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Dynamic Analysis of Multiagent Q-learning with ϵ -greedy Exploration

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- > Multiagent Learning (MAL) has become very active research area
- > MAL-based systems are finding application in a wide variety of domains
- > **Tools to understand and model the expected dynamics are necessary**

Motivation



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Multiagent Q -learning with ϵ -greedy exploration

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Multiagent Q -learning with ϵ -greedy exploration

- > Classic algorithm
- > It has been applied with success in several domains



Q-learning

- > Most studied Reinforcement Learning algorithm
- > Strong theoretical support and convergence guarantees



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- > ... only in the single-agent case



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Motivation



Q-learning

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- > ... **only in the single-agent case**

Multiagent Q-learning

- > Lack of theoretical support and convergence guarantees
- > Very dynamic environment
- > Co-adaptation effect
- > Rewards and state transitions depend on the joint actions
- > **Very hard to obtain the dynamics**





- > Researchers have explored links between RL and EGT
- > Same principles
 - Growth in one strategy's probability is directly proportional to its performance against the others
- > Model of Multiagent Q-learning with Boltzmann exploration



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 - Growth in one strategy's probability is directly proportional to its performance against the others
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- > **Cannot be applied because we have a semi-uniform distribution**

RL and Evolutionary Game Theory



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- > Same principles
 - Growth in one strategy's probability is directly proportional to its performance against the others
- > Model of Multiagent Q-learning with Boltzmann exploration
- > **Cannot be applied because we have a semi-uniform distribution**

ϵ – *greedy* mechanism

- > Selects the best action with probability $1 - \epsilon$
- > Selects a random action with probability ϵ



Multiagent Q-learning

- > Each agent applies the standard Q-learning algorithm
- > The agents learn independently
- > Rewards and state transitions depend on their joint strategies

Background



Multiagent Q-learning

- > Each agent applies the standard Q-learning algorithm
- > The agents learn independently
- > Rewards and state transitions depend on their joint strategies

- > Each agent maintains a table of Q-values
 - $Q(s, i)$ represents how good it is to take action i at state s
- > They update the Q-values as they gather experience in the environment
$$Q(s, i) = Q(s, i) + \alpha(r(s, i) + \gamma \max_{i'} Q(s', i') - Q(s, i))$$
 - $r(s, i)$ is the reward for taking action i at state s
 - α is the learning rate
 - γ is the discount rate



Exploration - exploitation problem

- > exploit actions known to be good
- > explore new actions

ϵ -greedy

- > chose the currently best action with probability $1 - \epsilon$
- > chose a random action with probability ϵ



Exploration - exploitation problem

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 ϵ -greedy

- > chose the currently best action with probability $1 - \epsilon$
- > chose a random action with probability ϵ

$$x(s, i) = \begin{cases} (1 - \epsilon) + (\epsilon/n), & \text{if } Q(s, i) \text{ is currently the highest} \\ \epsilon/n, & \text{otherwise} \end{cases}$$

Modelling the algorithm



Modelling the algorithm



- > Build a continuous-time version of the Q-learning update rule

Modelling the algorithm



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- > Analyse the limits of this equation for the single-learner case

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- > Investigate how the ϵ -greedy affects the shape of the function



- > Build a continuous-time version of the Q-learning update rule
- > Analyse the limits of this equation for the single-learner case
- > Show how they change dynamically in the multi-learner case
- > Investigate how the ε -greedy affects the shape of the function
- > Develop a system of difference equations to obtain the expected behaviour of the agents





Single-state scenarios composed of 2 agents with 2 actions each

Notation



Single-state scenarios composed of 2 agents with 2 actions each

The reward functions can be described as payoff tables

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

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Q-learning rule can be simplified to

$$Q_{a_i} \leftarrow Q_{a_i} + \alpha(r_{a_i} - Q_{a_i})$$

Q_{a_i} is the Q -value of agent a for action i

r_{a_i} is the immediate reward that agent a receives for playing action i

Continuous-time version



$$Q_{a_j} \leftarrow Q_{a_j} + \alpha(r_{a_j} - Q_{a_j})$$

Q-learning rule

Continuous-time version



$$Q_{a_i} \leftarrow Q_{a_i} + \alpha(r_{a_i} - Q_{a_i})$$

Q-learning rule

$$Q_{a_i}(k+1) = Q_{a_i}(k) + \alpha(r_{a_i}(k+1) - Q_{a_i}(k))$$



Continuous-time version

$$Q_{a_j} \leftarrow Q_{a_j} + \alpha(r_{a_j} - Q_{a_j})$$

Q-learning rule

$$Q_{a_j}(k+1) = Q_{a_j}(k) + \alpha(r_{a_j}(k+1) - Q_{a_j}(k))$$

$$Q_{a_j}(k+1) - Q_{a_j}(k) = \alpha(r_{a_j}(k+1) - Q_{a_j}(k))$$

discrete



Continuous-time version

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discrete

$$Q_{a_j}(k + \Delta t) - Q_{a_j}(k) \approx \Delta t \times \alpha(r_{a_j}(k + \Delta t) - Q_{a_j}(k))$$



Continuous-time version

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$$Q_{a_j}(k + \Delta t) - Q_{a_j}(k) \approx \Delta t \times \alpha(r_{a_j}(k + \Delta t) - Q_{a_j}(k))$$

$$\lim_{\Delta t \rightarrow 0} \frac{Q_{a_j}(k + \Delta t) - Q_{a_j}(k)}{\Delta t} \approx \alpha(r_{a_j}(k) - Q_{a_j}(k))$$



Continuous-time version

$$Q_{a_j} \leftarrow Q_{a_j} + \alpha(r_{a_j} - Q_{a_j})$$

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$$\frac{dQ_{a_j}(k)}{dt} \approx \alpha(r_{a_j}(k) - Q_{a_j}(k))$$

continuous

Limit of the equation



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continuous

$$Q_{a_i}(k) = Ce^{-\alpha t} + r_{a_i}$$

general solution

Limit of the equation



$$\frac{dQ_{a_i}(k)}{dt} \approx \alpha(r_{a_i}(k) - Q_{a_i}(k))$$

continuous

$$Q_{a_i}(k) = Ce^{-\alpha t} + r_{a_i}$$

general solution

$$\lim_{t \rightarrow \infty} Q_{a_i}(k) = \underbrace{\lim_{t \rightarrow \infty} Ce^{-\alpha t}}_0 + \underbrace{\lim_{t \rightarrow \infty} r_{a_i}}_{r_{a_i}} = r_{a_i}$$

Non-learning adversary with pure strategy

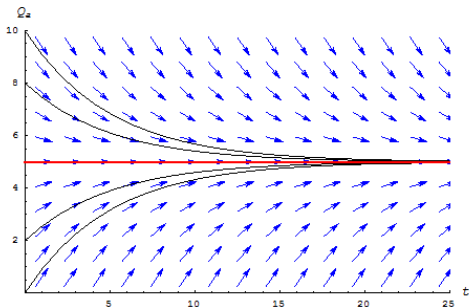


Q_{a_i} will monotonically increase or decrease towards r_{a_i}



Q_{a_i} will monotonically increase or decrease towards r_{a_i}

$\alpha = 0.2$ and $r_{a_i} = 5$; $Q_{a_i}(0) \in \{0, 2, 8, 10\}$







r_{a_i} can be replaced by $E[r_{a_i}] = \sum_j a_{ij}y_j$

0.8	0.2
1	5
0	3

$$E[r_{a_1}] = (0.8 * 1) + (0.2 * 5) = 1.8$$

$$E[r_{a_2}] = (0.8 * 0) + (0.2 * 3) = 0.6$$

$$\frac{dQ_{a_i}(t)}{dt} \approx \alpha(E[r_{a_i}(t)] - Q_{a_i}(t))$$



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$$Q_{a_i}(t) = Ce^{-\alpha t} + E[r_{a_i}]$$

$$\lim_{t \rightarrow \infty} Q_{a_i}(k) = \underbrace{\lim_{t \rightarrow \infty} Ce^{-\alpha t}}_0 + \underbrace{\lim_{t \rightarrow \infty} E[r_{a_i}]}_{E[r_{a_i}]} = E[r_{a_i}]$$

Non-learning adversary with mixed strategy



r_{a_i} can be replaced by $E[r_{a_i}] = \sum_j a_{ij}y_j$

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$$\frac{dQ_{a_i}(t)}{dt} \approx \alpha(E[r_{a_i}(t)] - Q_{a_i}(t))$$

$$Q_{a_i}(t) = Ce^{-\alpha t} + E[r_{a_i}]$$

$$\lim_{t \rightarrow \infty} Q_{a_i}(k) = \underbrace{\lim_{t \rightarrow \infty} Ce^{-\alpha t}}_0 + \underbrace{\lim_{t \rightarrow \infty} E[r_{a_i}]}_{E[r_{a_i}]} = E[r_{a_i}]$$

then Q_{a_i} will move in expectation towards $E[r_{a_i}]$ in a monotonic fashion



Learning adversary



Adversary can change its strategy during the learning
changing the expected rewards

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$$E[r_{a_1}] = (0.2 * 1) + (0.8 * 5) = 4.2$$

Learning adversary



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$$E[r_{a_1}] = (0.2 * 1) + (0.8 * 5) = 4.2$$

Each time the expected reward changes, it changes the limits
and direction fields

Learning adversary



Important to identify when the changes in the adversary's strategy will occur

Learning adversary



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ϵ -greedy updates the strategy whenever a new action becomes the one with highest Q -value

Need to find the intersection points in the adversary's functions

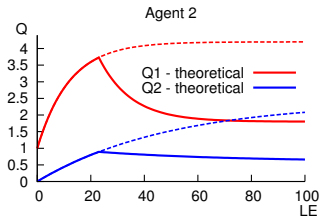
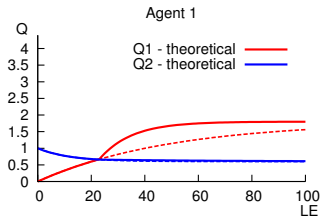
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The effects of the ϵ -greedy



The effects of the ε -greedy



Actions have different probabilities (x_i) of being played

e.g. if $\varepsilon = 0.2 \rightarrow x = [0.9, 0.1]$ or $x = [0.1, 0.9]$

they are updated at different *speeds*



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$$\frac{dQ_{a_i}(t)}{dt} \approx x_i(t) \alpha (E[r_{a_i}(t)] - Q_{a_i}(t))$$



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they are updated at different *speeds*

$$\frac{dQ_{a_i}(t)}{dt} \approx x_i(t)\alpha(E[r_{a_i}(t)] - Q_{a_i}(t))$$

$$Q_{a_i}(t) = Ce^{-x_i\alpha t} + E[r_{a_i}]$$

The effects of the ε -greedy

It does not change the limits of the equation

$$\lim_{t \rightarrow \infty} Q_{a_i}(t) = \underbrace{\lim_{t \rightarrow \infty} C e^{-x_i \alpha t}}_0 + \underbrace{\lim_{t \rightarrow \infty} E[r_{a_i}]}_{E[r_{a_i}]} = E[r_{a_i}]$$

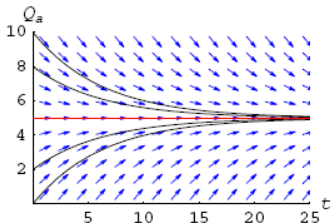
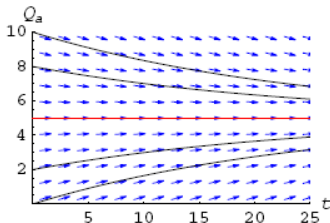
The effects of the ε -greedy



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$$\lim_{t \rightarrow \infty} Q_{a_i}(t) = \underbrace{\lim_{t \rightarrow \infty} C e^{-x_i \alpha t}}_0 + \underbrace{\lim_{t \rightarrow \infty} E[r_{a_i}]}_{E[r_{a_i}]} = E[r_{a_i}]$$

But changes the shape of the function and associated direction field



Summary of the analysis (roughly speaking)



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Expected Rewards

are the values to which the Q -values will converge to

Summary of the analysis (roughly speaking)



Expected Rewards

are the values to which the Q -values will converge to

Speeds

determine the paths that the Q -values will follow to get there

Summary of the analysis (roughly speaking)



Expected Rewards

are the values to which the Q -values will converge to

Speeds

determine the paths that the Q -values will follow to get there

Intersection points

define if the Q -values will ever get there

System of difference equations



System of difference equations



A and B payoff tables		X and Y strategy vectors		Q_a and Q_b Q-values vectors
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System of difference equations



A and B payoff tables	X and Y strategy vectors	Q_a and Q_b Q-values vectors
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$$Q_{a_i}(t+1) = Q_{a_i}(t) + x_i(t)\alpha(\sum_j a_{ij}y_j(t) - Q_{a_i}(t))$$

$$Q_{b_i}(t+1) = Q_{b_i}(t) + y_i(t)\alpha(\sum_j b_{ij}x_j(t) - Q_{b_i}(t))$$

$$x_i(t) = \begin{cases} (1 - \epsilon) + (\epsilon/n), & \text{if } Q_{a_i}(t) \text{ is currently the highest} \\ \epsilon/n, & \text{otherwise} \end{cases}$$

$$y_i(t) = \begin{cases} (1 - \epsilon) + (\epsilon/n), & \text{if } Q_{b_i}(t) \text{ is currently the highest} \\ \epsilon/n, & \text{otherwise} \end{cases}$$

Prisoner's Dilemma



$$A = \begin{bmatrix} 1 & 5 \\ 0 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 5 & 3 \end{bmatrix}$$

Prisoner's Dilemma



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$$Q_a = [0, 1], Q_b = [1, 0], \alpha = 0.1, \varepsilon = 0.4$$

$$X = [0.2, 0.8], Y = [0.8, 0.2].$$

Prisoner's Dilemma

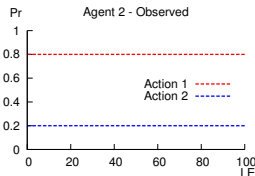
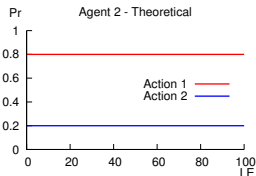
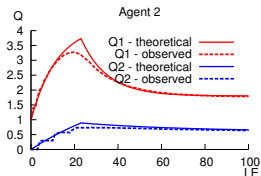
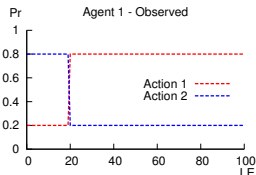
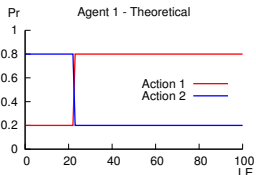
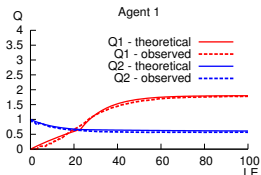


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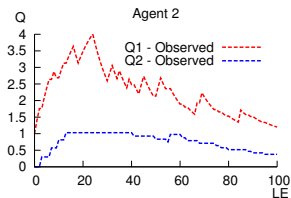
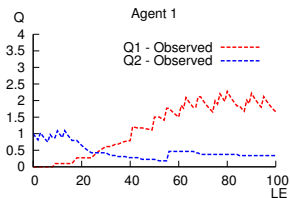
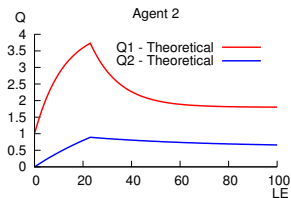
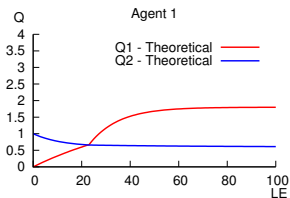
$$X = [0.2, 0.8], Y = [0.8, 0.2].$$



Prisoner's Dilemma



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Battle of the Sexes



$$A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

Battle of the Sexes



$$A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$Q_a = [2, 1], Q_b = [2, 4], \alpha = 0.1, \varepsilon = 0.1 \\ X = [0.95, 0.05], Y = [0.05, 0.95].$$

Battle of the Sexes

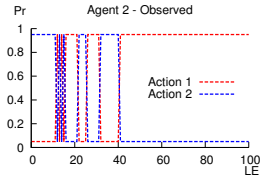
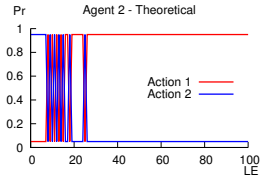
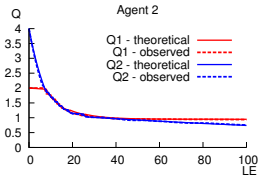
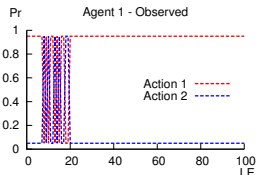
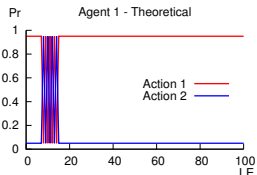
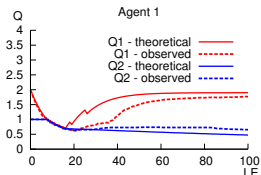


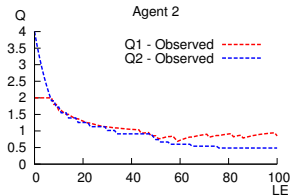
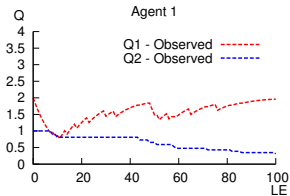
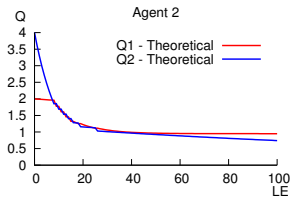
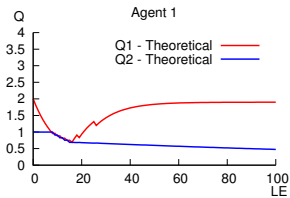
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$$X = [0.95, 0.05], Y = [0.05, 0.95].$$





A game with no equilibrium



$$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$$

A game with no equilibrium



$$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$$

$$Q_a = [0, 1], Q_b = [2, 3], \alpha = 0.1, \varepsilon = 0.1$$

$$X = [0.05, 0.95], Y = [0.05, 0.95].$$



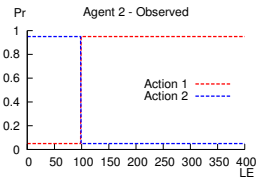
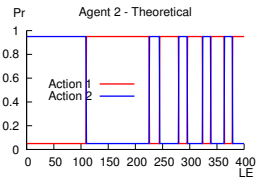
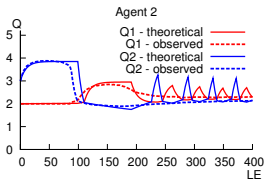
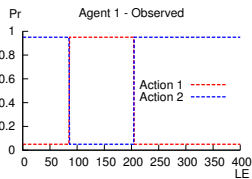
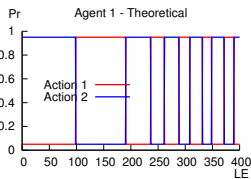
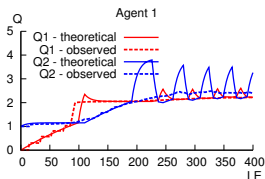
A game with no equilibrium

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$

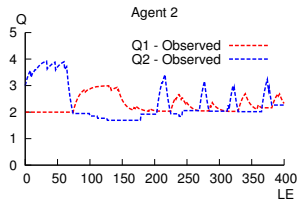
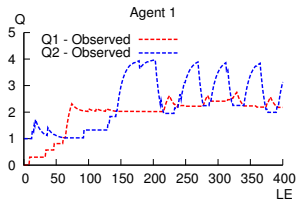
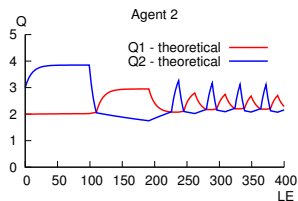
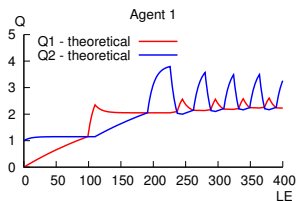
$$B = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$$

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A game with no equilibrium



Conclusions



- > Presented a model for the dynamics of Multiagent Q-learning with ϵ -greedy exploration
 - Studied a continuous-time version of the Q-learning update rule
 - Investigated how the presence of other agents and the ϵ -greedy mechanism affect it

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 - Studied a continuous-time version of the Q -learning update rule
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- > Defined a system of difference equations
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 - Derive the expected behaviour from the Q -values



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- > The evaluation of the model in typical games has shown its feasibility

Future Works



- > Extend the model to multi-state scenarios
- > Develop techniques for the visualization of the agents' behaviour

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Dynamic Analysis of Multiagent Q-learning with ϵ -greedy Exploration

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