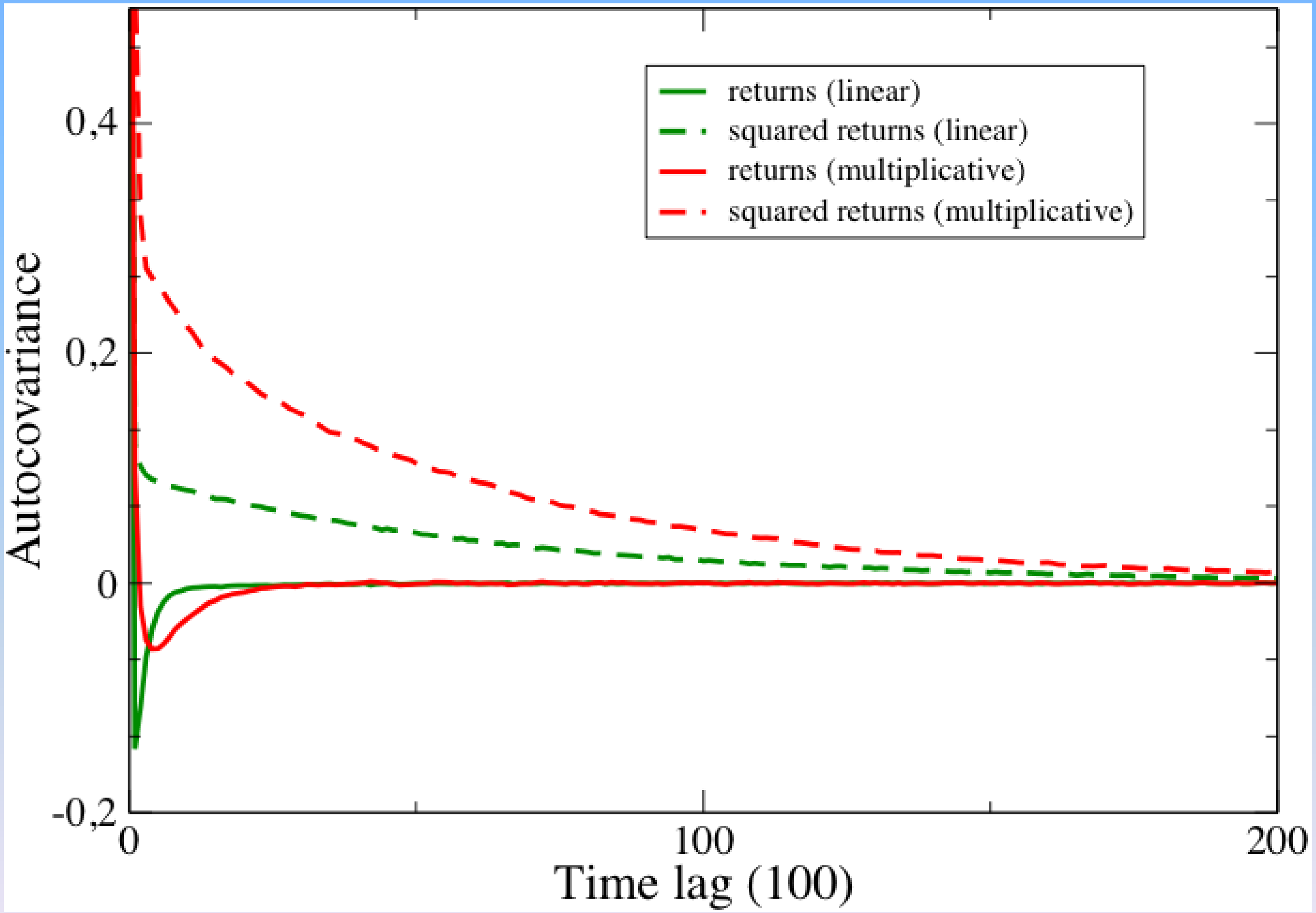
Paper2 fig. 16

Comparison between linear and multiplicative dynamics
Fat Tails are usually larger for the Multiplicative case



Paper2 fig. 17

Volatility clustering and market efficiency
For Linear and Multiplcative dynamics
Similar trends but amplification of
V.C. for the multiplicative case

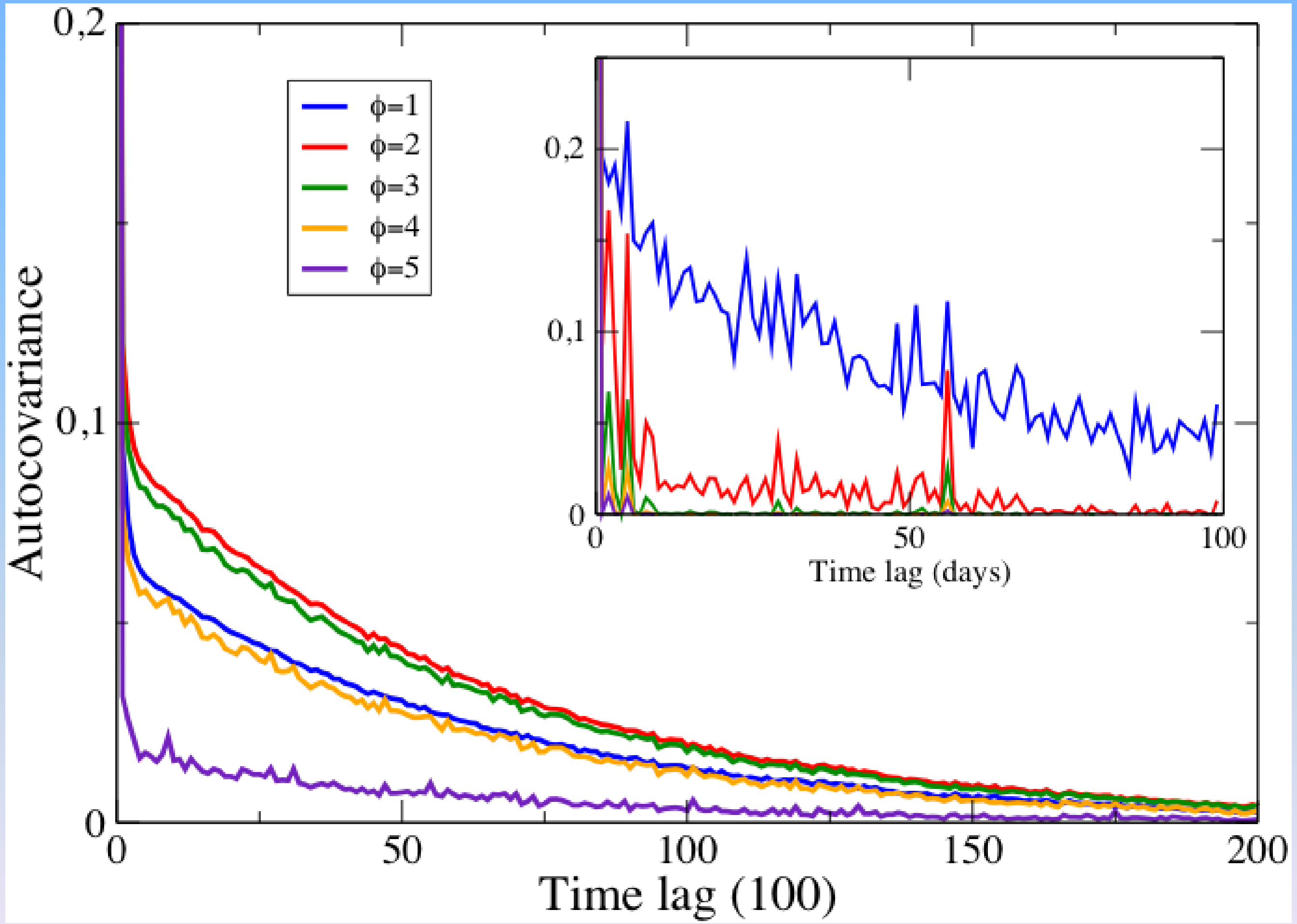


Paper2 fig. 18

Linear model:

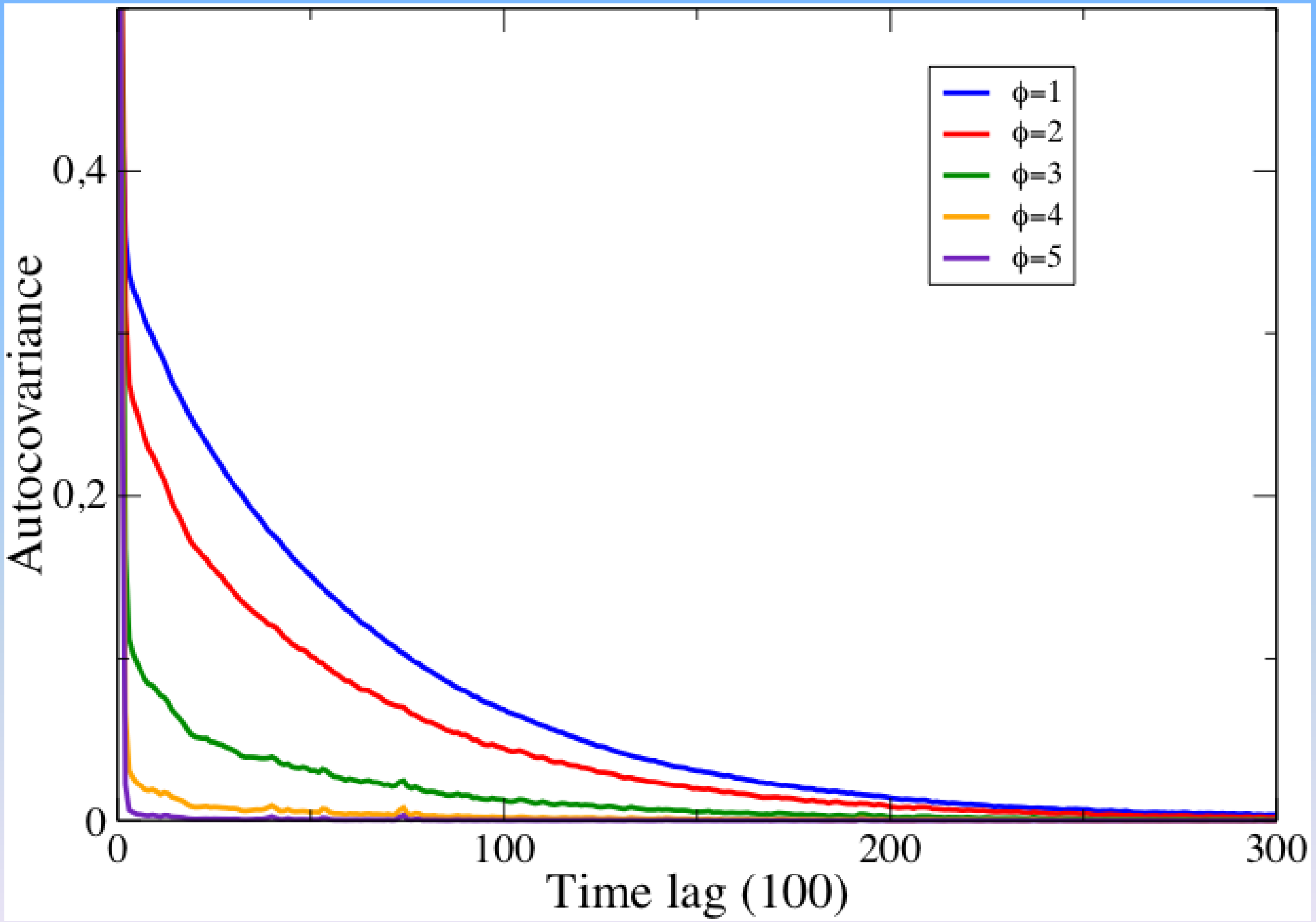
Correlations of absolute returns with respect to their power. The case ϕ =1 is weaker than ϕ =2

Opposite to observations



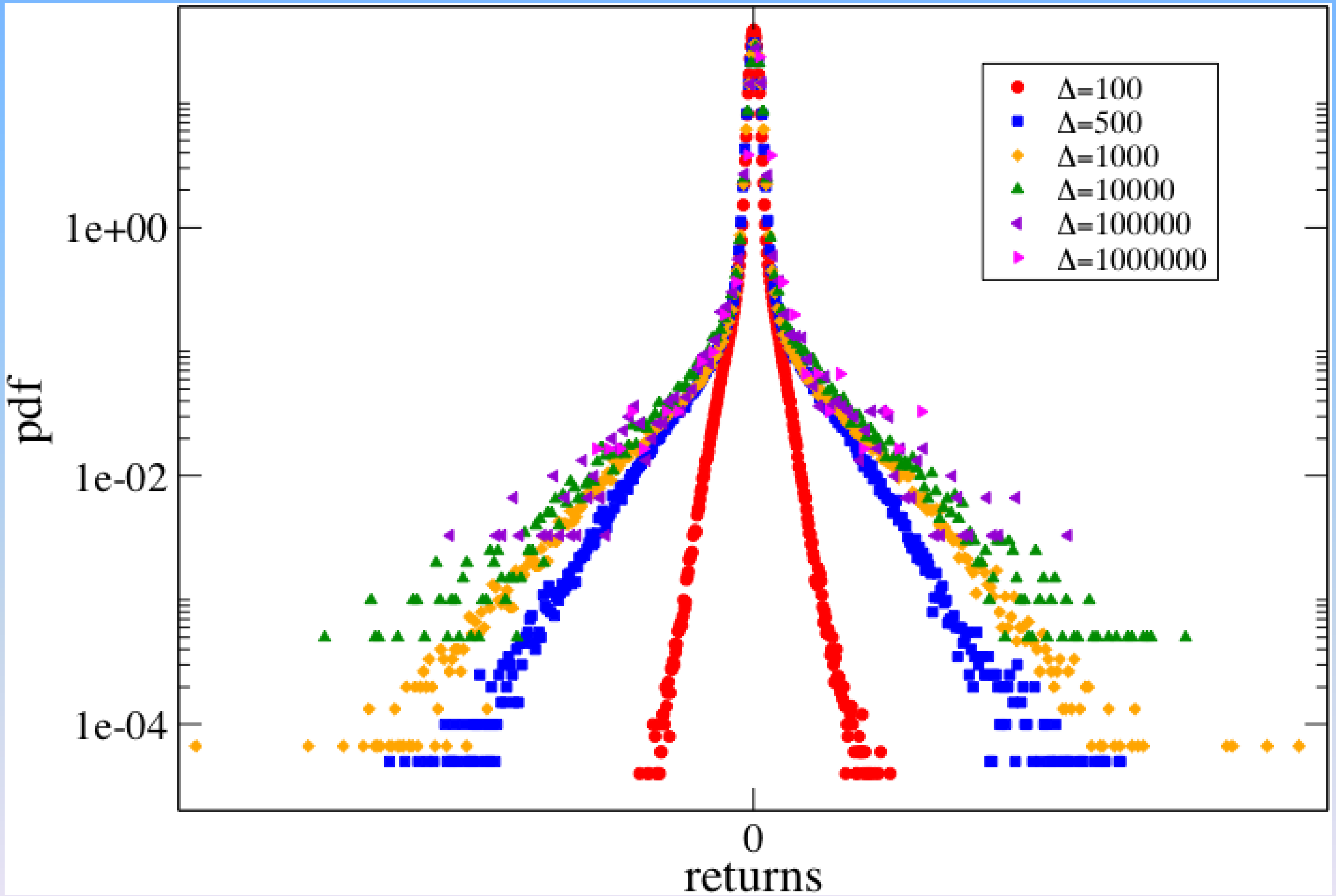
Paper2 fig. 19

Multiplicative model:
Correlations of absolute returns with respect to their power. The case ϕ =1 is stronger than ϕ =2 in agreement with observations



Paper2 fig. 20

Multiplicative case: Fat Tails as function of the time interval

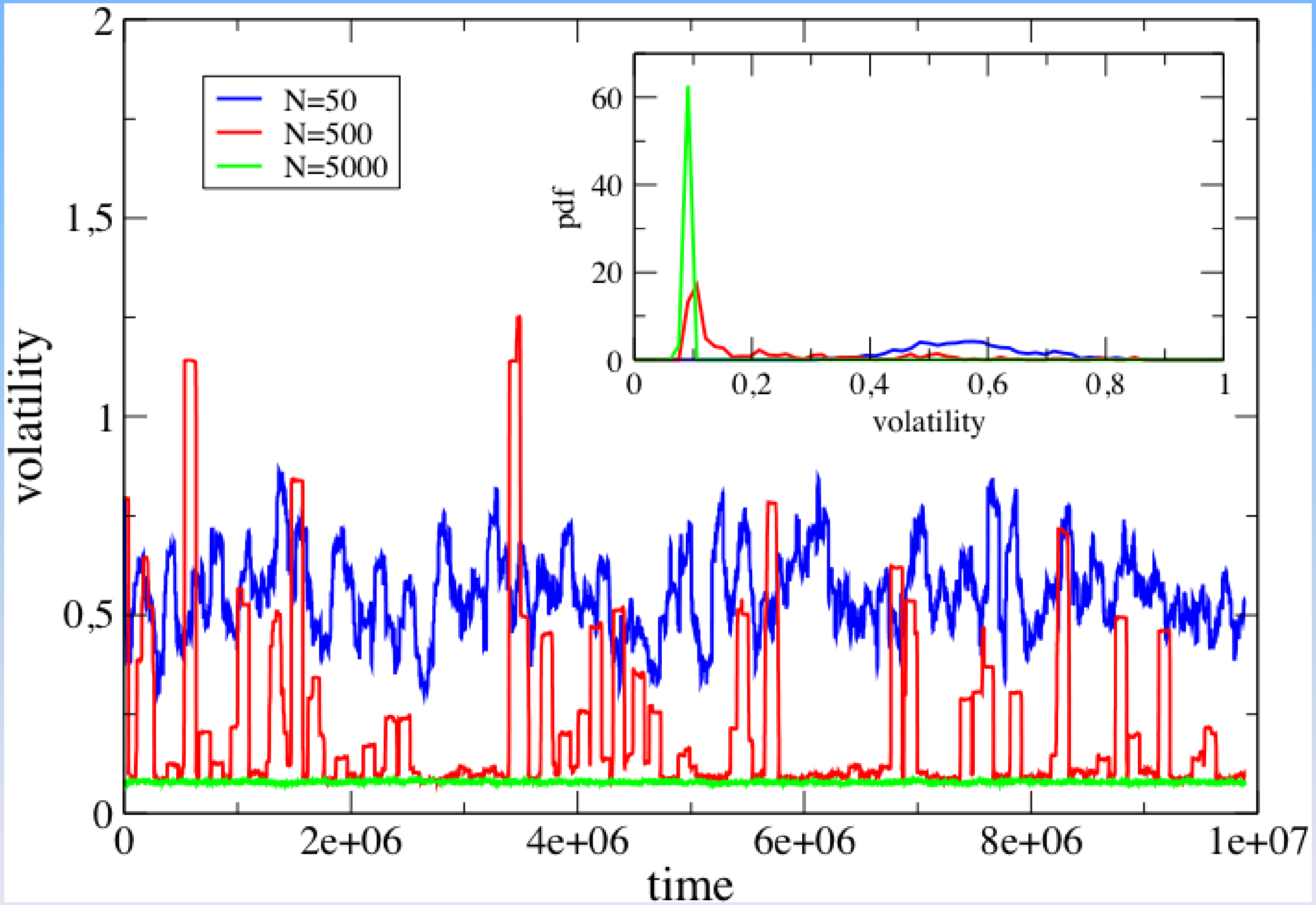


Paper2 fig. 22

Multiplicative case:

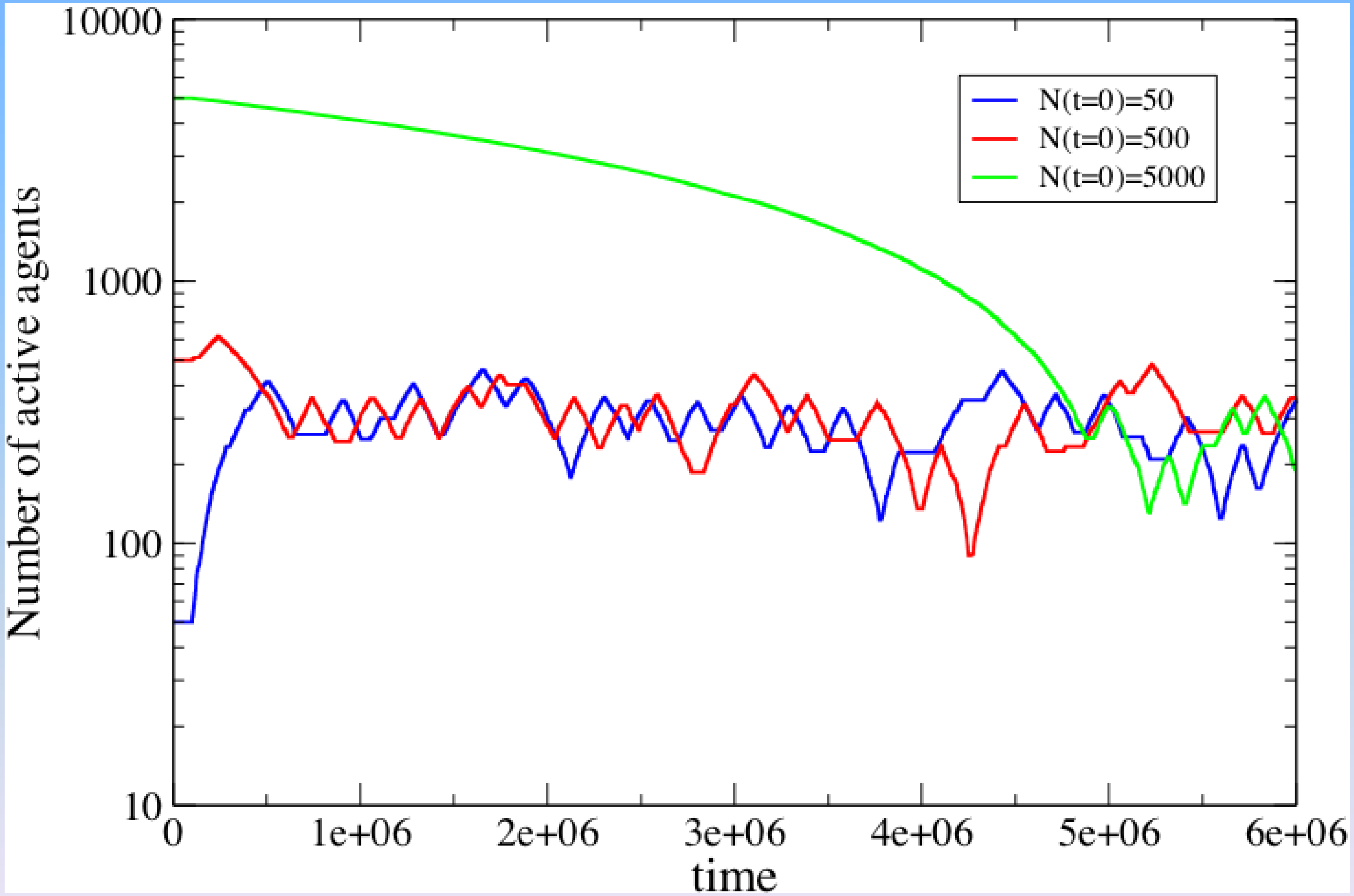
Volatility fluctuations for different values of N

General behavior is similar to the linear case



Paper2 fig. 24

Multiplicative case:
Self-Organized-Intermittency
(relatively slow convergence from large N)



Paper2 fig. 25

SOC perspective vs. SOI

- First possibility (SOC like):

  Try to get the SF in a model for N going to infinity and argue that for real data N is large.

  Problem: SOC with respect to the other parameters (?)

- Present scheme (Self-Organized-Intermittency):

  Given the various parametrs the system self-organizes at a finite value of N corresponding to the intermittent behavior (quasi critical)

  Stylized Facts correspond to finite size effects in N and in time.

  No universality, but similar behavior at different scales.

  A crucial problem is the identification of the effective value of N for a given system.

Minimal ABM

- Minimal Agent Based Model in which the role of each parameter is clarified.
- The model can be easily generalized to make it more realistic and to consider specific questions
- Possible applied side: identification of the nature of the agents from the time series p(t)
- Importance of Herding: possible test from data
- Introduction of different time scales, possibly interacting (apparent or real power laws)

Summary

- Price movement leads to increase of effective action (N*). Multiplicative cascade (avalanche like SOC, sandpile, absence of cause-effect relation): fat tails and volatility clustering
- The reason that price returns have much less correlations depends on the fact that they are functions of many more parameters (arbitrage).
- The specific structure of the fluctuations is due to the competition between stability and instability which is controlled by the rates of the changes of opinion
- Stylized Facts seem to correspond to finite size effects in N and in time. Conceptual and practical implications.
- Self-organization in a quasi-critical state arises from the agent's strategies with respect to price movements

V.Alfi, L. Pietronero and A. Zaccaria

http://arxiv.org/pdf/0807.1888

and other papers on the archive

THREE LEVELS OF UNDERSTANDING

1. *Penomenological - Geometric - Empirical*
   STYLIZED FACTS

2. *Microscopic: PHYSICAL MODELS*
   Computer Simulations: ***AGENT BASED MODELS***

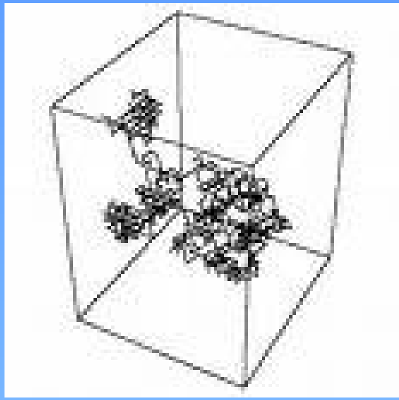
3. *Complete Theoretical Understanding*
   i.e. Renormalization Group for Critical Phenomena

- ***STYLIZED FACTS ARE STILL VERY FEW:***
  SKILLED DATA MINING IS VERY IMPORTANT

Bachelier and Random Walk (1900)

In 1900 Louis Bachelier, a student of Poincare', in his PhD Thesis: *Theorie de la Speculation,* developed a Random Walk model to explain the dynamics of the stocks exchanged in the Paris Stock Market. His model of Random Walk was theorized 5 years before the famous Einstein's interpretation of the Brownian Motion.

L. **Bachelier**, Ann. Sci. Ecolole Norm. Super. **17** (1900) 21



Random Walk

The price is the sum of independent and identically (Gaussian) distributed stochastic variable.

Efficient market hypothesis

Random Walk represents only a first approximation of what is observed in real data

Power Laws

Many of the probability density functions of economic quantity show a power laws behavior which is an asymptotic relations of the form:

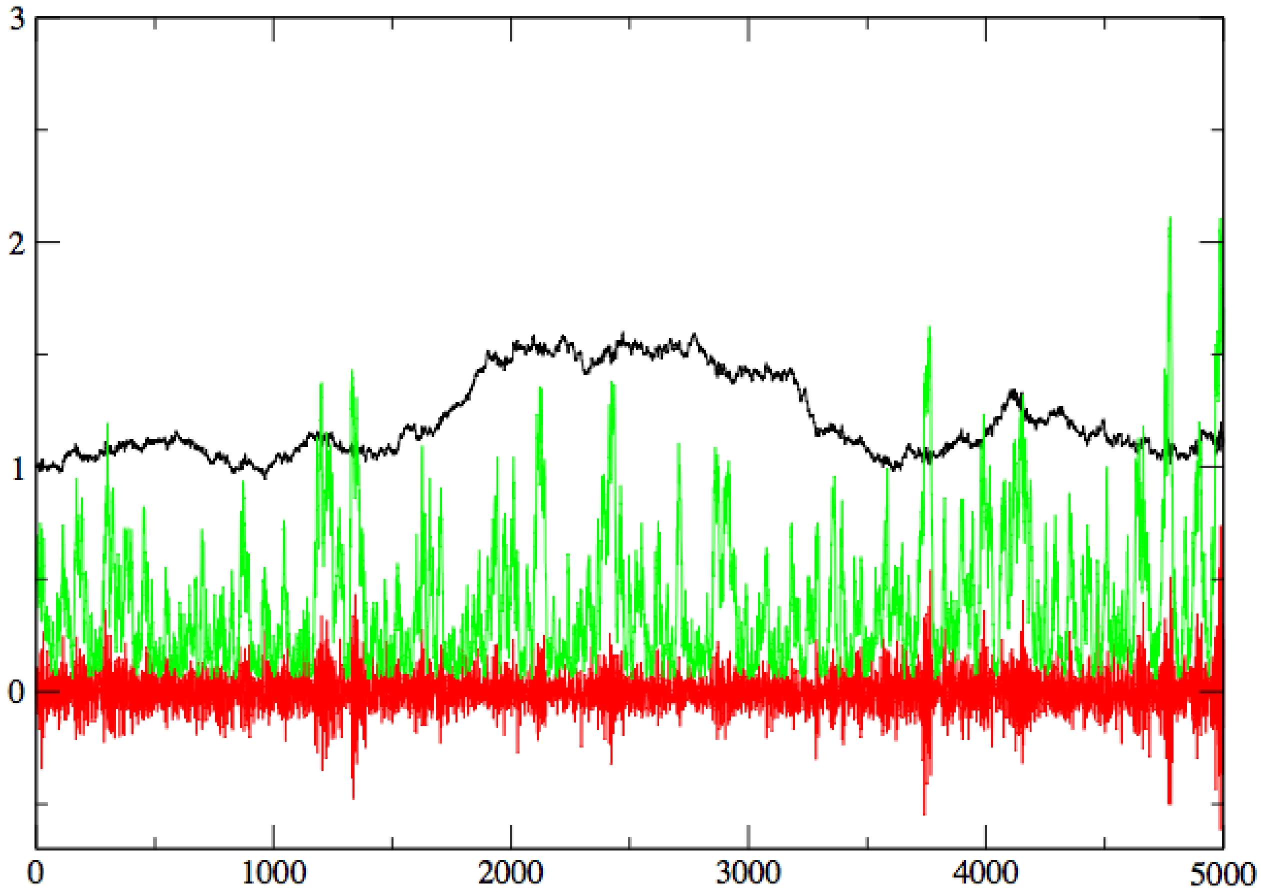
$$f(x) \to x^{-\alpha}$$

In 1963, B. Mandelbrot observed that the distribution of cotton price fluctuations follows a power law

$\longrightarrow$ scale invariance of the process.

Typical (good) Results

Stylized facts can be reproduced with some choices of parameters



price

Nc

returns

players

N=500

In the end 13 parameters with a strongly nonlinear dynamics (3 α, 2 ν, β, γ, tc , pf, r, R, s, σ)

Lower bounds:

One should artificially avoid that the number of chartists goes to zero (attractive state)

In such a case the system is locked in the state nf=500, nc=0

A minimal value of 4 is set for chartists

Fine tuning of parameters is crucial to get the Stylized Facts

The parameters below reproduce reasonably well the stylized facts

$$N = 500,\ v_1 = 3,\ v_2 = 2,\ \beta = 6,\ T_c(\equiv Nt_c) = 10,\ T_f(\equiv N\gamma) = 5,$$
$$\alpha_1 = 0.6,\ \alpha_2 = 0.2,\ \alpha_3 = 0.5,\ p_f = 10,\ r = 0.004,\ R = 0.0004,\ s = 0.75$$

But what does this mean in terms of

- Stability
- Self-organization, etc