ELP
Tractable Rules for OWL 2

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Where do we want to get to?

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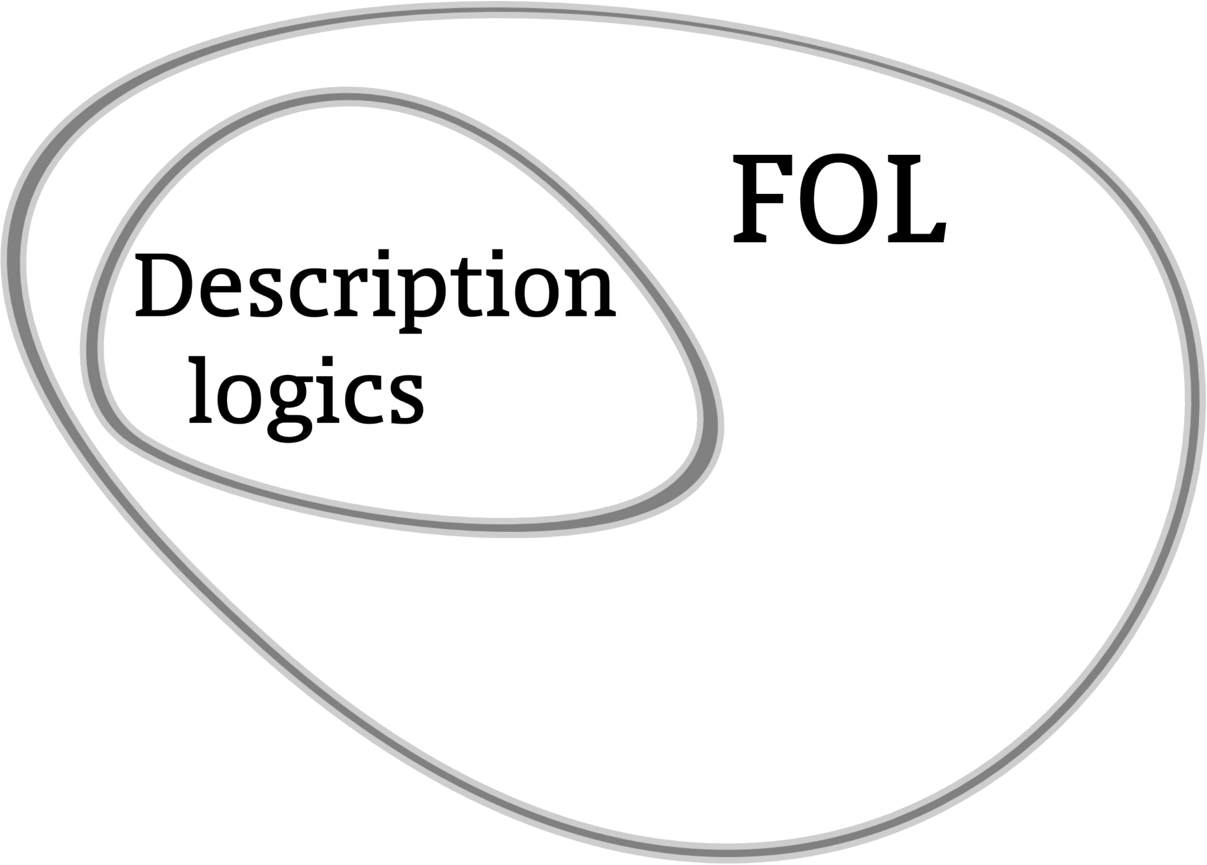
Semantic Web language that is …

- focussed on data,
- rule-oriented,
- able to express schema knowledge,
- easy to implement,
- of polynomial worst-case complexity,
- compatible with OWL.

OWL and Description Logics

OWL and Description Logics



OWL and Description Logics

“Sebastian ordered some Thai curry.”

$sebastian: \exists orderedDish.ThaiCurry$

$\exists x.\ orderedDish(sebastian, x)$
$\quad \wedge\ \mathrm{ThaiCurry}(x)$

OWL and Description Logics

“Everything ordered as a dish is actually a dish.”

$$\top \sqsubseteq \forall \mathit{orderedDish}.\mathit{Dish}$$

$$\forall x.\forall y.\ \mathit{orderedDish}(x,y) \rightarrow \mathrm{Dish}(y)$$

OWL and Description Logics

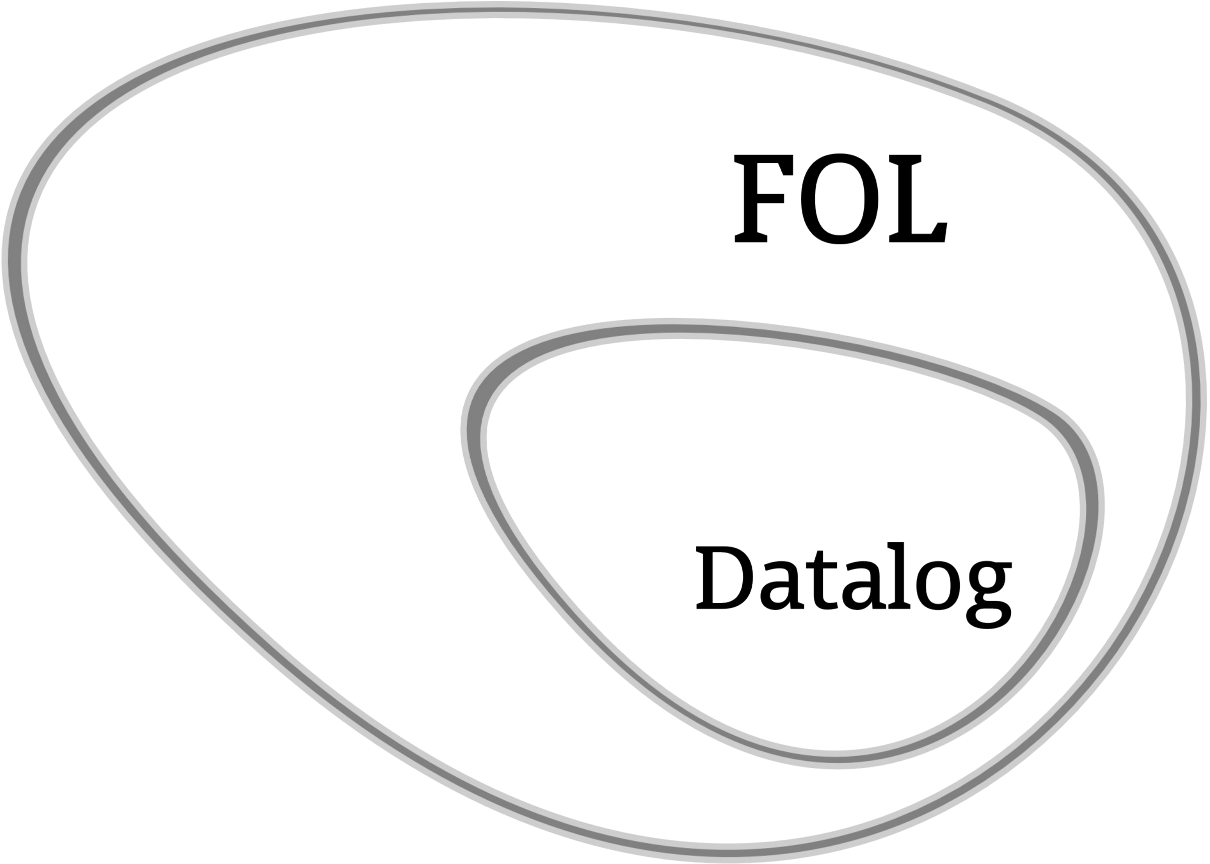
“Every Thai curry dish contains peanut oil.”

$$\mathit{ThaiCurry} \sqsubseteq \exists \mathit{contains}.\{\mathit{peanutOil}\}$$

$$\begin{aligned}&\forall x.\ \mathit{ThaiCurry}(x)\\ &\qquad \rightarrow \mathit{contains}(x,\mathit{peanutOil})\end{aligned}$$

Rules in First-Order Logic

Rules in First-Order Logic



Rules in First-Order Logic

“Nut allergics dislike nut products.”

$$\begin{aligned}&\mathit{NutAllergic}(x) \wedge \mathit{NutProduct}(y)\\ &\qquad\qquad\qquad\qquad \rightarrow \mathit{dislikes}(x,y)\end{aligned}$$

Rules in First-Order Logic

"People who order a dish they dislike are unhappy."

$$\mathit{orderedDish}(x,y) \wedge \mathit{dislikes}(x,y) \rightarrow \mathit{Unhappy}(x)$$

Rules in First-Order Logic

“If someone dislikes an ingredient of a dish, she will also dislike the dish.”

$$
\begin{aligned}
&\mathit{dislikes}(x,z) \land \mathit{Dish}(y) \land \mathit{contains}(y,z) \\
&\qquad\qquad\qquad \rightarrow \mathit{dislikes}(x,y)
\end{aligned}
$$

Rules in First-Order Logic

"Sebastian is a nut allergic, and peanut oil is a nut product."

→ *NutAllergic(sebastian)*

→ *NutProduct(peanutOil)*

Can we combine datalog rules and DL axioms?

“

$$\mathit{ThaiCurry} \sqsubseteq \exists \mathit{contains}.\{\mathit{peanutOil}\}$$

$$\top \sqsubseteq \forall \mathit{orderedDish}.\mathit{Dish}$$

$$\mathit{sebastian}: \exists \mathit{orderedDish}.\mathit{ThaiCurry}$$

$$\mathit{NutAllergic}(x) \wedge \mathit{NutProduct}(y) \rightarrow \mathit{dislikes}(x,y)$$

$$\mathit{orderedDish}(x,y) \wedge \mathit{dislikes}(x,y) \rightarrow \mathit{Unhappy}(x)$$

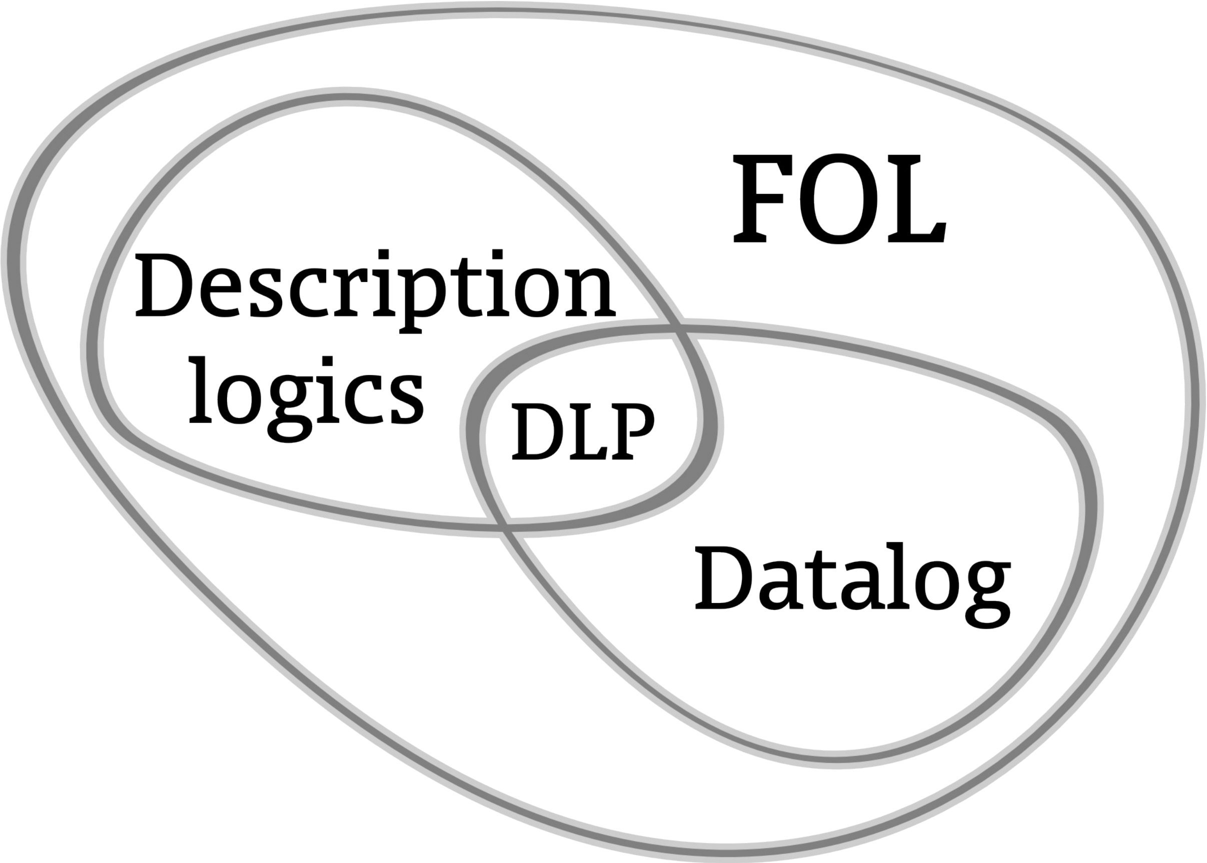
$$\mathit{dislikes}(x,z) \wedge \mathit{Dish}(y) \wedge \mathit{contains}(y,z) \rightarrow \mathit{dislikes}(x,y)$$

$$\rightarrow \mathit{NutAllergic}(\mathit{sebastian})$$

$$\rightarrow \mathit{NutProduct}(\mathit{peanutOil})$$

”

Combining OWL and Rules



DLP: “OWL ∩ datalog”

$ThaiCurry \sqsubseteq \exists contains.\{peanutOil\}$ ✓

$\top \sqsubseteq \forall orderedDish.Dish$ ✓

$sebastian: \exists orderedDish.ThaiCurry$ ✗

$NutAllergic(x) \land NutProduct(y) \rightarrow dislikes(x,y)$ ✗

$orderedDish(x,y) \land dislikes(x,y) \rightarrow Unhappy(x)$ ✗

$dislikes(x,z) \land Dish(y) \land contains(y,z) \rightarrow dislikes(x,y)$ ✗

$\rightarrow NutAllergic(sebastian)$ ✓

$\rightarrow NutProduct(peanutOil)$ ✓

SWRL: “OWL ∪ datalog”

$\mathit{ThaiCurry} \sqsubseteq \exists \mathit{contains}.\{\mathit{peanutOil}\}$ ✓

$\top \sqsubseteq \forall \mathit{orderedDish}.\mathit{Dish}$ ✓

$\mathit{sebastian}\colon \exists \mathit{orderedDish}.\mathit{ThaiCurry}$ ✓

$\mathit{NutAllergic}(x) \wedge \mathit{NutProduct}(y) \rightarrow \mathit{dislikes}(x,y)$ ✓

$\mathit{orderedDish}(x,y) \wedge \mathit{dislikes}(x,y) \rightarrow \mathit{Unhappy}(x)$ ✓

$\mathit{dislikes}(x,z) \wedge \mathit{Dish}(y) \wedge \mathit{contains}(y,z) \rightarrow \mathit{dislikes}(x,y)$ ✓

$\rightarrow \mathit{NutAllergic}(\mathit{sebastian})$ ✓

$\rightarrow \mathit{NutProduct}(\mathit{peanutOil})$ ✓

⚡

SWRL is undecidable.

DL-safe Rules

DL-safe Rules

Restrict rules to apply only to named individuals.

DL-safe Rules

$\mathit{ThaiCurry} \sqsubseteq \exists \mathit{contains}.\{\mathit{peanutOil}\}$ ✓

$\top \sqsubseteq \forall \mathit{orderedDish}.\mathit{Dish}$ ✓

$\mathit{sebastian}\colon \exists \mathit{orderedDish}.\mathit{ThaiCurry}$ ✓

$\mathit{NutAllergic}(x) \land \mathit{NutProduct}(y) \rightarrow \mathit{dislikes}(x,y)$ 🔒

$\mathit{orderedDish}(x,y) \land \mathit{dislikes}(x,y) \rightarrow \mathit{Unhappy}(x)$ 🔒

$\mathit{dislikes}(x,z) \land \mathit{Dish}(y) \land \mathit{contains}(y,z) \rightarrow \mathit{dislikes}(x,y)$ 🔒

$\rightarrow \mathit{NutAllergic}(\mathit{sebastian})$ ✓

$\rightarrow \mathit{NutProduct}(\mathit{peanutOil})$ ✓

DL Rules

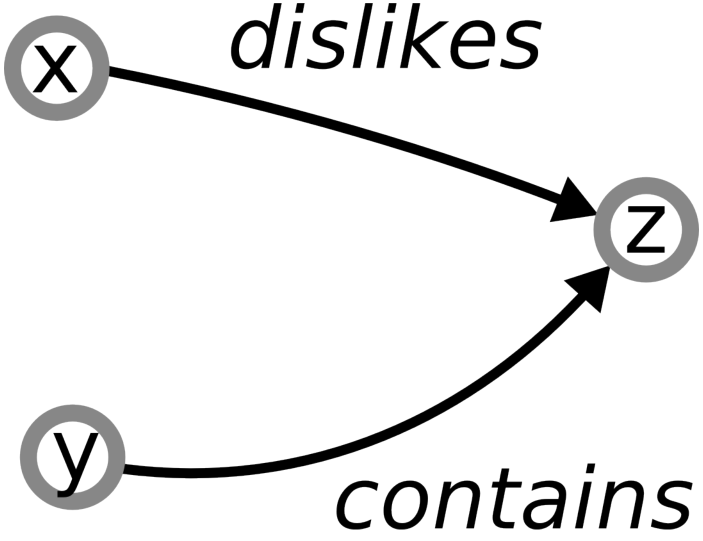
DL Rules

Restrict to rules that could (indirectly) be encoded with DL anway.*

*) rules with “tree-shaped” bodies

DL Rules*

dislikes(x,z) ∧ Dish(y) ∧ contains(y,z) → dislikes(x,y)



*) rules with "tree-shaped" bodies

DL Rules

$\mathit{ThaiCurry} \sqsubseteq \exists \mathit{contains}.\{\mathit{peanutOil}\}$ ✓

$\top \sqsubseteq \forall \mathit{orderedDish}.\mathit{Dish}$ ✓

$\mathit{sebastian}: \exists \mathit{orderedDish}.\mathit{ThaiCurry}$ ✓

$\mathit{NutAllergic}(x) \wedge \mathit{NutProduct}(y) \rightarrow \mathit{dislikes}(x,y)$ ✓

$\mathit{orderedDish}(x,y) \wedge \mathit{dislikes}(x,y) \rightarrow \mathit{Unhappy}(x)$ ✗

$\mathit{dislikes}(x,z) \wedge \mathit{Dish}(y) \wedge \mathit{contains}(y,z) \rightarrow \mathit{dislikes}(x,y)$ ✓

$\rightarrow \mathit{NutAllergic}(\mathit{sebastian})$ ✓

$\rightarrow \mathit{NutProduct}(\mathit{peanutOil})$ ✓

DL-safe rules + DL Rules

DL-safe rules + DL Rules

$\mathit{ThaiCurry} \sqsubseteq \exists \mathit{contains}.\{\mathit{peanutOil}\}$ ✓

$\top \sqsubseteq \forall \mathit{orderedDish}.\mathit{Dish}$ ✓

$\mathit{sebastian}\colon \exists \mathit{orderedDish}.\mathit{ThaiCurry}$ ✓

$\mathit{NutAllergic}(x) \wedge \mathit{NutProduct}(y) \rightarrow \mathit{dislikes}(x,y)$ ✓

$\mathit{orderedDish}(x,y) \wedge \mathit{dislikes}(x,y) \rightarrow \mathit{Unhappy}(x)$ 🔒

$\mathit{dislikes}(x,z) \wedge \mathit{Dish}(y) \wedge \mathit{contains}(y,z) \rightarrow \mathit{dislikes}(x,y)$ ✓

$\rightarrow \mathit{NutAllergic}(\mathit{sebastian})$ ✓

$\rightarrow \mathit{NutProduct}(\mathit{peanutOil})$ ✓

⚠ DL-safe rules + DL Rules ⚠

Desired conclusion does not follow

It is still computationally expensive

DL-safe rules: ExpTime

DL Rules: like DL, i.e. NExpTime for OWL DL

Tractable Profiles in OWL 2

Tractable Profiles in OWL 2

OWL RL: *Horn logic* fragment, similar to DLP, no existentials

OWL EL: includes existentials, based on DL *EL++*

Regaining Tractability: OWL 2 EL

$\mathit{ThaiCurry} \sqsubseteq \exists \mathit{contains}.\{\mathit{peanutOil}\}$ ✓

$\top \sqsubseteq \forall \mathit{orderedDish}.\mathit{Dish}$ ✓

$\mathit{sebastian}: \exists \mathit{orderedDish}.\mathit{ThaiCurry}$ ✓

$\mathit{NutAllergic}(x) \land \mathit{NutProduct}(y) \rightarrow \mathit{dislikes}(x,y)$ ✗

$\mathit{orderedDish}(x,y) \land \mathit{dislikes}(x,y) \rightarrow \mathit{Unhappy}(x)$ ✗

$\mathit{dislikes}(x,z) \land \mathit{Dish}(y) \land \mathit{contains}(y,z) \rightarrow \mathit{dislikes}(x,y)$ ✗

$\rightarrow \mathit{NutAllergic}(\mathit{sebastian})$ ✓

$\rightarrow \mathit{NutProduct}(\mathit{peanutOil})$ ✓

Regaining Tractability: OWL 2 RL

$\mathit{ThaiCurry} \sqsubseteq \exists \mathit{contains}.\{\mathit{peanutOil}\}$ ✓

$\top \sqsubseteq \forall \mathit{orderedDish}.\mathit{Dish}$ ✓

$\mathit{sebastian}: \exists \mathit{orderedDish}.\mathit{ThaiCurry}$ ✗

$\mathit{NutAllergic}(x) \wedge \mathit{NutProduct}(y) \rightarrow \mathit{dislikes}(x,y)$ ✗

$\mathit{orderedDish}(x,y) \wedge \mathit{dislikes}(x,y) \rightarrow \mathit{Unhappy}(x)$ ✗

$\mathit{dislikes}(x,z) \wedge \mathit{Dish}(y) \wedge \mathit{contains}(y,z) \rightarrow \mathit{dislikes}(x,y)$ ✗

$\rightarrow \mathit{NutAllergic}(\mathit{sebastian})$ ✓

$\rightarrow \mathit{NutProduct}(\mathit{peanutOil})$ ✓

OWL EL: PTime complete

OWL RL: PTime complete

OWL EL: PTime complete

OWL RL: PTime complete

OWL EL+RL:
N2ExpTime complete

Bringing it all together: ELP

Bringing it all together: ELP

DL Rules for OWL EL

+

Conjunctions of Roles

+

DL-safe *variables*

Theorem

Inferencing in ELP is PTime complete.

Bringing it all together: ELP

$\mathit{ThaiCurry} \sqsubseteq \exists \mathit{contains}.\{\mathit{peanutOil}\}$ ✓

$\top \sqsubseteq \forall \mathit{orderedDish}.\mathit{Dish}$ ✓

$\mathit{sebastian}: \exists \mathit{orderedDish}.\mathit{ThaiCurry}$ ✓

$\mathit{NutAllergic}(x) \land \mathit{NutProduct}(y) \rightarrow \mathit{dislikes}(x,y)$ ✓

$\mathit{orderedDish}(x,y) \land \mathit{dislikes}(x,y) \rightarrow \mathit{Unhappy}(x)$ ✓

$\mathit{dislikes}(x,z)$ 🔒 $\land \mathit{Dish}(y) \land \mathit{contains}(y,z)$ 🔒 $\rightarrow \mathit{dislikes}(x,y)$ ✓

$\rightarrow \mathit{NutAllergic}(\mathit{sebastian})$ ✓

$\rightarrow \mathit{NutProduct}(\mathit{peanutOil})$ ✓

Bringing it all together: ELP

→ *Unhappy(sebastian)*

Note

ELP supports inferencing in OWL EL and OWL RL.

Understanding DL-safety

$$\mathit{ThaiCurry} \sqsubseteq \exists \mathit{contains}.\boldsymbol{\mathit{FishProduct}}$$

$$\top \sqsubseteq \forall \mathit{orderedDish}.\mathit{Dish}$$

$$\boldsymbol{\mathit{markus}}\colon \exists \mathit{orderedDish}.\mathit{ThaiCurry}$$

$$\boldsymbol{\mathit{Vegetarian}}(x) \wedge \boldsymbol{\mathit{FishProduct}}(y) \rightarrow \mathit{dislikes}(x,y)$$

$$\mathit{orderedDish}(x,y) \wedge \mathit{dislikes}(x,y) \rightarrow \mathit{Unhappy}(x)$$

$$\mathit{dislikes}(x,z)\text{🔒} \wedge \mathit{Dish}(y) \wedge \mathit{contains}(y,z)\text{🔒} \rightarrow \mathit{dislikes}(x,y)$$

$$\rightarrow \boldsymbol{\mathit{Vegetarian}(\mathit{markus})}$$

Understanding DL-safety

Unhappy(markus)

cannot be concluded

Towards Implementation

Theorem

Inferencing in ELP can be
reduced in linear time
to inferencing in
3-variable datalog.

Reasoning through Datalog

- Transformation to datalog is completely syntactic.
- Each axiom/rule can be transformed individually.
- Datalog engines can be used as blackbox.
- Instance and subsumption checking directly in datalog.

Summary

ELP: DL-based tractable rule language

- Almost completely expressible in OWL 2
- Support for OWL EL and OWL RL
- Linear-time conversion to 3-var datalog
  → simple implementation strategy

Happy(markus) *Happy(sebastian)*

Happy(pascal)