How to quantify the influence of correlations on investment diversification

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Investor's story

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DIVERSIFICATION

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- ***mean-variance portfolio:*** minimizes V_P for a given R_P

This is the key slide

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$$V_P^*(R_P, M, \mathbf{C}) = V_P^*(R_P, m_{\text{ef}}, \mathbf{1}) \quad \Longrightarrow \quad m_{\text{ef}}$$

Effective portfolio size: properties

$$m_{\mathrm{ef}} = \sum_{i,j=1}^{M} \left(\mathbf{C}^{-1}\right)_{ij}$$

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- N groups of stocks with no inter-group correlations:

$$m_{\mathrm{ef}} = m_{\mathrm{ef}}(1) + \cdots + m_{\mathrm{ef}}(N)$$

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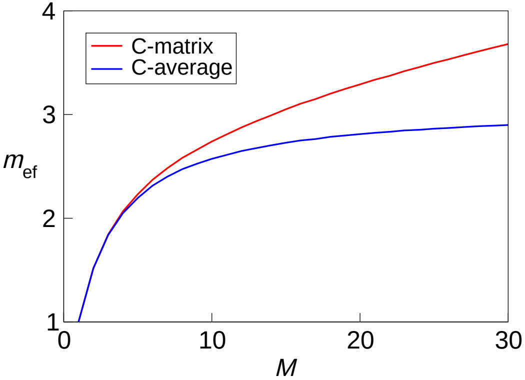
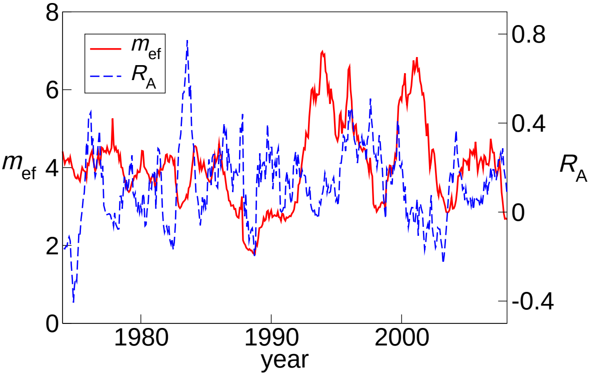


Effective portfolio size: evolution

20 current stocks from the DJIA (Jan 1973—Apr 2008)



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Thank you for your attention