

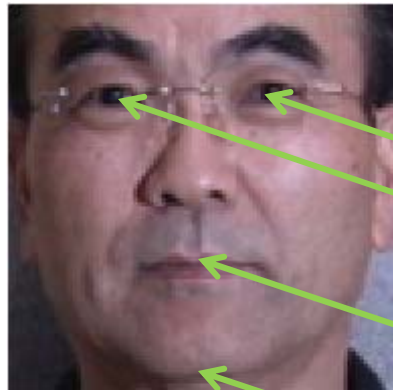
Local Minima Free Parameterized Appearance Models

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Joint work with Fernando de la Torre

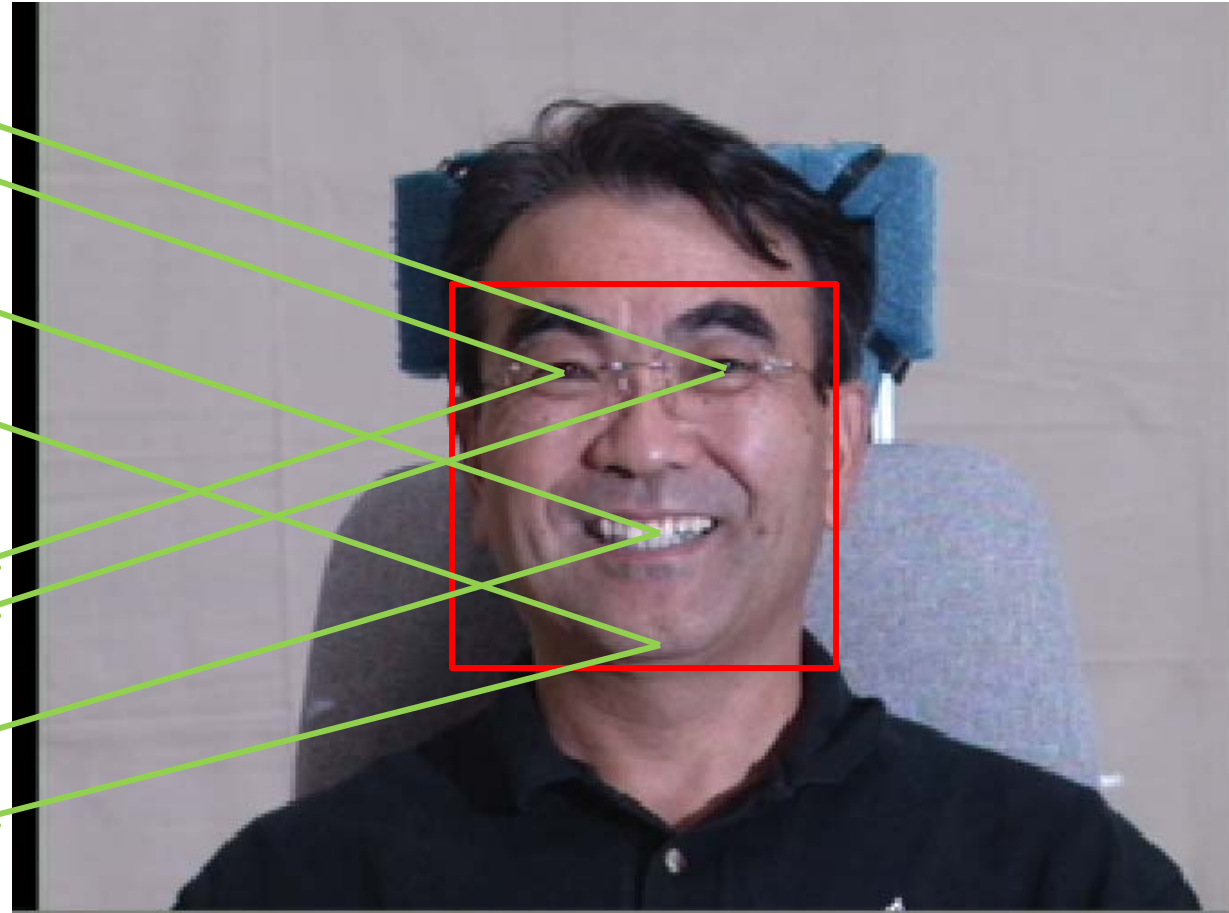
Image alignment



Template



AAM

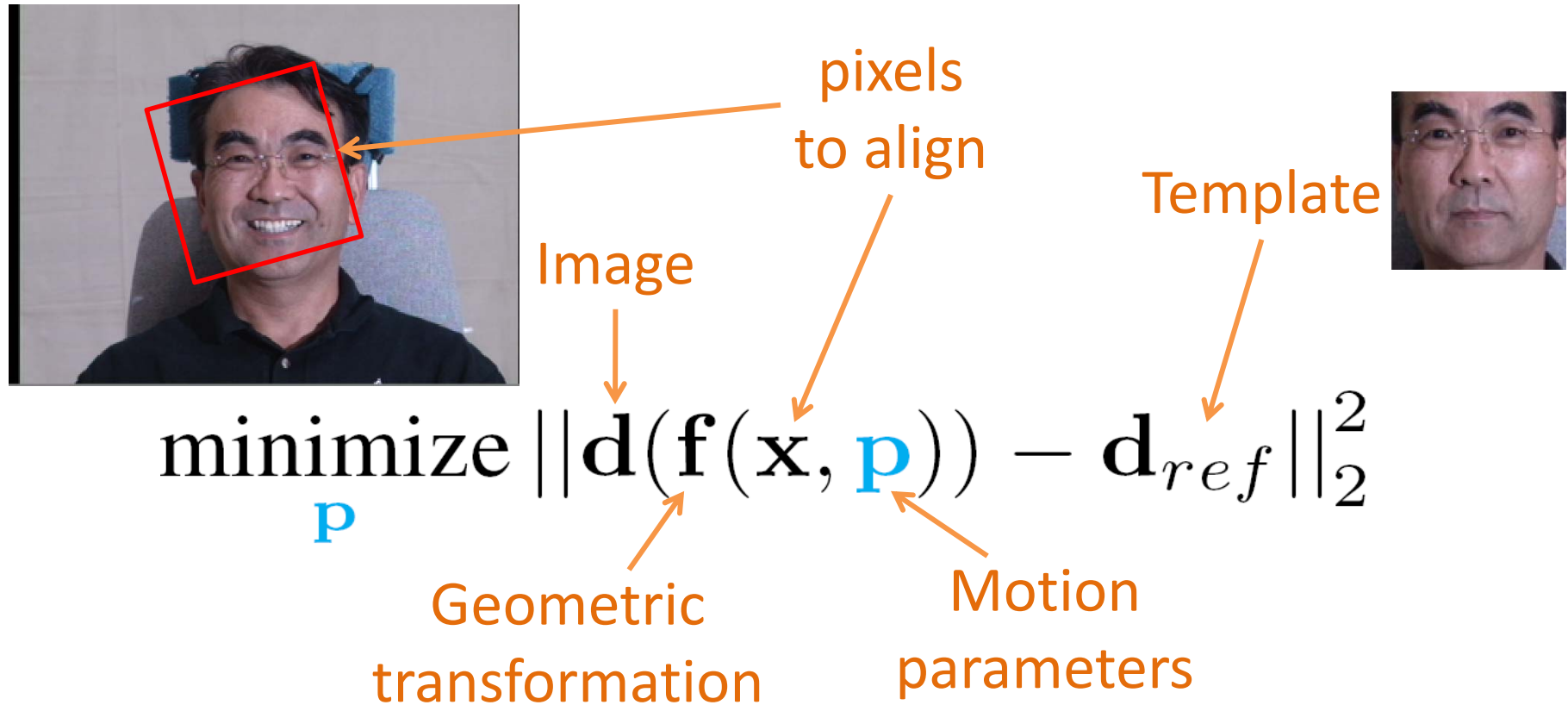


Image

Some work in image alignment

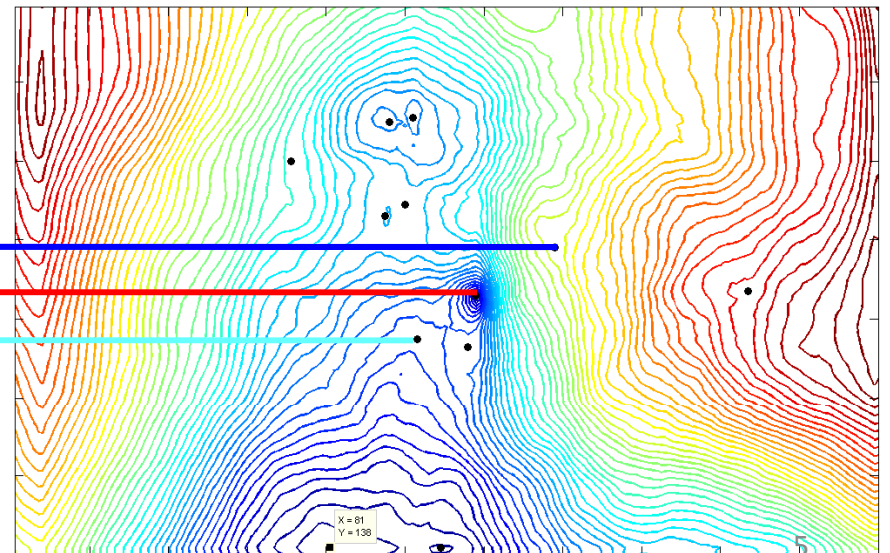
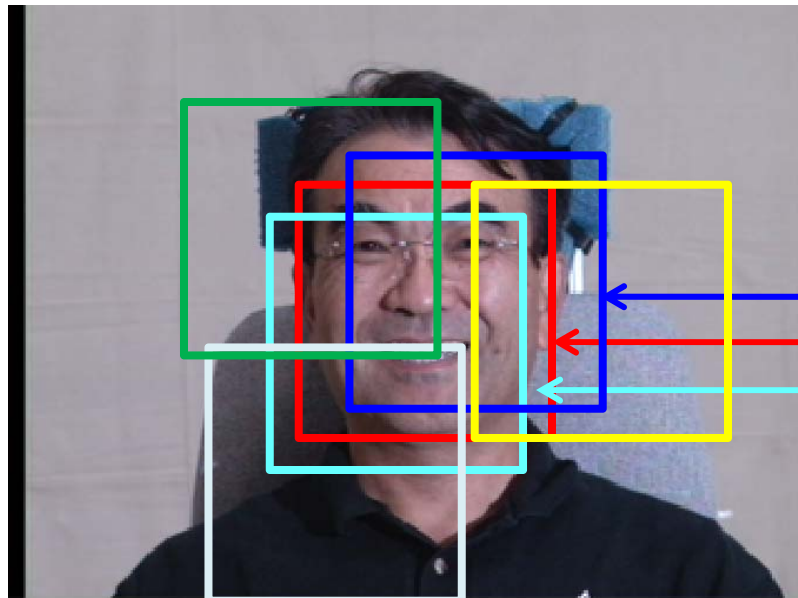
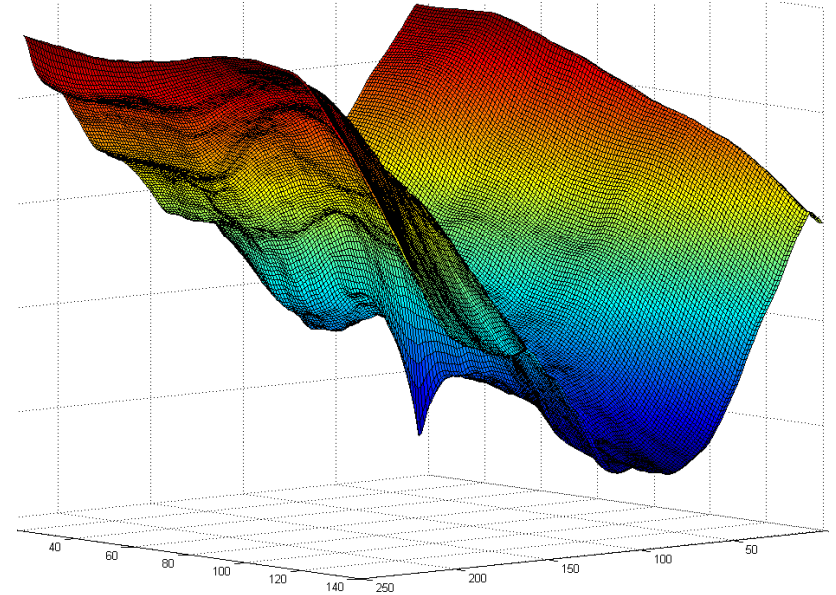
- **Direct methods (pixel domain)**
 - Hierarchical motion estimation [Quam84, Anandan89, Bergen et al 92]
 - Fourier-based [Kuglin & Hines 75, De Castro & Morandi 87, Brown 92, Fleet and Jepson 90, Oppenheim et al 99]
 - Parametric motions [Lucas-Kanade 81, Rehg & Witkin 91, Fuh & Maragos 91, Bergen et al 92, Baker & Matthews 04]
 - Robust metrics [Black & Anandan 96, Black & Rangarajan 96, Stewart 99, Wimmer et al 06]
- **Feature based**
 - Distinctive features [Hannah 74, Moravec 83, Zoghlami et al 97, Capel & Zisserman 98, Cham & Cipolla 98, Badra et al 98, Mclauchlan & Jaenicke 02, Brown & Lowe 03, Brown et al 05]
- **Model-based methods**
 - Generative
 - Active Appearance Models [Cootes et al 98, Matthews & Baker 04]
 - Morphable models [Jones & Poggio 98, Blanz & Vetter 99]
 - 3D [Movellan 03, Marks 06]
 - Kernel PCA [de la Torre & Nguyen 08]
 - Discriminative
 - SVM [Saragih & Goecke 07]
 - Boosting [Liu 07, Wu et al 08, Whitehill & Movellan 08]

Image alignment as an optimization problem

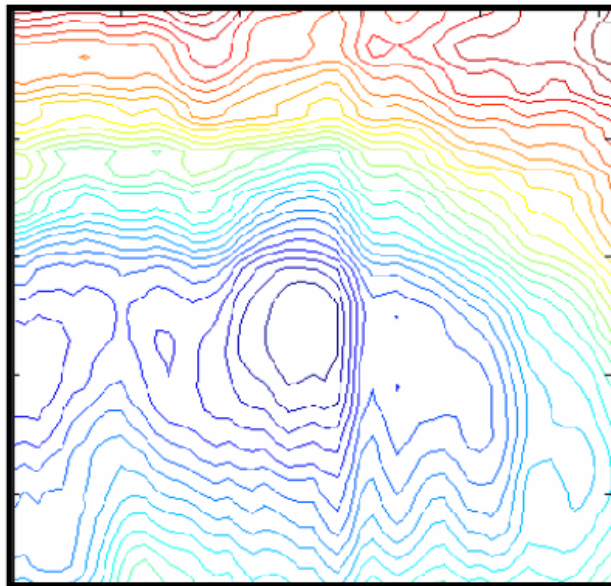
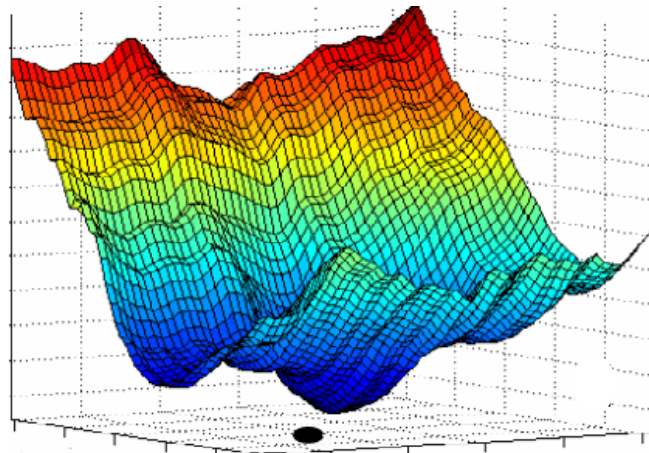


1. Search space (\mathbf{p})
2. Energy function (e.g. L_2)
3. Search strategy (e.g. exhaustive search, branch & bound, gradient-based)

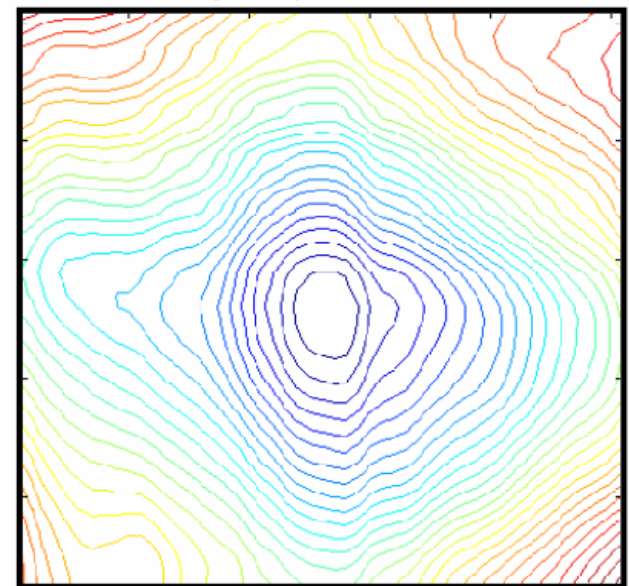
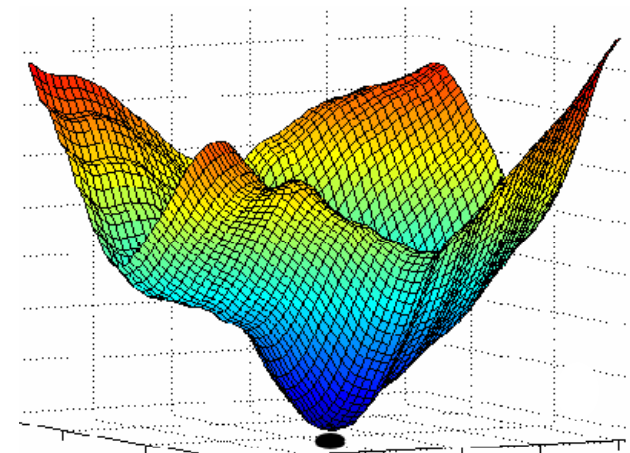
Issues of gradient-based optimization



Local minima problems in image alignment



Learn the
energy function →

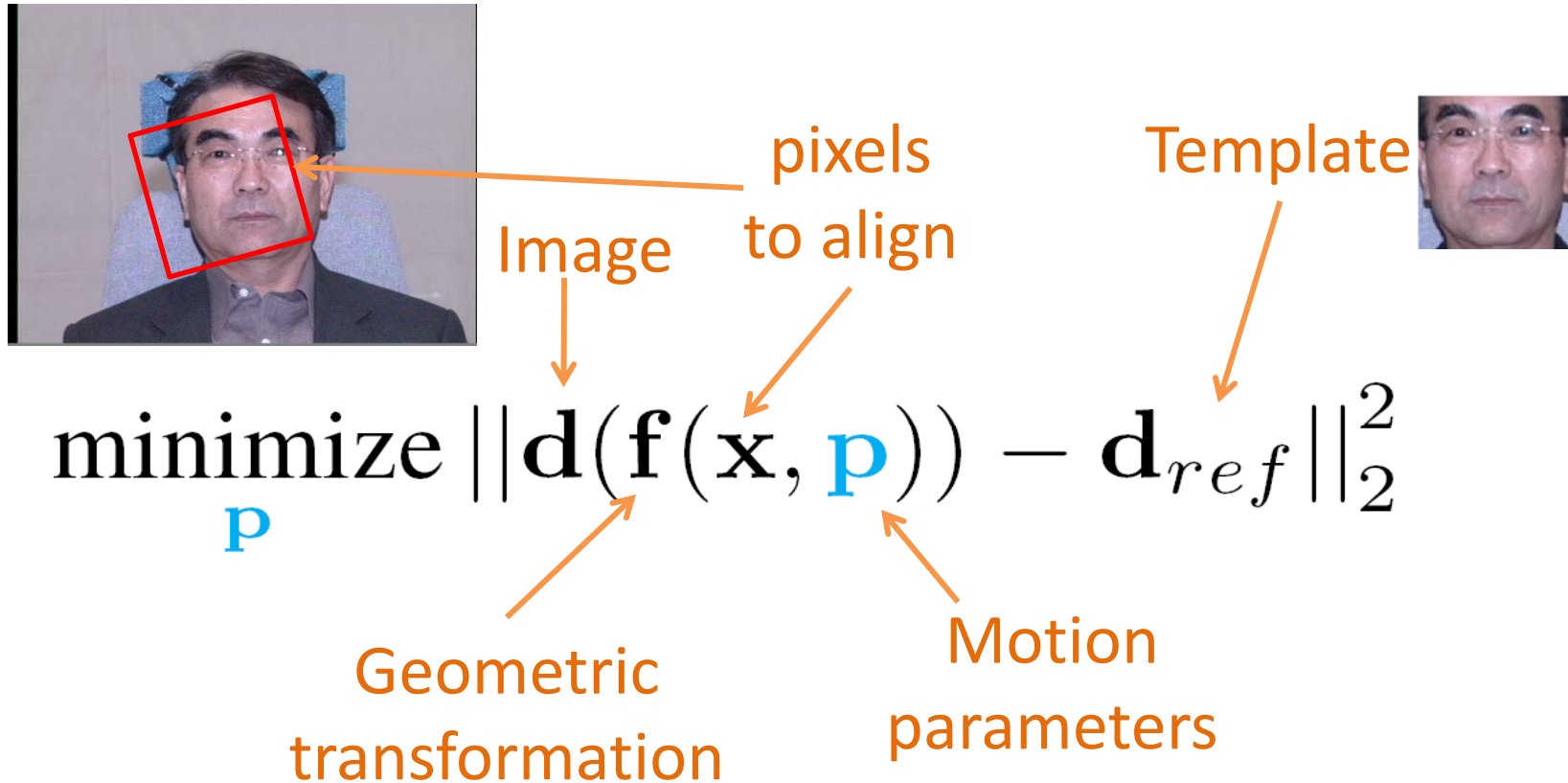


(Black & Jepson98, Cootes & Taylor 01,
Baker et al 04, de la Torre et al 07)

Local minima at only at right places

(Wimmer et al 06, Wu et al 08)

Quadratic cost function



$$E(\mathbf{d}, \mathbf{p}) = \mathbf{d}(\mathbf{f}(\mathbf{x}, \mathbf{p}))^T \mathbf{A} \mathbf{d}(\mathbf{f}(\mathbf{x}, \mathbf{p})) + 2\mathbf{b}^T \mathbf{d}(\mathbf{f}(\mathbf{x}, \mathbf{p}))$$

$$\mathbf{A} = \mathbf{I}_m \text{ and } \mathbf{b} = -\mathbf{d}_{ref}$$

Identity matrix

Quadratic cost function

Learn **A** and **b**

$$E(\mathbf{d}, \mathbf{p}) = \mathbf{d}(\mathbf{f}(\mathbf{x}, \mathbf{p}))^T \mathbf{A} \mathbf{d}(\mathbf{f}(\mathbf{x}, \mathbf{p})) + 2\mathbf{b}^T \mathbf{d}(\mathbf{f}(\mathbf{x}, \mathbf{p}))$$

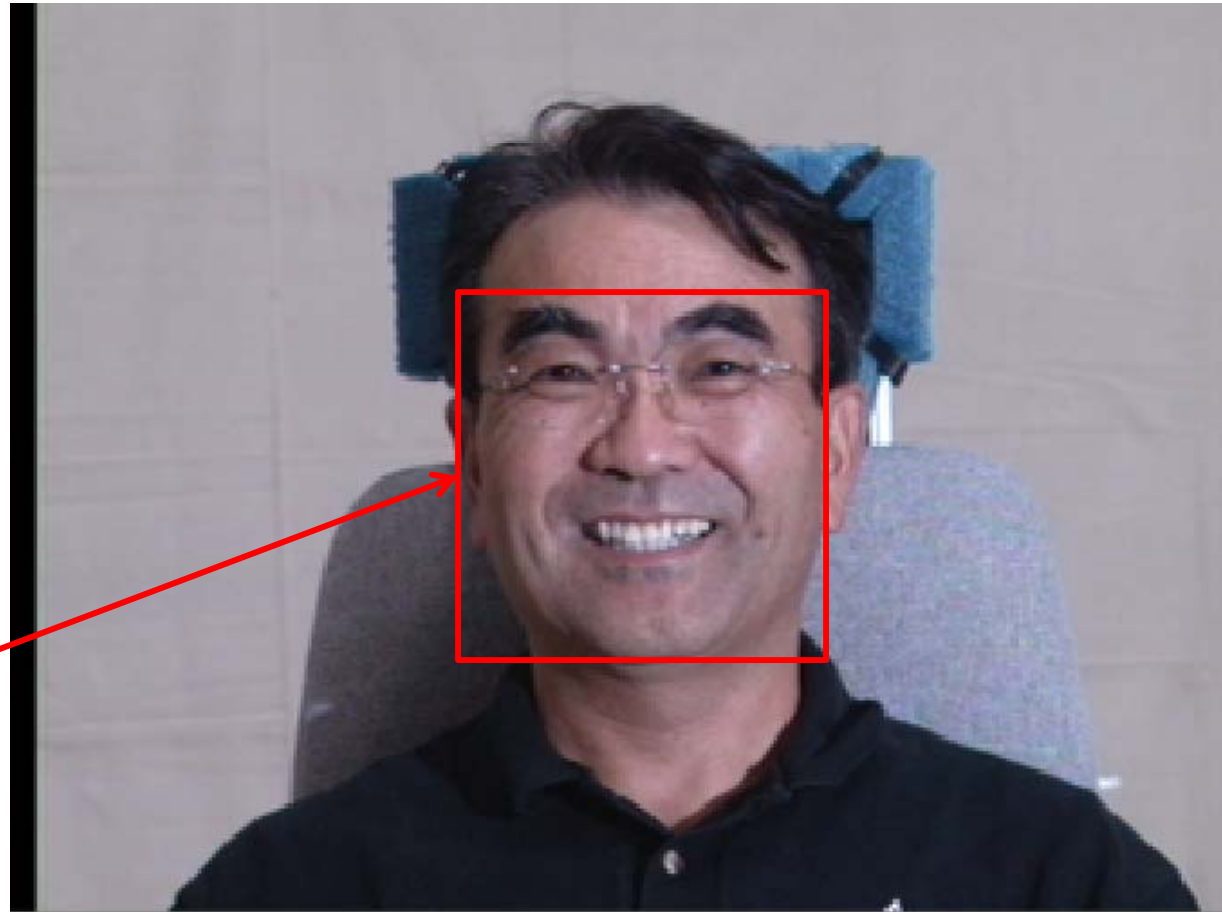
$$\mathbf{A} = \mathbf{I}_m \text{ and } \mathbf{b} = -\mathbf{d}_{ref}$$

Identity matrix

Training data



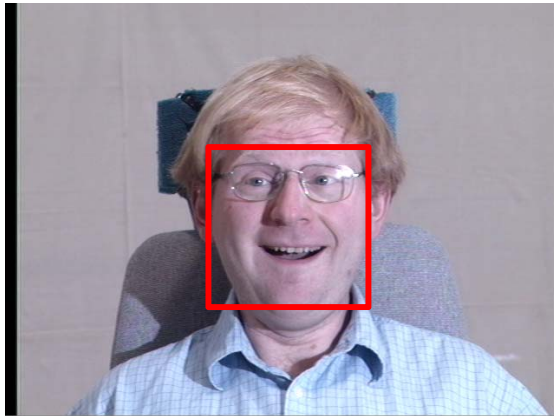
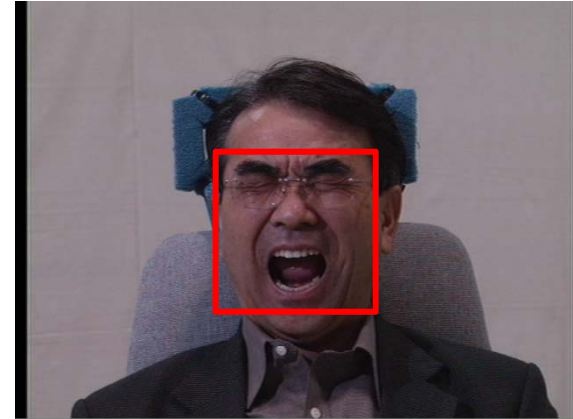
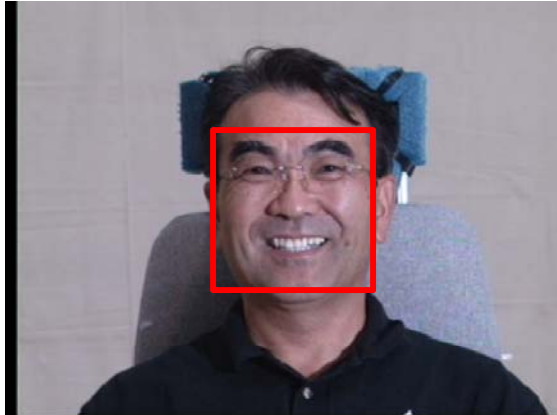
Template



Ground truth

Image

Multiple training images



...

$(\mathbf{d}_i, \mathbf{d}_i^{ref}, \mathbf{p}_i)$
Image Template Ground truth

1st desired property of the cost function

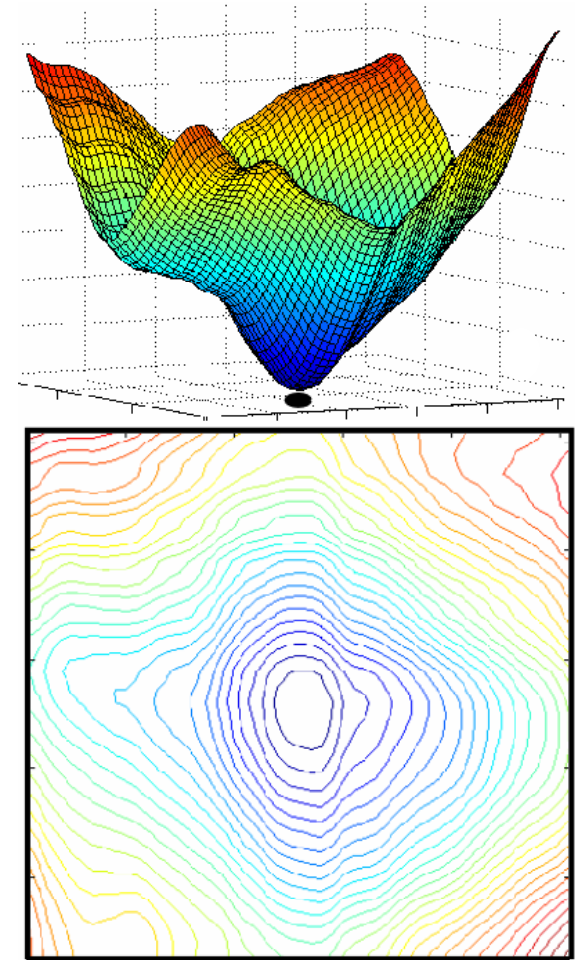
There is a local minimum at the right place!

Training image i^{th}

$$\left. \frac{\partial E(\mathbf{d}_i, \mathbf{p})}{\partial \mathbf{p}} \right|_{\mathbf{p}_i} = \vec{0}$$

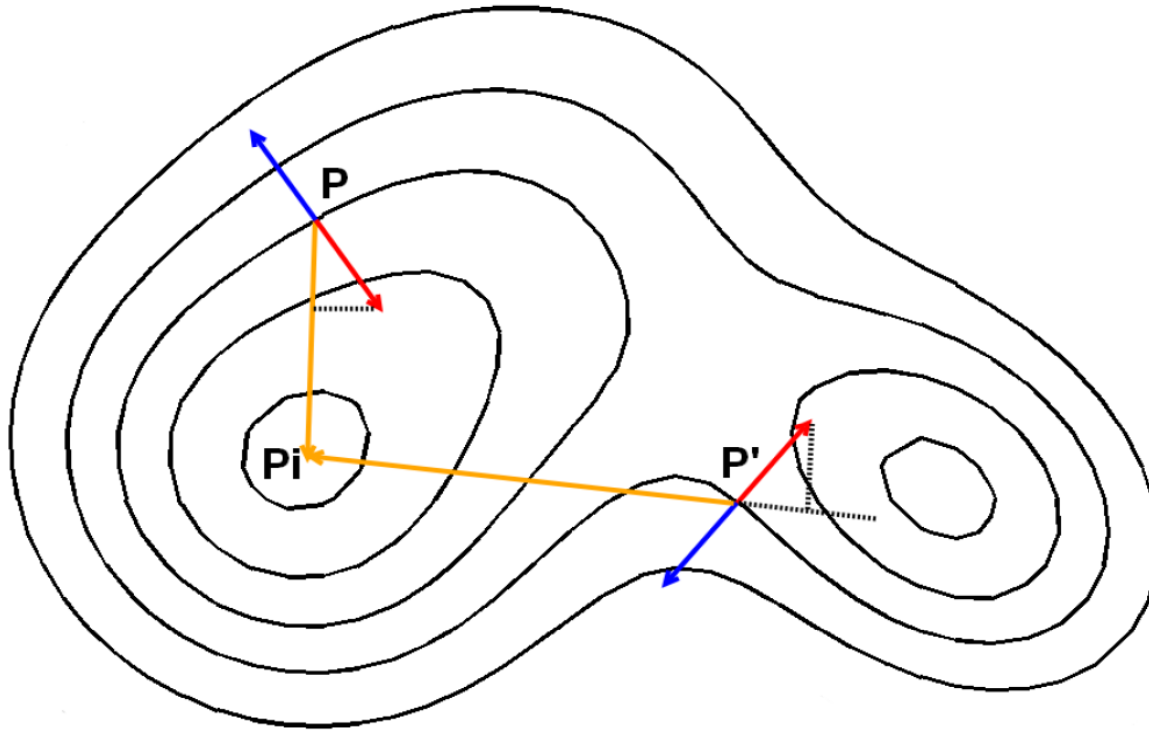
Correct alignment parameter

$$\left\| \left. \frac{\partial E(\mathbf{d}_i, \mathbf{p})}{\partial \mathbf{p}} \right|_{\mathbf{p}_i} \right\|_2^2 = 0$$



2nd desired property of the cost function

Gradient descent agrees with the optimal walking direction



P_i : desired location

At **P** , gradient descent moves closer to **P_i**

At **P'** , gradient descent moves away from **P_i**

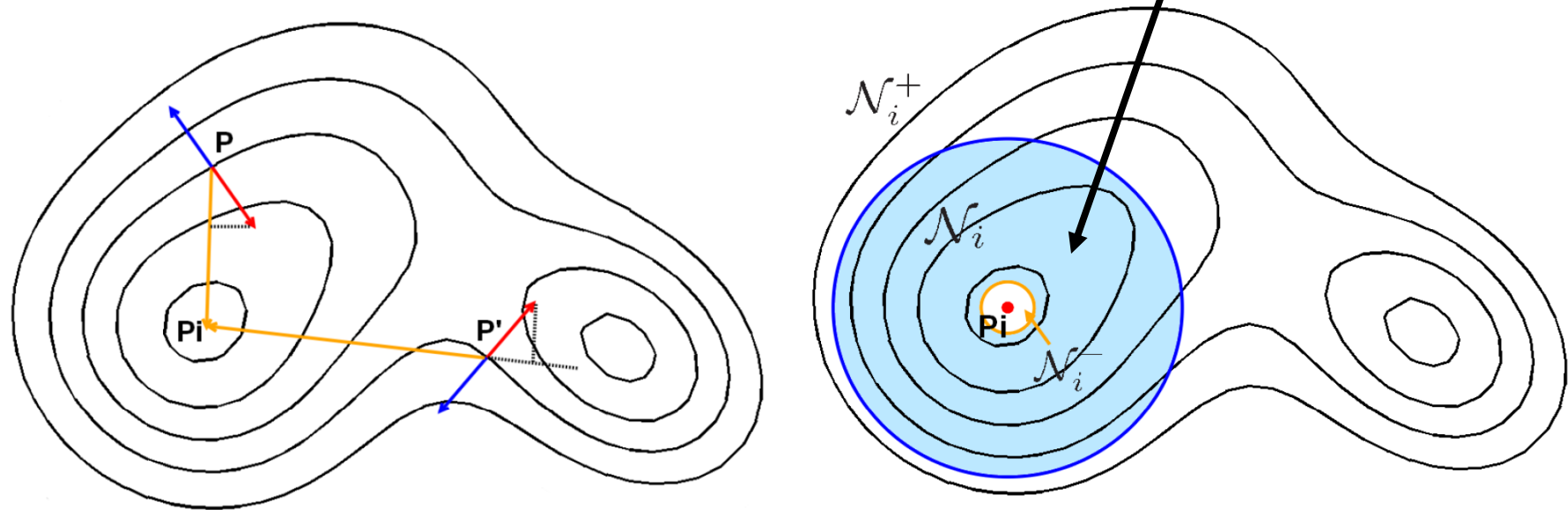
Enforcing the 2nd desired property:

$$\left\langle - \left(\frac{\partial E(\mathbf{d}_i, \mathbf{p})}{\partial \mathbf{p}} \right)^T, \frac{\mathbf{p}_i - \mathbf{p}}{\|\mathbf{p}_i - \mathbf{p}\|_2} \right\rangle > 0 \quad \forall \mathbf{p} \in \mathcal{N}_i$$

Gradient descent direction

Optimal direction

Appropriate neighbourhood



The learning problem

$$\left\| \frac{\partial E(\mathbf{d}_i, \mathbf{p})}{\partial \mathbf{p}} \right\|_{\mathbf{p}_i} \Big|_2^2 = 0 \quad \forall i \quad \text{1st desired property}$$

$$\left\langle - \left(\frac{\partial E(\mathbf{d}_i, \mathbf{p})}{\partial \mathbf{p}} \right)^T, \frac{\mathbf{p}_i - \mathbf{p}}{\|\mathbf{p}_i - \mathbf{p}\|_2} \right\rangle > 0 \quad \forall \mathbf{p} \in \mathcal{N}_i \quad \forall i \quad \text{2nd desired property}$$

$$\text{minimize}_{\mathbf{A}, \mathbf{b}, \xi} \sum_i \frac{1}{2} \left\| \frac{\partial E(\mathbf{d}_i, \mathbf{p})}{\partial \mathbf{p}} \right\|_{\mathbf{p}_i} \Big|_2^2 + C \sum_i \xi_i$$

$$\text{s.t.} \quad \left\langle - \left(\frac{\partial E(\mathbf{d}_i, \mathbf{p})}{\partial \mathbf{p}} \right)^T, \frac{\mathbf{p}_i - \mathbf{p}}{\|\mathbf{p}_i - \mathbf{p}\|_2} \right\rangle \geq M \xi_i \quad \forall \mathbf{p} \in \mathcal{N}_i, \forall i$$

$$\xi_i \geq 0 \quad \forall i$$

The learning problem

- Convex optimization problem
- Infinite number of constraints:
 - Subset of most violated constraints
 - Iteratively update

$$\begin{aligned} & \underset{\mathbf{A}, \mathbf{b}, \xi}{\text{minimize}} \sum_i \frac{1}{2} \left\| \left. \frac{\partial E(\mathbf{d}_i, \mathbf{p})}{\partial \mathbf{p}} \right|_{\mathbf{p}_i} \right\|_2^2 + C \sum_i \xi_i \\ & \text{s.t.} \left\langle - \left(\frac{\partial E(\mathbf{d}_i, \mathbf{p})}{\partial \mathbf{p}} \right)^T, \frac{\mathbf{p}_i - \mathbf{p}}{\|\mathbf{p}_i - \mathbf{p}\|_2} \right\rangle \geq M \xi_i \quad \forall \mathbf{p} \in \mathcal{N}_i, \forall i \\ & \xi_i \geq 0 \quad \forall i \end{aligned}$$

Weighted template alignment

$$\underset{\mathbf{p}}{\text{minimize}} \|\mathbf{d}(\mathbf{f}(\mathbf{x}, \mathbf{p})) - \mathbf{d}_{ref}\|_2^2$$

$$(\mathbf{d}(\mathbf{f}(\mathbf{x}, \mathbf{p})) - \mathbf{d}_{ref})^T \text{diag}(\mathbf{w}) (\mathbf{d}(\mathbf{f}(\mathbf{x}, \mathbf{p})) - \mathbf{d}_{ref})$$

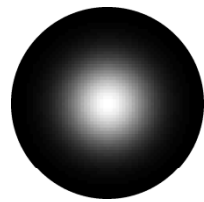
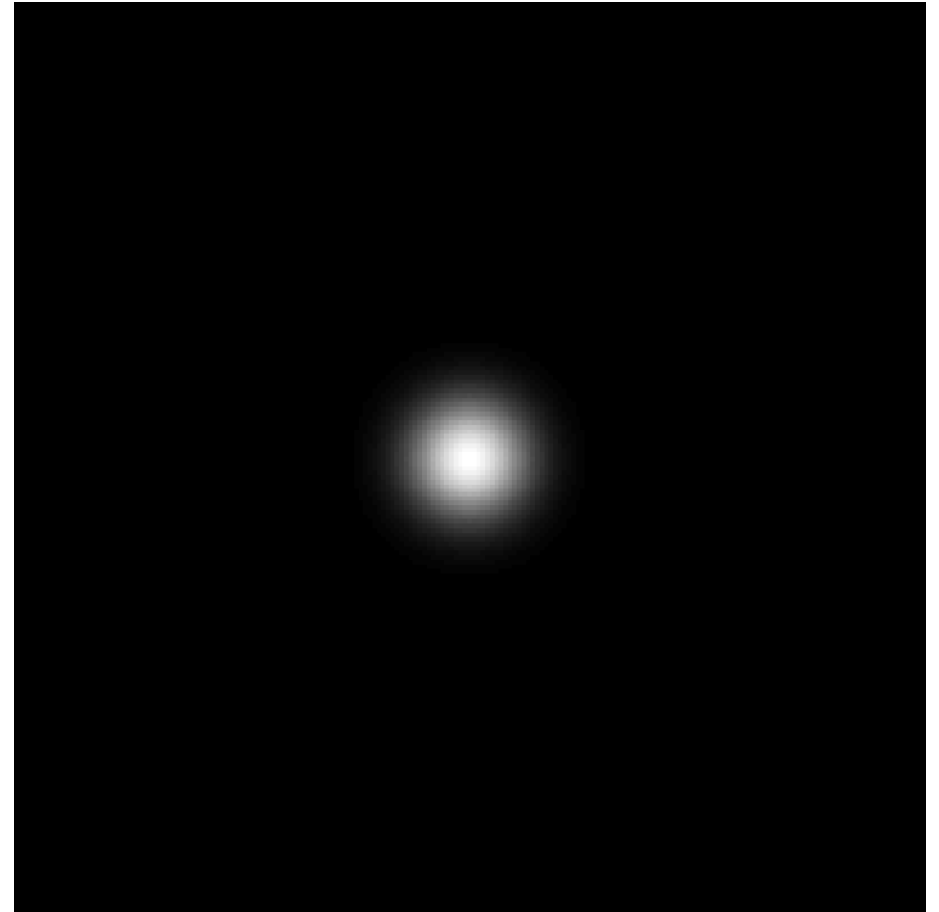
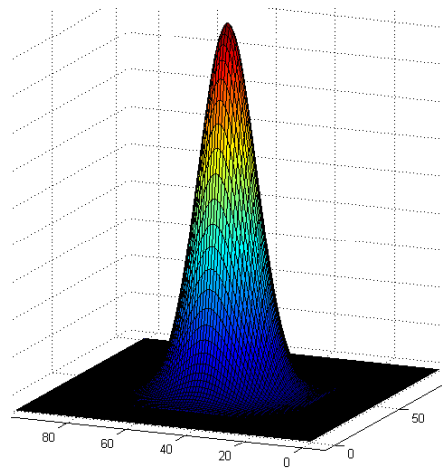
$$E(\mathbf{d}, \mathbf{p}) = \mathbf{d}(\mathbf{f}(\mathbf{x}, \mathbf{p}))^T \mathbf{A} \mathbf{d}(\mathbf{f}(\mathbf{x}, \mathbf{p})) + 2\mathbf{b}^T \mathbf{d}(\mathbf{f}(\mathbf{x}, \mathbf{p}))$$

$$\mathbf{A} = \text{diag}(\mathbf{w})$$

$$\mathbf{b} = -\text{diag}(\mathbf{w}) \mathbf{d}_{ref}$$

$$0 \leq w_i \leq 1$$

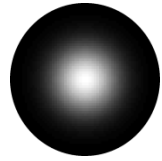
Experiment with the Gaussian



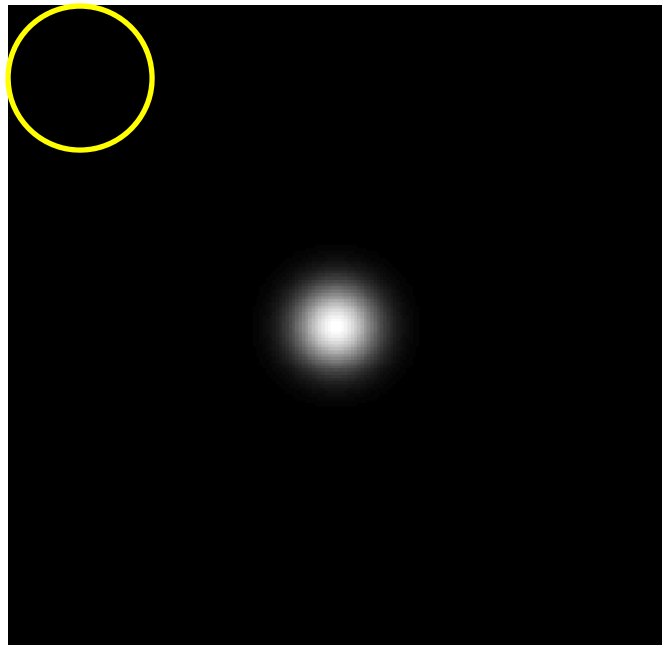
Template

Image

Error surfaces



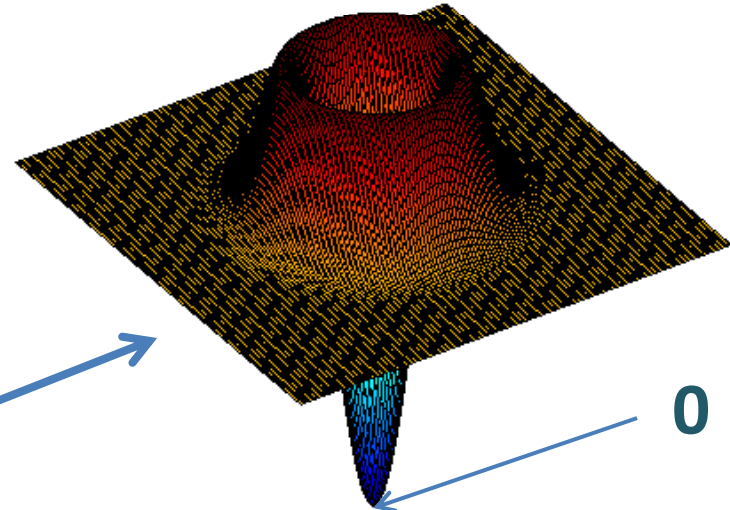
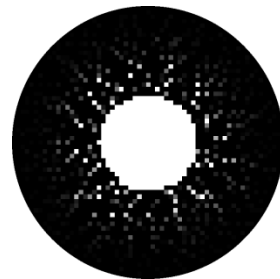
Template



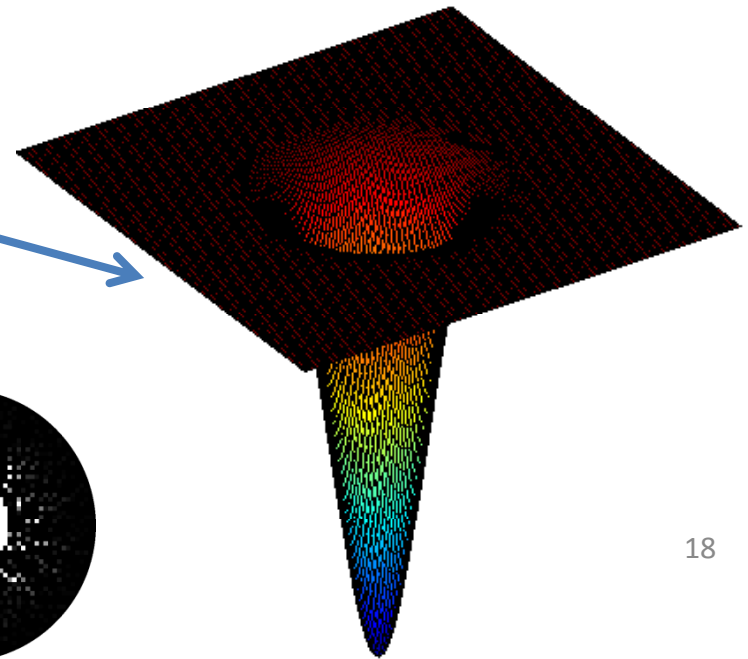
Image

Weighted
template

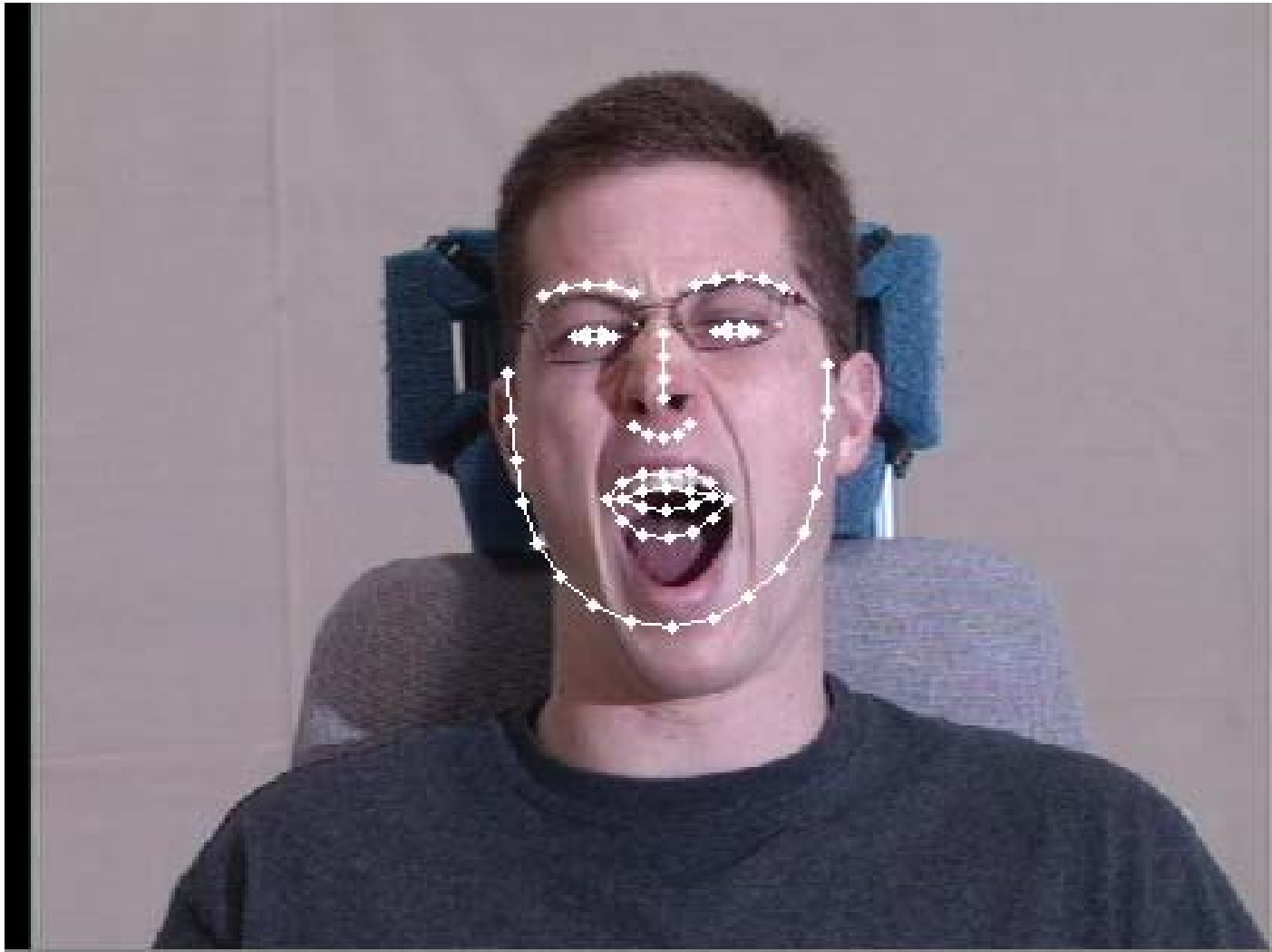
Weights



0



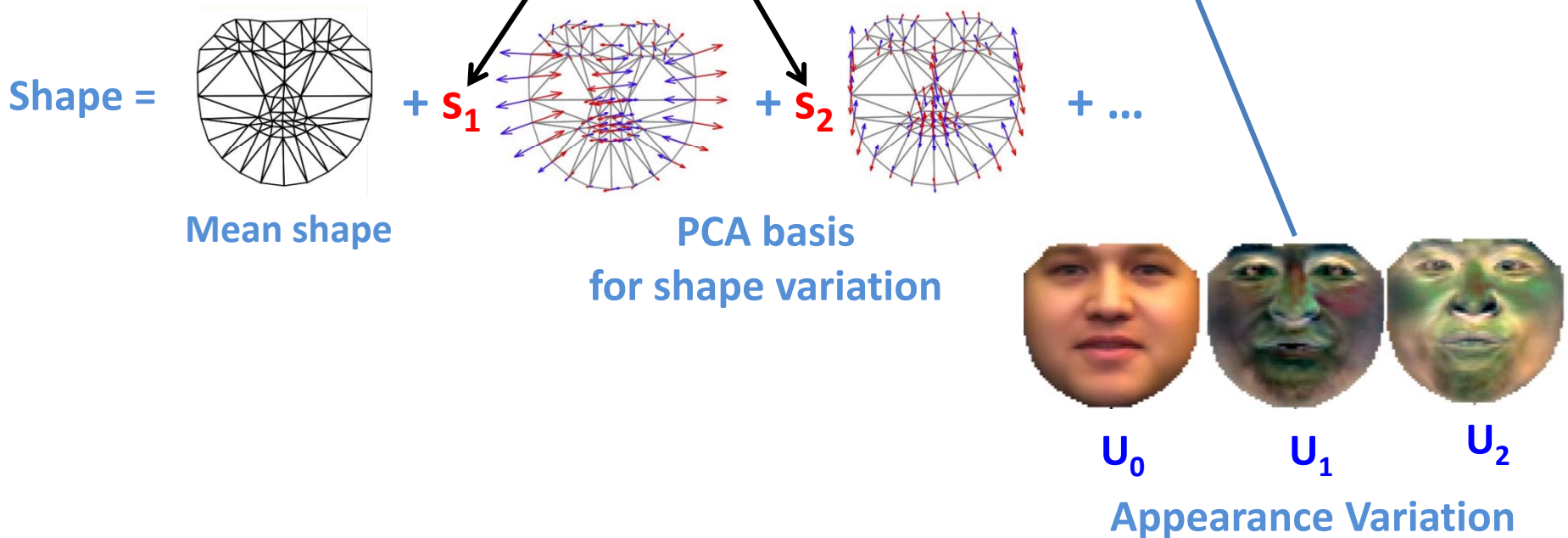
Active Appearance Models (AAMs)



Energy function in AAMs

$$\text{minimize}_{\mathbf{p}, \mathbf{c}} \left\| \mathbf{d}(\mathbf{f}(\mathbf{x}, \mathbf{p})) - \mathbf{U}\mathbf{c} \right\|_2^2$$

Affine parameters + Shape coefficients



(Cootes et al 98, Mathews & Baker 04, Gong et al 00)

Learning AAM energy function

$$\underset{\mathbf{p}, \mathbf{c}}{\text{minimize}} \|\mathbf{d}(\mathbf{f}(\mathbf{x}, \mathbf{p})) - \mathbf{U}\mathbf{c}\|_2^2$$

$$E(\mathbf{d}, \mathbf{p}) = \mathbf{d}(\mathbf{f}(\mathbf{x}, \mathbf{p}))^T \mathbf{A} \mathbf{d}(\mathbf{f}(\mathbf{x}, \mathbf{p})) + 2\mathbf{b}^T \mathbf{d}(\mathbf{f}(\mathbf{x}, \mathbf{p}))$$

$$\mathbf{A} = \mathbf{I}_m - \mathbf{U}\mathbf{U}^T = \mathbf{I}_m - \sum_{i=1}^k \mathbf{u}_i \mathbf{u}_i^T$$

$$\mathbf{b} = \mathbf{0}_m$$

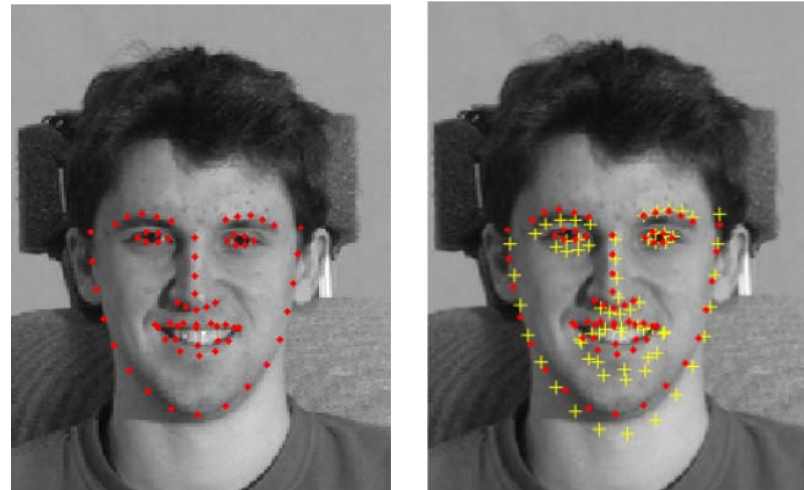
Weighted Basis AAM alignment:

$$\mathbf{A} = \mathbf{I}_m - \sum_{i=1}^K \lambda_i \mathbf{u}_i \mathbf{u}_i^T \quad 0 \leq \lambda_i \leq 1$$

\mathbf{b} No constraint

Experiments with Multi-PIE database

- Data:
 - 337 subjects
 - Frontal, directly illuminated
 - Expressions: smile, disgust, squint, surprise, scream
 - 68 hand labeled landmarks
 - Training, validation, test sets: 400/200/500
- Motion:
 - Affine + non-rigid
- Testing:
 - Random perturbation



Results of weighted PCA basis

Alignment problem becomes harder



Perturbation amount	0.75	1.00	1.25	1.5
Initial	1.50 ± 0.50	2.16 ± 0.76	2.74 ± 1.04	3.08 ± 1.08
PCA 90%	0.72 ± 0.40	0.86 ± 0.66	0.94 ± 0.72	1.20 ± 1.30
PCA 80%	0.80 ± 0.46	0.86 ± 0.68	0.98 ± 0.74	1.14 ± 1.00
PCA 70%	0.82 ± 0.40	0.86 ± 0.50	0.94 ± 0.60	1.10 ± 0.92
Ours	0.74 ± 0.38	0.80 ± 0.50	0.86 ± 0.58	0.96 ± 0.78

Mean error + std, the smaller the better!

Summary

- Learning an optimal metric (A , b of the cost function) for image alignment
 - Template alignment
 - Active Appearance Models (AAMs)
- Local minima at and only at the right places
- Convex quadratic formulation

Less on search strategies
More on what we are searching for

Thank you