Activized Learning:

Transforming Passive to Active with Improved Label Complexity

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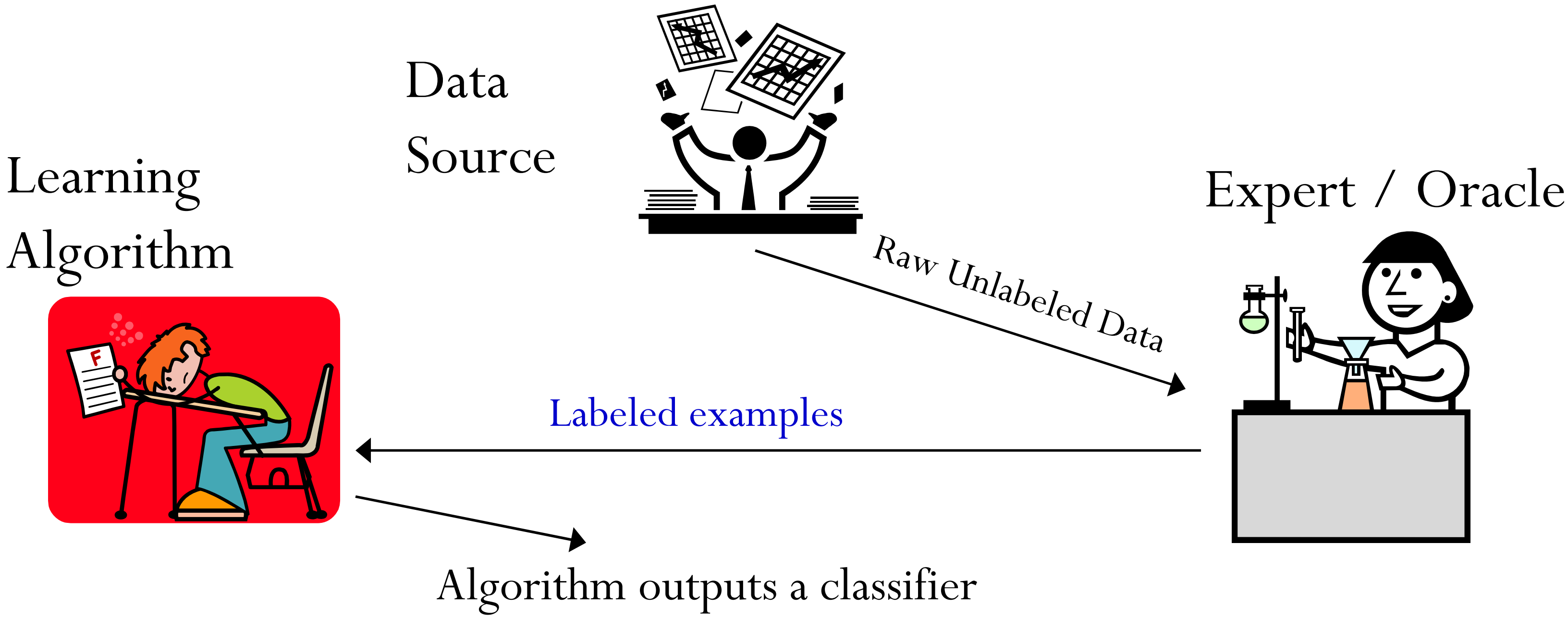
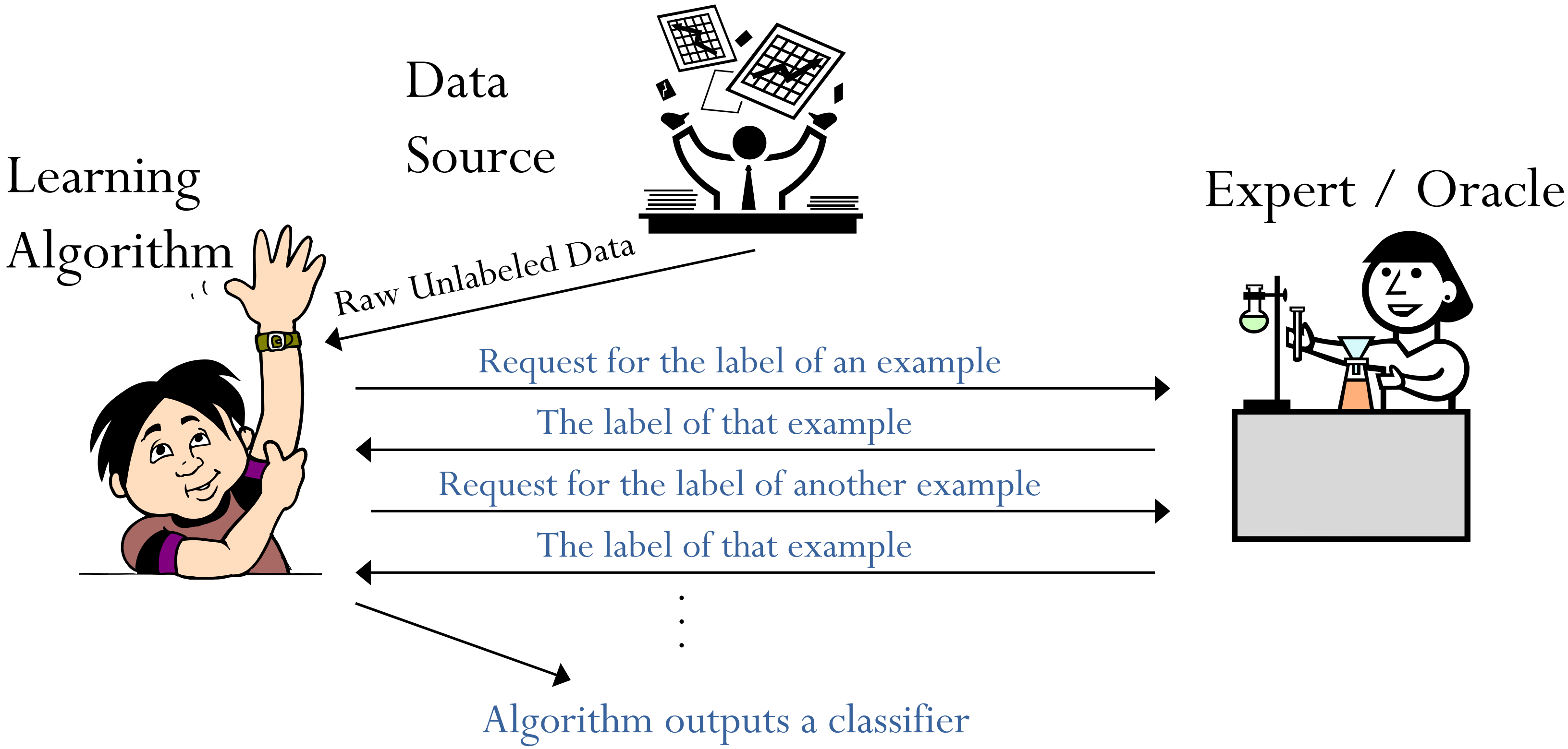
Passive Learning



Active Learning



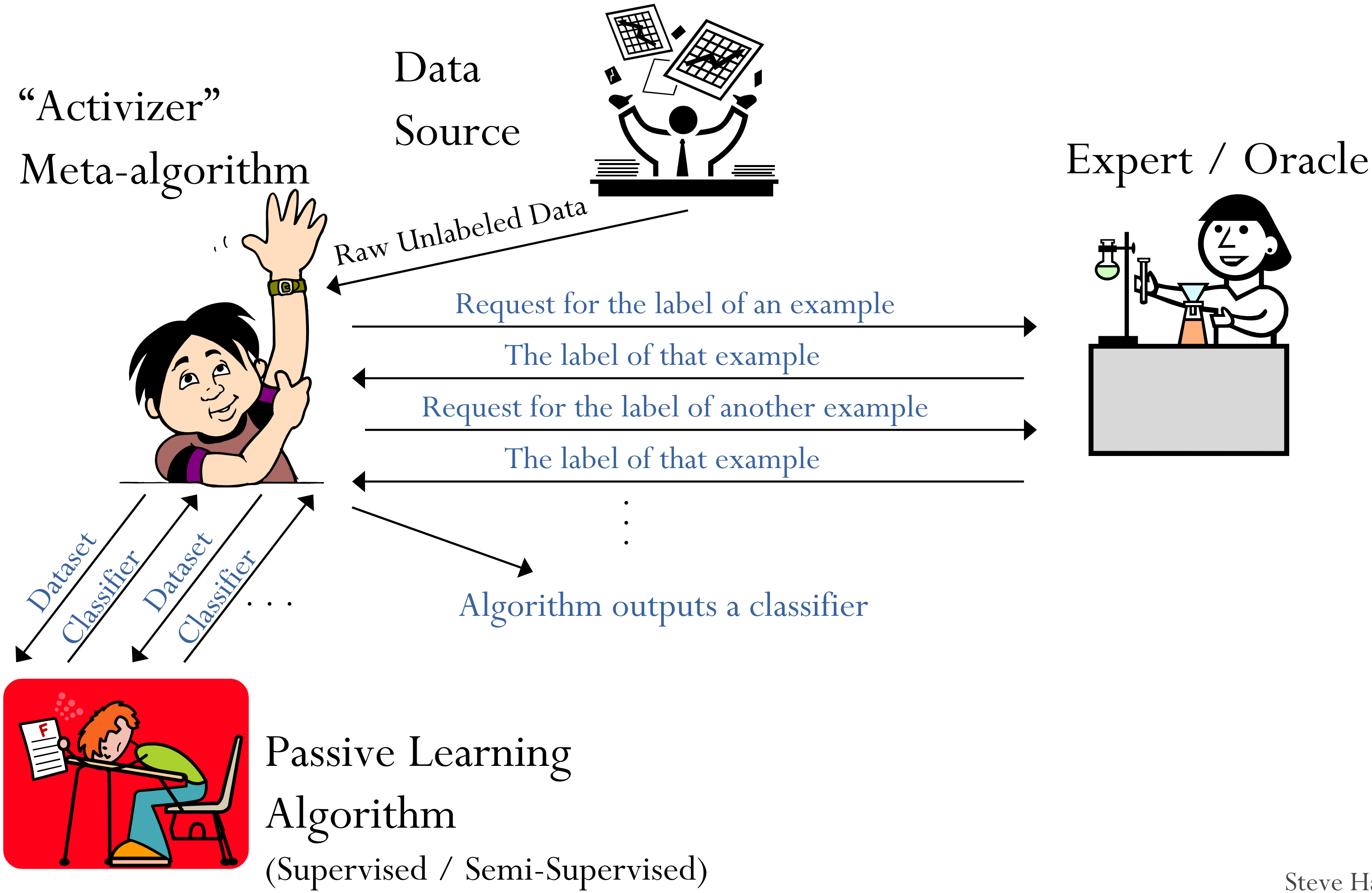
Active Learning

How many label requests are required to learn?

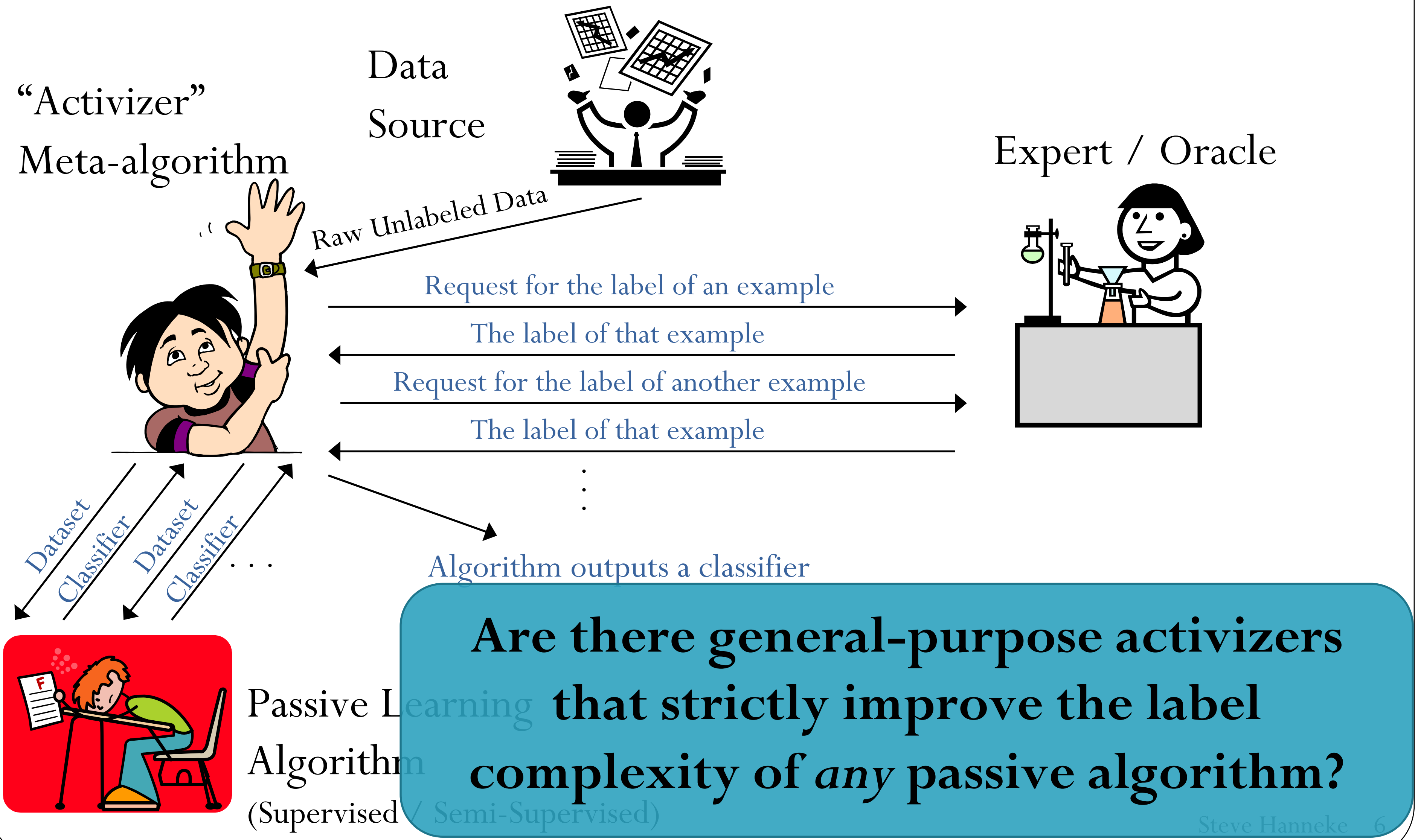
Label Complexity

e.g., Das04, Das05, DKM05, BBL06, Kaa06, Han07a&b, BBZ07, DHM07, BHW08

Activized Learning



Activized Learning



Are there general-purpose activizers that strictly improve the label complexity of *any* passive algorithm?

An Example: Threshold Classifiers

A simple activizer for any threshold-learning algorithm.

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A simple activizer for any threshold-learning algorithm.

Take n/2 unlabeled examples, request their labels

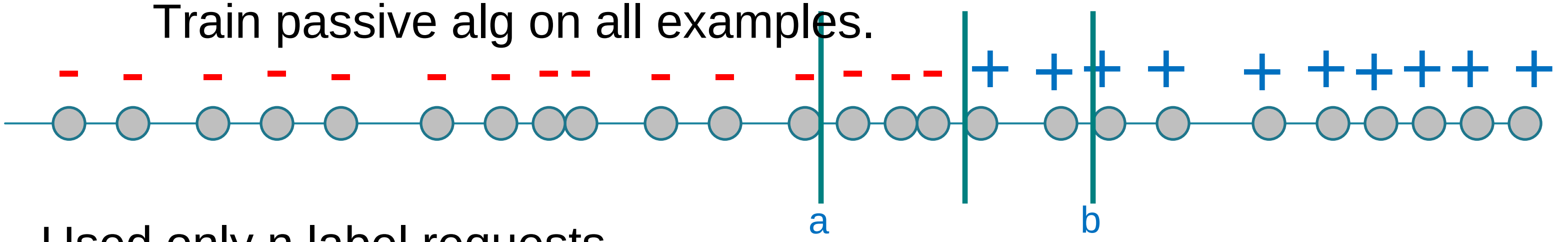
Locate the closest -/+ points: a,b

Estimate P([a,b]), and sample ≈ n/(4P([a,b])) unlabeled examples

Request the labels in [a,b]

Label rest ourselves.

Train passive alg on all examples.



Used only n label requests,

but get a classifier trained on $\Omega(n^2)$ examples!

Improvement in label complexity over passive.

(in this case, apply idea sequentially to get exponential improvement)

Outline

- Formal model
- Exciting New Results ☺
- Dealing with noise?
- Conclusions & open problems

Formal Model

$\mathcal{X}$: *Instance space*
$\mathbb{C}$: *Concept space* (a set of classifiers $h : \mathcal{X} \to \{-1, 1\}$)
d: VC dimension of $\mathbb{C}$ (assume $d < \infty$)
$\mathcal{D}$: *Distribution* over $\mathcal{X}$

Unknown *target function* $f \in \mathbb{C}$
$er(h) = \mathbb{P}_{X \sim \mathcal{D}}[h(X) \neq f(X)]$

Sequence of i.i.d. *training examples* $x_1, x_2, \ldots \sim \mathcal{D}$

Algorithm chooses any x_i, receives label $f(x_i)$, repeat

The objective is to produce some $h : \mathcal{X} \to \{-1, 1\}$ s.t. $er(h)$ is small.

Formal Model

Definition: An algorithm $A(n, \delta)$ achieves *label complexity* $\Lambda(\epsilon, \delta, f, \mathcal{D})$ for $\mathbb{C}$ if it outputs a classifier h_n after at most n label requests, and for any target function $f \in \mathbb{C}$, distribution $\mathcal{D}$, $\epsilon > 0$, $\delta > 0$, for any $n \geq \Lambda(\epsilon, \delta, f, \mathcal{D})$,

$$\mathbb{P}[er(h_n) \leq \epsilon] \geq 1 - \delta.$$

Definition: Suppose A_p is a passive algorithm achieving a label complexity $\Lambda_p(\epsilon, \delta, f, \mathcal{D})$ for $\mathbb{C}$. A (meta-)algorithm A_a *activizes* A_p for $\mathbb{C}$ if $A_a(A_p, n, \delta)$ achieves a label complexity $\Lambda_a(\epsilon, \delta, f, \mathcal{D})$ for $\mathbb{C}$, where $\exists c < \infty$ s.t. $\forall f \in \mathbb{C}, \mathcal{D}$: $1 \ll \Lambda_p(\epsilon, \delta, f, \mathcal{D}) \ll \infty$,

$$\Lambda_a(c\epsilon, c\delta, f, \mathcal{D}) = o(\Lambda_p(\epsilon, \delta, f, \mathcal{D})).$$

Recall $s(\epsilon) = o(t(\epsilon))$ iff $\lim\limits_{\epsilon \longrightarrow 0} \frac{s(\epsilon)}{t(\epsilon)} = 0$.

Naïve Approach

Algorithm: **NaiveActivizer**($\mathcal{A}_p$,n,δ)
0. Sample $n/2$ examples Q, request their labels
1. Let $V \leftarrow \{h \in \mathbb{C} : er_Q(h) = 0\}$
2. Estimate $\hat{\Delta} \approx \mathbb{P}(x : \exists h_1, h_2 \in V \text{ s.t. } h_1(x) \neq h_2(x))$
3. Sample $\approx n/(4\hat{\Delta})$ examples $\mathcal{L}$
4. Request label of all x s.t. $\exists h_1, h_2 \in V : h_1(x) \neq h_2(x)$
5. Label the rest ourselves
6. Return the output of $\mathcal{A}_p(\mathcal{L}, \delta)$

Produces a perfectly labeled data set, which we can feed into any passive algorithm!
So we get a natural fallback guarantee.

But does it always *improve* over the passive algorithm?

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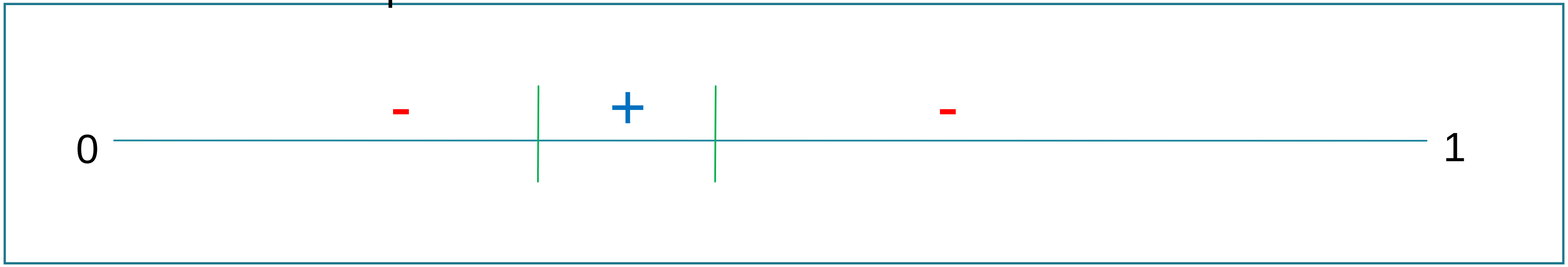
A more subtle example: Intervals



Naïve Approach

Algorithm: **NaiveActivizer**$(\mathcal{A}_p, n, \delta)$

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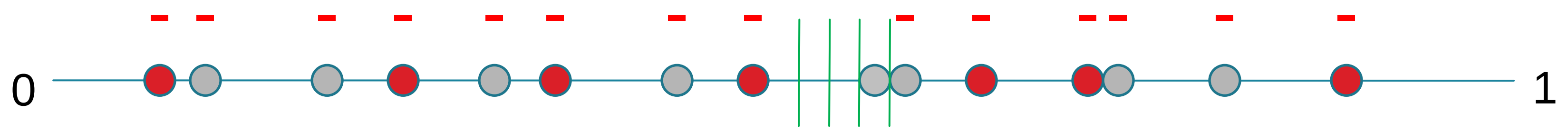

Suppose the target labels everything "-1"

Passive algorithm still trained with just O(n) examples. No improvements. ☹

A Simple Activizer

Algorithm: **SimpleActivizer**($\mathcal{A}_p$,n,δ)
0. Sample $n/3$ examples Q, request their labels
1. Let $V \leftarrow \{h \in \mathbb{C} : er_Q(h) = 0\}$, $S \leftarrow \{\}$
2. For $k = 1, 2, \ldots, d+1$ (where $d = VC(\mathbb{C})$)
3.   Estimate $\hat{\Delta} \approx \mathbb{P}(x : V$ shatters $S \cup \{x\})$
4.   Sample $\approx n/(6d\hat{\Delta})$ examples $\mathcal{L}_k$
5.   Request label of all x s.t. V shatters $S \cup \{x\}$
6.   Label the rest ourselves (opposite to unrealizable labels)
7.   Sample x_k s.t. V shatters $S \cup \{x_k\}$ (if exists), add to S
8. Return $ActiveSelect(\{\mathcal{A}_p(\mathcal{L}_1, \delta), \ldots, \mathcal{A}_p(\mathcal{L}_{d+1}, \delta)\}, n/3)$

Subroutine: **ActiveSelect**($\{h_1, h_2, \ldots, h_{d+1}\}$,$m$)
0. For each pair h_i, h_j
1.   Sample $m/(d+1)^2$ examples x s.t. $h_i(x) \neq h_j(x)$
2.   Let m_{ij} denote the number of mistakes h_i makes
3. Return $h_{\hat{i}}$, where $\hat{i} = \mathrm{argmin}_i \ \max_j \ m_{ij}$

A Simple Activizer

Algorithm: **SimpleActivizer**$(\mathcal{A}_p, n, \delta)$

0. Sample $n/3$ examples Q, request their labels
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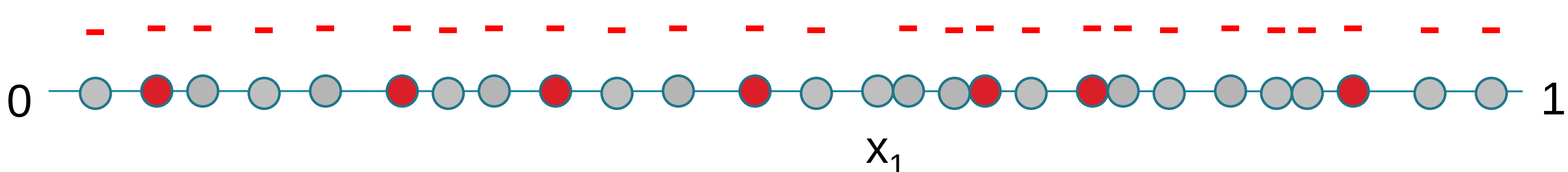
Intervals revisited



Again, suppose the target labels everything "-1"

Passive algorithm trained on $\Omega(n^2)$ samples. Improved label complexity. ☺

(can apply steps 0/1 and 5 sequentially, updating V after every label request, for more savings)

Does This Activize *Any* Passive Algorithm?

This Activizes *Any* Passive Algorithm!

Theorem: For any $\mathbb{C}$, SimpleActivizer activizes any passive learning algorithm.

Corollary: For any $\mathbb{C}$, there is an active learning algorithm that achieves a label complexity $\Lambda_a(\epsilon, \delta, f, \mathcal{D})$ such that $\forall f \in \mathbb{C}, \mathcal{D}$,

$$\Lambda_a(\epsilon, \delta, f, \mathcal{D}) = o(1/\epsilon).$$

[HLW94] passive algorithm has O(1/ε) sample complexity.

This Activizes *Any* Passive Algorithm!

Theorem: For any $\mathbb{C}$, SimpleActivizer activizes any passive learning algorithm.

Proof idea: if $\hat{\Delta} \to 0$ for $k = 1$, we're done.

Otherwise, $\lim_{n\to\infty} \mathbb{P}\{x : \exists h_1, h_2 \in V, h_1(x) \neq h_2(x)\} > c$, for some c.

For large enough n, x_1 will be in this limiting region.

In particular, $\inf_{h\in V:h(x)=+1} er(h) = \inf_{h\in V:h(x)=-1} er(h) = 0$.

So (w.p.1), for any x agreed upon by all $h \in V : h(x_1) = +1$ or all $h \in V : h(x_1) = -1$, the agreed upon label is correct.

So basically, we know the label of any x s.t. $\{x_1, x\}$ is not shattered.

Repeat the argument for $k > 1$ until we get a k where $\hat{\Delta} \to 0$, but then $|\mathcal{L}_k| \gg n$, so we're done.

Efficiency?

- Need to be able to test shatterability of a set of $\leq$ d points, subject to consistency with a set of O(n) labeled examples.

- For some concept spaces, could be exponential in d (or worse).
- But in many cases, it may be efficient. (e.g., linear separators?)

Dealing with Noise

Have an arbitrary distribution $\mathcal{D}_{XY}$ over $\mathcal{X} \times \{-1, +1\}$,
so label complexity for $\mathbb{C}$ is written $\Lambda(\epsilon, \delta, \mathcal{D}_{XY})$.
Now ϵ represents excess over best error rate in $\mathbb{C}$: want to guarantee

$$\mathbb{P}\left[er(h_n) - \inf_{f \in \mathbb{C}} er(f) \leq \epsilon\right] \geq 1 - \delta.$$

Dealing with Noise

Replace version space $V = \{h \in \mathbb{C} : er_Q(h) = 0\}$ with noise-robust version space

$$V = \{h \in \mathbb{C} : er_Q(h) - \min_{h' \in \mathbb{C}} er_Q(h') \leq O(n^{-1/2})\}.$$

Applied to a particular passive algorithm, this modification of SimpleActivizer achieves label complexity[1]

$$\Lambda_a(\epsilon, \delta, \mathcal{D}_{XY}) = o(1/\epsilon^2).$$

Under Tsybakov's noise conditions w/ exponent κ, a more careful variant achieves

$$\Lambda_a(\epsilon, \delta, \mathcal{D}_{XY}) = o(1/\epsilon^{2-1/\kappa}).$$

Open Question: Can we activize any passive algorithm, even with noise?

Open Question: Can we activize some empirical error minimizing algorithm?

[1]Technically, an additional slight modification is needed to handle the case where the Bayes optimal classifier is not in C. Details included in a forthcoming paper.

Conclusions & Open Questions

- Can activize any passive learning algorithm
  (in the zero-error, finite VC dimension case)

- Question: What about infinite VC dimension?
- Question: Can we give more detailed bounds on Λ_a?
- Question: Can we always activize, even when there is noise?

Thank You