

# When causality matters for prediction: Investigating the practical tradeoffs

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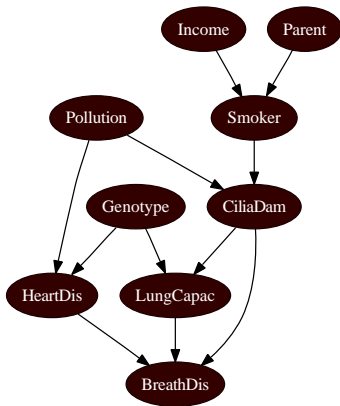
Machine Learning Department  
School of Computer Science

NIPS 2008 Workshop on Causality: Objectives and Assessment

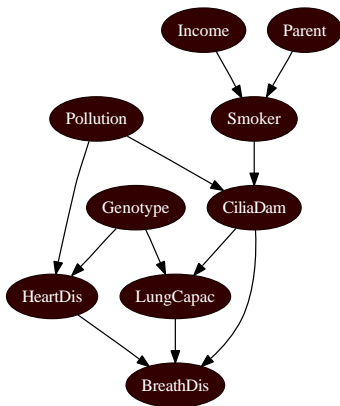
# Causal Discovery

## The Usual Setup:

- Unobserved data generating process
- i.i.d. sample



# Causal Discovery



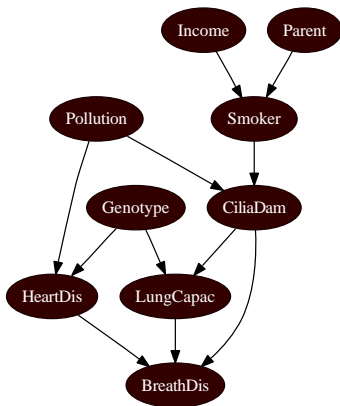
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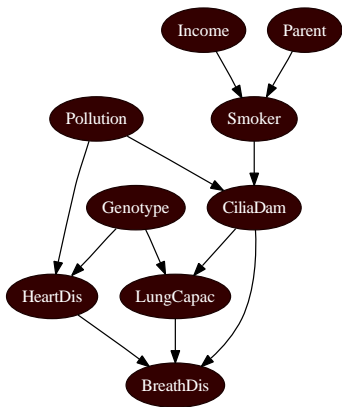
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- Compare to “ground truth”, i.e. simulations, experimental studies, expert knowledge

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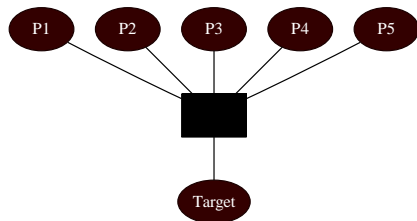
## Assessment:

- Compare to “ground truth”, i.e. simulations, experimental studies, expert knowledge

## Focus:

- Learn network models that accurately depict the data generating mechanism

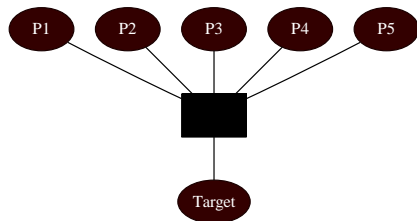
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## The Standard Problem:

- “Target” variable associated with “predictor” variables
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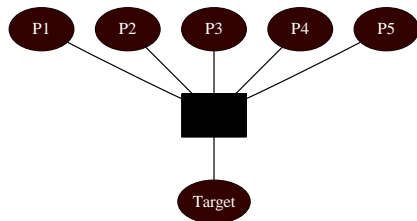
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- Compare predictions to known target values, i.e. testing data, cross validation

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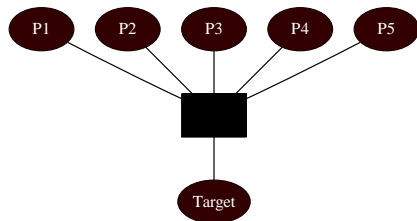
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## Focus:

- Train classifier/regression model that minimizes loss function, e.g. makes accurate predictions
- Model need not resemble the true data generating mechanism, i.e. Naive Bayes

## Causal Discovery and Prediction

Previous focus: predicting the effects of possible interventions:

- Specify the distribution for a manipulated population
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- System stabilizes

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Causation and Prediction Challenge:

- Training data from unmanipulated population
- (Structural) intervention is performed
- System stabilizes
- Draw i.i.d sample for predictors from manipulated population
- Predict target using predictor values from stabilized manipulated distribution

# Causation and Prediction Challenge

## Results:

- In some instances, noncausal methods outperformed causal methods

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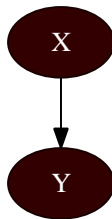
- Is causality useful for standard prediction tasks?
- Is it useful in practice?
- Is this a realistic scenario?

### Possible Explanations:

- Sampling error, overfitting
- Parametric assumptions do not hold, i.e. linearity, Gaussianity
- Prediction for target is invariant under the manipulation.

# Invariance of prediction under manipulations

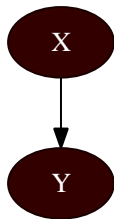
Simple example:



Bayes optimal prediction for  $Y$  is  $P(Y|X)$

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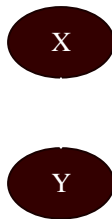


Bayes optimal prediction for  $Y$  is  $P(Y|X)$

- Manipulating  $X$  does not change distribution of  $P(Y|X)$ , still Bayes optimal
- Prediction (once system stabilizes) is invariant under manipulation

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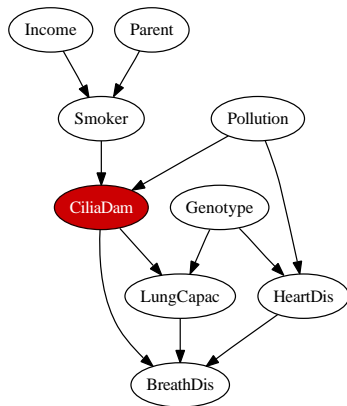
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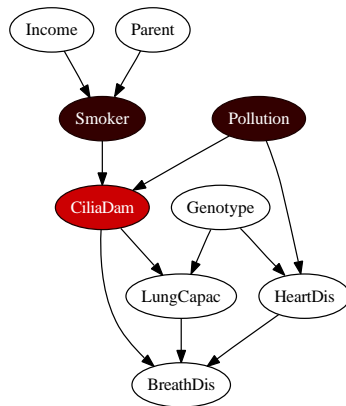
- Manipulating  $Y$  does change distribution of  $P(Y|X)$ ,  $Y$  depends on manipulation
- Incorrect predictions in stabilized manipulated population

# Terminology



Predict CiliaDam

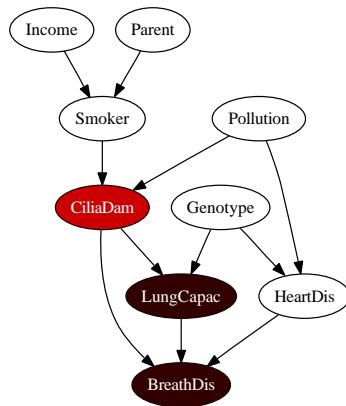
# Terminology



Parents of CiliaDam

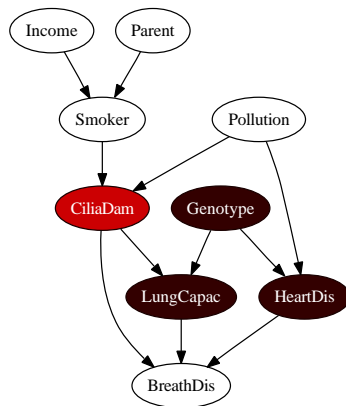


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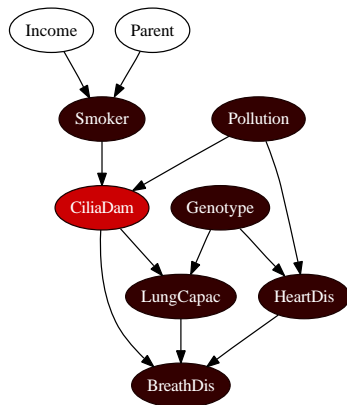
Children of CiliaDam

# Terminology



Coparents (spouses) of CiliaDam

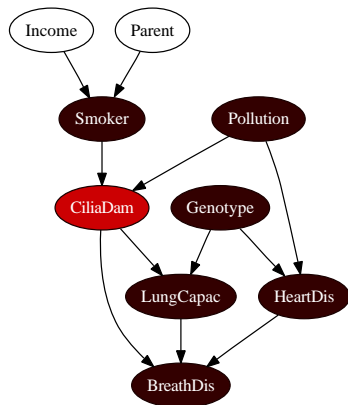
# Terminology



## Definition

In a causal Bayesian network  $\mathcal{B} = \langle \mathcal{G}, P \rangle$  over variables  $\mathbf{V}$ , the Markov Blanket for  $X \in \mathbf{V}$  is the minimal set of variables  $\mathbf{MB}_X^{\mathcal{G}} \subseteq \mathbf{V} / \{X\}$  such that  $X \perp\!\!\!\perp \mathbf{V} / \mathbf{MB}_X^{\mathcal{G}} \mid \mathbf{MB}_X^{\mathcal{G}}$ .

# Terminology



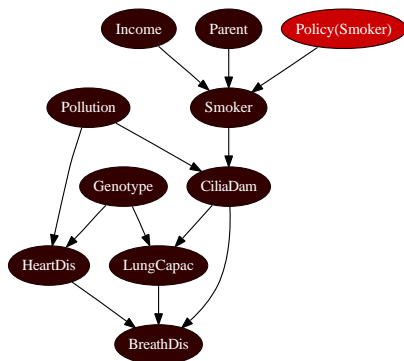
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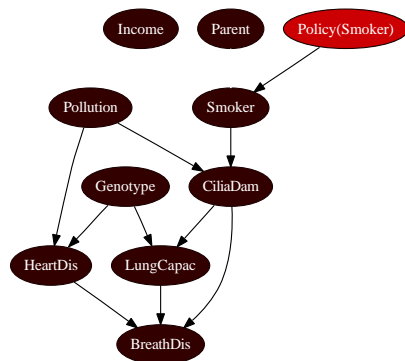
## Theorem (Pearl, 1988)

*The Markov blanket for  $X$  consists of the parents, children and coparents of  $X$  in  $\mathcal{G}$ .*

## Interventions

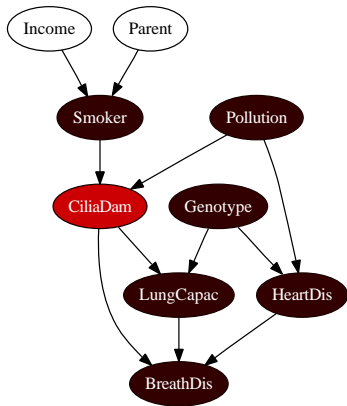


Policy(Smoker)=0



Policy(Smoker)=1

# Conditions for invariance of prediction under manipulations



## Theorem (Prediction invariance)

*In a causal Bayesian network  $\mathcal{B} = \langle \mathcal{G}, P \rangle$  over variables  $V$ , let  $T \in V$  be a target,  $X \subseteq V$  a set of predictor variables, and  $Y \subseteq V$  the set of manipulated variables. If  $X \supseteq \mathbf{MB}_T^{\mathcal{G}}$  and  $\forall Y \in Y, Y \neq T$  and  $Y \notin \mathbf{Children}(T)$ , then prediction of  $T$  using  $X$  is invariant under the manipulation.*

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in the Markov blanket subgraph

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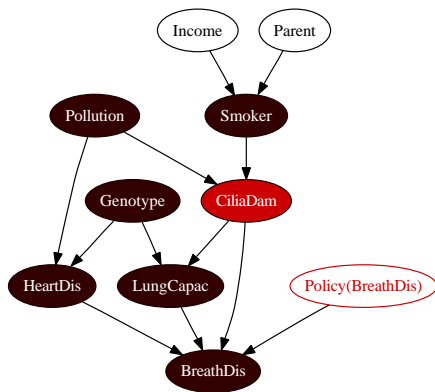
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# Correcting for manipulations



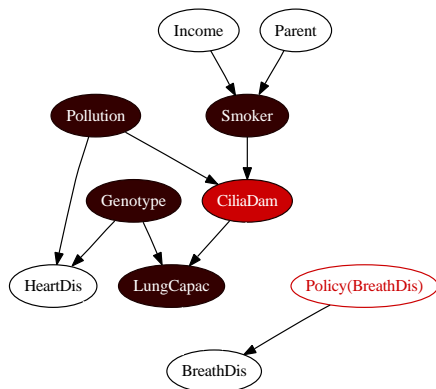
$$\text{Policy}(\text{BreathDis}) = 0$$

## Theorem (Causal correction)

In a causal Bayesian network  $\mathcal{B} = \langle \mathcal{G}, P \rangle$  over variables  $\mathbf{V}$ , let  $T$  be a target and  $\mathbf{Y} \subseteq \mathbf{V}$  the set of manipulated variables.

$P\left(T \mid \mathbf{MB}_T^{\mathcal{G}(\text{Policy}(\mathbf{Y}))}\right)$ , is invariant under the manipulation of  $\mathbf{Y}$  if  $\nexists Y \in \mathbf{Y}$ , such that  $Y \in \mathbf{Children}(T)$  and  $Y$  is an ancestor of some  $C \in \mathbf{Children}(T) \cap \mathbf{V}/\mathbf{Y}$ .

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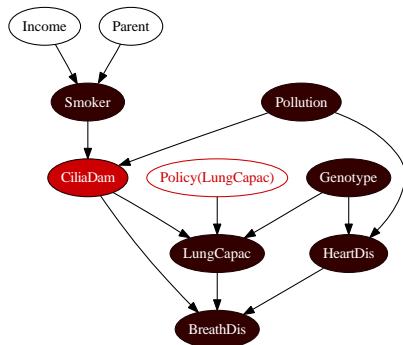
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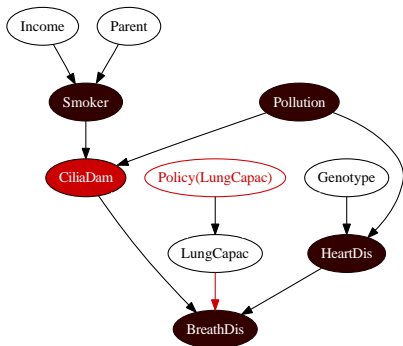
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$Policy(BreathDis) = 1$   
Make Correction!

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# Experiments

Hypotheses:

- Noncausal methods will be equivalent or better when no children are manipulated

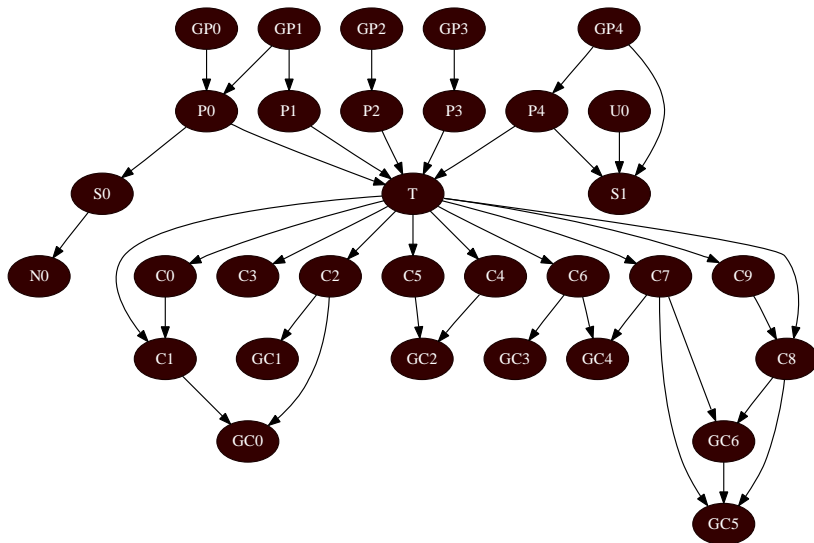
# Experiments

## Hypotheses:

- Noncausal methods will be equivalent or better when no children are manipulated
- Causal methods will do increasingly better than noncausal methods as more children are manipulated



# Model for experiments



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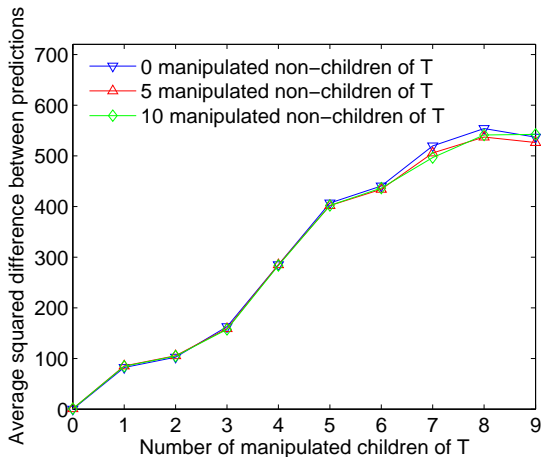
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- Predict  $T$  from manipulated distribution

## Differences between distributions



Squared difference between ground truth predictions for  $T$  using unmanipulated and manipulated model

## Prediction methods

### Noncausal Methods:

- LR-ALL** linear regression using all predictors
- LR-MB** linear regression using only the Markov blanket
- LASSO** “least absolute shrinkage and selection operator”
- SVR-RBF** support vector regression using radial kernel
- RVR-RBF** relevance vector regression using radial kernel

## Prediction methods

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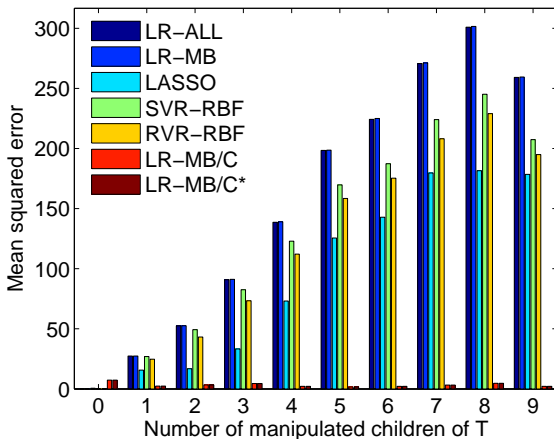
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### Causal Methods:

- LR-MB/C** linear regression with Markov blanket correcting for manipulated children
- LR-MB/C\*** linear regression with Markov blanket correcting for manipulated children and active paths to unmanipulated children

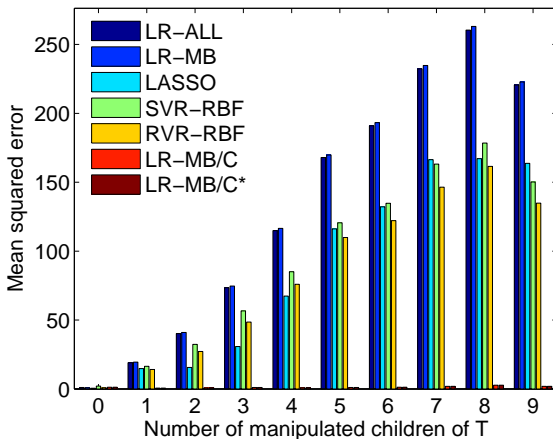


# Total prediction error



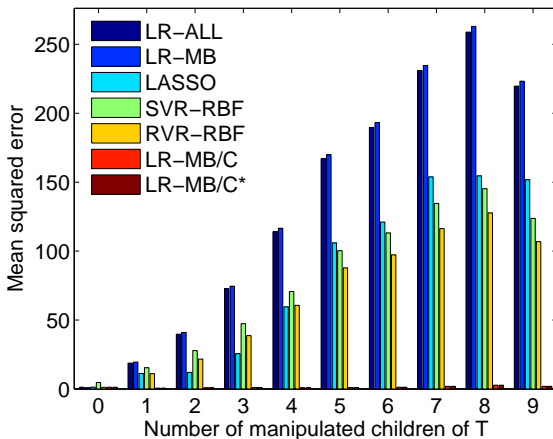
0 Manipulated Nonchildren of  $T$

# Total prediction error



5 Manipulated Nonchildren of  $T$

# Total prediction error



10 Manipulated Nonchildren of  $T$

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- Repeated previous simulations adding nonlinear dependencies
- Results so far inconclusive
- In general, nonparametric methods do best, though poor performance in all cases

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- Noncausal prediction may be frequently invariant (or *almost* invariant)
- Advantages of nonparametric methods and methods which deal with overfitting well may cancel out errors related to causality
- Many other variables involved, analysis incomplete