Granger Causality and Dynamic Structural Systems[1]

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Objective

Relate Granger causality to a notion of structural causality

- Granger (G) causality

  Granger, 1969 and Granger and Newbold, 1986

- Structural causality

  White and Kennedy, 2008 and
  White and Chalak "Settable Systems," JMLR 2009

Outline

1. Granger causality, a dynamic DGP and structural causality
2. Granger causality and time-series natural experiments
3. Granger causality and structural vector autoregressions (VARs)
4. Testing finite-order Granger causality
5. Conditional exogeneity
6. Applications
7. Conclusions

1. Granger causality, a dynamic DGP and structural causality

Granger causality

Notation

- subscript$_t$ denotes a variable at time t.
- superscript^t denotes a variable's "t-history",
  (e.g., $Y^t \equiv \{Y_0, Y_1, ..., Y_t\}$).

Definition 2.1: Granger non-causality
Let $\{Q_t, S_t, Y_t\}$ *be a sequence of random vectors. Suppose that*

$$Y_t \perp Q^t \mid Y^{t-1}, S^t \quad t = 1, 2, ... \; .$$

Then Q **does not** $G-$**cause** Y **w.r.t.** S.
Else Q $G-$**causes** Y **w.r.t.** S.

Data generating process (DGP)

Assumption A.1 (White and Kennedy, 2009)

Let $\{D_t, U_t, W_t, Y_t, Z_t;\ t = 0, 1, ...\}$ be a stochastic process. Further, suppose that

$$\begin{aligned} D_t &\Leftarrow (D^{t-1}, U^t, W^t, Z^t), \\ Y_t &\Leftarrow (Y^{t-1}, D^t, U^t, W^t, Z^t) \end{aligned}$$

where, for an unknown measurable $k_y \times 1$ function q_t, $\{Y_t\}$ is structurally generated as

$$Y_t = q_t(Y^{t-1}, D^t, Z^t, U^t), \quad t = 1, 2, ...$$

Data generating process (DGP)

- $$Y_t = q_t(Y^{t-1}, D^t, Z^t, U^t), \quad t = 1, 2, \ldots.$$
- $\{D_t, W_t, Y_t, Z_t\}$ observable; $\{U_t\}$ unobservable
- Interested in
  - effects of D^t on Y_t (time-series natural experiment)
  - with $Y_t = (Y'_{1,t}, Y'_{2,t})'$, effects of Y_2^{t-1} on $Y_{1,t}$ (structural VAR)

Structural causality

Definition 3.1 (Direct causality: structural VAR)

Given A.1, for given $t > 0$, $j \in \{1, ..., k_y\}$, *and* $\mathbf{s}$, *suppose*

(i) *for all admissible values of* $y_{(\mathbf{s})}^{t-1}, d^t, z^t$, *and* u^t,

$$y_{\mathbf{s}}^{t-1} \longrightarrow q_{j,t}(y^{t-1}, d^t, z^t, u^t) \text{ is constant in } y_{\mathbf{s}}^{t-1}.$$

Then $Y_{\mathbf{s}}^{t-1}$ **does not directly structurally cause** $Y_{j,t}$:

$$Y_{\mathbf{s}}^{t-1} \overset{d}{\not\Rightarrow}_{\mathcal{S}} Y_{j,t}$$

Else $Y_{\mathbf{s}}^{t-1}$ **directly structurally causes** $Y_{j,t}$: $Y_{\mathbf{s}}^{t-1} \overset{d}{\Rightarrow}_{\mathcal{S}} Y_{j,t}$

Notation:

- $y_{\mathbf{s}}^{t-1}$: sub-vector of y^{t-1} with elements indexed by non-empty set $\mathbf{s} \subseteq \{1, ..., k_y\} \times \{0, ..., t-1\}$
- $y_{(\mathbf{s})}^{t-1}$: sub-vector of y^{t-1} with elements of $\mathbf{s}$ excluded.

Structural causality

Definition 3.1 (Direct causality: time-series natural experiment)

Given A.1, for given $t > 0$, $j \in \{1, ..., k_y\}$, *and* $\mathbf{s}$, *suppose that*

(ii) *for all admissible values of* $y^{t-1}, d^t_{(\mathbf{s})}, z^t$, *and* u^t,
$d^t_{\mathbf{s}} \to q_{j,t}(y^{t-1}, d^t, z^t, u^t)$ *is constant in* $d^t_{\mathbf{s}}$.

Then $D^t_{\mathbf{s}}$ **does not directly structurally cause** Y_{jt}: $D^t_{\mathbf{s}} \overset{d}{\not\Rightarrow}_{\mathcal{S}} Y_{j,t}$
Else $D^t_{\mathbf{s}}$ **directly structurally causes** $Y_{j,t}$: $D^t_{\mathbf{s}} \overset{d}{\Rightarrow}_{\mathcal{S}} Y_{j,t}$

Notation:

- $d^t_{\mathbf{s}}$: sub-vector of d^t with elements indexed by non-empty set $\mathbf{s} \subseteq \{1, ..., k_d\} \times \{0, ..., t\}$
- $d^t_{(\mathbf{s})}$: sub-vector of d^t with the elements of $\mathbf{s}$ excluded

Structural causality

- Recursive substitution of

$$Y_t = q_t(Y^{t-1}, D^t, Z^t, U^t), \quad t = 1, 2, ...$$

yields

$$Y_t = r_t(Y_0, D^t, Z^t, U^t), \quad t = 1, 2, ...,$$

Definition 3.2 (Total causality: time-series natural experiment)

Given A.1, suppose for all admissible values of y_0, z^t, and u^t,
$d^t \to r_t(y_0, d^t, z^t, u^t)$ *is constant in d^t.*

Then D^t **does not structurally cause** Y_t: $D^t \not\Rightarrow_{\mathcal{S}} Y_t$
Else D^t **structurally causes** Y_t : $D^t \Rightarrow_{\mathcal{S}} Y_t$

2. Granger causality and time-series natural experiments

G-causality, conditional exogeneity, and direct causality

- Let $X_t \equiv (W_t, Z_t), t = 0, 1, ...$.

Assumption A.2(a) (conditional exogeneity)

$$D^t \perp U^t | \ Y^{t-1}, X^t, t = 1, 2, ...$$

Proposition 4.1 *Let A.1 and A.2(a) hold. If* $D^t \overset{d}{\not\Rightarrow}_{\mathcal{S}} Y_t$, $t = 1, 2, ...,$ *then D does not G−cause Y w.r.t. X.*

Definition 4.3

Suppose A.1 holds and that for each $y \in supp(Y_t)$ there exists a measurable mapping $(y^{t-1}, x^t) \to f_{t,y}(y^{t-1}, x^t)$ such that w.p.1

$$\int 1\{q_t(Y^{t-1}, D^t, Z^t, u^t) < y\} \, dF_t(u^t \mid Y^{t-1}, X^t) = f_{t,y}(Y^{t-1}, X^t)$$

Then D^t **does not directly cause** Y_t **w.p.1 w.r.t.** (Y^{t-1}, X^t) :

$$D^t \overset{d}{\not\Rightarrow}_{\mathcal{S}(Y^{t-1}, X^t)} Y_t.$$

Else D^t **directly causes** Y_t **with pos. prob. w.r.t.** (Y^{t-1}, X^t) :

$$D^t \overset{d}{\Rightarrow}_{\mathcal{S}(Y^{t-1}, X^t)} Y_t.$$

Theorem 4.4

Let A.1 and A.2(a) hold. Then $D^t \overset{d}{\not\Rightarrow}_{\mathcal{S}(Y^{t-1},X^t)} Y_t$, $t = 1, 2, ...,$
if and only if *D does not G−cause Y w.r.t. X.*

Finite-order G-causality and Markov structures

Notation: finite histories
$\mathbf{Y}_{t-1} \equiv (Y_{t-\ell}, ..., Y_{t-1})$ and $\mathbf{Q}_t \equiv (Q_{t-k}, ..., Q_t)$.

Definition 4.8 *Let $\{Q_t, S_t, Y_t\}$ be a sequence of random variables, and let $k \geq 0$ and $\ell \geq 1$ be given finite integers. Suppose*

$$Y_t \perp \mathbf{Q}_t \mid \mathbf{Y}_{t-1}, S_t, \quad t = 1, 2, ...$$

Then **Q** *does not* **finite-order** *$G-$cause Y w.r.t. S.*
Else **Q** **finite-order** *$G-$causes Y w.r.t. S.*

Finite-order G-causality and Markov structures

Notation: finite histories
$\mathbf{D}_t \equiv (D_{t-k}, ..., D_t)$, $\mathbf{Z}_t \equiv (Z_{t-m}, ..., Z_t)$, $\mathbf{X}_t \equiv (X_{t-\tau_1}, ..., X_{t+\tau_2})$

Assumption B.1 A.1 holds, and for $k, \ell, m \in \mathbb{N}$, $\ell \geq 1$,

$$Y_t = q_t(\mathbf{Y}_{t-1}, \mathbf{D}_t, \mathbf{Z}_t, U_t), t = 1, 2, ...$$

Assumption B.2 For k, ℓ, and m as in B.1 and for $\tau_1 \geq m$, $\tau_2 \geq 0$, suppose

$$\mathbf{D}_t \perp U_t \mid \mathbf{Y}_{t-1}, \mathbf{X}_t, t = 1, ..., T - \tau_2.$$

Finite-order G-causality and Markov structures

Definition 4.9

Suppose B.1 holds and that for given $\tau_1 \geq m, \tau_2 \geq 0$ *and for each* $y \in supp(Y_t)$ *there exists a* $\sigma(\mathbf{Y}_{t-1}, \mathbf{X}_t)-$*measurable version of*

$$\int 1\{q_t(\mathbf{Y}_{t-1}, \mathbf{D}_t, \mathbf{Z}_t, u_t) < y\} \, dF_t(u_t \mid \mathbf{Y}_{t-1}, \mathbf{X}_t).$$

Then $\mathbf{D}_t \overset{d}{\not\Rightarrow}_{\mathcal{S}(\mathbf{Y}_{t-1}, \mathbf{X}_t)} Y_t$ (**direct non-causality**$-\sigma(\mathbf{Y}_{t-1}, \mathbf{X}_t)$ $w.p.1$).

Else $\mathbf{D}_t \overset{d}{\Rightarrow}_{\mathcal{S}(\mathbf{Y}_{t-1}, \mathbf{X}_t)} Y_t$.

Finite-order G-causality and Markov structures

Theorem 4.10

Let B.1 and B.2 hold. Then $\mathbf{D}_t \overset{d}{\nRightarrow}_{\mathcal{S}(\mathbf{Y}_{t-1},\mathbf{X}_t)} Y_t, t = 1, ..., T - \tau_2,$
if and only if

$$Y_t \perp \mathbf{D}_t \mid \mathbf{Y}_{t-1}, \mathbf{X}_t, \quad t = 1, ..., T - \tau_2,$$

i.e., **D** *does not finite-order* $G-$*cause* Y *w.r.t.* **X**.

3. Granger causality and structural VARs

G-causality and structural VARs

- Special case of A.1: structural VARs (set $k_d = 0$)
- The DGP becomes

$$Y_t = q_t \left(Y^{t-1}, Z^t, U^t \right).$$

- Letting $Y_t \equiv (Y'_{1,t}, Y'_{2,t})'$,

$$\begin{aligned} Y_{1,t} &= q_{1,t} \left(Y_1^{t-1}, Y_2^{t-1}, Z^t, U^t \right) \\ Y_{2,t} &= q_{2,t} (Y_1^{t-1}, Y_2^{t-1}, Z^t, U^t). \end{aligned}$$

G-causality and structural VARs

Notation:
$\mathbf{Y}_{1,t-1} \equiv (Y_{1,t-\ell}, \ldots, Y_{1,t-1})$, $\mathbf{Y}_{2,t-1} \equiv (Y_{2,t-\ell}, \ldots, Y_{2,t-1})$,
$\mathbf{Z}_t \equiv (Z_{t-m}, \ldots, Z_t)$, and $\mathbf{X}_t \equiv (X_{t-\tau_1}, \ldots, X_{t+\tau_2})$.

Assumption C.1 A.1 holds, and for $\ell, m, \in \mathbb{N}, \ell \geq 1$, suppose that $Y_t = q_t(\mathbf{Y}_{t-1}, \mathbf{Z}_t, U_t)$, $t = 1, 2, \ldots$, such that, with $Y_t \equiv (Y'_{1,t}, Y'_{2,t})'$ and $U_t \equiv (U'_{1,t}, U'_{2,t})'$,

$$Y_{1,t} = q_{1,t}(\mathbf{Y}_{t-1}, \mathbf{Z}_t, U_{1,t}) \qquad Y_{2,t} = q_{2,t}(\mathbf{Y}_{t-1}, \mathbf{Z}_t, U_{2,t}).$$

Assumption C.2 For ℓ and m as in C.1 and for $\tau_1 \geq m, \tau_2 \geq 0$, suppose that

$$\mathbf{Y}_{2,t-1} \perp U_{1t} \mid \mathbf{Y}_{1,t-1}, \mathbf{X}_t, t = 1, \ldots, T - \tau_2.$$

Definition 5.2

Suppose C.1 holds and that for given $\tau_1 \geq m, \tau_2 \geq 0$ *and for each* $y \in supp(Y_{1,t})$ *there exists a* $\sigma(\mathbf{Y}_{1,t-1}, \mathbf{X}_t)-$*measurable version of*

$$\int 1\{q_{1,t}(\mathbf{Y}_{t-1}, \mathbf{Z}_t, u_{1,t}) < y\}\ dF_{1,t}(u_{1,t} \mid \mathbf{Y}_{1,t-1}, \mathbf{X}_t).$$

Then $\mathbf{Y}_{2,t-1} \overset{d}{\not\Rightarrow}_{\mathcal{S}(\mathbf{Y}_{1,t-1},\mathbf{X}_t)} Y_{1,t}$ (**direct non-causality**$-\sigma(\mathbf{Y}_{1,t-1}, \mathbf{X}_t)$ *w.p.*1).

Else $\mathbf{Y}_{2,t-1} \overset{d}{\Rightarrow}_{\mathcal{S}(\mathbf{Y}_{1,t-1},\mathbf{X}_t)} Y_{1,t}$.

G-causality and structural VARs

Theorem 5.3

Let C.1 and C.2 hold. Then $\mathbf{Y}_{2,t-1} \overset{d}{\not\Rightarrow}_{\mathcal{S}(\mathbf{Y}_{1,t-1},\mathbf{X}_t)} Y_{1,t}$, $t = 1, ..., T - \tau_2$, **if and only if**

$$Y_{1,t} \perp \mathbf{Y}_{2,t-1} \mid \mathbf{Y}_{1,t-1}, \mathbf{X}_t, \quad t = 1, ..., T - \tau_2,$$

i.e., $\mathbf{Y}_2$ *does not finite-order* $G-$*cause* Y_1 *w.r.t.* $\mathbf{X}$.

4. Testing finite-order Granger causality

Testing finite-order Granger causality

$$\text{Test}: Y_t \perp Q_t \mid Y_{t-1}, S_t.$$

- Test conditional mean independence with linear regression

$$Y_t = \alpha_0 + Y_{t-1}\rho_0 + Q_t'\beta_0 + S_t'\beta_1 + \varepsilon_t.$$

- Test conditional mean independence with neural nets

$$Y_t = \alpha_0 + Y_{t-1}\rho_0 + Q_t'\beta_0 + S_t'\beta_1 + \sum_{j=1}^{r} \psi(Y_{t-1}\gamma_{0,j} + S_t'\gamma_j)\beta_{j+1} + \varepsilon_t.$$

- Test conditional independence with nonlinear transforms

$$\psi_{y,1}(Y_t) = \alpha_0 + \psi_{y,2}(Y_{t-1})\rho_0 + \psi_q(Q_t)'\beta_0 + S_t'\beta_1 + \sum_{j=1}^{r} \psi(Y_{t-1}\gamma_{0,j} + S_t'\gamma_j)\beta_{j+1} + \eta_t.$$

5.Conditional exogeneity

The crucial role of conditional exogeneity

Proposition 7.1 *Given C.1, suppose* $\mathbf{Y}_{2,t-1} \overset{d}{\not\Rightarrow}_{\mathcal{S}} Y_{1,t}$, $t = 1, 2, \ldots$. *If C.2 (cond exog) does not hold, then for each t there exists* $q_{1,t}$ *such that* $Y_{1,t} \perp \mathbf{Y}_{2,t-1} \mid \mathbf{Y}_{1,t-1}, \mathbf{X}_t$ *does not hold.*

Corollary 7.2 *Given C.1 with* $\mathbf{Y}_{2,t-1} \overset{d}{\not\Rightarrow}_{\mathcal{S}} Y_{1,t}$, $t = 1, 2, \ldots$, *suppose that* $q_{1,t}$ *is invertible in the sense that*

$$Y_{1,t} = q_{1,t}(\mathbf{Y}_{1,t-1}, \mathbf{Z}_t, U_{1,t})$$

implies the existence of $\xi_{1,t}$ *such that*

$$U_{1,t} = \xi_{1,t}(\mathbf{Y}_{1,t-1}, \mathbf{Z}_t, Y_{1,t}), t = 1, 2, \ldots.$$

If C.2 fails, then $Y_{1,t} \perp \mathbf{Y}_{2,t-1} \mid \mathbf{Y}_{1,t-1}, \mathbf{X}_t$ *fails,* $t = 1, 2, \ldots$.

Separability and finite-order conditional exogeneity

Proposition 7.3

Given C.1, suppose that $E(Y_{1,t}) < \infty$ *and*

$$q_{1,t}(\mathbf{Y}_{t-1}, \mathbf{Z}_t, U_{1,t}) = \zeta_t(\mathbf{Y}_{t-1}, \mathbf{Z}_t) + \upsilon_t(\mathbf{Y}_{1,t-1}, \mathbf{Z}_t, U_{1,t}),$$

where ζ_t *and* υ_t *are unknown measurable functions. Let*

$$\varepsilon_t \equiv Y_{1,t} - E(Y_{1,t}|\mathbf{Y}_{t-1}, \mathbf{X}_t).$$

If C.2 holds, then

$$\varepsilon_t = \upsilon_t(\mathbf{Y}_{1,t-1}, \mathbf{Z}_t, U_{1,t}) - E(\upsilon_t(\mathbf{Y}_{1,t-1}, \mathbf{Z}_t, U_{1,t}) \mid \mathbf{Y}_{1,t-1}, \mathbf{X}_t),$$

$$E(\varepsilon_t|\mathbf{Y}_{t-1}, \mathbf{X}_t) = E(\varepsilon_t|\mathbf{Y}_{1,t-1}, \mathbf{X}_t) = 0,$$

and

$$\mathbf{Y}_{2,t-1} \perp \varepsilon_t \mid \mathbf{Y}_{1,t-1}, \mathbf{X}_t. \qquad (*)$$

An indirect test for structural causality

1. *Reject* structural non-causality if either:

   (i) the conditional exogeneity test *fails to reject* and the $G-$causality test *rejects*; or

   (ii) the conditional exogeneity test *rejects* and the $G-$causality test *fails to reject*.

2. *Fail to reject* structural non-causality if the conditional exogeneity and Granger non-causality tests both *fail to reject;*

3. *Make no decision* as to structural non-causality if the conditional exogeneity and Granger non-causality tests both *reject.*

6. Applications

- Crude oil prices and gasoline prices (White and Kennedy, 2008)
- Monetary policy and industrial production (Angrist and Kuersteiner, 2004)
- Economic announcements and stock returns (Flannery and Protopapadakis, 2002)

Economic announcements and stock returns

- Decompose macro announcements into news and expected changes:
  - Announcements: A_t
  - Announcement expectations: A_t^e
  - Decomposition:
    $A_t - A_{t-1} = (A_t - A_t^e) + (A_t^e - A_{t-1}) = Z_t + D_t$
  - News: $Z_t = A_t - A_t^e$
  - Expected change: $D_t = A_t^e - A_{t-1}$
- A_t : eight major macro announcements:
  (1) real GDP (advanced) (2) core CPI
  (3) core PPI (4) unemployment rate
  (5) new home sales (6) nonfarm payroll employment
  (7) consumer confidence (8) capacity utilization rate
- A_t^e : Money Market Service survey data.

Economic announcements and stock returns

- Y_t : CRSP value-weighted NYSE-AMEX-NASDAQ index daily returns
- W_t : drivers of D_t and responses to unobservable causes

  (1) three month T-Bill yield
  (2) term structure premium
  (3) corporate bond premium
  (4) daily change in the Index of the Foreign Exchange Value of the Dollar
  (5) daily change in crude oil price

  These variables represent macro fundamentals.
- Covariates : $X_t = (Z_t, W_t)$.

Economic announcements and stock returns

- Test retrospective conditional exogeneity (CE) by testing

$$D_t \perp \varepsilon_t \mid Y_{t-1}, \mathbf{X}_t$$

where

$$\varepsilon_t = Y_t - E\left(Y_t \middle| D_t, Y_{t-1}, \mathbf{X}_t\right)$$

- Test finite-order retrospective G non-causality (GN) by testing

$$Y_t \perp D_t \mid Y_{t-1}, \mathbf{X}_t$$

Economic announcements and stock returns

Results

- Fail to reject GN for all $\tau = 0, \pm1, ..., \pm8$.
- Fail to reject CE for all $\tau = 0, \pm1, ..., \pm8$.
- Suggests no structural effects of expected macro announcements on stock returns.
- Consistent with both weak market efficiency and absence of other distributional impacts.
- Non-retrospective GN and CE tests (conditioning on lags only) yield exhibit identical pattern.

7. Conclusions

This paper

- Links G non-causality with structural non-causality
- Links G non-causality with conditional exogeneity
- Provides explicit guidance as to how to choose S so G non-causality gives structural insight
- Provides new tests of G non-causality, conditional exogeneity, and structural non-causality