Spiraling toward complete markets and financial instability

Matteo Marsili
Abdus Salam ICTP, Trieste

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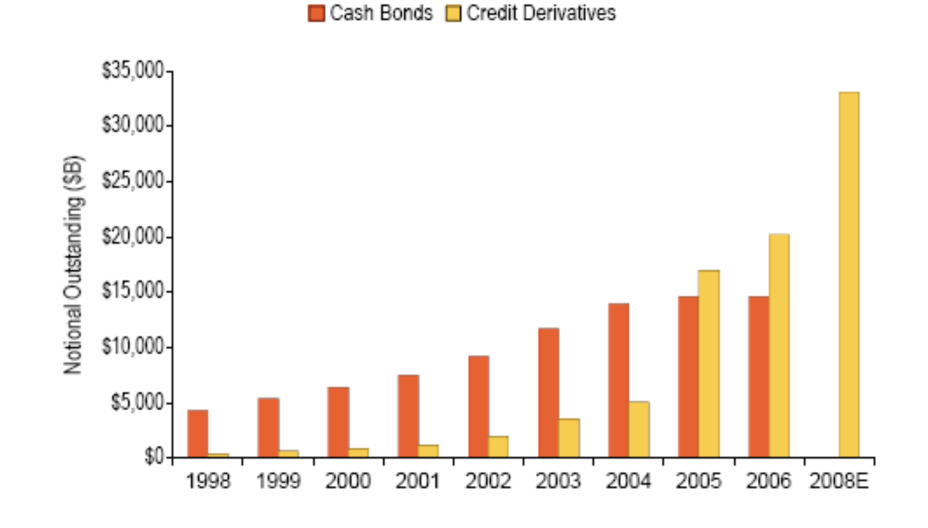
Bad guys or bad theories?

- *"... very frequently the "world images" that have been created by "ideas" have, like switchmen, determined the tracks along which action has pushed the dynamic of interest." (M.Weber)*



Sources: British Bankers' Association Credit Derivatives Report 2006, Bank for International Settlements and ISDA.
Note: Cash bonds through June 2006.

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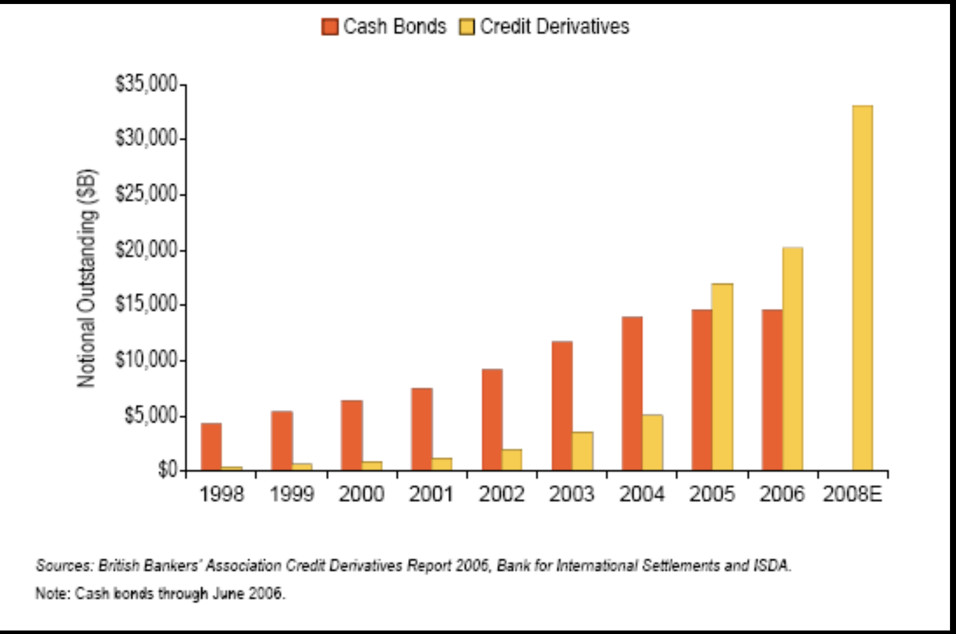

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  a) consumers in a risky world
  b) the financial industry: engineer new trading instruments
  General Equilibrium Theory: optimality with complete markets

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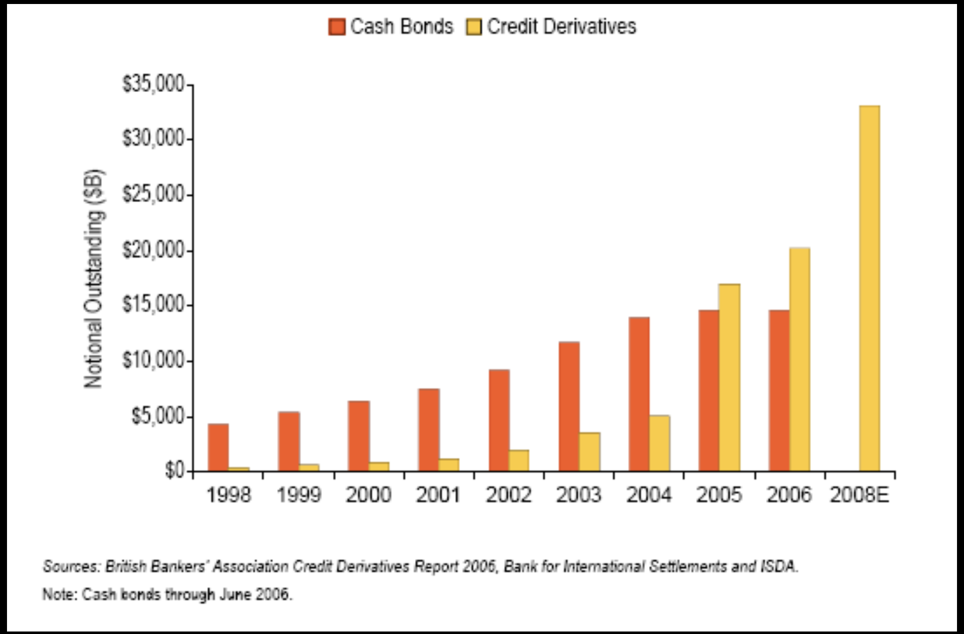

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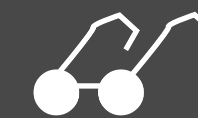
- Results:
  - in an ideal world: i) completeness = instability
    ii) trading volumes in interbank market diverges
  - in non-ideal world: i) derivative markets destabilize underlying markets
    ii) from supply limited to demand limited equilibria

Outline

- The General Equilibrium Theory perspective: What is the role of financial markets?
- A simple model of a complex market
- Spiraling toward market completeness in ideal markets
- Non-ideal markets: some preliminary results
- Conclusions

The perspective of General Equilibrium Theory:

- Tomorrow: rain or sun?
  wait and buy sunglasses or umbrella
  Inefficient, if e.g. tomorrow
  price of sunglasses > price of umbrella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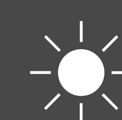	No	Yes

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		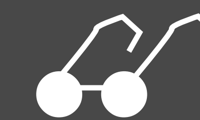
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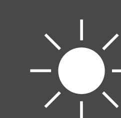	No	Yes

- Contingent commodity markets:
  markets and prices, open today for
  (sunglasses if rain), (sunglasses if sun), (umbrella if rain), (umbrella if sun)
  Today: shopping in contingency commodity markets
  Tomorrow: delivery and consumption

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- Optimal allocation under perfect competition

What if contingent commodity markets do not exist?

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- Financial market: 1 riskless B_t and 1 risky S_t assets
  Today $B_0=S_0=1$
  Tomorrow $B_1=1$, $S_1=1+u$ if sun, $S_1=1-d$ if rain

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$$C_0 = z_B + z_S = \frac{d}{u+d}C^{\text{sun}} + \frac{u}{u+d}C^{\text{rain}} = E_q[C_{t=1}]$$

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- Assumptions:
  i) perfect competition
  ii) full information
  iii) no-arbitrage: ud>0
  iv) complete market: what if there are three states? (e.g. sun, cloud, rain)

The financial innovation spiral

(Merton and Bodie 2005)

"As products such as futures, options, swaps, and securitized loans become standardized [...] the producers (typically, financial intermediaries) trade in these new markets and volume expands; increased volume reduces marginal transaction costs and thereby makes possible further implementation of more new products and trading strategies by intermediaries, which in turn leads to still more volume [...] and so on it goes, spiraling toward the theoretically limiting case of zero marginal transactions costs and dynamically complete markets."

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"When particular transaction costs or behavioral patterns produce large departures from the predictions of the ideal frictionless neoclassical equilibrium for a given institutional structure, new institutions tend to develop that partially offset the resulting inefficiencies. In the longer run, after institutional structures have had time to fully develop, the predictions of the neoclassical model will be approximately valid for asset prices and resource allocations."

(see also R. J. Shiller, "The Subprime Solution" 2008)

A simple model of a complex financial market

	consumers	market	banks
Today	max E[u(c)] buy assets	portfolio $\Rightarrow$	sell financial instruments
Tomorrow (?)	buy and consume	payoff $\Leftarrow$	state dependent return

The game: N assets, Ω states

The market

$$\begin{matrix} r_1^1 & \cdots & r_1^\omega & \cdots & r_1^\Omega \\ \vdots & \ddots & \vdots & & \vdots \\ r_k^1 & \cdots & r_k^\omega & \cdots & r_k^\Omega \\ \vdots & & \vdots & \ddots & \vdots \\ r_N^1 & \cdots & r_N^\omega & \cdots & r_N^\Omega \end{matrix}$$

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\begin{matrix}
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\vdots & \ddots & \vdots & & \vdots \\
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\vdots & & \vdots & \ddots & \vdots \\
r_N^1 & \cdots & r_N^\omega & \cdots & r_N^\Omega
\end{matrix}
\quad \Leftarrow
$$

Financial industry (banks)

The game: N assets, Ω states

The market

demand

Investors $\Leftarrow$
$$\begin{matrix} z_1 & & r_1^1 & \cdots & r_1^\omega & \cdots & r_1^\Omega \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ z_k & \Leftarrow & r_k^1 & \cdots & r_k^\omega & \cdots & r_k^\Omega \\ \vdots & & \vdots & & \vdots & \ddots & \vdots \\ z_N & & r_N^1 & \cdots & r_N^\omega & \cdots & r_N^\Omega \end{matrix}$$
$\Leftarrow$ Financial industry (banks)

$$\max_{\vec{z} \geq 0} E\left[u\left(c(\vec{z})\right)\right]$$

The game: N assets, Ω states

The market

Investors $\Leftarrow$ demand $\Leftarrow$ The market $\Leftarrow$ Financial industry (banks)

$$
\begin{array}{c}
\text{demand} \\
z_1 \\ \vdots \\ z_k \\ \vdots \\ z_N
\end{array}
\;\Leftarrow\;
\begin{array}{ccccc}
r_1^1 & \cdots & r_1^\omega & \cdots & r_1^\Omega \\
\vdots & \ddots & \vdots & & \vdots \\
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\vdots & & \vdots & \ddots & \vdots \\
r_N^1 & \cdots & r_N^\omega & \cdots & r_N^\Omega
\end{array}
\;\Leftarrow
$$

$$\Downarrow$$

$$\max_{\vec{z} \geq 0} E\left[u\left(c(\vec{z})\right)\right]$$

$$r_{N+1}^1 \quad \cdots \quad r_{N+1}^\omega \quad \cdots \quad r_{N+1}^\Omega$$

financial innovation

The game: N assets, Ω states

The market

demand

Investors

Financial industry (banks)

$$
\text{Investors} \Leftarrow
\begin{array}{c}
z_1 \\ \vdots \\ z_k \\ \vdots \\ z_N
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\end{array}
\Leftarrow \text{Financial industry (banks)}
$$

$$\max_{\vec{z}\geq 0} E\left[u\left(c(\vec{z})\right)\right]$$

$$N, \Omega \to \infty, \quad n = \frac{N}{\Omega}$$

Optimizing consumers

Solution of optimal consumption

$$\frac{\partial}{\partial z_i} E_\pi \left[u\left(c^\omega\right)\right] = \sum_\omega \pi^\omega \frac{u'(c^\omega)}{p^\omega} r_i^\omega \quad \begin{cases} = 0 & \Leftrightarrow \quad z_i > 0 \\ < 0 & \Leftrightarrow \quad z_i = 0 \end{cases}$$

i) investors select the assets which are traded $z_i > 0$

ii) they determine the Equivalent Martingale Measure (EMM)

$$q^\omega = \pi^\omega \frac{u'(c^\omega)}{Q p^\omega}, \qquad Q = \sum_\omega \pi^\omega \frac{u'(c^\omega)}{p^\omega}$$

A creative financial sector

- Financial instruments are drawn at random from a probability distribution with

$$E_\pi\left[r_i\right] = \sum_\omega \pi^\omega r_i^\omega = -\frac{\epsilon}{\Omega}, \qquad \mathrm{Var}\left[r_i\right] = \frac{1}{\Omega}, \quad i = 1, \ldots, N$$

- Successful innovations (z_i>0) are not independent draws!

Theory: statistical mechanics

Typical behavior of self-averaging quantities

(De Martino et al. Macroecon. Dyn. 2007)

$$\lim_{\Omega\to\infty}\left\langle \max_{\vec{z}\geq 0} E[u(c^\omega)]\right\rangle_{\vec{p},\hat{a}} = \lim_{\beta\to\infty}\lim_{\Omega\to\infty}\frac{1}{\beta}\langle \log Z(\beta)\rangle_{\vec{p},\hat{a}}$$

1- The partition function $Z(\beta)=\sum_{\{\vec{z}\geq 0\}} e^{\beta u[c^\omega(\vec{z})]}$

2- The replica trick $\langle \log Z\rangle_{\vec{p},\hat{a}} = \lim_{r\to 0}\frac{1}{r}\log\langle Z^r\rangle_{\vec{p},\hat{a}}$

3- For integer r

$$\langle Z^r\rangle_{\vec{p},\hat{a}} = \sum_{\{\vec{z}_1\geq 0\}}\cdots\sum_{\{\vec{z}_r\geq 0\}}\left\langle e^{\beta\sum_{a=1}^r u[c^\omega(\vec{z}_a)]}\right\rangle_{\vec{p},\hat{a}}$$

$$= \int d\hat{\Phi} e^{r\beta\nu(r,\beta,\hat{\Phi})} \qquad \hat{\Phi} = \text{order parameters}$$

4- Saddle point:

$$\lim_{\Omega\to\infty}\left\langle \max_{\vec{z}\geq 0} E[u(c^\omega)]\right\rangle_{\vec{p},\hat{a}} = \lim_{\beta\to\infty}\lim_{r\to 0}\max_{\hat{\Phi}}\nu(r,\beta,\hat{\Phi})$$

The typical behavior

- Observables:

response function $\chi = \lim_{\beta\to\infty} \frac{\beta}{2N} \sum_{i=1}^{N} (z_{i,a} - z_{i,b})^2 = \frac{1}{N} \sum_i \frac{\delta z_i}{\delta p_i^0}$

EMM dispersion $\sigma = |q - \pi|$

market completeness $\phi = |\{i : \ z_i > 0\}|/\Omega$

volume (or revenue) $V = \sum_i z_i$

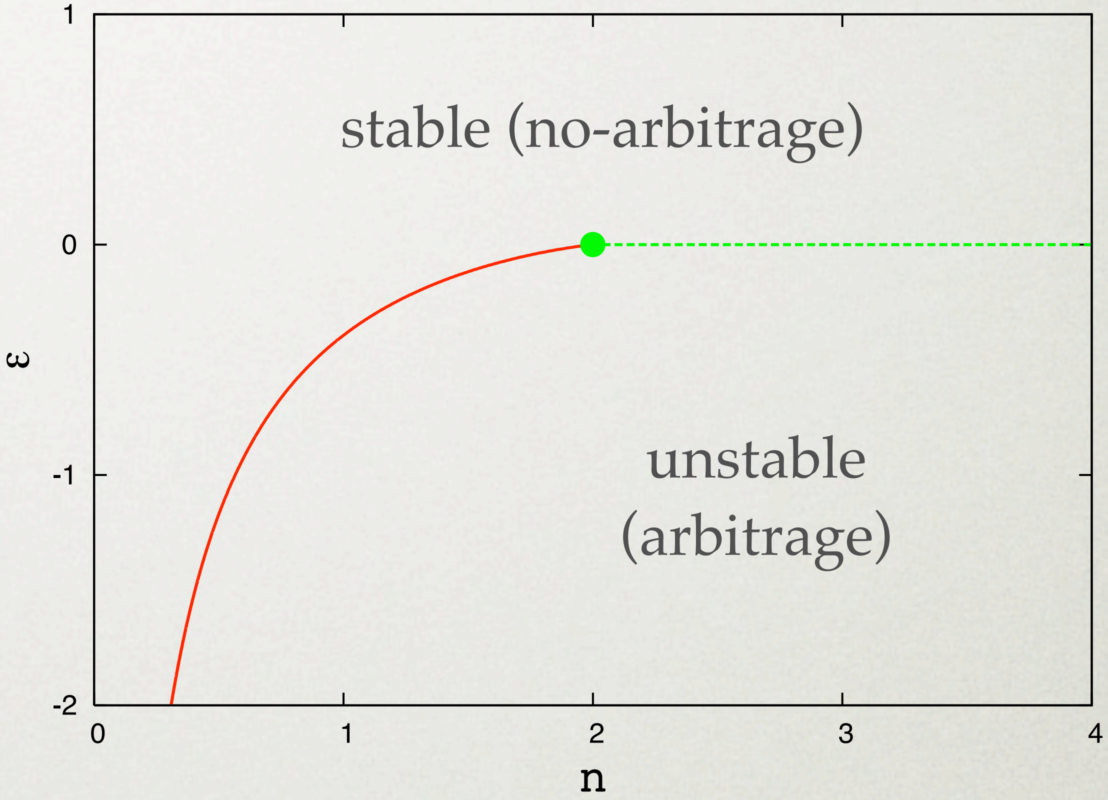
- Consistency relations

Conservation $1 = \langle c^* p \rangle_{t,p} + \epsilon n \langle z^* \rangle_t$

no-arbitrage $E_q[c^\omega p^\omega] = E_q[1] = 1$

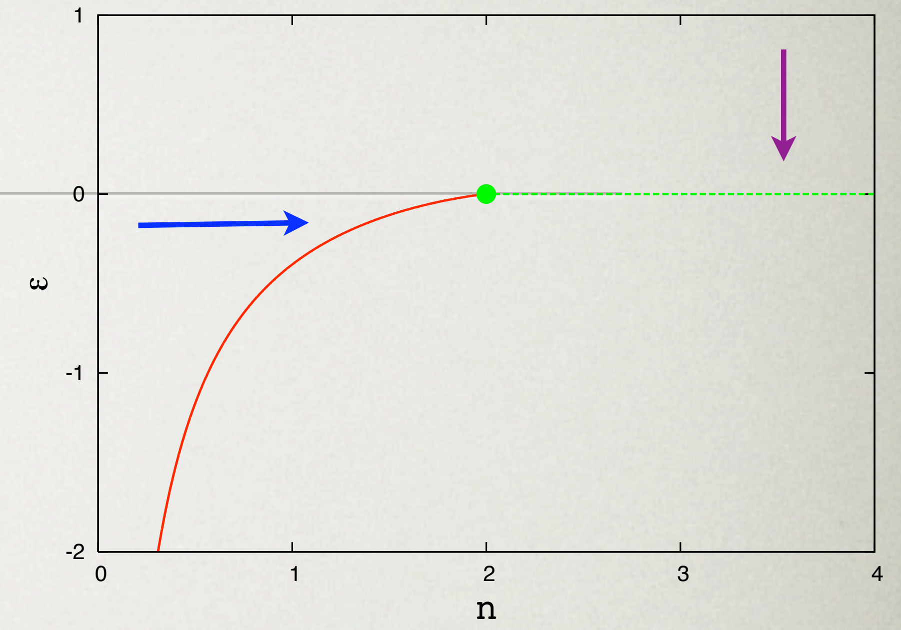
Phase diagram

Independent of u(c) & p



- $\chi \rightarrow \infty \ \forall \ \varepsilon$
- $\sigma \rightarrow 0$ for $\varepsilon > 0$
- $\sigma \rightarrow \infty$ for $\varepsilon < 0$
- For $\varepsilon > 0$
  singularity = complete market ($\varepsilon = 0,\ n > 2$)
- For $\varepsilon < 0$
  singularity < complete market

Distribution of c



$\sigma \to \infty$ for $\varepsilon < 0$

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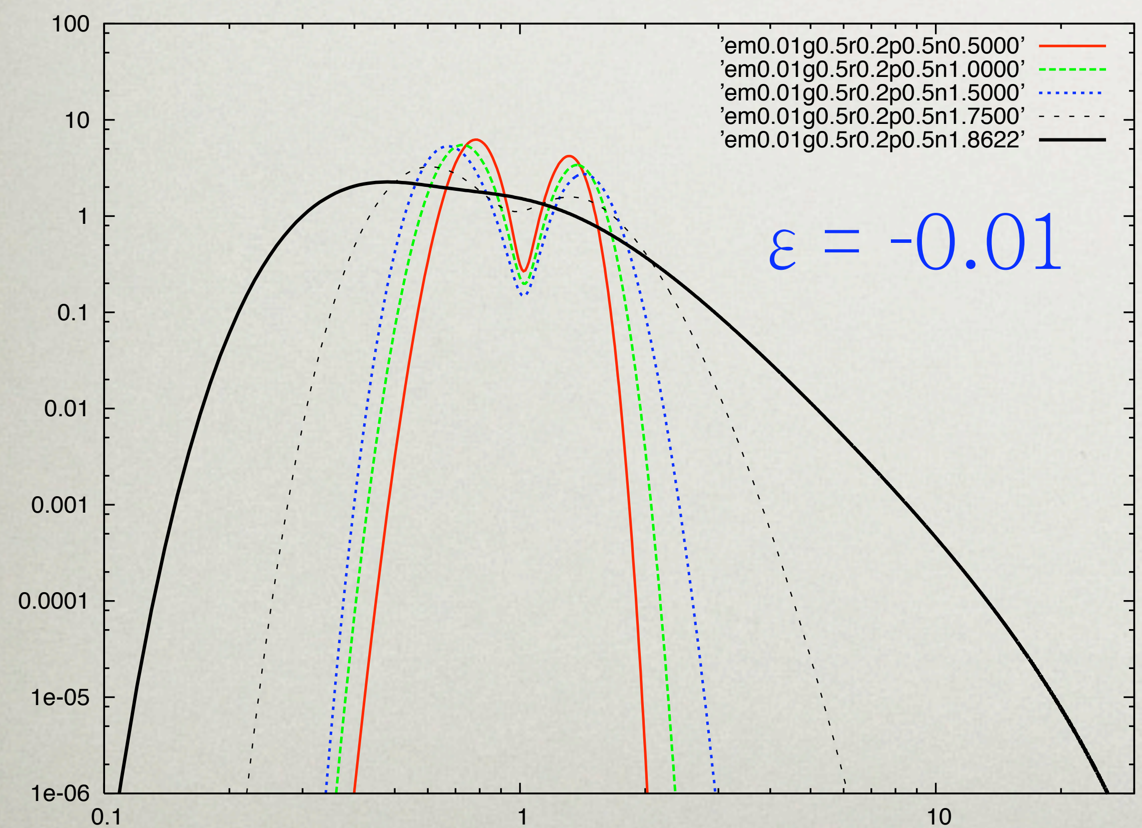
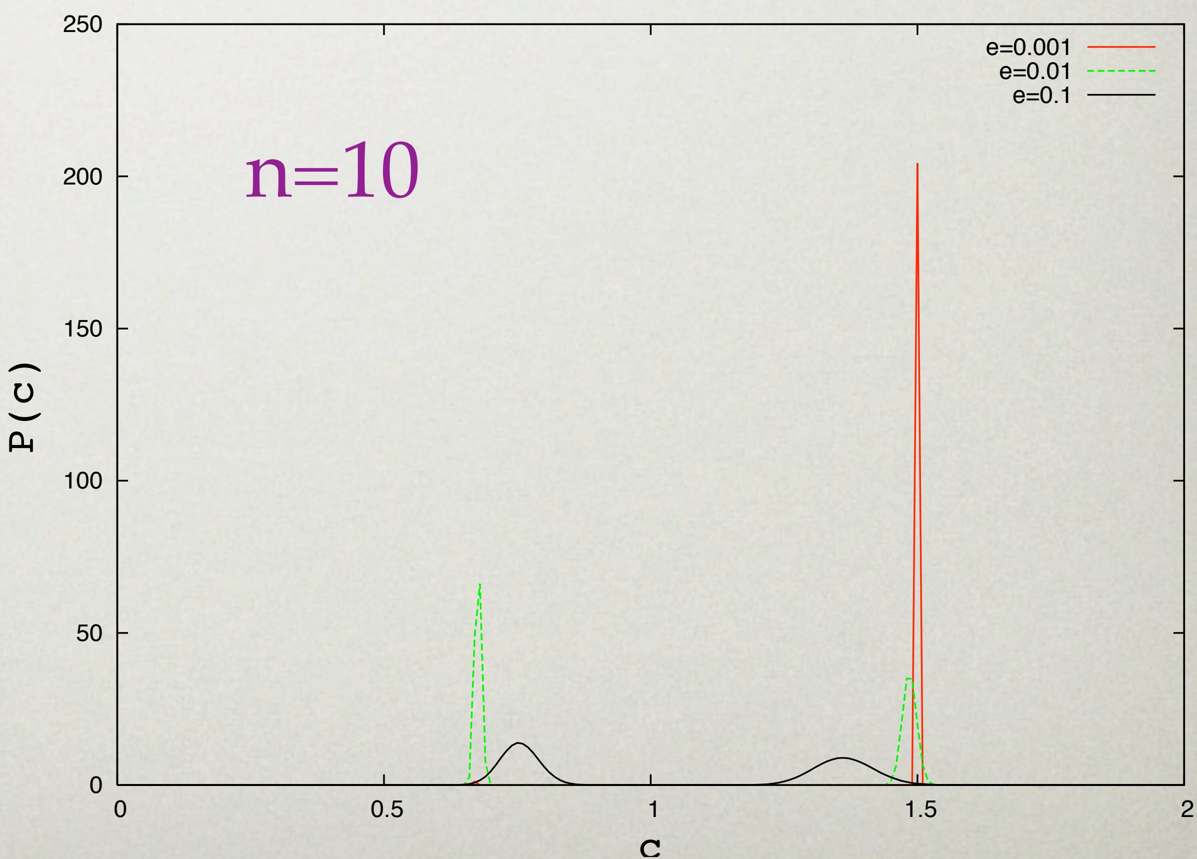
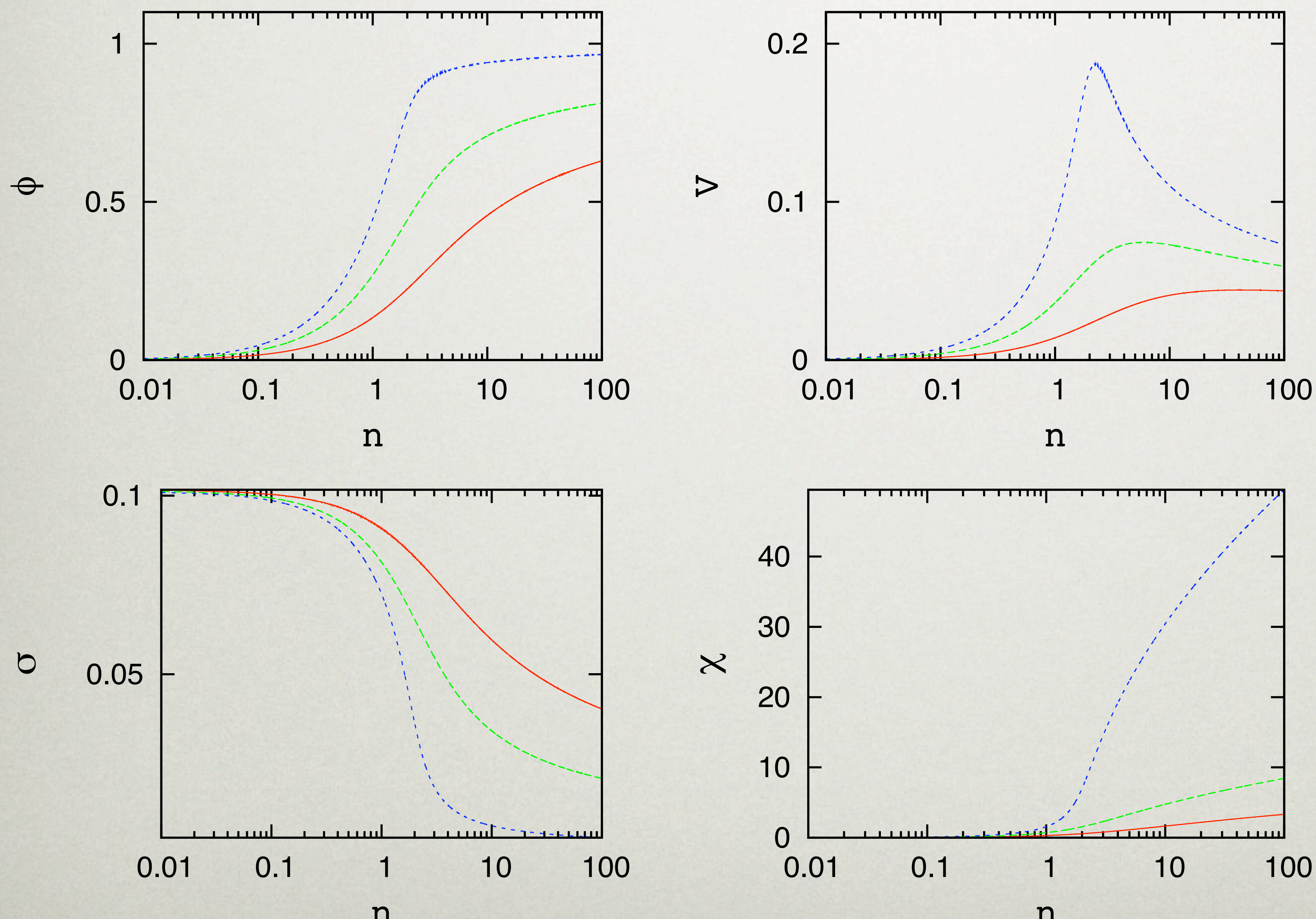
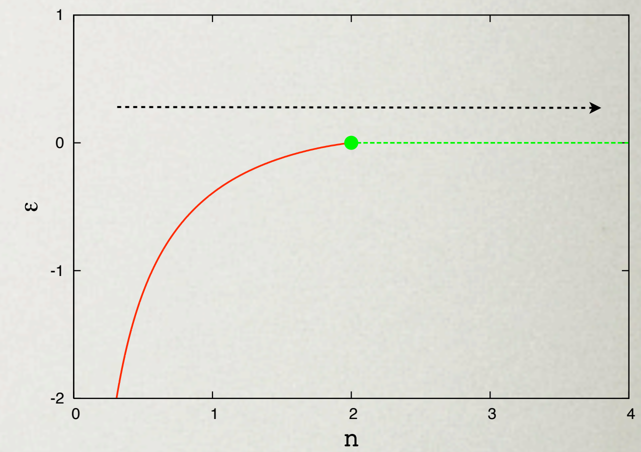




Increasing financial complexity

$\epsilon = 0.01, 0.05, 0.10$



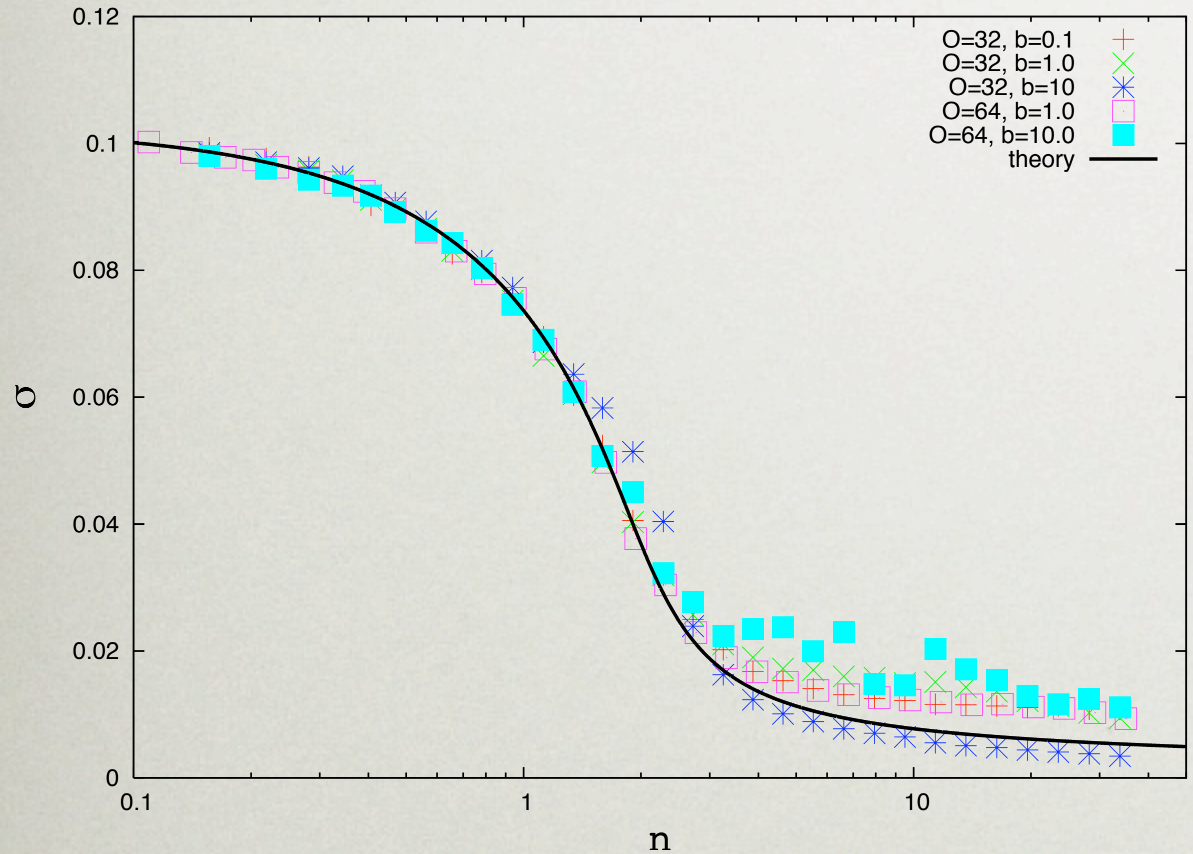


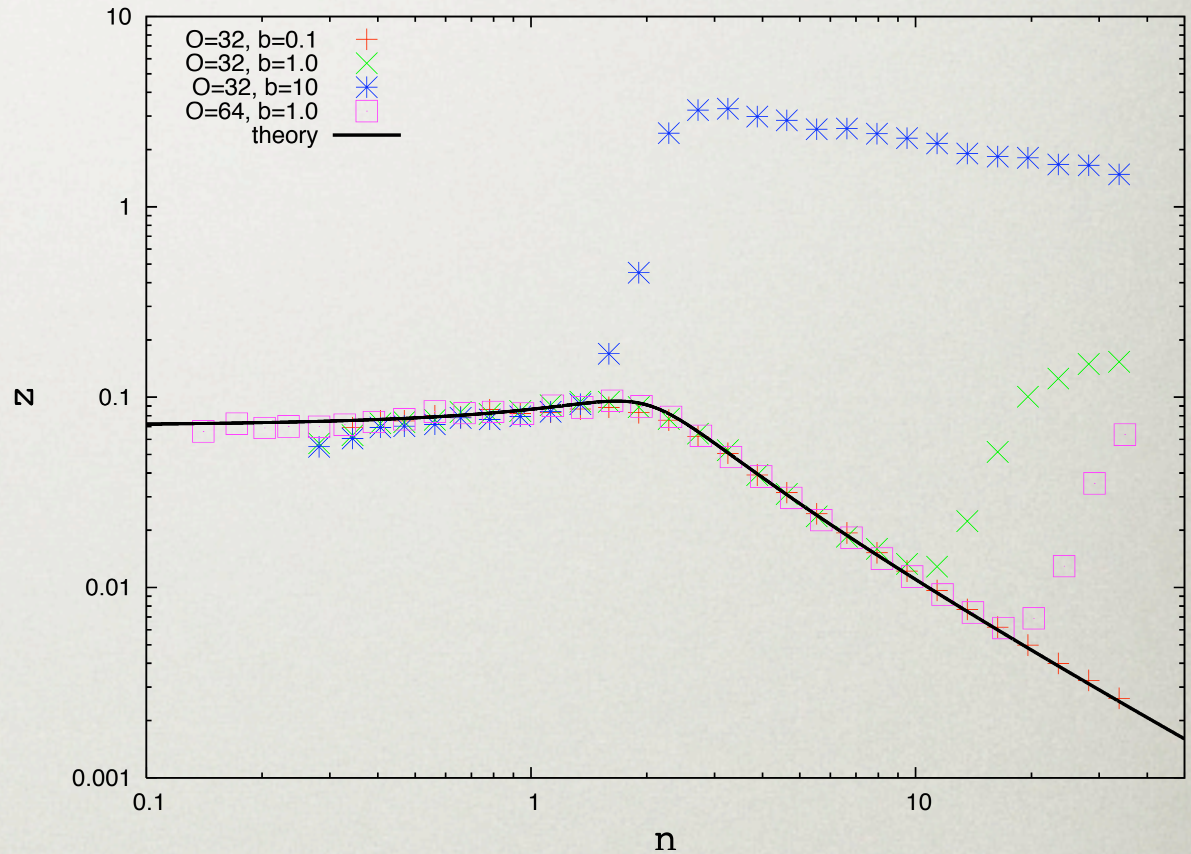
Learning to invest

$$\epsilon = 0.01, \quad \gamma = 0.5, \quad \Omega = 32$$

Hard to learn when market is nearly complete

(cfr Brock, Hommes, Wagener, 2006)





$$\sigma^2 = \frac{1}{\Omega} \sum_{\omega} (q^{\omega} - \bar{q})^2$$

$$\langle z \rangle = \frac{1}{N} \sum_{i} z_{i}$$

A competitive Financial Industry

- Part of the risk of a new instrument can be hedged buying existing instruments
- Residual risk

$$\Sigma = \min_{\vec{u}} \mathrm{Var}\left[r^{\omega}_{\mathrm{new}} - \sum_i v_i r^{\omega}_i\right] = 1 - \phi$$

- Risk premium vanishes as markets become complete e.g. Mean Variance profit function

$$\Rightarrow \quad \epsilon = \frac{\gamma}{2}(1-\phi)$$

- The weights of portfolios used to hedge each instrument diverges as $\phi \to 1$

$$\sum_i v_i^2 = \frac{\phi}{1-\phi}$$

- Susceptibility in the interbank market also diverges

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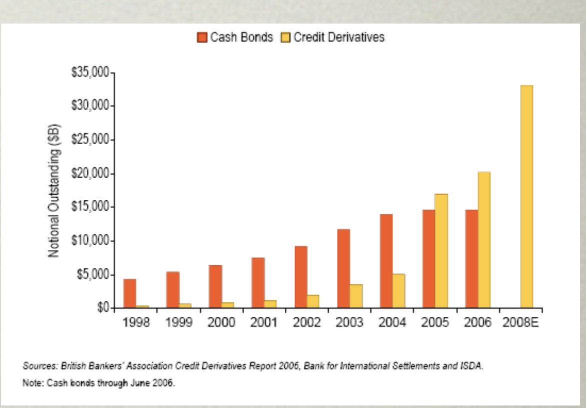

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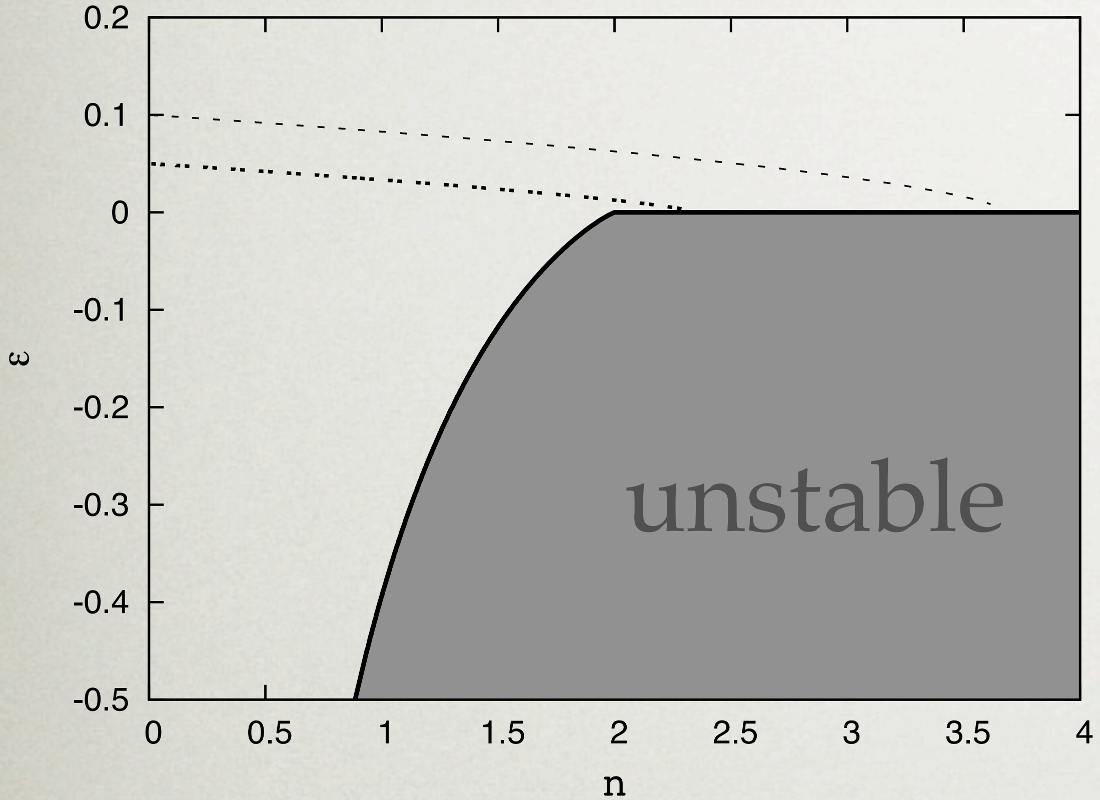
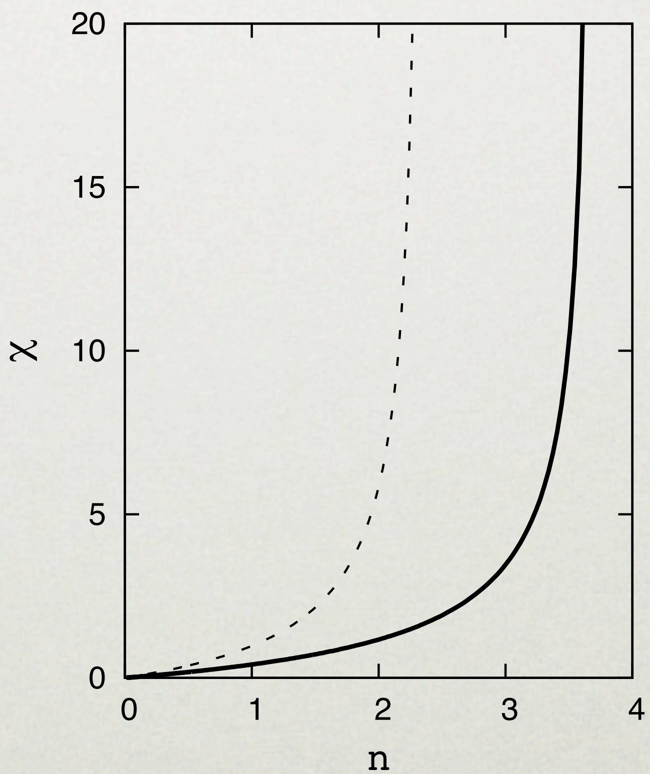
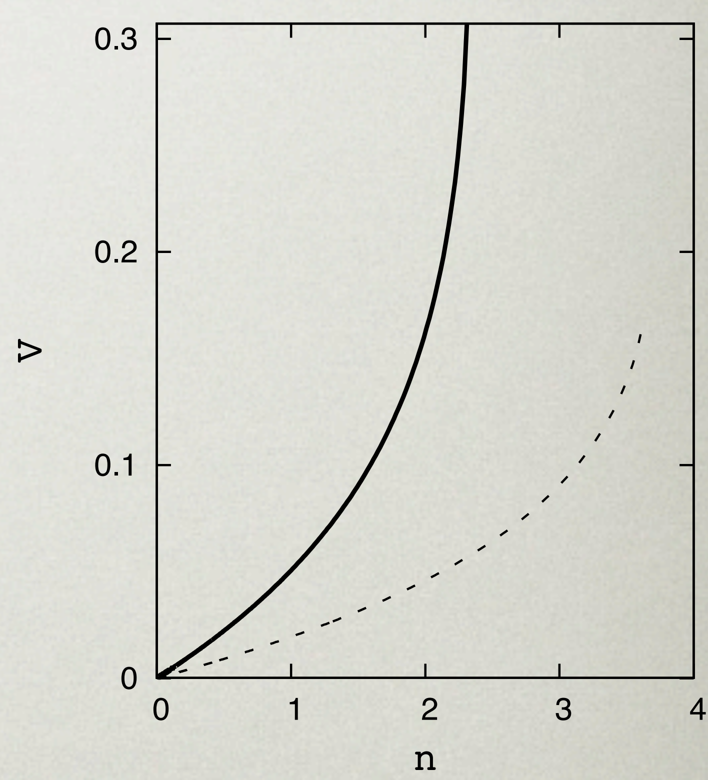
Mean variance banks $\epsilon = \frac{\gamma}{2}\Sigma$



Consumer market:
infinite susceptibility, finite volume





Interbank market:
both susceptibility and volumes diverge as $\phi \to 1$

Conclusions I

- The proliferation of financial instruments, even in an ideal world (perfect competition and full information), is problematic
  - Complete markets lie on a critical line with infinite susceptibility
  - A competitive financial sector is expected to converge to this singularity
  - The volume generated by banks to hedge financial instruments they sell diverges as market approaches completeness
- Learning to invest optimally is hard (as in Brock, Hommes, Wagener 2006)
- Market imperfections amplified close to complete markets: institution size grows with financial complexity

Illiquid markets: underlying and derivatives

demand

$$
\begin{array}{cccccc}
z_1 & r_1^1 & \cdots & r_1^\omega & \cdots & r_1^\Omega \\
\vdots & \vdots & \ddots & \vdots & & \vdots \\
z_k & r_k^1 & \cdots & r_k^\omega & \cdots & r_k^\Omega \\
\vdots & \vdots & & \vdots & \ddots & \vdots \\
z_N & r_N^1 & \cdots & r_N^\omega & \cdots & r_N^\Omega \\
\zeta_1 & f_1^1 & \cdots & f_1^\omega & \cdots & f_1^\Omega \\
\vdots & \vdots & & \vdots & & \vdots \\
\zeta_H & f_H^1 & \cdots & f_H^\omega & \cdots & f_H^\Omega
\end{array}
$$

Derivatives:

$$f_h^\omega = F_h(r_1^\omega, \ldots, r_N^\omega) - f_h^0$$

Return of underlying:

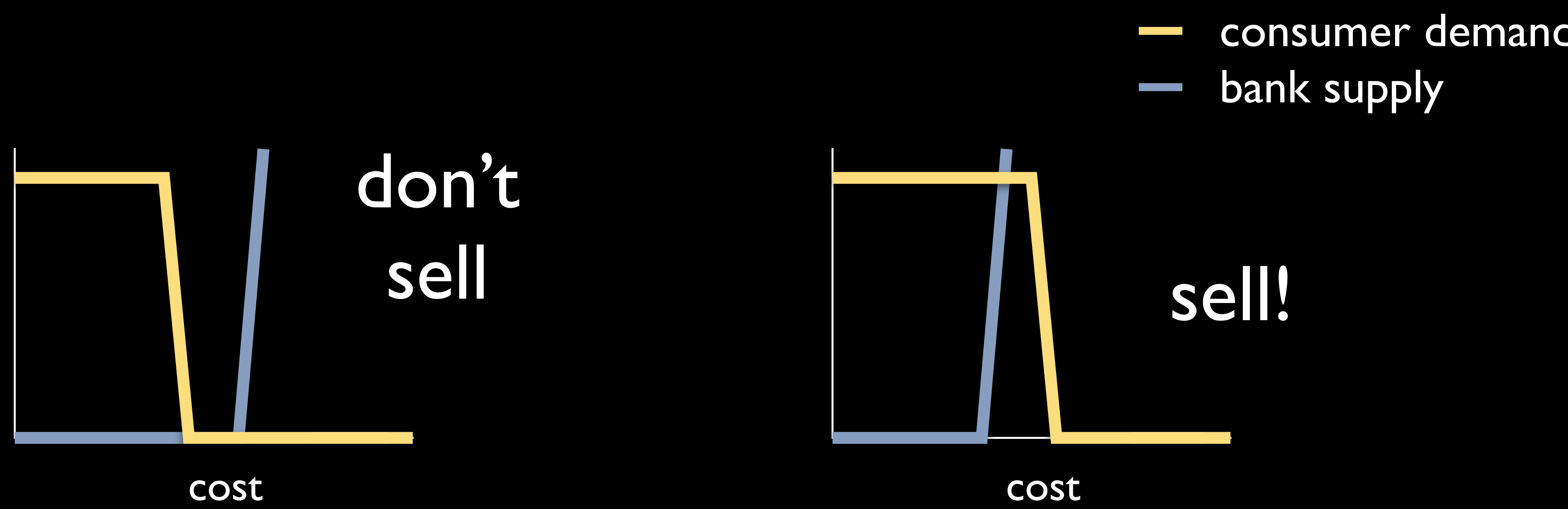
$$r_k^\omega = \rho(z_k, \zeta_1, \ldots, \zeta_H)$$

Price of derivatives:

$$f_h^0(z_1, \ldots, z_N, \zeta_1, \ldots, \zeta_H)$$

Illiquid markets: N derivatives on 1 underlying

- derivative:
  pay c today ⇒ a^{ω} units of asset in state $\omega=1,..,\Omega$ tomorrow



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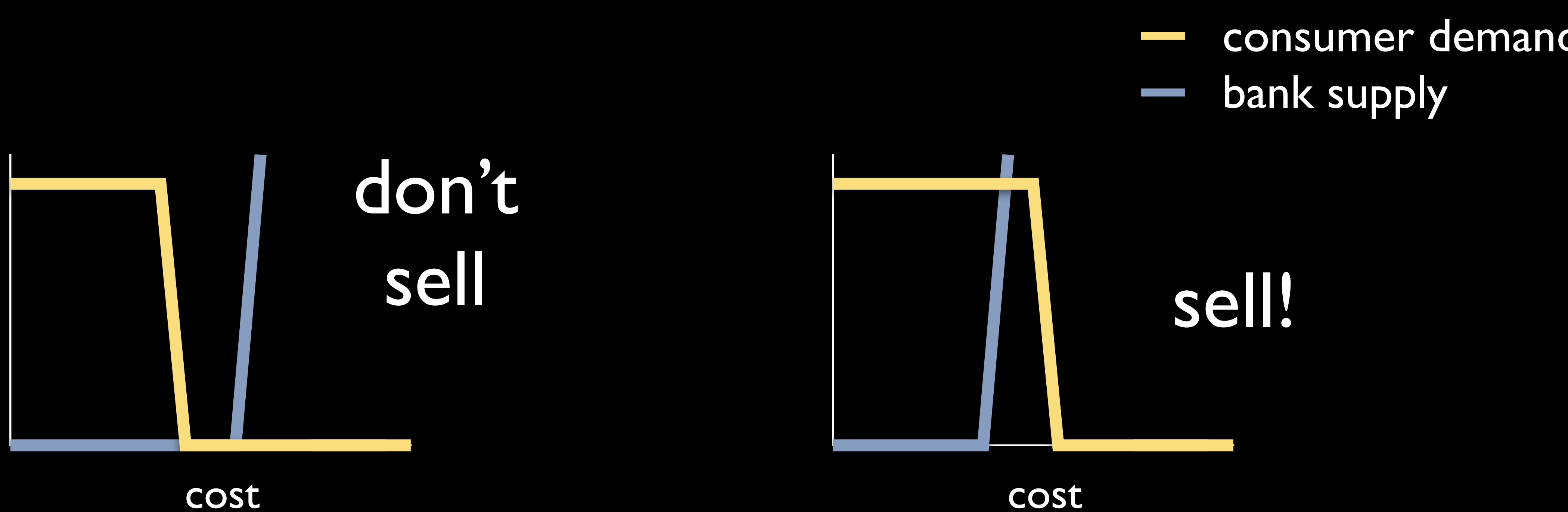


$$N, \Omega \to \infty, \quad n = \frac{N}{\Omega}$$

The price of the underlying

$$p^\omega(t=1) \equiv 1 + r^\omega = D^\omega + \sum_{i=1}^{N} s_i a_i^\omega$$

s_i = supply of derivative i

> 0 if E[profit] > risk premium

Competitive equilibria

- For general demand functions
- Banks supply a quantity of derivative contracts $\{s_i, i=1,\dots,N\}$ which is given by the minima of the function

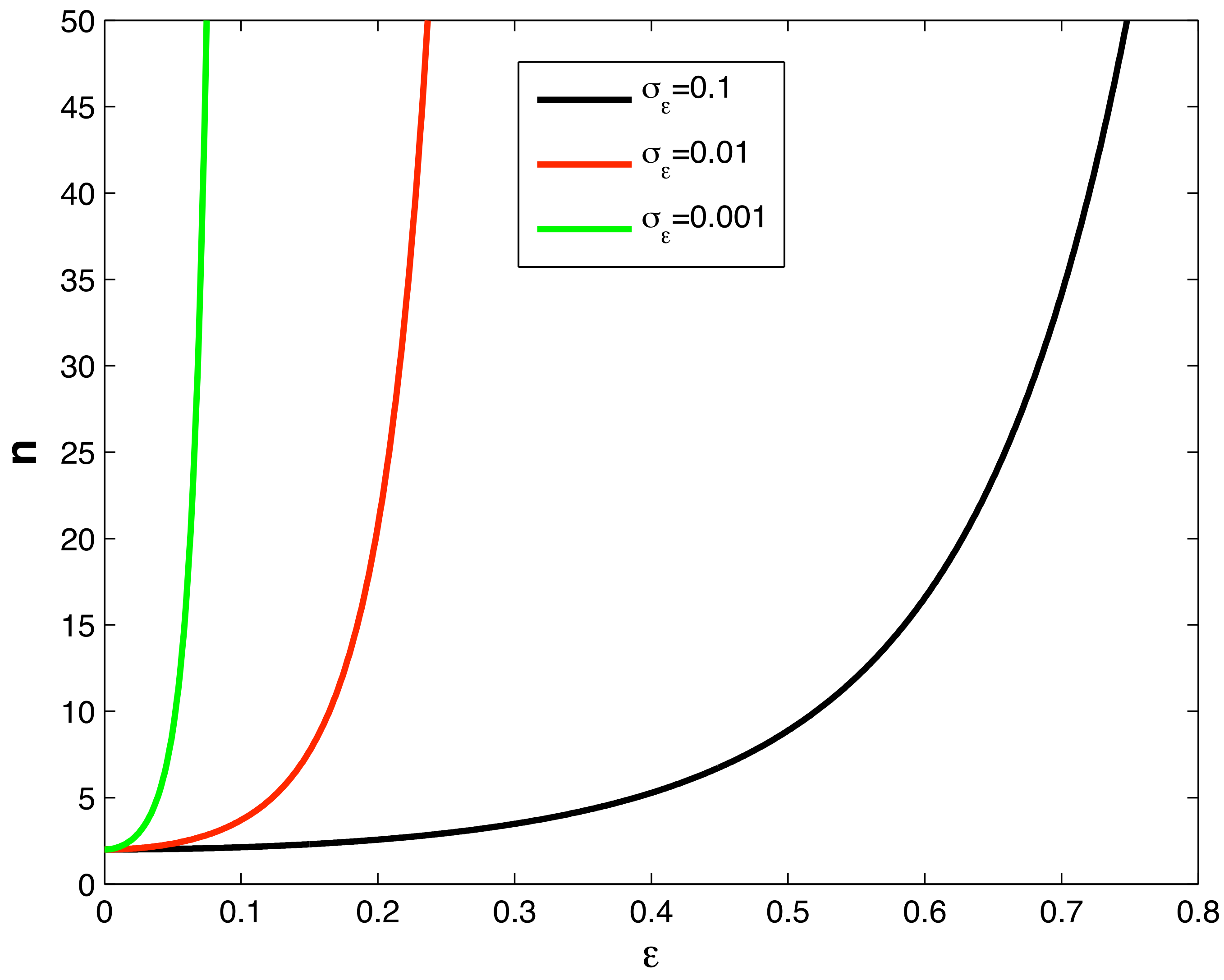
$$H = \frac{1}{2}\sum_{\omega=1}^{\Omega}\pi^{\omega}\left(d^{\omega}+\sum_{i=1}^{N}s_i a_i^{\omega}\right)^2+\sum_{i=1}^{N}g(s_i)$$

return^2

g related to inverse demand function

= GC Minority Game

Phase diagram



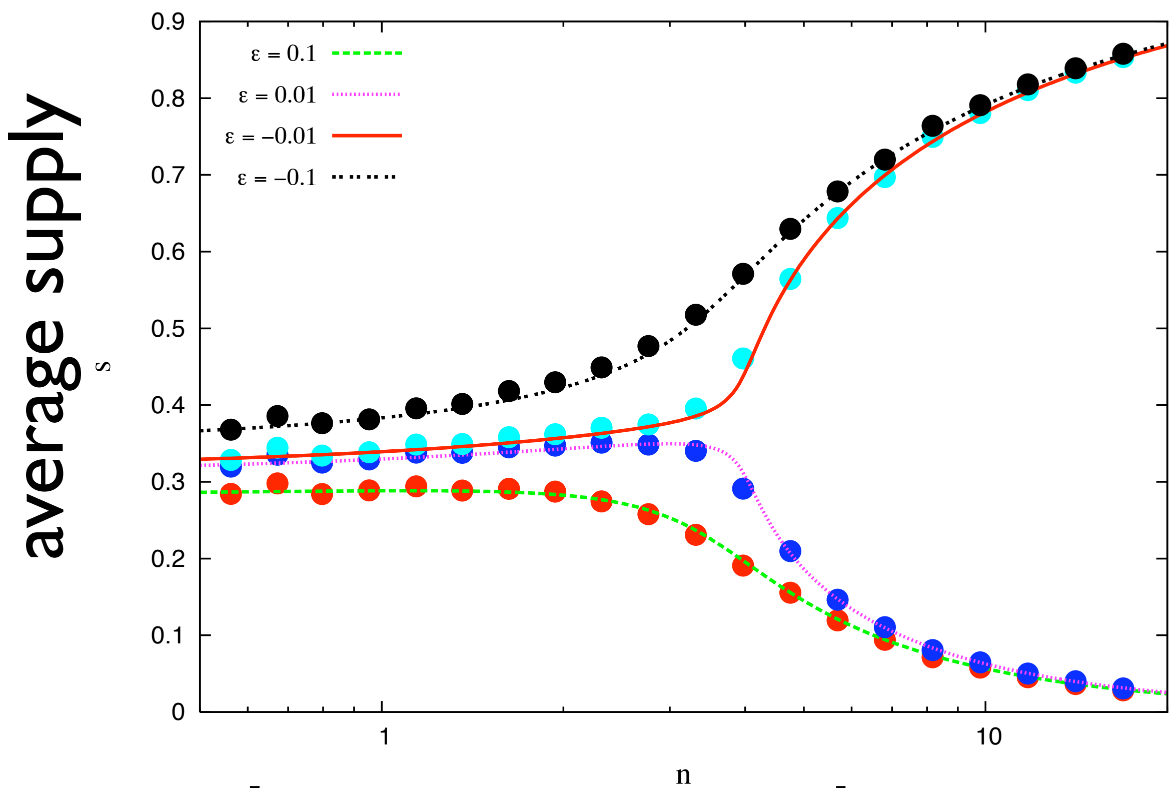
Susceptibility

$\chi \to \infty$

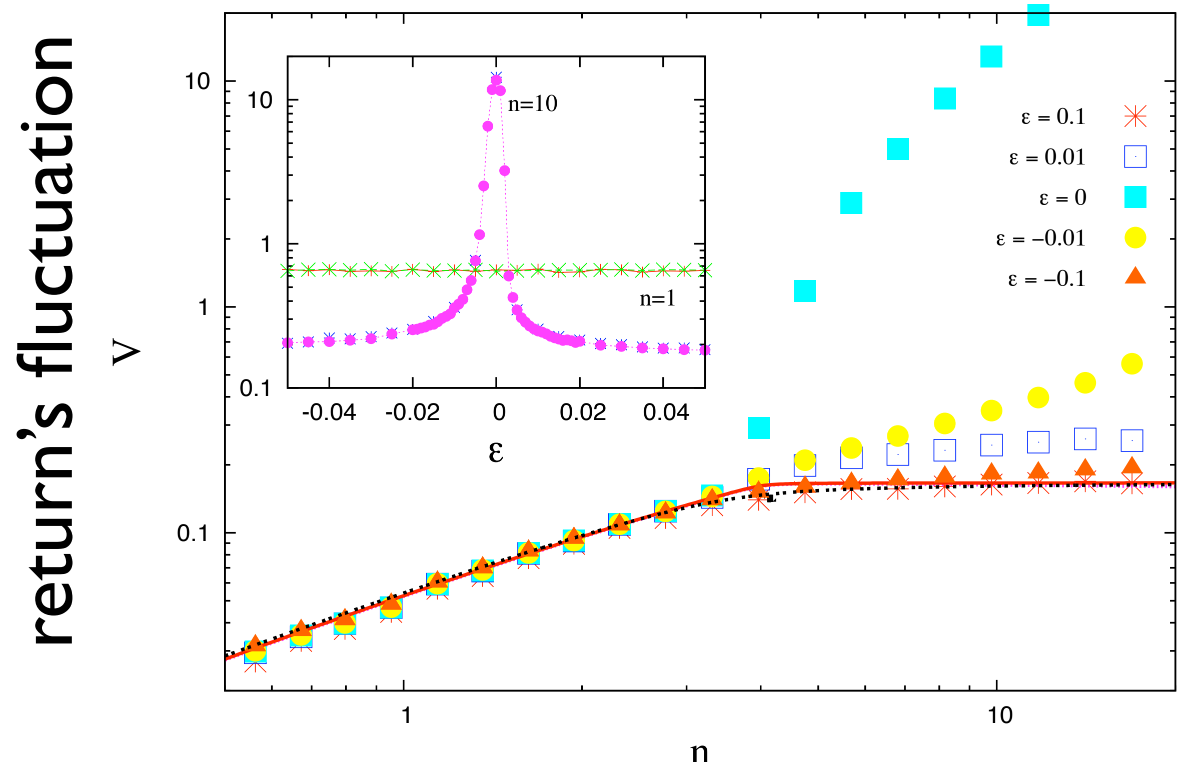
on phase boundary

Increasing financial complexity

$\epsilon = c_i - c_i^{(0)} - \rho_i$ ~ risk premium



derivative market



underlying market

Derivative markets destabilize underlying markets

Conclusions

- System-wide picture of complex markets as large random economies
- Quantifying financial stability

$$\chi = \frac{\delta \text{equilibrium}}{\delta \text{parameters}}$$

  fragility when repertoire of instruments expands

- Asset Pricing Theory for illiquid markets

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