A Tableau Algorithm for Handling Inconsistency in OWL

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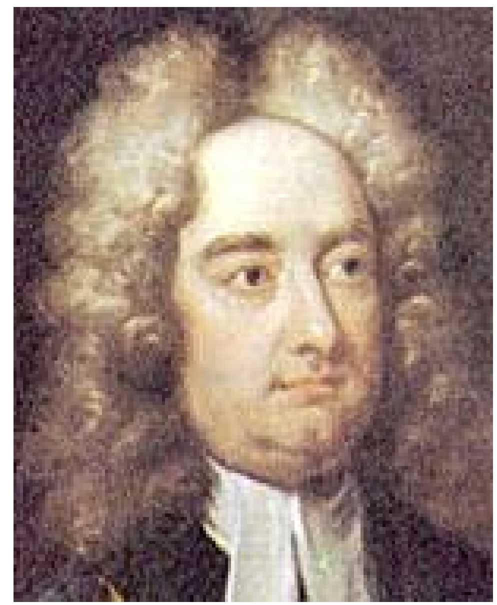
Presented by Liping Zhou

One cannot live without inconsistency.

Carl Jung (1875-1961)



There is nothing constant in this world but inconsistency.

Jonathan Swift (1667-1745)

Outline

- Motivation
- Quasi-classical description Logic $\mathcal{ALCNQ}$
- A Tableau Algorithm for $\mathcal{ALCNQ}$
- Conclusions and our future works

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$$\begin{array}{rl} \text{modus ponens (MP)} & \{C(a), C \sqsubseteq D\} \models D(a) \\ \text{modus tollens (MT)} & \{\neg D(a), C \sqsubseteq D\} \models \neg C(a) \\ \text{disjunctive syllogism (DS)} & \{\neg C(a), C \sqcup D\} \models D(a) \end{array}$$

Motivation

We try to find a new semantics for description logics to **handle inconsistency** with satisfying the **three inference rules.**

Motivation

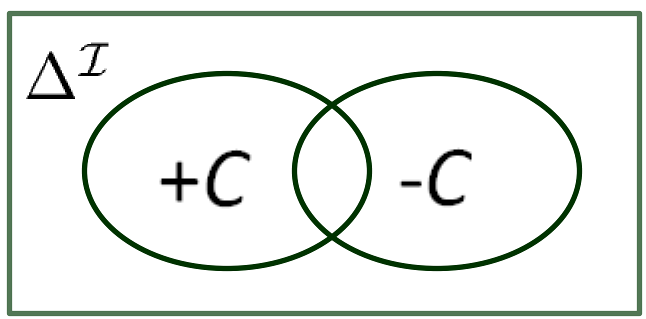
Quasi-classical semantics by presented ***Besnard*** and ***Hunter*** (1995) has those good features.

Quasi-classical DL $\mathcal{ALCNQ}$

- Syntax
  1. The language of QC $\mathcal{ALCNQ}$ is almost the same as that of $\mathcal{ALCNQ}$.
  2. A new concept constructor ($\overline{C}$) called ***complement of a concept*** is introduced.
  3. A concept *C* is in **QC NNF**, if concept *C* is in NNF and complement only occurs over a concept name or negation of a concept name.

Quasi-classical DL $\mathcal{ALCNQ}$

- Semantics
  1. **QC semantics** contains two semantics, namely, **QC weak semantics** and **QC strong semantics**.
  2. A concept *C* is interpreted over domain as a pair ‹+*C*,-*C*›.



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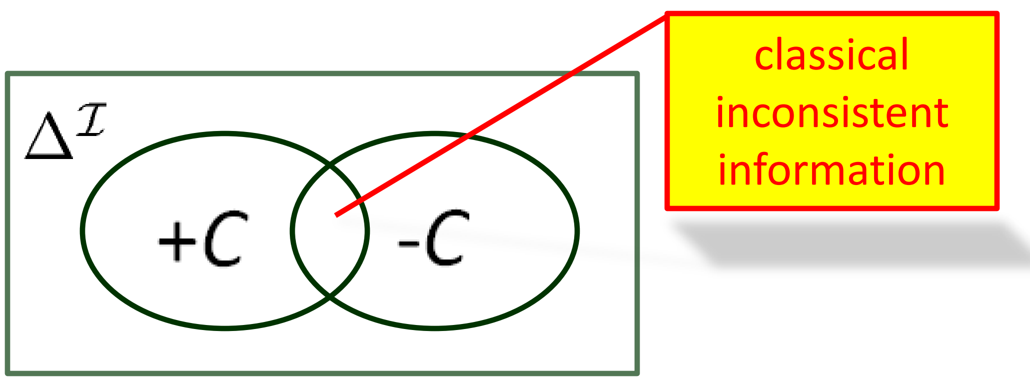

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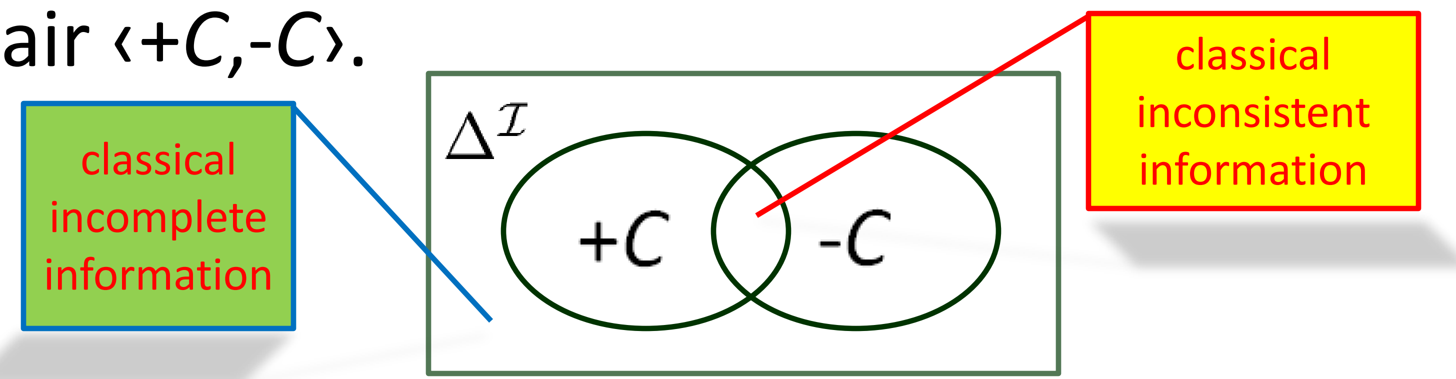

Quasi-classical DL $\mathcal{ALCNQ}$

- *QC weak interpretations*

Constructor Syntax	Weak Semantics
A	$A^{\mathcal{I}} = \langle +A, -A \rangle$, where $+A, -A \subseteq \Delta^{\mathcal{I}}$
R	$R^{\mathcal{I}} = \langle +R, -R \rangle$, where $+R, -R \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
o	$o^{\mathcal{I}} \in \Delta^{\mathcal{I}}$
$\top$	$\langle \Delta^{I}, \emptyset \rangle$
$\bot$	$\langle \emptyset, \Delta^{\mathcal{I}} \rangle$
$C_1 \sqcap C_2$	$\langle +C_1 \cap +C_2, -C_1 \cup -C_2 \rangle$
$C_1 \sqcup C_2$	$\langle +C_1 \cup +C_2, -C_1 \cap -C_2 \rangle$
$\neg C$	$\langle -C, +C \rangle$
$\overline{C}$	$\langle \Delta^{\mathcal{I}} \setminus (-C), \Delta^{\mathcal{I}} \setminus (+C) \rangle$
$\exists R.C$	$\langle \{x \mid \exists y, (x,y) \in +R \text{ and } y \in +C\}, \{x \mid \forall y, (x,y) \in +R \text{ implies } y \in -C\} \rangle$
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$\geq nR$	$\langle \{x \mid \sharp(\{y.(x,y) \in +R\}) \geq n\}, \{x \mid \sharp(\{y.(x,y) \in +R\}) < n\} \rangle$
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Quasi-classical DL $\mathcal{ALCNQ}$

- *QC weak satisfaction*
  - $\mathcal{I} \models_w C(a)$ *iff* $a^{\mathcal{I}} \in +C, C^{\mathcal{I}} = \langle +C, -C\rangle$;
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- *QC entailment relationship*
  An ontology $\mathcal{O}$ ***quasi-classically entails*** an axiom ϕ iff for each interpretation $\mathcal{I}$ if $\mathcal{I}$ is a QC strong model of $\mathcal{O}$ then $\mathcal{I} \models_w \phi$. $(\mathcal{O} \models_Q \phi)$

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- *QC entailment features*
  - $\{B(a), \neg B(a)\} \not\models_Q C(a)$.
  - $\{C \sqcup D(a), \neg C \sqcup E(a)\} \models_Q D \sqcup E(a)$.
  - If $\mathcal{O} \models_Q \phi$ then $\mathcal{O} \models \phi$.
  - If $\mathcal{O} \models_4 \phi$ then $\mathcal{O} \models_Q \phi$.

Quasi-classical DL $\mathcal{ALCNQ}$

- ***QC consistency***
  - A concept C is *QC satisfiable* w.r.t. a TBox $\mathcal{T}$ if there exists a QC model $\mathcal{I}$ of $\mathcal{T}$ such that $+C \neq \emptyset$ where $C^{\mathcal{I}} = \langle +C, -C \rangle$; and *QC unsatisfiable* w.r.t. $\mathcal{T}$ otherwise.
  - An ABox $\mathcal{A}$ is *QC consistent* if there exists a QC model $\mathcal{I}$ of $\mathcal{A}$, and *QC inconsistent* otherwise.
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- ***QC inference problems***
  - ***Instance checking:*** an individual a is called a *QC instance* of a concept C w.r.t. an ABox $\mathcal{A}$ iff for any QC model $\mathcal{I}$ of $\mathcal{A}$, $\mathcal{I}$ is a QC model of $C(a)$.
  - ***Subsumption*** a concept C *QC subsumes* a concept D w.r.t. a TBox $\mathcal{T}$ iff for any QC model $\mathcal{I}$ of $\mathcal{T}$, $\mathcal{I}$ is a QC model of $C \sqsubseteq D$.

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Reducing QC inference problems to QC consistency problem

- $\mathcal{O} \models_Q C(a)$ iff $\mathcal{O} \cup \{\overline{C}(a)\}$ is QC inconsistent.
- $\mathcal{O} \models_Q C \sqsubseteq D$ iff $\mathcal{O} \cup \{C \sqcap \overline{D}(b)\}$ is QC inconsistent for some new individual b not occurring in $\mathcal{O}$.

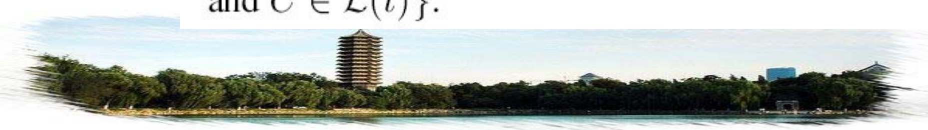
A Tableau Algorithm for $\mathcal{ALCNQ}$

- ***QC Tableau*** (Based on *Ian Horrocks*'s tableau)

Given an ABox $\mathcal{A}$, $T = (S, \mathcal{L}, \mathcal{E}, \mathcal{J})$ is a QC tableau for $\mathcal{A}$ iff

- S is a non-empty set;
- $\mathcal{L} : S \to 2^{clos(\mathcal{A})}$ maps each element in S to a set of concepts;
- $\mathcal{E} : R_{\mathcal{A}} \to 2^{S \times S}$ maps each role to a set of pairs of elements in S;
- $\mathcal{J} : U_{\mathcal{A}} \to S$ maps individuals occurring in $\mathcal{A}$ to elements in S.

Furthermore, for all $s, t \in S, C, C_1, C_2 \in clos(\mathcal{A})$ and T satisfies:
(P1) if $C \in \mathcal{L}(s)$, then $\overline{C} \notin \mathcal{L}(s)$,
(P2) if $C_1 \sqcap C_2 \in \mathcal{L}(s)$, then $C_1 \in \mathcal{L}(s)$ and $C_2 \in \mathcal{L}(s)$,
(P3) if $C_1 \sqcup C_2 \in \mathcal{L}(s)$, then
(a) if $\sim C_i \in \mathcal{L}(s)$ for some ($i \in \{1, 2\}$), then $\otimes(C_1 \sqcup C_2, C_i) \in \mathcal{L}(s)$,
(b) else $C_1 \in \mathcal{L}(s)$ or $C_2 \in \mathcal{L}(s)$,
(P4) if $\forall R.C \in \mathcal{L}(s)$ and $\langle s, t\rangle \in \mathcal{E}(R)$, then $C \in \mathcal{L}(t)$,
(P5) if $\exists R.C \in \mathcal{L}(s)$, then there is some $t \in S$ such that $\langle s, t\rangle \in \mathcal{E}(R)$ and $C \in \mathcal{L}(t)$,
(P6) if $\leq nR.C \in \mathcal{L}(s)$, then $\sharp R^T(s, C) \leq n$,
(P7) if $\geq nR.C \in \mathcal{L}(s)$, then $\sharp R^T(s, C) \geq n$,
(P8) if $(\bowtie nR.C) \in \mathcal{L}(s)$ and $\langle s, t\rangle \in \mathcal{E}(R)$ then $C \in \mathcal{L}(t)$ or $\overline{C} \in \mathcal{L}(t)$,
(P9) if $a : C \in \mathcal{A}$, then $C \in \mathcal{L}(\mathcal{J}(a))$,
(P10) if $(a, b) : R \in \mathcal{A}$, then $\langle \mathcal{J}(a), \mathcal{J}(b)\rangle \in \mathcal{E}(R)$,
(P11) if $a \neq b \in \mathcal{A}$, then $\mathcal{J}(a) \neq \mathcal{J}(b)$,
where $\bowtie$ is a place-holder for both $\leq$ and $\geq$, and $R^T(s, C) = \{t \in S \mid \langle s, t\rangle \in \mathcal{E}(R)$ and $C \in \mathcal{L}(t)\}$.



A Tableau Algorithm for $\mathcal{ALCNQ}$

- ***QC Tableau*** (Based on *Ian Horrocks*'s tableau)

Given an ABox $\mathcal{A}$, $T = (S, \mathcal{L}, \mathcal{E}, \mathcal{J})$ is a QC tableau for $\mathcal{A}$ iff

- S is a non-empty set;
- $\mathcal{L} : S \to 2^{clos(\mathcal{A})}$ maps each element in S to a set of concepts;
- $\mathcal{E} : R_{\mathcal{A}} \to 2^{S \times S}$ maps each role to a set of pairs of elements in S;
- $\mathcal{J} : U_{\mathcal{A}} \to S$ maps individuals occurring in $\mathcal{A}$ to elements in S.

Furthermore, for all $s, t \in S, C, C_1, C_2 \in clos(\mathcal{A})$ and T satisfies:

A QC ABox is <u>QC consistent</u> if and only if it has a <u>QC tableau</u>.

(P8) if $(\bowtie nR.C) \in \mathcal{L}(s)$ and $\langle s, t \rangle \in \mathcal{E}(R)$ then $C \in \mathcal{L}(t)$ or $\overline{C} \in \mathcal{L}(t)$,
(P9) if $a : C \in \mathcal{A}$, then $C \in \mathcal{L}(\mathcal{J}(a))$,
(P10) if $(a, b) : R \in \mathcal{A}$, then $\langle \mathcal{J}(a), \mathcal{J}(b) \rangle \in \mathcal{E}(R)$,
(P11) if $a \neq b \in \mathcal{A}$, then $\mathcal{J}(a) \neq \mathcal{J}(b)$,
where $\bowtie$ is a place-holder for both $\leq$ and $\geq$, and $R^T(s, C) = \{t \in S \mid \langle s, t \rangle \in \mathcal{E}(R)$ and $C \in \mathcal{L}(t)\}$.

A Tableau Algorithm for $\mathcal{ALCNQ}$

1. The $\rightarrow_{\sqcap}$-rule
Condition: $C_1 \sqcap C_2 \in \mathcal{L}(x)$, x is not indirectly blocked, and $\{C_1, C_2\} \not\subseteq \mathcal{L}(x)$.
Action: $\mathcal{L}(x) := \mathcal{L}(x) \cup \{C_1(x), C_2(x)\}$.

2. The $\rightarrow_{\sqcup}$-rule
Condition: $C_1 \sqcup C_2 \in \mathcal{L}(x)$, x is not indirectly blocked, and $\{C_1, C_2, \sim C_1, \sim C_2\} \cap \mathcal{L}(x) = \emptyset$.
Action: $\mathcal{L}(x) := \mathcal{L}(x) \cup \{E\}$ for some $E \in \{C_1, C_2\}$.

3.The $\rightarrow_{QC}$-rule
Condition: $C_1 \sqcup C_2 \in \mathcal{L}(x)$, x is not indirectly blocked, and $\sim C_i \in \mathcal{L}(x)$(for some $i \in \{1, 2\}$).
Action: $\mathcal{L}(x) := \mathcal{L}(x) \cup \{\otimes(C_1 \sqcup C_2, C_i)\}$.

4. The $\rightarrow_{\exists}$-rule
Condition: $\exists R.C \in \mathcal{L}(x)$, x is not blocked, and x has no R-neighbor y with $C \in \mathcal{L}(y)$.
Action: create a new node y with $\mathcal{L}(\langle x, y\rangle) := \{R\}$ and $\mathcal{L}(y) := \{C\}$.

5. The $\rightarrow_{\forall}$-rule
Condition:
(1) $\forall R.C \in \mathcal{L}(x)$, x is not indirectly blocked, and
(2) there is an R-neighbor y of x with $C \in \mathcal{L}(y)$.
Action: $\mathcal{L}(y) := \mathcal{L}(y) \cup \{C\}$.

6. The choose-rule
Condition: $(\bowtie nR.C) \in \mathcal{L}(x)$, x is not indirectly blocked, and there is an R-neighbor y of x with $\{C, \overline{C}\} \cap \mathcal{L}(y) = \emptyset$.
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Other six rules are based on Ian Horrocks's expansion rules for DLs.

A Tableau Algorithm for $\mathcal{ALCNQ}$

- *QC Tableau Algorithm*

  1. All concepts is in ***QC NNF***.

  E.g. $A, \neg A, \overline{A}, \overline{\neg A}$

  2. $clos(\mathcal{A})$:a ***closure*** of concepts occurring in $\mathcal{A}$

  3. **Node**: $\mathcal{L}(x)$, $\mathcal{L}(x) \subseteq clos(\mathcal{A})$

  4. R **-Edge**: $\mathcal{L}(\langle x, y\rangle), \mathcal{L}(\langle x, y\rangle) \in R$

  5. A **QC forest** is a collection of QC trees with nodes and edges.

  6. **Closed condition**: $\{C, \overline{C}\} \subseteq \mathcal{L}(x)$

A Tableau Algorithm for $\mathcal{ALCNQ}$

7. A QC tree is **closed** if it satisfies the closed condition.
8. A QC forest is **closed** if all trees of it are closed.

A Tableau Algorithm for $\mathcal{ALCNQ}$

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Soundness and Completeness

A QC ABox has a <u>*QC tableau*</u> if and only if the QC forest of it is <u>*closed*</u>.

A Tableau Algorithm for $\mathcal{ALCNQ}$

- ***Example***: *given an ABox* ***A*** *and a query* **q**,

1.***A****={Bird(a),Penguin(a),¬Fly(a), ¬Bird⊔Fly(a), ¬Bird⊔Haswing(a) }.*

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8. **Return** ***true*** *about query **q w.r.t. A**. That is, **A** $\models_Q$ **q.***

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- Our approach ensures **stronger inference power** than those based four-valued semantics.
- Our approach has **approximate ability** to handle consistent DL-ontologies.
- Our approach **localizes inconsistent information** in whole knowledgebase to some extent.

Future works

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- We will employ indirectly some classical reasoners such as ***Pellet*** and ***KAON2*** to implement paraconsistent reasoning.*(under consideration)*
- We will build our **QC reasoner** based on QC tableau algorithm presented in this paper.

Thank you for your attention!

Ευχαριστώ

謝謝

Questions?