Efficient and Decentralized PageRank Approximation in a P2P Network

Josiane Xavier Parreira, Debora Donato, Sebastian Michel,
Gerhard Weikum

Max Planck Institute for Computer Science, Universita di Roma

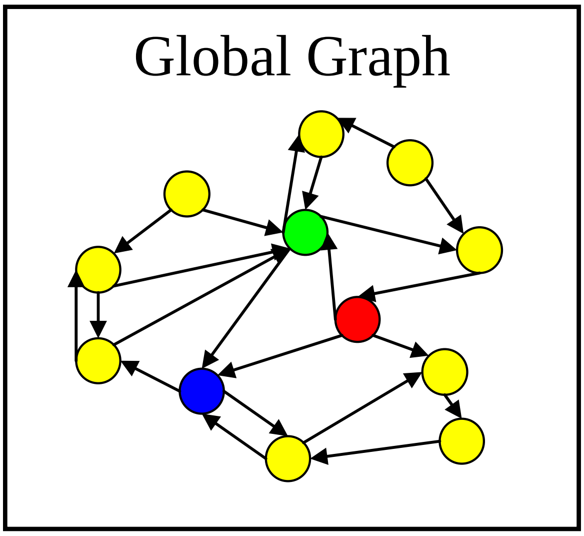
May 20^{th}, 2006

Outline

1. Introduction
2. Related Work
3. The JXP Algorithm
4. Mathematical Analysis
5. Experimental Results
6. Conclusions and Ongoing Work

Motivation

Scenario: A graph...



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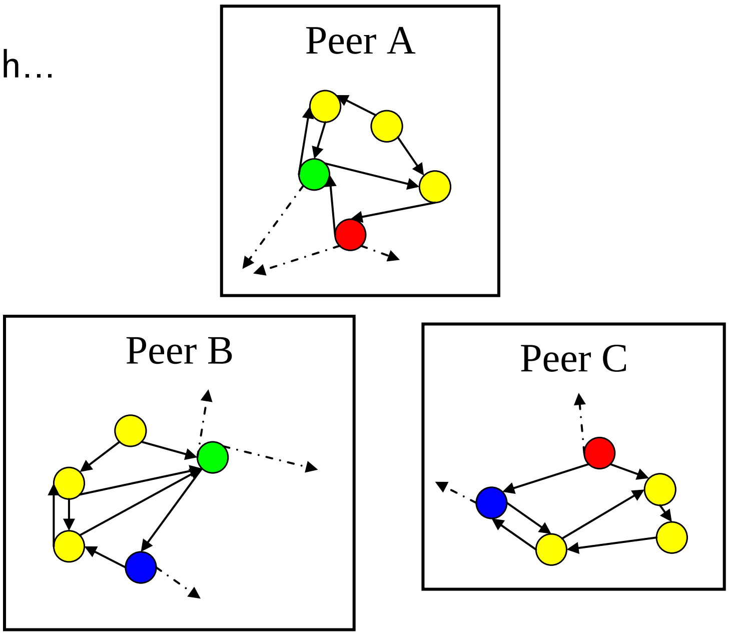

...distributed into peers in a P2P network.

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Goal

Compute “global” authority scores of pages in the network.

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Problems

- Peers have only local (incomplete) information
- Pages might link to or be linked by pages at other peers
- No control over overlaps between local graphs

Related Work

Efficient PR

- Graph Aggregation [Broder et al., WWW'04]
- Iterative Aggregation [Langville & Meyer, WWW'04]

Decentralized PR

- *Local PageRank & ServerRank* [Wang & DeWitt, VLDB'04]
- *BlockRank* [Kamvar et al., Stanford Tech. Report'03]

Markov Chains Aggregation/Disaggregation Techniques

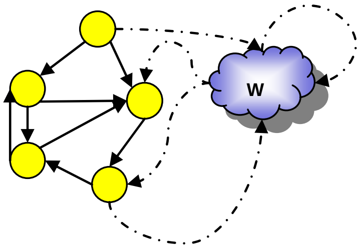
- Kemeny & Snell [1963]
- Stewart [1994]
- Meyer [2000]

JXP Algorithm

JXP Algorithm

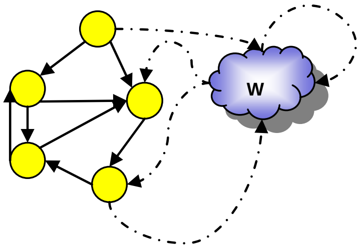
- Decentralized algorithm for computing authority scores of pages in a P2P Network
- Runs locally at every peer
- Combines local PageRank computations + Meetings between peers
- JXP scores converge to the true global PageRank scores

World Node



World Node

- Special node added to each local graph



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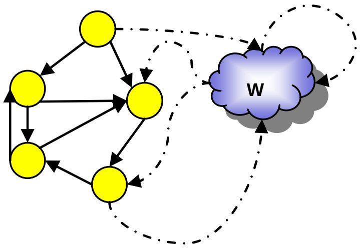

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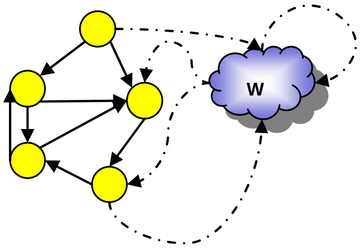

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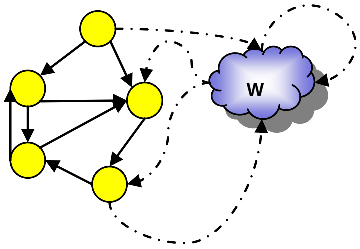

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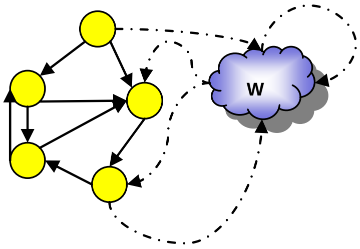

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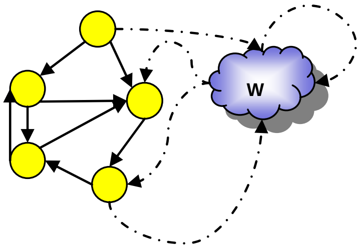

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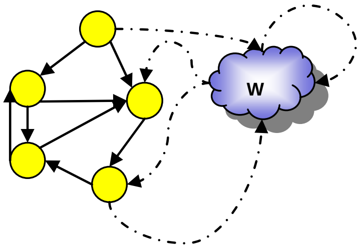

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  - Self-loop link to represent transitions among external pages

The Algorithm

Initialization step

- Local graph is extended by adding the world node;
- PageRank is computed in the extended graph → JXP scores

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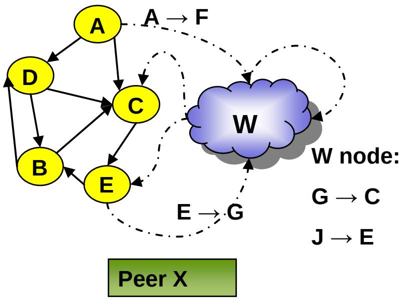
Main Algorithm (for every P_i in the network)

- Select P_j to meet
- Update world node
  - Add links from P_j that point to pages in P_i
  - If a link already exists at the world node, the score of the source page is updated by taking the highest of both scores
- Compute PageRank → JXP scores

Example



A → F

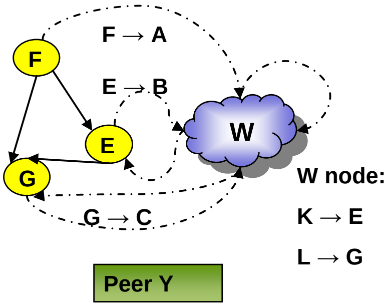
E → G

W node:

G → C

J → E

Peer X



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L → G

Peer Y

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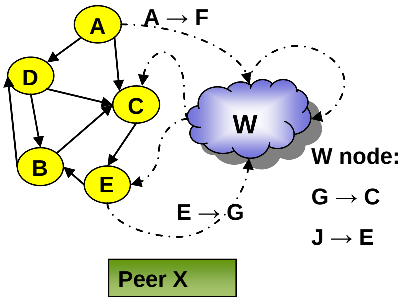

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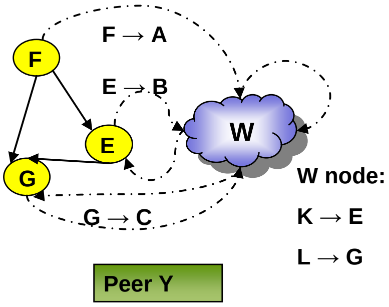

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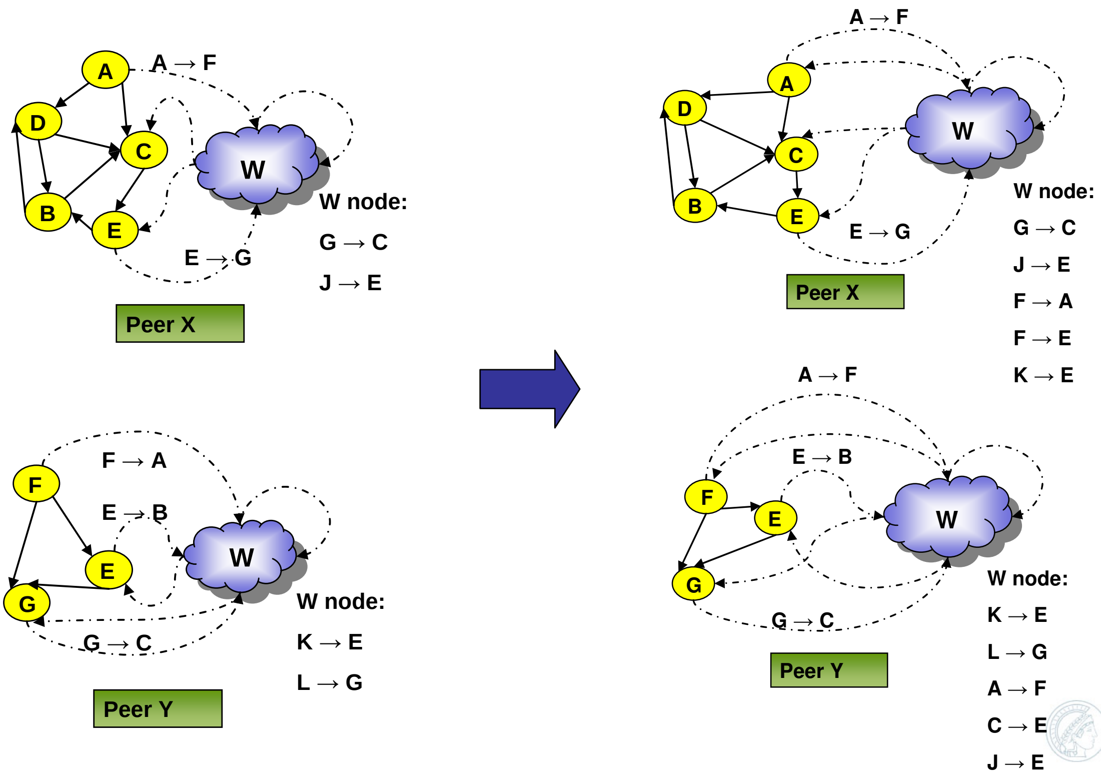

Peer Selection Strategy

Motivation

- Peers' contribution for the convergence are different
- Finding most promising peers would speed up convergence
- "Quality indicator": Amount of outgoing links of a peer in the network that are also incoming links in the local graph

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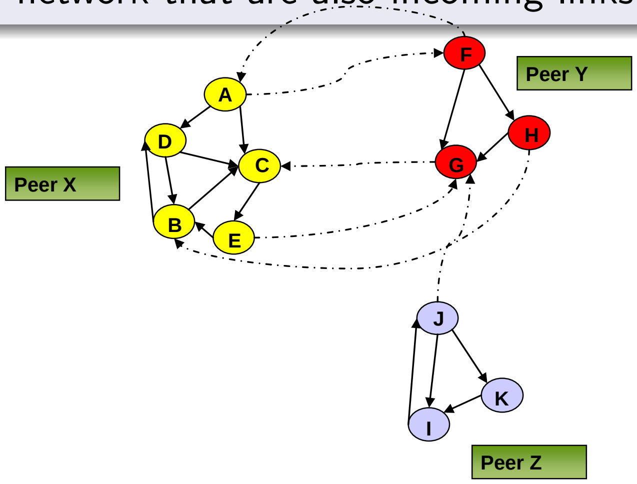

Peer Selection Strategy

Good strategy

Find promising peers without increasing much bandwidth consumption

- Caching + statistical synopses

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Statistical synopses

Approximation technique for comparing data of different peers without explicitly transferring their contents.

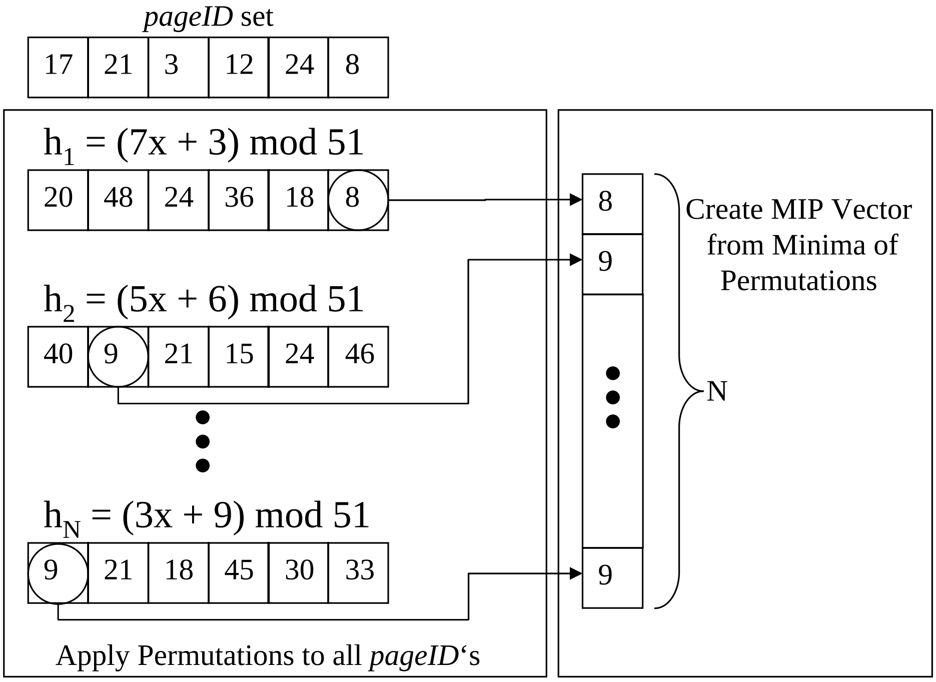
- Compact representation of sets
- Can be used to estimate cardinality of the intersection between two sets
- JXP uses Min-Wise Independent Permutations (MIPs)

MIPs

MIPs [Broder et al., 2000]

- Computes N permutations of the set's elements
- Permutation uses linear hash function $h_i(x) := a_i * x + b_i \ mod \ U$
- For each of N permutations, it determines the minimum hash value and stores it in a N-dimensional vector
- Idea: Each element has same probability of becoming the minimum element
- MIPs vector are then used to estimate the intersection between two sets

MIPs Example



pageID set

17	21	3	12	24	8

$h_1 = (7x + 3) \bmod 51$

20	48	24	36	18	8

$h_2 = (5x + 6) \bmod 51$

40	9	21	15	24	46

$h_N = (3x + 9) \bmod 51$

9	21	18	45	30	33

Apply Permutations to all *pageID*'s

8
9
⋮
9

Create MIP Vector from Minima of Permutations

N

Figure: Example of Min-Wise Permutations.

Pre-meetings Strategy

- Each peer P_i computes $local(P_i)$ and $successors(P_i)$ MIPs vectors
- When P_i meets P_j
  - Uses MIPs vectors to estimate percentage of local pages pointed by pages in P_j
  - If percentage above threshold, P_i caches P_j's ID
  - Uses MIPs again to estimate overlap between overlap between the two local graphs
  - If there is high overlap, peers exchange their list of cached ID's and store them in a temporary list
  - Idea: Peers on the temporary list are potential candidates for the next meeting

Pre-meetings Strategy

Pre-meetings phase

- P_j contacts peers on the temporary list and ask for their MIPs vectors
- Assign scores to each peer
- For next (real) meeting, P_i chooses P_k where
  - P_k is best scored peer in temporary list, with prob. α
  - P_k is one of the already cached peers, with prob. β
  - P_k is a random peer, with prob. $(1 - \alpha - \beta)$

Mathematical Analysis

Assumptions

Global transition matrix $\mathbf{C}_{N \times N}$ and global stationary distribution vector $\boldsymbol{\pi}$

Local transition matrix and local stationary distr. (JXP scores)

$$\mathbf{P} = \left(\begin{array}{ccc|c} p_{11} & \dots & p_{1n} & p_{1w} \\ \vdots & \dots & \vdots & \vdots \\ p_{n1} & \dots & p_{nn} & p_{nw} \\ \hline p_{w1} & \dots & p_{wn} & p_{ww} \end{array} \right)$$

$$p_{ij} = \begin{cases} \frac{1}{out(i)} & \text{if } \exists\ i \rightarrow j \\ 0 & \text{otherwise} \end{cases}$$

$$p_{iw} = \sum_{\substack{i \rightarrow r \\ r \notin G}} \frac{1}{out(i)}$$

$$\boldsymbol{\alpha} = \left(\begin{array}{ccc|c} \alpha_1 & \dots & \alpha_n & \alpha_w \end{array} \right)^T$$

for every i, j, $1 \leq i, j \leq n$. (G is the set of local pages)

Mathematical Analysis

World Node transitions prob.

$$p^t_{wi} = \sum_{\substack{r \to i \\ r \in W^t}} \frac{\alpha(r)^t}{out(i)} \cdot \frac{1}{\alpha_w^{t-1}} \qquad p^t_{ww} = 1 - \sum_{i=1}^{n} p^t_{wi}$$

W^t: Set of pages represented at the World Node during meeting t

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Random Jumps

$$\mathbf{P}' = \epsilon\,\mathbf{P} + (1-\epsilon)\frac{1}{N}\left(\begin{array}{ccc|c} 1 & \ldots & 1 & (N-n) \end{array}\right)$$

Mathematical Analysis

Meeting Step

Considering one link addition/update at a time

$$\mathbf{P}^{t} = \mathbf{P}^{t-1} + \mathbf{E} \qquad \mathbf{E} = \left(\begin{array}{ccccccc|c} 0 & & & \ldots & & & 0 & 0 \\ \vdots & & & \ldots & & & \vdots & \vdots \\ 0 & & & \ldots & & & 0 & 0 \\ \hline 0 & \ldots & 0 & \delta & 0 & \ldots & 0 & -\delta \end{array} \right)$$

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Theorem 1

The JXP score of the world node, at every peer in the network, is monotonically non-increasing.

Mathematical Analysis

Theorem 2

Consider the true stationary probabilities (PR scores) of pages $i \in G$ and the world node w, π_i and π_w, and their JXP scores after t meetings α_i^t and α_w^t. The following holds throughout all JXP meetings:
$0 < \alpha_i^t \leq \pi_i$ *for* $i \in G$ *and* $\pi_w \leq \alpha_w^t < 1$.

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Theorem 3

In a fair series of JXP meetings, the JXP scores of all nodes converge to the true global PR scores.

Setup

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Amazon collection

- 55,196 pages
- 237,160 links
- 10 categories

Web collection

- 103,591 pages
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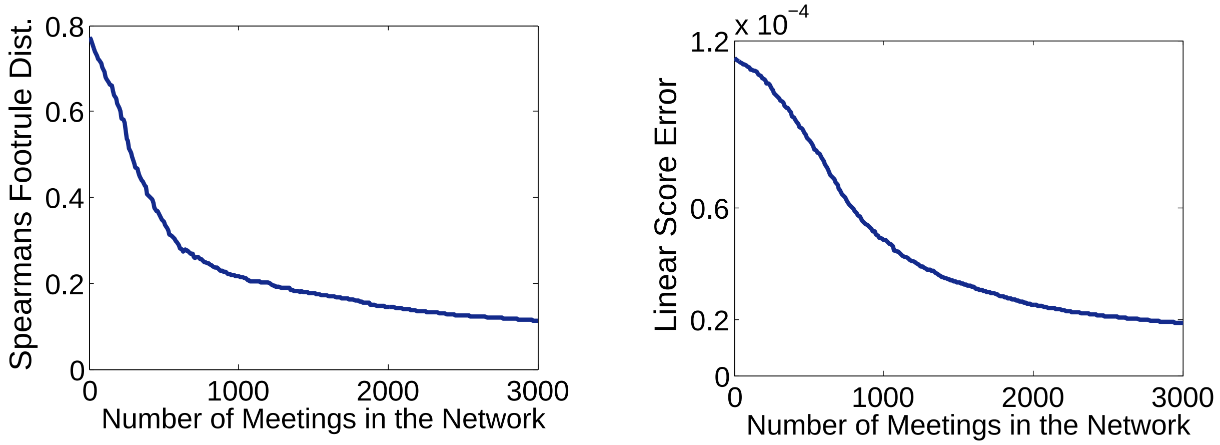
Evaluation Measures

- “Global” JXP ranking vs. Global PageRank ranking
- Spearman’s Footrule Distance at top-k
- Linear Score Error at top-k

Overall performance comparison

Amazon Collection, top-1,000



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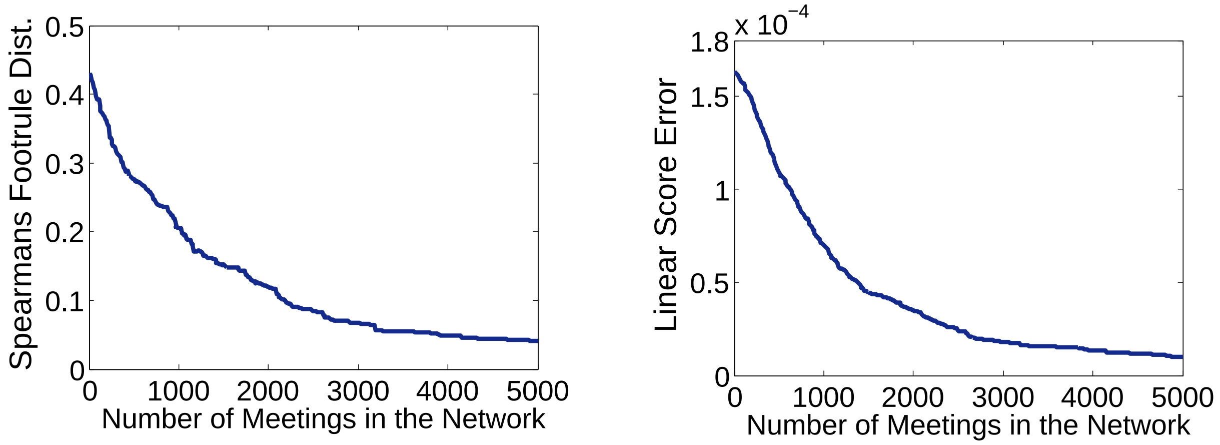

JXP in P2P Search

JXP in P2P Search

JXP integrated into the Minerva search engine for P2P networks. (Minerva Project Website: http://www.minerva-project.org)

Setup

- Bigger set of Web Collection (250,760 docs and 3,123,993 links)
- 40 peers, high overlap
- 15 queries, using the Minerva query routing mechanism
- Results were ranked in two ways:
  - tf*idf only
  - weighted sum of tf*idf and JXP scores
- Precision at top-10 measured

Results

Query	tf*idf	(0.6 tf*idf + 0.4 JXP)
affirmative action	40%	40%
amusement parks	60%	60%
armstrong	20%	**80%**
basketball	20%	**60%**
blues	20%	20%
censorship	**30%**	20%
cheese	40%	**60%**
iraq war	**50%**	30%
jordan	40%	40%
moon landing	**90%**	70%
movies	30%	**100%**
roswell	30%	**70%**
search engines	20%	**60%**
shakespeare	60%	**80%**
table tennis	50%	**70%**
Average	40%	**57%**

Conclusions and Ongoing Work

Conclusions

- JXP algorithm for dynamically computing authority scores of pages distributed in a P2P network
- Combines local PageRank computation with meetings between peers
- JXP scores are proved to converge to global PageRank scores

Ongoing Work

- Integrate JXP into the query routing mechanism
- JXP in dynamic networks