

Transfer Learning for Collaborative Filtering via a Rating-Matrix Generative Model

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Outline

1 Motivation

- Sparsity Problem
- Knowledge Sharing

2 Rating-Matrix Generative Model

- Model Construction
- Learning & Prediction

3 Experiments

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Sparsity Problem in Collaborative Filtering

- Most CF methods are based on the idea of discovering latent user/item clusters and sharing rating knowledge within clusters
- However, in real-world recommender systems, the rating-matrix is usually too sparse to well cluster

| | a | b | c | d | e | f |
|---|---|---|---|---|---|---|
| 1 | ? | 3 | ? | 3 | 2 | 3 |
| 2 | 3 | 1 | 2 | 2 | ? | 1 |
| 3 | 3 | ? | 2 | ? | 3 | 1 |
| 4 | 3 | ? | 1 | 1 | ? | 2 |
| 5 | 2 | 3 | 3 | ? | 2 | ? |
| 6 | 3 | 2 | ? | 1 | 3 | 2 |

Sparse rating-matrix

→
Clustering

| | a | e | b | f | c | d |
|---|---|---|---|---|---|---|
| 2 | 3 | 3 | 1 | 1 | 2 | 2 |
| 3 | 3 | 3 | 1 | 1 | 2 | 2 |
| 1 | 2 | 2 | 3 | 3 | 3 | 3 |
| 5 | 2 | 2 | 3 | 3 | 3 | 3 |
| 4 | 3 | 3 | 2 | 2 | 1 | 1 |
| 6 | 3 | 3 | 2 | 2 | 1 | 1 |

Expected clustering result

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Main Idea

Pool the rating data from **multiple related** CF domains to discover a better latent structure

| | | Item Group | | | | | | | |
|---------------|----|--------------|-----|-----|--|---|--|--|--|
| | | A | | B | | | | | |
| | | User Group I | 3 3 | 1 1 | | | | | |
| User Group I | A | | | | | This can be from either MOVIE or BOOK website | | | |
| | B | | | | | | | | |
| User Group II | I | | 3 3 | 1 1 | | A Romance movies | | | |
| | II | | 2 2 | 3 3 | | B Sci-Fi movies | | | |
| | I | | 2 2 | 3 3 | | I Girls in IMDB | | | |
| | II | | 2 2 | 3 3 | | OR II Boys in IMDB | | | |
| | | | | | | Romance books | | | |
| | | | | | | Sci-Fi books | | | |
| | | | | | | OR Girls in Amazon | | | |
| | | | | | | Boys in Amazon | | | |

What “relatedness”

Users are related in **Interest**; Items are related in **Genre**

Cluster-Level Rating Matrix as Knowledge Sharing

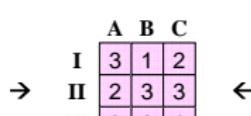
| | a | b | c | d | e | f |
|---|---|---|---|---|---|---|
| 1 | ? | 3 | ? | 3 | 2 | 3 |
| 2 | 3 | 1 | 2 | 2 | ? | 1 |
| 3 | 3 | ? | 2 | ? | 3 | 1 |
| 4 | 3 | ? | 1 | 1 | ? | 2 |
| 5 | 2 | 3 | 3 | ? | 2 | ? |
| 6 | 3 | 2 | ? | 1 | 3 | 2 |

| | a | b | c | d | e |
|---|---|---|---|---|---|
| 1 | 3 | 3 | ? | ? | 1 |
| 2 | 2 | ? | 2 | 1 | ? |
| 3 | ? | 1 | 2 | 1 | 1 |
| 4 | ? | 3 | 3 | 3 | 1 |
| 5 | 2 | ? | 2 | 1 | ? |
| 6 | ? | 1 | 2 | 1 | 3 |
| 7 | 1 | 2 | ? | 2 | 2 |

| | a | b | c | d | e | f | g |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 1 | ? | 3 | 3 | ? | 1 |
| 2 | ? | ? | 3 | 2 | 1 | 2 | 2 |
| 3 | 2 | 1 | 2 | ? | 3 | 3 | ? |
| 4 | 1 | 3 | ? | 1 | 2 | 1 | 3 |
| 5 | 3 | 2 | 3 | ? | ? | 2 | ? |

Permute rows & cols
→

| | a | e | b | f | c | d |
|---|---|---|---|---|---|---|
| 2 | 3 | ? | 1 | 1 | 2 | 2 |
| 3 | 3 | 3 | ? | 1 | 2 | ? |
| 1 | ? | 2 | 3 | 3 | ? | 3 |
| 5 | 2 | 2 | 3 | ? | 3 | ? |
| 4 | 3 | ? | ? | 2 | 1 | 1 |
| 6 | 3 | 3 | 2 | 2 | ? | 1 |



Cluster-level Rating Matrix

| | A | B | C | D |
|-----|---|---|---|---|
| I | 3 | 1 | 2 | 1 |
| II | 2 | 3 | 3 | 1 |
| III | 3 | 2 | 1 | 2 |
| IV | 1 | 1 | 2 | 3 |



Permute rows & cols
→

| | b | d | a | c | e |
|---|---|---|---|---|---|
| 5 | ? | 1 | 2 | 2 | ? |
| 3 | 1 | 1 | ? | 2 | 1 |
| 1 | 3 | ? | 3 | ? | 1 |
| 4 | 3 | 3 | ? | 3 | 1 |
| 7 | 2 | 2 | 1 | ? | 2 |
| 2 | ? | 1 | 2 | 2 | ? |
| 6 | 1 | 1 | ? | 2 | 3 |

| | B | C | D |
|-----|---|---|---|
| I | 1 | 2 | 1 |
| II | 3 | 3 | 1 |
| III | 2 | 1 | 2 |
| IV | 1 | 2 | 3 |

Cluster-level Rating Matrix

| | A | B | C | D |
|-----|---|---|---|---|
| I | 3 | 1 | 2 | 1 |
| II | 2 | 3 | 3 | 1 |
| III | 3 | 2 | 1 | 2 |
| IV | 1 | 1 | 2 | 3 |



Permute rows & cols
→

| | a | c | d | f | e | b | g |
|---|---|---|---|---|---|---|---|
| 1 | 2 | ? | 3 | ? | 3 | 1 | 1 |
| 3 | 2 | 2 | ? | 3 | 3 | 1 | ? |
| 2 | ? | 3 | 2 | 2 | 1 | ? | 2 |
| 5 | 3 | 3 | ? | 2 | ? | 2 | ? |
| 4 | 1 | ? | 1 | 1 | 2 | 3 | 3 |

| | A | B | C | D |
|-----|---|---|---|---|
| II | 2 | 3 | 3 | 1 |
| III | 3 | 2 | 1 | 2 |
| IV | 1 | 1 | 2 | 3 |

Cluster-level Rating Matrix

| | A | B | C | D |
|-----|---|---|---|---|
| I | 3 | 1 | 2 | 1 |
| II | 2 | 3 | 3 | 1 |
| III | 3 | 2 | 1 | 2 |
| IV | 1 | 1 | 2 | 3 |



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Problem Setting

Given Z rating matrices in **related** domains, in the z -th domain

- User set $U_z = \{u_1^{(z)}, \dots, u_{n_z}^{(z)}\} \subset \mathcal{U}$
- Item set $V_z = \{v_1^{(z)}, \dots, v_{m_z}^{(z)}\} \subset \mathcal{V}$
- Rating data $D_z = \{(u_1^{(z)}, v_1^{(z)}, r_1^{(z)}), \dots, (u_{s_z}^{(z)}, v_{s_z}^{(z)}, r_{s_z}^{(z)})\}$

Assume $\bigcap_z U_z = \emptyset$ and $\bigcap_z V_z = \emptyset$ and ratings in $\{D_1, \dots, D_Z\}$ should be in the same rating scales R (e.g., 1 – 5)

Goal

To learn a rating-matrix generative model (RMGM) for the given related tasks on the pooled rating data $\bigcup_z D_z$ and predict missing values

User-Item Joint Mixture Model

Users/Items can **simultaneously** belong to multiple clusters

- Users may have multiple **Personalities**
- Items may have multiple **Attributes**

Suppose there are K user clusters $\{c_u^{(1)}, \dots, c_u^{(K)}\}$ and L item clusters $\{c_v^{(1)}, \dots, c_v^{(L)}\}$, the marginal distributions for users and items are

$$P_u(u) = \sum_k P(c_u^{(k)})P(u|c_u^{(k)}), \quad P_v(v) = \sum_l P(c_v^{(l)})P(v|c_v^{(l)})$$

User-Item Joint Mixture Model

$$(u_i^{(z)}, v_i^{(z)}) \sim \sum_{k,l} P(c_u^{(k)})P(c_v^{(l)})P(u|c_u^{(k)})P(v|c_v^{(l)}) \quad (1)$$

Cluster-Level Rating Model

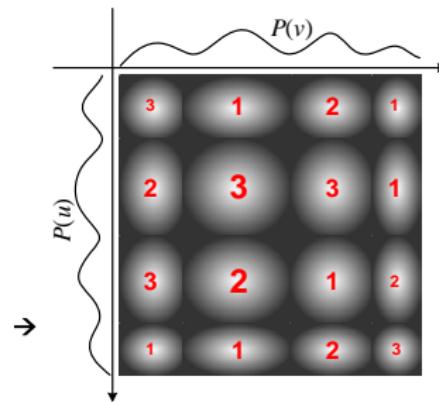
Cluster-Level Rating Model

$$r_i^{(z)} \sim P(r|c_u^{(k)}, c_v^{(l)}) \quad (2)$$

4x4 Cluster-level Rating Matrix

| | A | B | C | D |
|-----|---|---|---|---|
| I | 3 | 1 | 2 | 1 |
| II | 2 | 3 | 3 | 1 |
| III | 3 | 2 | 1 | 2 |
| IV | 1 | 1 | 2 | 3 |

Extended to a cluster-level rating model



Combining (1) and (2) gives rating-matrix generative model (RMGM)

Rating-Matrix Generating Process

CF Task I

| | a | e | b | f | c | d |
|---|---|---|---|---|---|---|
| 2 | 3 | ? | 1 | 1 | 2 | 2 |
| 3 | 3 | 3 | ? | 1 | 2 | ? |
| 1 | ? | 2 | 3 | 3 | ? | 3 |
| 5 | 2 | 2 | 3 | ? | 3 | ? |
| 4 | 3 | ? | ? | 2 | 1 | 1 |
| 6 | 3 | 3 | 2 | 2 | ? | 1 |

CF Task II

| | b | d | a | c | e |
|---|---|---|---|---|---|
| 5 | ? | 1 | 2 | 2 | ? |
| 3 | 1 | 1 | ? | 2 | 1 |
| 1 | 3 | ? | 3 | ? | 1 |
| 4 | 3 | 3 | ? | 3 | 1 |
| 7 | 2 | 2 | 1 | ? | 2 |
| 2 | ? | 1 | 2 | 2 | ? |
| 6 | 1 | 1 | ? | 2 | 3 |

CF Task III

| | a | c | d | f | e | b | g |
|---|---|---|---|---|---|---|---|
| 1 | 2 | ? | 3 | ? | 3 | 1 | 1 |
| 3 | 2 | 2 | ? | 3 | 3 | 1 | ? |
| 2 | ? | 3 | 2 | 2 | 1 | ? | 2 |
| 5 | 3 | 3 | ? | 2 | ? | 2 | ? |
| 4 | 1 | ? | 1 | 1 | 2 | 3 | 3 |



Draw users and items from the **same user-item joint mixture model** for **related tasks**

| | a | e | b | f | c | d |
|---|---|---|---|---|---|---|
| 2 | 3 | | 1 | | 2 | 1 |
| 3 | | | | | | |
| 1 | 2 | | 3 | | 3 | 1 |
| 5 | | | | | | |
| 4 | 3 | | 2 | | 1 | 2 |
| 6 | | | | | | |
| | 1 | | 1 | | 2 | 3 |

| | b | d | a | c | e |
|---|---|---|---|---|---|
| 5 | 3 | | 1 | | 2 |
| 3 | | | | | |
| 1 | 2 | | 3 | | 3 |
| 4 | | | | | |
| 7 | 3 | | 2 | | 1 |
| 2 | | | | | |
| 6 | 1 | | 1 | | 2 |

| | a | c | d | f | e | b | g |
|---|---|---|---|---|---|---|---|
| 1 | 3 | | | 1 | | 2 | 1 |
| 3 | | | | | | | |
| 2 | 2 | | 3 | | 3 | 1 | ? |
| 5 | | | | | | | |
| 4 | 3 | | 2 | | 1 | 2 | 3 |
| 6 | 1 | | 1 | | 2 | 3 | 3 |

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Learning the RMGM

Five sets of parameters in RMGM need to learn:

$$P(c_{\mathcal{U}}^{(k)}), P(c_{\mathcal{V}}^{(l)}), P(u|c_{\mathcal{U}}^{(k)}), P(v|c_{\mathcal{V}}^{(l)}), \text{ and } P(r|c_{\mathcal{U}}^{(k)}, c_{\mathcal{V}}^{(l)})$$

for $k = 1, \dots, K$; $l = 1, \dots, L$; $u \in \bigcup_z U_z$; $v \in \bigcup_z V_z$; and $r \in R$

Expectation Maximization (EM) Algorithm

- **E-Step:** the joint posterior probability $P(c_{\mathcal{U}}^{(k)}, c_{\mathcal{V}}^{(l)} | u_i^{(z)}, v_i^{(z)}, r_i^{(z)})$ is computed using the five sets of parameters
- **M-Step:** the five sets of parameters are updated based on $P(c_{\mathcal{U}}^{(k)}, c_{\mathcal{V}}^{(l)} | u_i^{(z)}, v_i^{(z)}, r_i^{(z)})$

Note: all the parameters are computed on the pooled rating data $\bigcup_z D_z$

RMGM-Based Prediction

Predicting Missing Values for An Existing User

$$f_R(u_i^{(z)}, v_i^{(z)}) = \sum_r r \sum_{k,l} P(r|c_u^{(k)}, c_v^{(l)}) P(c_u^{(k)}|u_i^{(z)}) P(c_v^{(l)}|v_i^{(z)})$$

Predicting Missing Values for A New User

Solve a quadratic optimization problem to estimate the user-cluster membership $\mathbf{p}_{u^{(z)}} \in \mathbb{R}^K$ for $u^{(z)}$ based on the given ratings $\mathbf{r}_{u^{(z)}}$

$$\min_{\mathbf{p}_{u^{(z)}}} \|[\mathbf{B}\mathbf{P}_{V_z}]^\top \mathbf{p}_{u^{(z)}} - \mathbf{r}_{u^{(z)}}\|_{\mathbf{W}_{u^{(z)}}}^2, \quad \text{s.t. } \mathbf{p}_{u^{(z)}}^\top \mathbf{1} = 1$$

where $\mathbf{B}_{kl} = \sum_r r P(r|c_u^{(k)}, c_v^{(l)})$ and $[\mathbf{P}_{V_z}]_{li} = P(c_v^{(l)}|v_i^{(z)})$

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Experimental Setup

Compared Methods

- Pearson Correlation Coefficients (**PCC**) - Baseline
- Flexible Mixture Model (**FMM**) - Single-task
- Rating-Matrix Generative Model (**RMGM**) - **Multi-task**

Data Sets

Randomly select 500 users and 1000 items from each of the following real-world data sets: 1) **MovieLens** (Movie); 2) **EachMovie** (Movie); 3) **Book-Crossing** (Book) - Three 500×1000 rating matrices

Evaluation Protocol

- First 100/200/300 users for training; last 200 users for testing
- Given 5/10/15 observable ratings for each test user
- Evaluation metric: Mean Absolute Error (**MAE**)

Experimental Results

Table: MAE Comparison on MovieLens (ML)

| Train | Method | Given5 | Given10 | Given15 |
|-------|--------|--------|---------|---------|
| ML100 | PCC | 0.930 | 0.908 | 0.895 |
| | FMM | 0.908 | 0.868 | 0.846 |
| | RMGM | 0.868 | 0.822 | 0.808 |
| ML200 | PCC | 0.934 | 0.899 | 0.888 |
| | FMM | 0.890 | 0.863 | 0.847 |
| | RMGM | 0.859 | 0.821 | 0.806 |
| ML300 | PCC | 0.935 | 0.896 | 0.888 |
| | FMM | 0.885 | 0.868 | 0.846 |
| | RMGM | 0.857 | 0.820 | 0.804 |

Experimental Results

Table: MAE Comparison on EachMovie (EM)

| Train | Method | Given5 | Given10 | Given15 |
|-------|--------|--------|---------|---------|
| EM100 | PCC | 0.996 | 0.952 | 0.936 |
| | FMM | 0.969 | 0.937 | 0.924 |
| | RMGM | 0.942 | 0.908 | 0.895 |
| EM200 | PCC | 0.983 | 0.943 | 0.930 |
| | FMM | 0.955 | 0.933 | 0.923 |
| | RMGM | 0.934 | 0.905 | 0.890 |
| EM300 | PCC | 0.976 | 0.937 | 0.933 |
| | FMM | 0.952 | 0.930 | 0.924 |
| | RMGM | 0.934 | 0.906 | 0.890 |

Experimental Results

Table: MAE Comparison on Book-Crossing (BX)

| Train | Method | Given5 | Given10 | Given15 |
|-------|--------|--------|---------|---------|
| BX100 | PCC | 0.617 | 0.599 | 0.600 |
| | FMM | 0.619 | 0.592 | 0.583 |
| | RMGM | 0.612 | 0.583 | 0.573 |
| BX200 | PCC | 0.621 | 0.612 | 0.620 |
| | FMM | 0.617 | 0.602 | 0.596 |
| | RMGM | 0.615 | 0.591 | 0.583 |
| BX300 | PCC | 0.621 | 0.619 | 0.630 |
| | FMM | 0.615 | 0.604 | 0.596 |
| | RMGM | 0.612 | 0.590 | 0.581 |

Summary

Summary

- Relate in cluster-level rating patterns
- Bridge via a cluster-level rating model
- Transfer rating knowledge
- Benefit from one another
- Alleviate sparsity problem

Future Work

- Quantify the “relatedness”
- Asymmetric setting: Dense → Sparse