Algebras of ontology alignment relations

Jérôme Euzenat

INRIA RHÔNE-ALPES & LIG
Grenoble, France
`Jerome.Euzenat@inrialpes.fr`

Outline

Outline

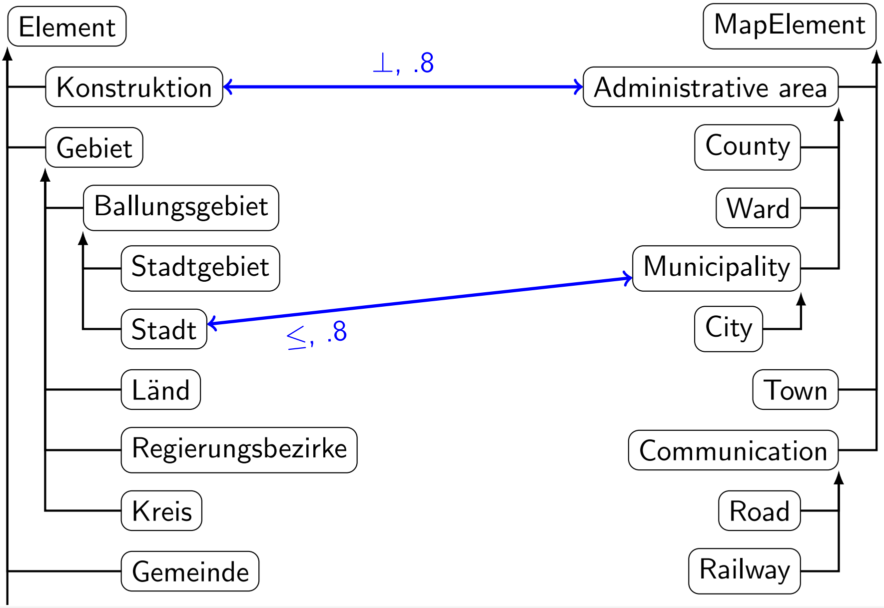
Motivation: disjunctive alignments

Algebra of alignment relations

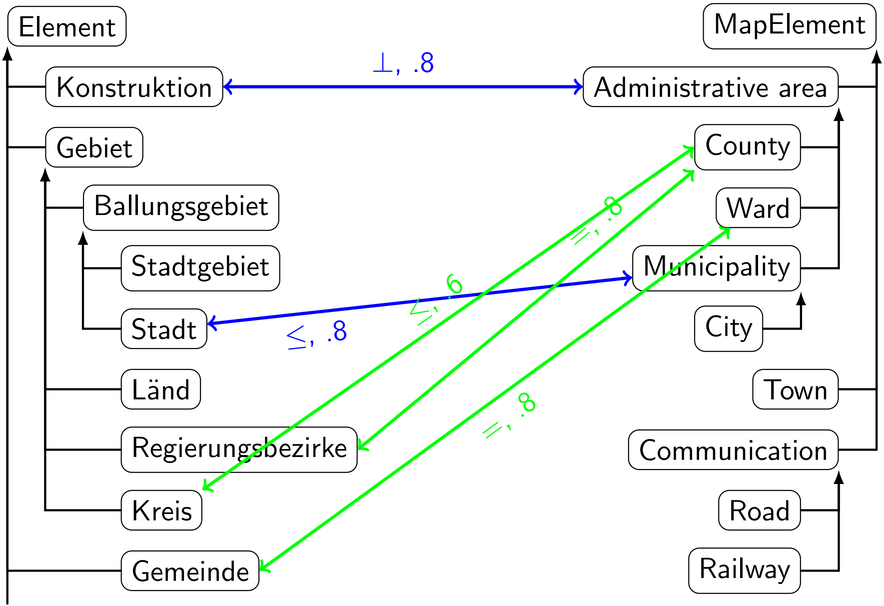
Benefits

Ontology alignments

Ontology alignments



Ontology alignments



Ontology alignments

An alignment is a set of correspondences:

Definition (Correspondence)

Given two ontologies o and o' with associated entity languages Q_L and $Q_{L'}$, a set of alignment relations Θ, and a confidence structure over Ξ, a correspondence is a quadruple: $\langle e, e', r, n \rangle$, such that

- $e \in Q_L(o)$ and $e' \in Q_{L'}(o')$;
- $r \in \Theta$;
- $n \in \Xi$.

We will first restrict ourselves to correspondences as $\langle e, e', r \rangle$.

Motivating example

- expressing disjunctions of relations: Stadt and a Town are similar things but we may not know exactly the nature of the overlaps, maybe equals, maybe one is more general that the other but at least the intersection is not empty.

Stadt ? Town

Motivating example

- expressing disjunctions of relations: Stadt and a Town are similar things but we may not know exactly the nature of the overlaps, maybe equals, maybe one is more general that the other but at least the intersection is not empty.

$$\text{Stadt} \ ? \ \text{Town}$$

- merging independent positive alignments:

$$\text{Stadtgebiet} \between \text{Municipality} \qquad \text{Stadtgebiet} \leq \text{Municipality}$$

Motivating example

- expressing disjunctions of relations: Stadt and a Town are similar things but we may not know exactly the nature of the overlaps, maybe equals, maybe one is more general that the other but at least the intersection is not empty.

$$\text{Stadt} \; ? \; \text{Town}$$

- merging independent positive alignments:

$$\text{Stadtgebiet} \between \text{Municipality} \qquad \text{Stadtgebiet} \leq \text{Municipality}$$

$$\text{Stadtgebiet} \; ? \; \text{Municipality}$$

Motivating example

- expressing disjunctions of relations: Stadt and a Town are similar things but we may not know exactly the nature of the overlaps, maybe equals, maybe one is more general that the other but at least the intersection is not empty.

  Stadt ? Town

- merging independent positive alignments:

  Stadtgebiet $\between$ Municipality   Stadtgebiet $\leq$ Municipality

  Stadtgebiet ? Municipality

What is Θ?

Matching process

The matching process can be achieved by aggregating:

- independent sources of matching along different dimensions (based on instances rather than definitions);
- competing sources of matching (providing different opinions on the same correspondences);
- indirect sources of matching (using an intermediate ontology and composing the alignments).

This is becoming more common with the availability of alignments on the web.

How to combine these alignments?

Outline

Relation algebra

Definition (Relation algrebra (Tarski 1941))

An algebra of binary relations is a structure $\langle\Theta, \wedge, \vee, *, 1, 0, 1', \neg\rangle$ such that

- $\langle\Theta, \wedge, \vee, 1, 0\rangle$ is a Boolean algebra;
- $*$ is an associative internal composition law with (left and right) unity element $1'$, that distributes over $\vee$;
- $\neg$ is an internal involutive unary operator, that distributes over $\vee$, $\wedge$ and $*$.

$$((\leq * \leq) \wedge \geq)$$
$$= (\leq \wedge \geq)$$
$$==$$
$$\leq \vee \geq \ ?$$

Relation algebra

Γ is a set of jointly exhaustive and pairwise disjoint (JEPD) relations between two entities.

Definition (Powerset relation algrebra)

A powerset algebra of binary relations is a structure $\langle 2^\Gamma, \cap, \cup, \cdot, \Gamma, \varnothing, \{=\}, \cdot^{-1}\rangle$ such that

- $\langle 2^\Gamma, \cap, \cup, \Gamma, \varnothing\rangle$ is a Boolean algebra;
- $\cdot$ is an associative internal composition law with (left and right) unity element $\{=\}$, that distributes over $\cup$;
- $\cdot^{-1}$ is an internal involutive unary operator, that distributes over $\cup$, $\cap$ and $\cdot$.

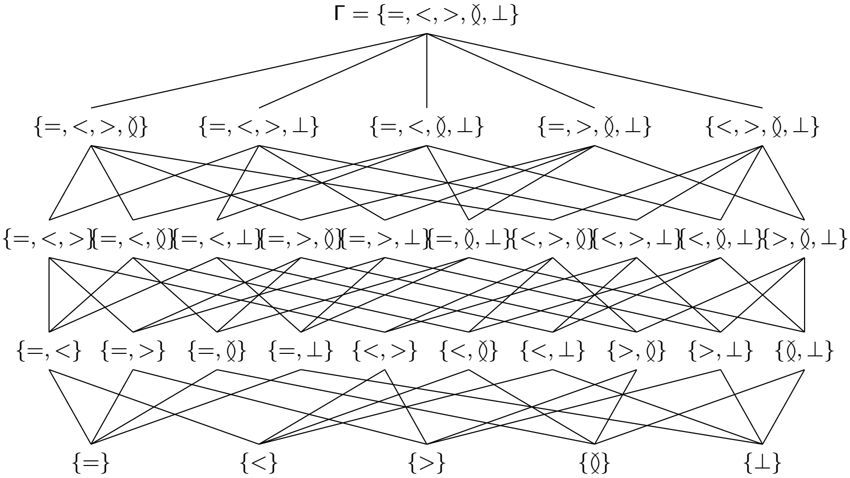
$$((\{<,=\}\cdot(\{<,=\})\cap(\{>,=\})$$
$$=(\{<,=\})\cap(\{>,=\})$$
$$=\{=\}$$
$$(\{<,=\}\cup\{>,=\})=\{<,>,=\}$$

One simple algebra

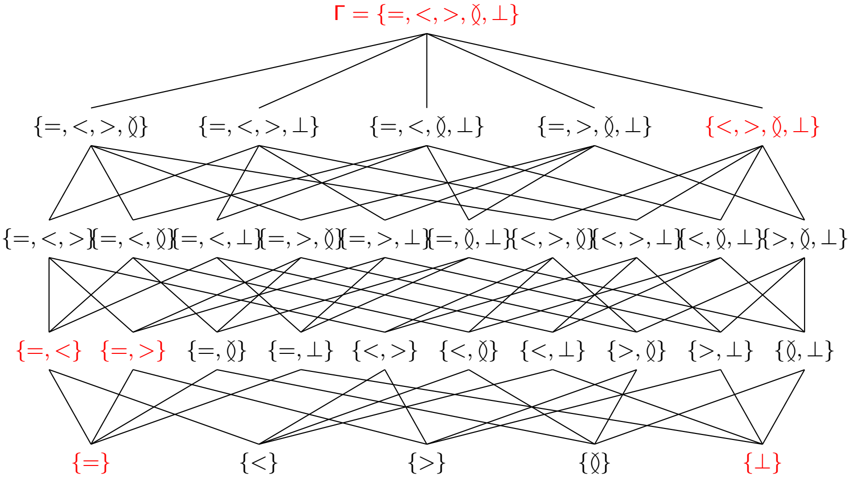
Because of the usual semantics of classes in ontology languages, it is possible to create a simple algebra corresponding to relations between sets:

- $=$ classes are equivalent;
- $<$ the first class is subsumed by the second one;
- $>$ the first class subsumes the second one;
- $\between$ classes overlap but none subsumes the other;
- $\perp$ classes are disjoint.

Lattice of relations among classes (or properties)



Lattice of relations among classes (or properties)



Other relations

Using alignment algebra is not restricted to these very classical types of relations:

Adding instances:

- relation between instances: $=$, $\perp$;
- relations between instances and classes: $\in$, $\not\in$;
- relations between classes and instances: $\ni$, $\not\ni$.

Adding other relations:

- part-of relations
- application relations

But beware of the JEPD constraint: these are obtained by product, not addition.

Outline

Expressing disjunction

- expressing disjunctions of relations: Stadt and a Town are similar things but we may not know exactly the nature of the overlaps, maybe equals, maybe one is more general that the other but at least the intersection is not empty.

Stadt ? Town

Expressing disjunction

- expressing disjunctions of relations: Stadt and a Town are similar things but we may not know exactly the nature of the overlaps, maybe equals, maybe one is more general that the other but at least the intersection is not empty.

$$\text{Stadt } \{=, <, >, \between\} \text{ Town}$$

Expressing disjunction

- expressing disjunctions of relations: Stadt and a Town are similar things but we may not know exactly the nature of the overlaps, maybe equals, maybe one is more general that the other but at least the intersection is not empty.

$$\text{Stadt } \{=, <, >, \between\} \text{ Town}$$

- merging independent positive alignments:

$$\text{Stadtgebiet } \between \text{ Municipality} \qquad \text{Stadtgebiet } \leq \text{ Municipality}$$

Expressing disjunction

- expressing disjunctions of relations: Stadt and a Town are similar things but we may not know exactly the nature of the overlaps, maybe equals, maybe one is more general that the other but at least the intersection is not empty.

$$\text{Stadt } \{=, <, >, \between\} \text{ Town}$$

- merging independent positive alignments:

Stadtgebiet $\between$ Municipality   Stadtgebiet $\leq$ Municipality

Stadtgebiet $\{\between\}$ Municipality   Stadtgebiet$\{=, <\}$Municipality

Expressing disjunction

- expressing disjunctions of relations: Stadt and a Town are similar things but we may not know exactly the nature of the overlaps, maybe equals, maybe one is more general that the other but at least the intersection is not empty.

$$\text{Stadt } \{=, <, >, \between\} \text{ Town}$$

- merging independent positive alignments:

$$\text{Stadtgebiet } \between \text{ Municipality} \qquad \text{Stadtgebiet } \leq \text{ Municipality}$$

$$\text{Stadtgebiet } \{\between\} \text{ Municipality} \qquad \text{Stadtgebiet}\{=, <\}\text{Municipality}$$

$$\text{Stadtgebiet } \{=, <, \between\} \text{ Municipality}$$

Aggregating results

Now we have several choices for aggregating alignments:

- disjunction is natural for combining independent alignments;
- conjunction is useful for aggregating competing alignments.

Alignment disjunction example

$\mathbf{A_6}: \; o \rightarrow o'$

Konstruktion$\{\perp\}$Municipality

Stadt$\{<\}$Town

Stadtgebiet$\{\between\}$Municipality

$\mathbf{A_4}: \; o \rightarrow o'$

Stadt$\{=, <, >, \between\}$Town

Stadtgebiet$\{\perp\}$Municipality

$\mathbf{A_8} = A_6 \vee A_4 : \; o \rightarrow o'$

Stadt$\{=, <, >, \between\}$Town

Stadtgebiet$\{\between, \perp\}$Municipality

Alignment disjunction example

$\mathbf{A_6}: \; o \rightarrow o'$

Konstruktion$\{\perp\}$Municipality

Stadt$\{<\}$Town

Stadtgebiet$\{\between\}$Municipality

$\mathbf{A_4}: \; o \rightarrow o'$

Konstruktion$\{=, <, >, \between, \perp\}$Municipality

Stadt$\{=, <, >, \between\}$Town

Stadtgebiet$\{\perp\}$Municipality

$\mathbf{A_8} = A_6 \vee A_4 : \; o \rightarrow o'$

Konstruktion$\{=, <, >, \between, \perp\}$Municipality

Stadt$\{=, <, >, \between\}$Town

Stadtgebiet$\{\between, \perp\}$Municipality

Alignment conjunction example

$\mathbf{A_3}: \; o \rightarrow o'$

Konstruktion$\{\perp\}$Municipality

Stadtgebiet$\{>, \between\}$Municipality

$\mathbf{A_5}: \; o \rightarrow o'$

Stadt$\{<\}$Town

Stadtgebiet$\{\perp, \between\}$Municipality

$\mathbf{A_6} = A_3 \wedge A_5 : \; o \rightarrow o'$

Konstruktion$\{\perp\}$Municipality

Stadt$\{<\}$Town

Stadtgebiet$\{\between\}$Municipality

Alignment conjunction example

$\mathbf{A_3}: \; o \rightarrow o'$

Konstruktion$\{\perp\}$Municipality

Stadt$\{=, <, >, \between, \perp\}$Town

Stadtgebiet$\{>, \between\}$Municipality

$\mathbf{A_5}: \; o \rightarrow o'$

Konstruktion$\{=, <, >, \between, \perp\}$Municipality

Stadt$\{<\}$Town

Stadtgebiet$\{\perp, \between\}$Municipality

$\mathbf{A_6} = A_3 \wedge A_5: \; o \rightarrow o'$

Konstruktion$\{\perp\}$Municipality

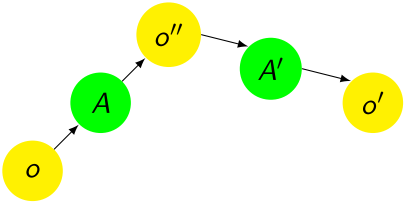
Stadt$\{<\}$Town

Stadtgebiet$\{\between\}$Municipality

Alignment composition

Alignment composition is very useful (and algebra of relations are very natural):

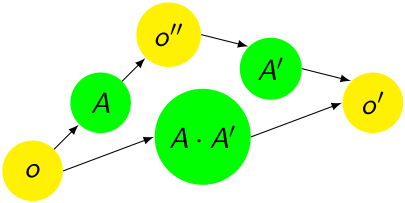
- when one has no direct alignments between two ontologies but intermediate alignments;



Alignment composition

Alignment composition is very useful (and algebra of relations are very natural):

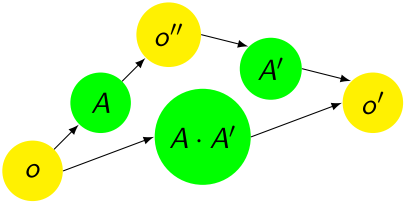
- when one has no direct alignments between two ontologies but intermediate alignments;



Alignment composition

Alignment composition is very useful (and algebra of relations are very natural):

- when one has no direct alignments between two ontologies but intermediate alignments;
- when one wants to saturate a network of ontologies and alignments.



Composition table

	$=$	$>$	$<$	$\between$	$\perp$
$=$	$=$	$>$	$<$	$\between$	$\perp$
$>$	$>$	$>$	$><=\between$	$>\between$	$>\between\perp$
$<$	$<$	Γ	$<$	$<\between\perp$	$\perp$
$\between$	$\between$	$>\between\perp$	$<\between$	Γ	$>\between\perp$
$\perp$	$\perp$	$\perp$	$<\between\perp$	$<\between\perp$	Γ

$$
\begin{aligned}
& \{<, \between\} \cdot \{<, =\} \\
= \; & \{<\} \cdot \{<\} \cup \{<\} \cdot \{=\} \cup \{\between\} \cdot \{<\} \cup \{\between\} \cdot \{=\} \\
= \; & \{<\} \cup \{<\} \cup \{<, \between\} \cup \{\between\} \\
= \; & \{<, \between\}
\end{aligned}
$$

$$\mathsf{A}\{<, \between\}\mathsf{B}\{<, =\}\mathsf{C} \Rightarrow \mathsf{A}\{<, \between\}\mathsf{C}$$

Composition example

$\mathbf{A_1}: \; o \longrightarrow o''$

Konstruktion$\{\perp\}$Commune

Stadtgebiet$\{>\}$Ville

$\mathbf{A_2}: \; o'' \longrightarrow o'$

Commune$\{>, =\}$Municipality

Ville$\{\between\}$Municipality

$\mathbf{A_3} = A_1 \cdot A_2 : \; o \longrightarrow o'$

Konstruktion$\{\perp\} \cdot \{>, =\}$Municipality

Stadtgebiet$\{>\} \cdot \{\between\}$Municipality

Composition example

$\mathbf{A_1}: \; o \rightarrow o''$

Konstruktion$\{\perp\}$Commune

Stadtgebiet$\{>\}$Ville

$\mathbf{A_2}: \; o'' \rightarrow o'$

Commune$\{>, =\}$Municipality

Ville$\{\between\}$Municipality

$\mathbf{A_3} = A_1 \cdot A_2: \; o \rightarrow o'$

Konstruktion$\{\perp\}$Municipality

Stadtgebiet$\{>, \between\}$Municipality

Composition example

$\mathbf{A_1} : \; o \rightarrow o''$

Konstruktion$\{\perp\}$Commune

Stadtgebiet$\{>\}$Ville

Konstruktion$\{\perp\}$Ville

$\mathbf{A_2} : \; o'' \rightarrow o'$

Commune$\{>, =\}$Municipality

Ville$\{\between\}$Municipality

$\mathbf{A_3} = A_1 \cdot A_2 : \; o \rightarrow o'$

Konstruktion$\{\perp\} \cdot \{>, =\} \cap \{\perp\} \cdot \{\between\}$Municipality

Stadtgebiet$\{>\} \cdot \{\between\}$Municipality

Composition example

$\mathbf{A_1}: \; o \rightarrow o''$

Konstruktion$\{\perp\}$Commune

Stadtgebiet$\{>\}$Ville

Konstruktion$\{\perp\}$Ville

$\mathbf{A_2}: \; o'' \rightarrow o'$

Commune$\{>, =\}$Municipality

Ville$\{\between\}$Municipality

$\mathbf{A_3} = A_1 \cdot A_2: \; o \rightarrow o'$

Konstruktion$\{\perp\} \cap \{<, \between, \perp\}$Municipality

Stadtgebiet$\{>, \between\}$Municipality

Composition example

$\mathbf{A_1}: o \rightarrow o''$

Konstruktion$\{\perp\}$Commune

Stadtgebiet$\{>\}$Ville

Konstruktion$\{\perp\}$Ville

$\mathbf{A_2}: o'' \rightarrow o'$

Commune$\{>, =\}$Municipality

Ville$\{\between\}$Municipality

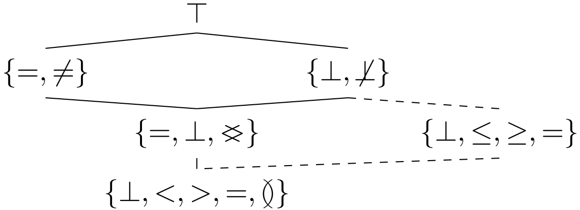
$\mathbf{A_3} = A_1 \cdot A_2 : o \rightarrow o'$

Konstruktion$\{\perp\}$Municipality

Stadtgebiet$\{>, \between\}$Municipality

Granularitity

The algebra of relations between the same set of entities is not abslolute. There can be many different set of base relations which depends on the granularity or the point of view of the representation.



Problem: can we deal with alignments at different granularities?

Algebra granularity example

$$\mathbf{A_7}[\{\perp, \not\perp\}] : \; o \rightarrow o'$$

$$\text{Stadt}\{\not\perp\}\text{Town}$$

$$\text{Stadtgebiet}\{\perp\}\text{Municipality}$$

$$\mathbf{A_5} : \; o \rightarrow o'$$

$$\text{Stadt}\{<\}\text{Town}$$

$$\text{Stadtgebiet}\{\perp, \between\}\text{Municipality}$$

$$\mathbf{A_4} = \Uparrow A_7 : \; o \rightarrow o'$$

$$\text{Stadt}\{=, <, >, \between\}\text{Town}$$

$$\text{Stadtgebiet}\{\perp\}\text{Municipality}$$

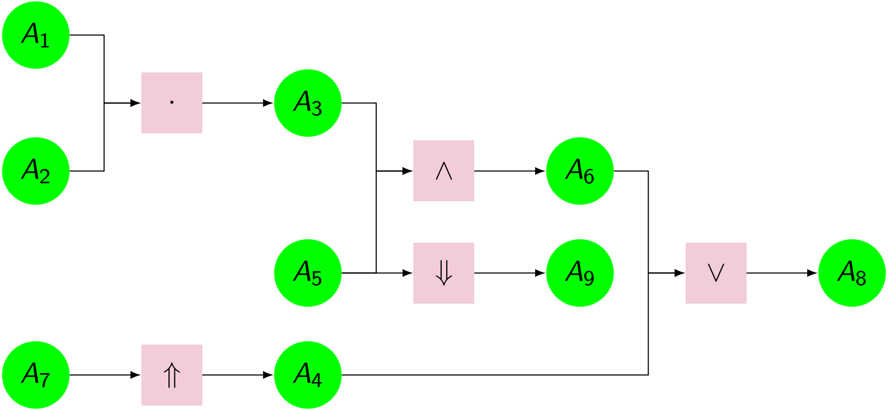
$$\mathbf{A_9}[\{\perp, \not\perp\}] = \Downarrow A_5$$

$$\text{Stadt}\{\not\perp\}\text{Town}$$

$$\text{Stadtgebiet}\{\perp, \not\perp\}\text{Municipality}$$

Thanks to the use of compatible algebras.

Summary



Back to confidence measures

Definition (Confidence structure)

A confidence structure is an ordered set of degrees $\langle \Xi, \leq \rangle$ for which there exists a greatest element $\top$ and a smallest element $\bot$.

A correspondence $\langle e, e', r, n \rangle$ entails another correspondence $\langle e, e', r', n' \rangle$ if and only if $r \subseteq r'$ and $n \geq n'$.

$$\text{Stadt}\{<\}_{.9}\text{Town} \models \text{Stadt}\{<, =\}_{.8}\text{Town}$$

So the order between correspondances is the order induced by $\subseteq, \geq$.

Back to confidence measures

Definition (Confidence structure)

A confidence structure is an ordered set of degrees $\langle \Xi, \leq \rangle$ for which there exists a greatest element $\top$ and a smallest element $\bot$.

A correspondence $\langle e, e', r, n \rangle$ entails another correspondence $\langle e, e', r', n' \rangle$ if and only if $r \subseteq r'$ and $n \geq n'$.

$$\text{Stadt}\{<\}_{.9}\text{Town} \models \text{Stadt}\{<,=\}_{.8}\text{Town} \models \text{Stadt } \Gamma_{\bot}\text{Town}$$

So the order between correspondances is the order induced by $\subseteq, \geq$.

Back to confidence measures

Definition (Confidence structure)

A confidence structure is an ordered set of degrees $\langle \Xi, \leq \rangle$ for which there exists a greatest element $\top$ and a smallest element $\bot$.

A correspondence $\langle e, e', r, n \rangle$ entails another correspondence $\langle e, e', r', n' \rangle$ if and only if $r \subseteq r'$ and $n \geq n'$.

$$\text{Stadt}\{<\}_{.9}\text{Town} \models \text{Stadt}\{<, =\}_{.8}\text{Town} \models \text{Stadt}\ \Gamma_{\bot}\text{Town}$$

$$\models \text{Stadt}\ \Gamma_{\top}\text{Town}$$

So the order between correspondances is the order induced by $\subseteq, \geq$.

Logical operator with confidence example

$$\text{Stadt}\{=,<\}_{.5}\text{Town} \qquad \text{Stadt}\{=\}_{1.}\text{Town}$$

$$\text{Stadt}\{=,\perp\}_{.2}\text{Town}$$

$$\text{Stadtgebiet}\{\perp\}_{.1}\text{Municipality} \qquad \text{Stadtgebiet}\{\perp,\between\}_{.2}\text{Municipality}$$

Applying basic boolean constraints ($A \wedge B \models A \models A \vee B$):

$\vee = \cup^{\min}$ yields:

$$\text{Stadt}\{=,<,\perp\}_{.2}\text{Town}$$

$$\text{Stadtgebiet}\{\perp,\between\}_{.1}\text{Municipality}$$

$\wedge = \cap^{\max}$ yields:

$$\text{Stadt}\{=\}_{1.}\text{Town}$$

$$\text{Stadtgebiet}\{\perp\}_{.2}\text{Municipality}$$

Logical operator with confidence example (relaxed)

$$\text{Stadt}\{=,<\}_{.5}\text{Town} \qquad \text{Stadt}\{=\}_{1.}\text{Town}$$

$$\text{Stadt}\{=,\perp\}_{.2}\text{Town}$$

$$\text{Stadtgebiet}\{\perp\}_{.1}\text{Municipality} \qquad \text{Stadtgebiet}\{\perp,\between\}_{.2}\text{Municipality}$$

$\vee = \cap^{\text{min}}$ yields:

$$\text{Stadt}\{=\}_{1.}\text{Town}$$

$$\text{Stadtgebiet}\{\perp,\between\}_{.2}\text{Municipality}$$

$$\text{Stadtgebiet}\{\perp\}_{.1}\text{Municipality}$$

$\wedge = \cup^{weightedsum}$ yields:

$$\text{Stadt}\{<,=\}_{.83}\text{Town}$$

$$\text{Stadt}\{\perp,=\}_{.4}\text{Town}$$

$$\text{Stadtgebiet}\{\perp,\between\}_{.16}\text{Municipality}$$

Conclusion

- Alignment relation algebras are an adequate tool for representing disjunctions of relations;
- Moreover, they are naturally useful for many operation: aggregation, composition, granularity;
- They can be made compatible with confidence measures;
- They usually provide a better way to compare alignments;

Conclusion

- Alignment relation algebras are an adequate tool for representing disjunctions of relations;
- Moreover, they are naturally useful for many operation: aggregation, composition, granularity;
- They can be made compatible with confidence measures;
- They usually provide a better way to compare alignments;

- Made for using in alignment management;
- Not implemented yet feature (likely in Alignment API 4.0).

Jerome.Euzenat@inria.fr

http://exmo.inrialpes.fr